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NON-EQUILIBRIUM DYNAMICS FROM FEW- TO  
MANY-BODY SYSTEMS

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# Abstract

We study different nonequilibrium phenomena of isolated quantum systems ranging from few- to many-body interacting bosons. Firstly, we have suggested the dynamics of the center-of-mass motion to sensitively detect unconverged numerical many-body dynamics in potential with separable quantum motion of the center of mass. As an example, we consider the time evolution of attractive bosons in a homogenous background and use it to benchmark a specific numerical method based on variational multimode expansion of the many-body wave function - the Multiconfigurational time-dependent Hartree for bosons (MCTDHB). We demonstrate that the simplified convergence criterion based on a threshold value for the least occupied mode function fails to assure qualitatively correct result while our suggested convergence test based on the center-of-mass motion correctly detects the deviation of numerical results from the exact results.

Recent technological progress in manipulating low-entropy quantum states has motivated us to study the phenomenon of interaction blockade in bosonic systems. We propose an experimental protocol to observe the expected bosonic enhancement factor in this blockade regime. Specifically, we suggest the use of an asymmetric double-well potential constructed by superposition of multiple optical tweezer laser beams. Numerical simulations using the MCTDHB method predict that the relevant states and the expected enhancement factor can be observed.

In the second half of the thesis, we have investigated the onset of quantum thermalization in a two-level generalization of the Bose-Hubbard dimer. To this end, the relaxation dynamics following a quench is studied using two numerical methods:

(1) full quantum dynamics and (2) semiclassical phase-space method. We rely on arguments based on the eigenstate thermalization hypothesis (ETH), quantum chaos as seen from the distribution of level spacings, and the concept of chaotic eigenstates in demonstrating equilibration dynamics of local observables in the system after an integrability-breaking quench. The same issue on quantum thermalization can be viewed from a different perspective using semiclassical phase-space methods. In particular, we employ the truncated Wigner approximation (TWA) to simulate the quantum dynamics. In this case, we show that the marginal distributions of the individual trajectories which sample the initial Wigner distribution are in good agreement with the corresponding microcanonical distribution.



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