On the Acoustical Theory of the Trumpet:

Is it Sound?

A thesis presented in partial fulfilment of the requirements

for the degree of Master of Science in Mathematics

at Massey University, New Zealand

Grant Redhead

March 1997

Abstract

Newton's Second Law of Motion for one-dimensional inviscid flow of an incompressible fluid, in the absence of external forces, is often expressed in a form known as Bernoulli's equation:

$$\int_{x_1}^{x_2} \frac{\partial u}{\partial t} dx + \frac{1}{2} u^2 \Big|_{x_1}^{x_2} + \frac{p}{\rho} \Big|_{x_1}^{x_2} = 0$$

There are two distinct forms of Bernoulli's equation used in the system of equations which is commonly considered to describe sound production in a trumpet.

The flow *between the trumpeter's lips* is, in the literature, assumed to be *quasi-steady*. From this assumption, the first term of the above Bernoulli equation is omitted, since it is then small in comparison to the other two terms.

The flow *within the trumpet* itself is considered to consist of small fluctuations about some mean velocity and pressure. A *linearized* version of Bernoulli's equation (as used in the equations of linear acoustics) is then adequate to describe the flow. In this case it is the second term of the above equation which is neglected, and the first term is retained.

Given that the flow between the trumpeter's lips is that same flow which enters the trumpet itself, a newcomer to the field of trumpet modelling might wonder whether the accepted model is really correct when these two distinct versions of the Bernoulli Equation are used side by side.

This thesis addresses this question, and raises others that arise from a review of the standard theory of trumpet physics. The investigation comprises analytical and experimental components, as well as computational simulations.

No evidence has been found to support the assumption of quasi-steady flow between the lips of a trumpeter. An alternative flow equation is proposed, and conditions given for its applicability.

Acknowledgements:

Preface

The work discussed in this thesis germinated, in February of 1992, from a project idea for a one-year masters program in Electrical and Electronic Engineering at the University of Canterbury: to synthesise trumpet sounds by implementing, in real-time, a mathematical model of trumpet physics.

After two years at the University of Canterbury, I summarised my findings in a report (Redhead, 1993) and sent copies to four researchers who I thought may be interested. *Chapter One* of this thesis represents (mainly) the findings of my two years at the University of Canterbury.

My report was received favourably by Shigeru Yoshikawa (Japan), who cited it in an article published by the Journal of the Acoustical Society of America (Yoshikawa, 1995), and by Douglas Keefe (U.S.), whose response led to my visit to the University of Washington, Seattle. *Chapter Five* describes the results of an experiment performed when I visited the University of Washington, Seattle, for three weeks of 1994.

Chapters Two to Four, *Chapter Six* and *Chapter Seven* have originated from my subsequent two years at Massey University.

None of the content of this thesis, in whole or in part, has been presented before for examination at any university.

Contents

1

Modelling of

Sound-Production in the Trumpet

The exact means by which sound oscillations are produced and maintained in many wind instruments is, to date, uncertain. A large obstacle to the pursuit of such knowledge has been the difficulty of making detailed physical measurements while the instrument is being played. Many parameters and variables of the system (such as lung pressure and diaphragm force, vocal tract shape and facial muscular tension) are, to a large extent, unmeasurable. But some useful results have been obtained in spite of these difficulties (Martin, 1942; Henderson, 1942; Stubbins, Lillya & Frederick, 1956; Weast, 1963; Bouhuys, 1965; Vivona, 1968; Elliott & Bowsher, 1982; Barbenal, Davies & Kenny, 1986; Yoshikawa & Plitnik, 1993; Copley & Strong, 1994; Yoshikawa, 1995).

It is a far simpler matter to investigate the acoustical response of a particular wind instrument shape in isolation from the musician. By appropriate coupling of an electro-acoustic transducer to the musical instrument a frequency response may be determined for the instrument alone (Webster, 1947; Igarashi & Koyasu, 1953; Benade, 1973; Backus, 1974, 1976; Smith & Daniell, 1976; Pratt, Elliott & Bowsher, 1977; Elliott, Bowsher & Watkinson, 1982). Other less specific studies also have relevance to musical instrument analysis (Jansson & Benade, 1974; Silcox & Lester, 1982; Davies, 1988).

A number of researchers have directed their efforts towards developing apparatus which sounds a wind instrument artificially from a supply of compressed air (Webster, 1919a; Martin, 1942; Backus, 1963, 1964; Backus & Hundley, 1971; Wilson & Beavers, 1974; Fletcher, Silk & Douglas, 1982; Idogawa et al., 1988).

In recent years, the ever-increasing speed, capacity and availability of computational resources has seen mathematical modelling become an invaluable tool in the investigation of wind instrument operation (Pyle, 1969; Stewart & Strong, 1980; Saneyoshi, Teramura & Yoshikawa, 1987; Park & Keefe, 1988; Sommerfeldt & Strong, 1988; Yoshikawa,

1988; Barjau & Agulló, 1989; Keefe, 1990, 1992; Strong, 1990b; Brown & Schumacher, 1990; Strong & Dudley, 1993; Adachi & Sato, 1995). Mathematical modelling offers significant advantages over experimental procedures, particularly in the separate analysis of subsystems of the overall system (e.g. Dudley & Strong, 1990, 1993).

The idea of mathematical modelling of musical instruments has been received eagerly by Electronic Engineers hoping to implement such models using fast and efficient algorithms for real-time synthesis of musical instrument sounds (Smith, 1991a; Valimaki et al.,1992; Cook, 1992; Borin, De Poli & Sarti, 1992; Rodet & Doval, 1992). But while providing additional motivation for the study of musical instrument operation, it is the present author's view that the birth of *physical modelling synthesis* has also brought to the field a sense of urgency which, perhaps, has not been completely advantageous inasmuch as attention to model detail is concerned.

Sound synthesis via a mathematical model of a real musical instrument promises numerous and significant advantages over other synthesis techniques (Smith, 1991a). However, these synthesisers can only yield accurate reproduction of traditional musical instrument sounds if based upon sufficiently accurate mathematical models of those instruments. Some engineers state that "... in general, a serious comparison between reality and simulated results cannot give satisfactory results" (Borin, De Poli & Sarti, 1992). Others have turned their attention away from applying the known models and towards improving the models themselves (Redhead, 1993).

Although this thesis focuses especially upon improving the model of sound-production in the trumpet, some of the conclusions are more broadly applicable to other wind instruments, and particular to other members of the brass family.

1.1   Fundamentals of Trumpet-Playing Technique:

To sound a note, the trumpet player draws his or her lips taut and places them against the mouthpiece of the instrument. Air from the lungs is used to set the lips into vibration, as depicted in Figure 1.1. By thus *buzzing* the lips against the trumpet mouthpiece, a stable acoustic oscillation may result - though not necessarily at the same pitch as for the lips alone (Dale, 1965).
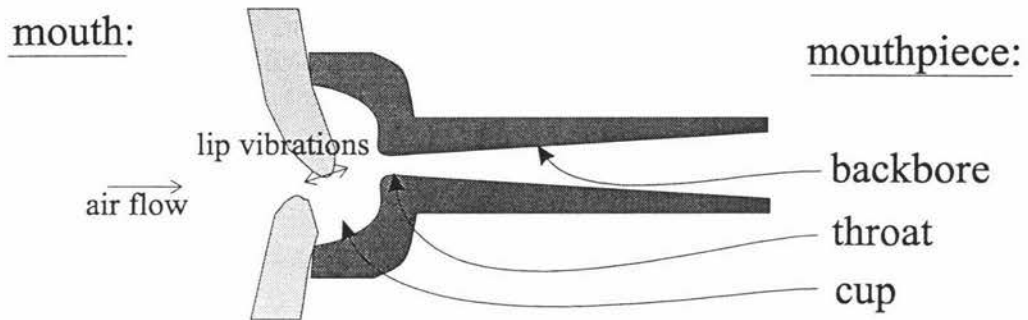
Figure 1.1:   Lips Against The Trumpet Mouthpiece

Valve Action:

The modern trumpet is fitted with three two-way *valves*. Depressing a valve adds a short detour to the path of any travelling sound wave within the instrument. There are a total of $2^3 = 8$ *fingering positions* possible; the player thus has access to eight different horn geometries from the one instrument. The first, second and third valves lower the pitch by approximately two, one and three semitones respectively. This gives seven possible notes (one of which has alternative fingering) as shown in Figure 1.2.

4

Figure 1.2:   Lower Notes Are Produced As More Valves Are Depressed

Selection of Higher Harmonics:

Most trumpet tunes contain more than the seven pitches shown in Figure 1.2. The trumpeter is able to extend this range by varying the tautness of the lips. Several stable oscillations are possible for each fingering position, and 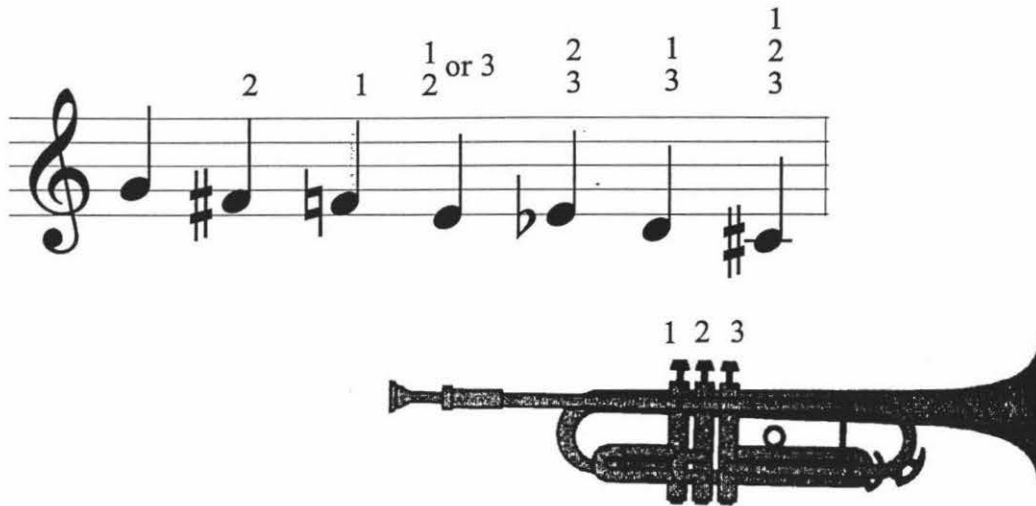the lip parameters determine which one of these is excited. Ideally, the geometry of the instrument is such that the fundamentals of these stable oscillations (for any one fingering position) are related to each other through membership of the same harmonic series (Figure 1.3).

Figure 1.3: Some Of The Notes That May Be Sounded With No Valves Depressed

The mouth position and lip parameters of the player define his or her *embouchure*. Using variations of both embouchure and fingering position, the practised trumpeter will often be capable of producing a full complement of notes within a range of around two-and-a-half octaves.

Lipping:

The player is also able vary the embouchure to effect small variations to the frequency of a note without swapping to another stable oscillation; the player is then said to be *lipping* the note up. This technique may be used by a skilled player to overcome inherent tuning

deficiencies of an instrument, and also for *bending* a note for musical effect (useful for playing jazz).

Dynamic Level:

The trumpet player varies the loudness or *dynamic level* of the music primarily through control of the diaphragm and upper abdominal muscles, thereby causing changes to the flow-rate of air from the lungs. Concurrent adjustment to the embouchure may be necessary to prevent the oscillation from jumping to another harmonic of the system.

Tone Control:

The origins of the steady-state *tone* of a trumpet sound are perhaps a little more unclear. A player will often prefer the tone of one instrument over another, and certainly the geometry of the instrument plays a significant role in the tonal quality of the sound it produces. The trumpet, cornet and flugel horn are three brass instruments which all have the same bore length. Their characteristic differences in tone quality are attributed to the different proportions of cylindrical and conical ducting along the length of the instruments.

Significant differences in tonal quality of the instrument can also be afforded by means of a *mute*. Several different shapes of mute are readily available. A mute is designed to sit in the flared end (the *bell*) of the trumpet, thereby affecting its acoustic reflection and transmission properties.

Aside from these more concrete factors, two players will often produce notes of different tonal quality upon playing the same instrument. This indicates that the musician, as well as the instrument, influence the tone. Furthermore, an individual trumpeter is able to deliberately vary the tone of a note while maintaining a constant dynamic level. These effects have been attributed to variations of parameters of the player's oral cavity and vocal tract (Stauffer, 1968). Differences of geometry (determined by tongue position, for example) and perhaps also of the resilience of the muscle walls of the oral cavity and vocal tract (determined by the state of muscular tension) could be responsible for changes in trumpet tone.

Articulation:

The manner in which the player controls the attack of a note is known as *articulation*. If the flow of air into the trumpet is obstructed briefly by the tongue before starting a new note, then that note is said to be *tongued*. Pressure builds behind the tongue and the lips are unable to oscillate. When the tongue is released, there is a sudden burst of air available to aid the initiation of the new oscillation.

The process of tonguing a note is analogous to the pronunciation of unvoiced plosives in speech production - especially /t/. A similar function can be formed further back in the mouth by forming the syllable /k/. The /k/ may be usefully alternated with /t/ in very fast passages; this technique is known as *double-tonguing*.

When a smooth or *slurred* transition between two notes is required, the second note is not tongued. The change of note is effected by the variation of embouchure and/or fingering position only, without interruption of the air supply to the instrument and without a subsequent burst of air to aid the initiation the new oscillation.

1.2 The Traditional Trumpet Model:

Any musical instrument which is capable of generating sustained notes of stable loudness is an example of a *self-excited oscillator*.

A self-excited oscillator is characterised by the presence of an energy supply which is either static (i.e. it has constant properties throughout time) or *quasi-static* (i.e. its properties do vary in time, but upon a time scale much longer that the period of the oscillation). It is the motion of the oscillator itself which regulates the rate of transfer of energy from the energy reservoir to the oscillation. The *forcing function* of the oscillator thus depends upon time only through the *state* of the system and its time derivatives.
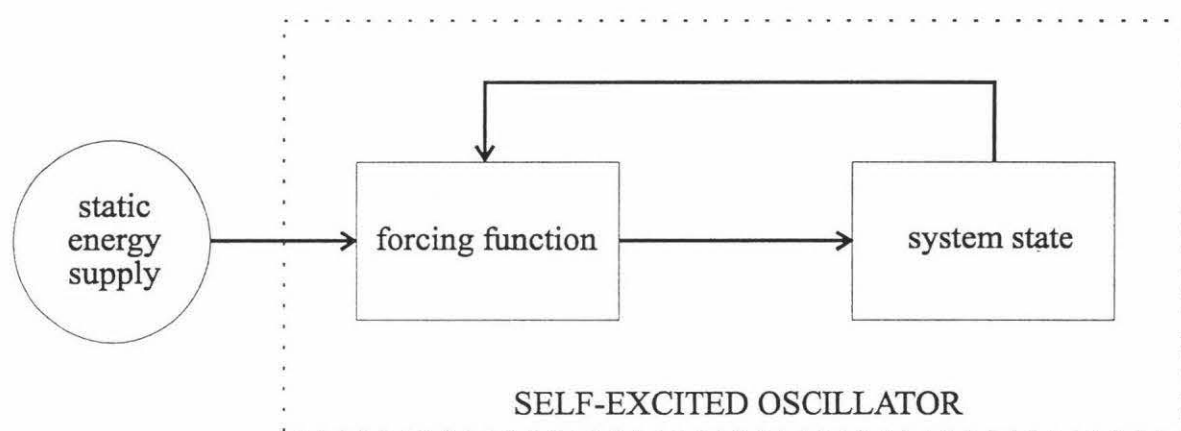

Figure 1.4:   Feedback Structure of Self-Excited Oscillator

When, during each cycle of system oscillation, more energy is received from the reservoir that is lost from the oscillator through other means, the amplitude of the oscillation increases. When less energy is received from the reservoir than is lost from the oscillator through other means, the amplitude of the oscillation decreases. An oscillation of stable amplitude results when the energy received per cycle balances that lost from the oscillator.

Wind instruments and bowed string instruments are examples of self-excited musical oscillators. For most wind instruments, a quasi-static energy supply is provided by the force exerted by the musician's diaphragm muscles upon the air held by the lungs. (The pipe organ and the bagpipes are exceptions.) For bowed string instruments, the quasi-static energy supply is in the form of the musician's drawing of the bow steadily across the string(s).

Percussion instruments (e.g. drums, piano) and plucked string instruments are examples of musical oscillators which are not self-excited. These receive energy once, at the beginning of each a note, and the energy of the oscillation begins to decay from that moment.

Energy is lost from a musical oscillator in two important ways. Of most interest is the energy of the system oscillation which is transformed into sound energy. The energy is then propagated away. Secondly, a proportion of the kinetic energy of motion of any kind is always converted irreversibly into heat energy, so that it is no longer useful to the system. This loss of useful energy is known as *damping*. There is damping associated with all moving parts of an oscillator, and there is also damping associated with (both acoustic and non-acoustic) fluid motions.

Generator - Resonator Decomposition:

Musically useful self-excited oscillators, though overall strongly non-linear, can often be described in terms of conceptually separate linear and non-linear subsystems which react with each other within a feedback loop (McIntyre, Schumacher & Woodhouse, 1983). This decomposition (see Figure 1.5 below) proves very useful in the analysis of the complicated overall-nonlinear system.
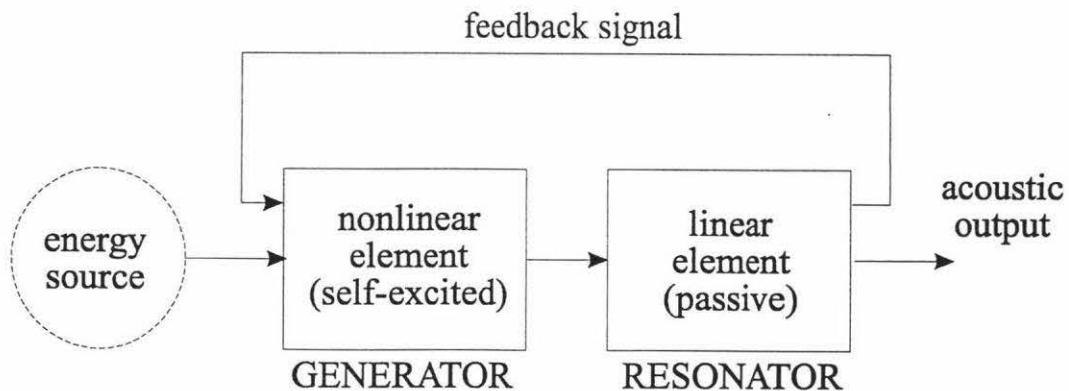

Figure 1.5: Basic Components of a Self-Excited Musical Oscillator

The generator subsystem receives energy from a constant energy source and releases it as an energy oscillation into the resonator. The resonator receives oscillatory energy presented by the generator, and combines this (linearly) with the energy it has already.

Energy oscillations of certain frequencies are favoured by the resonator. Whereas the generator acts as an energy *modulator*, the resonator acts as a linear energy *filter*.

Often feedback between the resonator and the generator provides a means for the generator to "lock in" to a frequency favoured by the resonator. This gives stability to the frequency of the resulting oscillation.

Wind Instruments

For wind instruments, the source of energy is most often provided by the diaphragm muscles of the musician, exerting a pressure force upon the air contained by the lungs.

The resonator subsystem of a wind instrument is generally considered to be the column of air inside the instrument, in which acoustic standing waves build up. The resonator is provided with acoustic energy by the generator subsystem, and it loses acoustic energy in the form of sound radiated from its open end. In addition to the energy lost in the form of acoustic radiation, there is additional energy loss through the irreversible conversion of kinetic energy to heat energy. This is an inevitable result of motion of any kind; energy is lost from the acoustic motion within the air column via internal fluid friction called *viscosity*. Any wall vibrations of the resonator, in response to the enclosed acoustic oscillations, are responsible for additional radiation and dissipation losses.

The generator subsystem of wind instruments is responsible for the conversion of a steady pressure force, provided by diaphragm of the musician, into acoustic flow energy (which is oscillatory in nature), for provision to the resonator subsystem. Thus it can be said that the generator subsystem, for wind instruments, has a role of *flow modulation*. For all wind instruments, very little is known about the flow modulation performed by the generation subsystem as compared with the amount of knowledge and understanding of the (linear) resonator subsystem.

The vast majority of wind instruments modulate the airflow by means of a vibrating body in a narrow passage somewhere within the system. (Exceptions are the flute and recorder families, whistles and flue organ pipes - these utilise aerodynamic instability to make the conversion from steady to oscillatory flow.) Flow modulation is caused by a vibrating body near a flow constriction in all reed instruments, the brass family, and in the voiced sounds of human speech.

The traditional model of sound-production in the trumpet is based upon a proposition by Webster (1919a) that the trumpeter's lips act as a *pressure-controlled valve*: "A spring of variable tension holds the valve in place and the proper pressure can cause a puff of air, which generates a sound in the horn which on reflection arrives at the valve in the proper phase to maintain vibration."

Resonator Equations:

The resonator subsystem of the trumpet is depicted below in Figure 1.6.
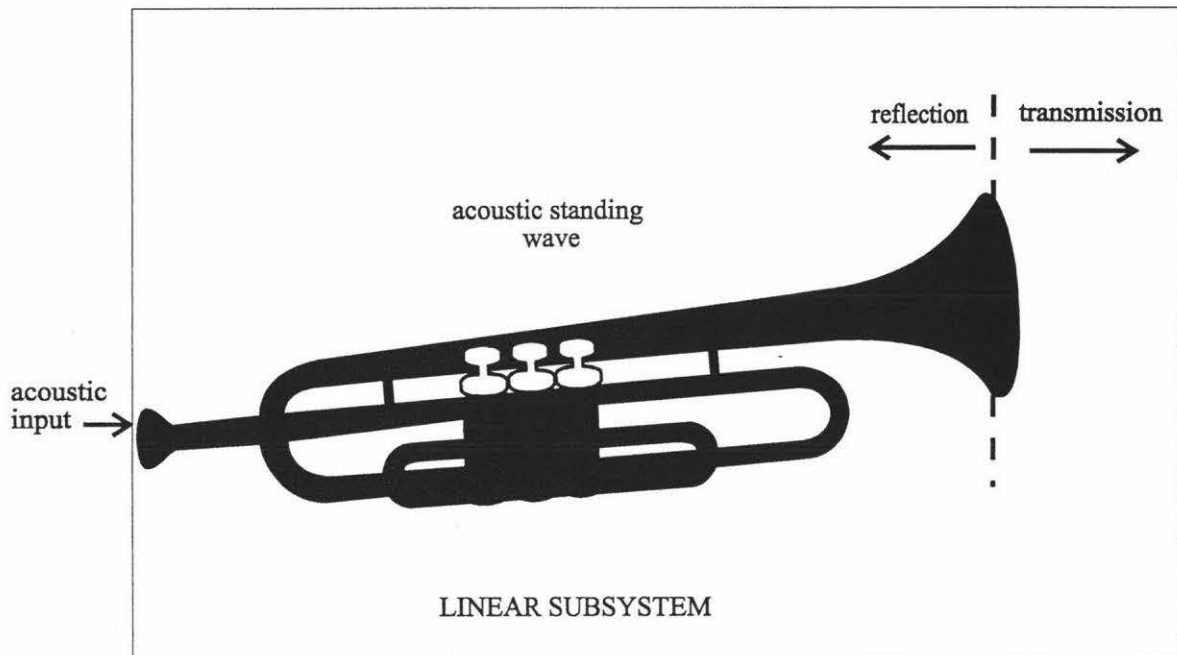
Figure 1.6   Resonator Subsystem for the Trumpeter-Trumpet System

The resonator behaviour is assumed linear and time-invariant, and therefore can be conveniently described in terms of its *acoustical impedance* (Webster, 1919b). The acoustical impedance relates a *pressure difference* $P(\omega)$ to an *acoustic volume velocity* $Q(\omega)$ at each frequency ω:

$$Z(\omega) = P(\omega) / Q(\omega).$$

Experimental data are available for the *acoustical input impedance* of trumpets (e.g. Backus, 1976; Elliott, Bowsher & Watkinson, 1982).

Many theoretical results are available for the mathematical description of linear acoustics within the trumpet and also at its open end (Levine & Schwinger, 1948; Young, 1966; Benade, 1968; Benade & Jansson, 1974).

The treatment of sound propagation in a duct, as an analogue to electrical current in a transmission line, is well-known (Morse & Ingard, 1968, §9.1). The basic theory has been refined considerably for application to woodwind instruments especially (Keefe, 1990b).

The idea of discretising the whole duct (transmission line) into a number of short lengths, each having lumped parameters, has found favour among modellers. The method of approximating a duct shape by a concatenation of several short cylindrical tubes has been popular for many years with modellers of the processes of human speech (Flanagan, 1972; Flanagan, Ishikaza & Shipley, 1975; Rabiner & Schafer, 1978; Bonder, 1983). A related method, which gives a better approximation for a given elemental length of duct, but which has not been extensively exploited because of the added numerical complexities, involves the use of truncated cones as duct elements instead (Plitnik & Strong, 1979; Caussé, Kergomard & Lurton, 1984).

When it is desirable to describe the resonator in the time domain the inverse Fourier transform of the acoustical input impedance $Z(\omega)$ will give the *impulse response* $z(t)$. The system response to an arbitrary input signal is then calculated by convolution of the impulse response with that input signal. A more computationally convenient method is to use the *reflection function*, usually denoted $r(t)$. This may be thought of as the disturbance found at the mouthpiece after an impulse is sent at $t = 0$ and the tube then terminated there by a perfect absorber. Using the reflection function, the pressure in the mouthpiece is conceptualised as the sum of incoming and outgoing waves (Equation 1.1 below), related through convolution (Equation 1.2 below) (Schumacher, 1978, 1981; McIntyre, Schumacher & Woodhouse, 1983).

$$p_2(t) = p_{in}(t) + p_{out}(t) \tag{1.1}$$

$$p_{in}(t) = r(t) * p_{out}(t) \tag{1.2}$$

The resonator description is then completed by taking the volume velocity between the lips as the input acoustic volume velocity for the trumpet.

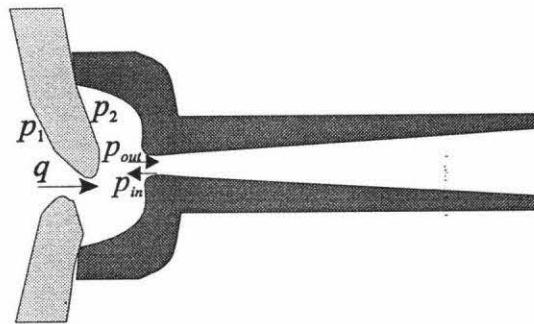$$q = \frac{1}{Z_0}\left[p_{out}(t) - p_{in}(t)\right] \tag{1.3}$$

Figure 1.7:   Pressures In The Mouthpiece

In Equation (1.3), Z_0 is the *characteristic acoustic impedance* at the entryway to the trumpet, given by $Z_0 = \rho c / A_{in}$, where A_{in} is the cross-sectional area at the entry, and ρ and c are the fluid density and sonic speed, respectively, for the fluid at rest conditions.

Alternatively, a *digital waveguide* description of the resonator is also possible (Smith, 1986, 1991b, 1992).

Little further discussion of the resonator description will be made in this thesis. The linearity of the resonator makes its mathematical description straightforward. The choice of the method of description is largely a matter of convenience.

Generator Equations:

The following description is of the generator subsystem of a basic trumpet model. Modifications to this basic model structure have been proposed by various authors from time to time, but are not considered in detail here. The model described here has been implemented before for the purpose of physical modelling synthesis of brass instrument sounds (Cook, 1991).

It is common in the literature to consider only the upper lip of the player as a vibrating body in the formulation of the model, the lower lip being treated as a fixed boundary. Experimental evidence seems to support such an approach: trumpet tones can be produced without lower lip vibrations by using a partially filled-in mouthpiece (Henderson, 1942), and some brass players have been observed to play without movement of their lower lip even while using a standard mouthpiece (Weast, 1963).

The lip motion is represented by the displacement x of a single *mass-spring-damper oscillator*. The mass m of the vibrating body is asssumed constant, the restoring force $-sx$

is assumed linear and the damping $-b\dot{x}$ is assumed *viscous* (proportional to the speed of motion). The forcing function for the oscillator is taken as the product of the pressure difference $p_1 - p_2$ across the lip and its area A^T normal to the flow (the superscript 'T' being used to denote the *transverse* area), as shown in Figure 1.8 below.

$$m\ddot{x} + b\dot{x} + sx = (p_1 - p_2)A^T \tag{1.4}$$
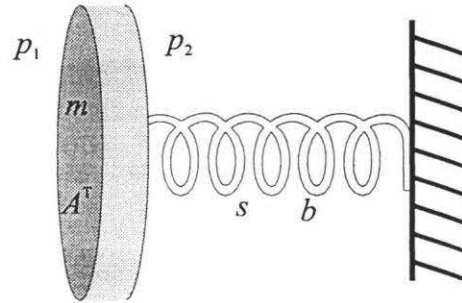
Figure 1.8:   The Lip As A Mass-Spring-Damper Oscillator

The pressure in the mouth p_1 is assumed constant and the pressure in the trumpet mouthpiece p_2 is related to the mouth pressure by an equation referred to by some modellers as a *Bernoulli Equation*:

$$p_1 - p_2 = \frac{\rho}{2}u^2 \tag{1.5}$$

Here u is the speed of the flow between the player's lips, and ρ is its density.

The area of the orifice formed by the lips is taken to be proportional to the displacement x of the oscillating body, in accordance with the results of Martin's stroboscopic observations of a trumpeter's lips (Martin, 1942; figs. 4 - 6). Martin's graphs collectively suggest that the horizontal lip displacement and the aperture area are roughly proportional. Thus the *volume velocity* of the flow, q (cubic metres per second), may be defined by

$$q = k\,ux \tag{1.6}$$

Equations (1.1) to (1.6) give a system of six equations in six unknowns (assuming that $r(t)$ is known, as well as the values of the various system parameters). These are repeated here:

$$p_2(t) = p_{in}(t) + p_{out}(t)$$

$$p_{in}(t) = r(t) * p_{out}(t)$$

$$q = \frac{1}{Z_0}\left[p_{out}(t) - p_{in}(t)\right]$$

$$m\ddot{x} + b\dot{x} + sx = \left(p_1 - p_2\right)A^{\mathrm{T}}$$

$$p_1 - p_2 = \frac{\rho}{2}u^2$$

$$q = k\,ux$$

1.3   Discussion of Some Concerns Regarding the Traditional Model:

The traditional trumpet model presented in the previous section fails to provide a convincing description of how sound is produced in the trumpet. Although the acoustical theory of the (linear) resonator subsystem is well understood, there is cause for concern in the mathematical description of the generator subsystem.

1.3.1 Concern over the Accepted Bernoulli Equation:

Equation (1.5), referred to by trumpet modellers as "Bernoulli's Equation", is repeated below:

$$p_1 - p_2 = \frac{\rho}{2}u^2$$

It is common in the literature to replace the particle velocity u using the relation $q = uA$ where q is the *volume velocity*, and A is the duct cross-sectional area at that position. (This replacement assumes that the flow does not separate from the walls of the duct between positions '1' and '2'.) Equation (1.5) becomes:

$$p_1 - p_2 = \frac{\rho q^2}{2A^2} \tag{1.7}$$

Actually, this equation is only an approximation to a very special form of Bernoulli's equation. Equation (1.8) below is that special Bernoulli equation; it is valid for steady, *quasi-1D flow* (flow is quasi-1D when velocity u and thermodynamic properties of the fluid are constant over any flow cross-section) of a fluid which has zero fluid *viscosity* (i.e. zero internal fluid friction), and which moves with constant density ρ, as it moves between two places (designated '1' and '2' in Figure 1.9).
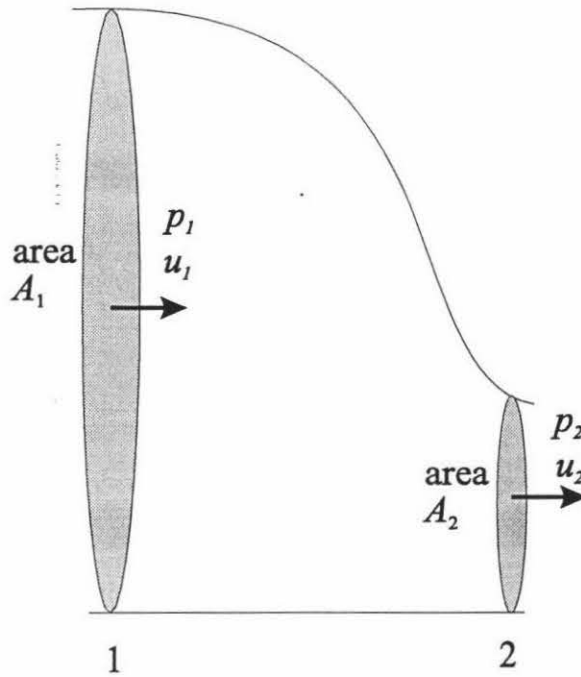
Figure 1.9:   Bernoulli Equation Variables at a Constriction

Furthermore, Equation 1.8 requires that there is no momentum transfer between the fluid and its surroundings (including gravitational effects) as it moves.

$$p_1 - p_2 = \frac{1}{2}\rho\left[\left(\frac{q_2}{A_2}\right)^2 - \left(\frac{q_1}{A_1}\right)^2\right] \tag{1.8}$$

There is an unwholesome number of unjustified assumptions in the use of even Equation (1.7) to describe the flow between the lips of a trumpeter. Comparison between Figures 1.7 and 1.9 shows that q and A of Equation (1.7) corresponds to q_2 and A_2 of Equation (1.8). Consequently a further assumption required for this Equation (1.7) to be valid, if Equation (1.8) is already known to be valid, is

$$\left(\frac{q_1}{A_1}\right)^2 << \left(\frac{q_2}{A_2}\right)^2$$

It can be concluded that Equation (1.7) will adequately describe the flow between the lips of the trumpeter if:

- the flow is steady (or varies so slowly in time that the steady form of the Bernoulli equation gives an acceptable approximation to the motion),

- the velocity u and thermodynamic properties of the fluid are constant over any flow cross-section (or such variations as so small as to have no effect upon the motion),

- the fluid viscosity is zero (or is so small that it has negligible effect upon the transport of fluid momentum between the lips),

- the fluid density is constant (or varies so little that these variations have no noticeable effect upon the overall motion of the fluid),

- the fluid is unaffected by gravity (or the effect of gravity upon the fluid is negligible with respect to the overall motion),

- there no momentum transfer to or from the fluid at its boundaries by external forces acting upon the fluid as it moves,

- the flow does not separate from the lips as it flows,

- $$\left(\frac{q_1}{A_1}\right)^2 << \left(\frac{q_2}{A_2}\right)^2$$

1.3.2 Further Concern Over the Flow Description Near the Lips:

Somewhere in the vicinity of the trumpeter's vibrating lips, there is some region of space where new acoustic energy is being generated, to be radiated outward as soundwaves. Such a region is said to contain an *acoustic source*. The actions of the source are responsible for the acoustic pressure fluctuations experienced at places far away from that source. But in the *near field* of an acoustic source, there are also other pressure fluctuations present which do not propagate as sound. The traditional model incorrectly assumes that all time-varying pressures at the lips can be regarded as acoustic.

Confusion between *unsteady convective fluid motion* and *acoustic fluid motion* sometimes arises when such fluctuations occur at audio frequencies, on account of both types of motion being accompanied by pressure fluctuations. The term *pseudosound* has been coined by workers in the field of *aero-acoustics* in an attempt to clarify the distinction between fluctuating pressures associated with unsteady convective flow and the sound

proper (Ffowcs Williams, 1969). Pseudosound refers to the pressure fluctuations which exist in the fluid as a result of local fluid accelerations; pseudosound does not propagate at all.

Texts upon the subject of acoustics often refer to non-propagating pressures as being *reactive* (Morse & Ingard, 1968). There are always reactive pressures in the near field of an acoustic source. But, because the reactive pressures do not propagate, only the *radiated* (sound) pressures are significant at places far from the source (i.e. in the *far field*).

In the case of the vibration of the trumpeter's lips, the reader will agree that there is some sort of acoustic source in the vicinity of the lips. Because the propagation medium (the player's breath) is moving relative to this acoustic source, the reactive pressures near the source will be convected away. Thus the convected flow into the trumpet will be characterised by spatial pressure gradients. The pressure fluctuations experienced downtream of the lips will then be a combination of convected pressure gradients and sound propagation.

Implications for Experimental Measurements:

The presence of pseudosound fluctuations has implications for the interpretation of pressure measurements made in the vicinity of the lips, since a microphone will respond to all pressure fluctations, not discriminating between sound and pseudosound (the ear does too - Ffowcs Williams & Lighthill, 1971).

Consider a flow-control valve at $x = 0$ that opens and closes at a frequency of 680 Hz (which lies within the range of the trumpet). The wavelength of the sound follows from

$$\lambda_s = \frac{c}{f} \approx \frac{340 \text{ ms}^{-1}}{680 \text{ s}^{-1}} = 0.5 \text{ m}$$

A microphone placed at a distance of, say, 10 mm from the flow control will measure a sound pressure which lags the pressure at the control mechanism by one-fiftieth of the period of the oscillation. Such a phase difference is barely significant.

Now consider a pseudosound fluctuation, coincident with the sound fluctuation at the flow-control, but which is convected with a mean velocity of 10 ms^{-1}. (Comparable flow speeds have been measured in the throat of a trumpet mouthpiece at approximately this

frequency - Elliott & Bowsher, 1982.) During one period of the flow-control mechanism, the pseudosound is convected by approximately the distance

$$\lambda_{ps} = \frac{\overline{u}}{f} = \frac{10 \text{ ms}^{-1}}{680 \text{ s}^{-1}} \approx 15 \text{ mm}$$

The microphone, 10 mm from the flow-control mechanism, will measure a pseudosound pressure which lags that at the control mechanism by two-thirds of an oscillation period.

The total pressure fluctuation as measured by the microphone cannot be assumed to give a faithful indication of fluctuations that are occurring at the control mechanism since the pressure signal detected by the microphone has an acoustic component and a pseudosound component that correspond to different instants of time in the history of the flow controller. Redhead (1993) has proposed an experiment in which an indication of the relative importance of convected and propagated pressure fluctuations, at a particular location, could be obtained through correlating the microphone output signal with that of another microphone placed a small distance downstream. The results of such an experiment appear in Chapter Five of this thesis.

Consequences for the Model:

It is clear that there is no consideration of pseudosound in the traditional trumpet model. The pressure in the player's mouth is assumed static while the pressure within the trumpet mouthpiece is assumed to be acoustic only. The traditional model assumes that all unsteady pressure is sound by equating the whole of the volume flow of air between the trumpeter's lips to the acoustic volume flow into the trumpet. Determination of the relative importance of the pseudosound and acoustical pressures in the vicinity of the lips is integral to the determination of the acoustic energy being produced there, and also to the determination of the fluid forces upon the lips.

Separation of the acoustic contribution to the overall flow field is a problem within the domain of *aero-acoustics*. When the *Mach number* of the flow remains small, i.e. $u \ll c$, this separation is simplified by knowledge that the pseudosound is quite uninfluenced by fluid compressibility (Ffowcs Williams, 1969). For this reason, the non-acoustic contribution to an overall flow field is sometimes referred to as the *hydrodynamic* component of the motion.

Little work has surfaced describing fluid-dynamical aspects of flow modulation within wind instruments (but see St. Hilaire, Wilson & Beavers, 1971). Fortunately, this is beginning to change (Hirschberg et al., 1990a, 1990b). Modellers of human speech production seem to have taken the fluid dynamics of the system more seriously (van den Berg, Zantemna & Doomenbal, 1957; Gupta, Wilson & Beavers, 1973; Kaiser, 1983; Teager & Teager, 1983; Pelorson et al., 1994).

1.3.3 What Excitation Mechanism for the Flow-Induced Vibration?

Flow modulation is caused by a vibrating body near a flow constriction in all reed instruments, the brass family, and in the voiced sounds of human speech. The action of flow modulation, by the vibrating body upon the passing flow, is coupled to the forcing action of the passing flow upon the oscillating body. The phenomena of *flow modulation* and *flow-induced vibration* are coupled within a feedback loop, as shown in Figure 1.10 below.

```
                    ┌─────────────────────┐
                    │     BODY MOTION     │
                    └─────────────────────┘
                         ▲          │
  flow-induced vibration │          │ flow modulation
  (by fluid forces on the body)     (by movement of the fluid's boundaries)
                         │          ▼
  upstream           ┌─────────────────────┐        downstream
  flow  ───────────▶ │     FLUID MOTION    │ ──────▶ flow
  conditions         └─────────────────────┘        conditions
```
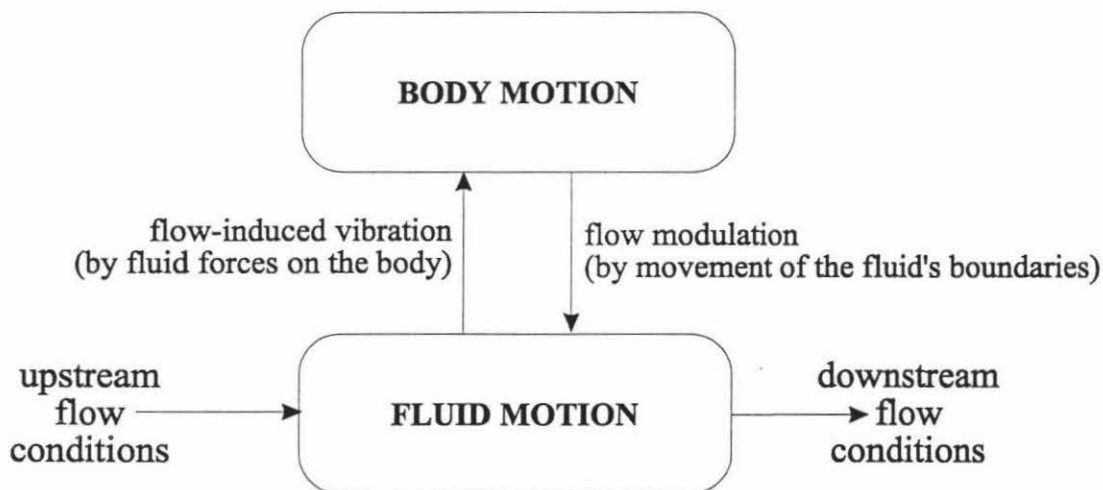
Figure 1.10:   Coupling Of Flow Modulation And Flow-Induced Vibration

Many wind instruments (e.g. clarinet, saxophone, oboe, bassoon and some organ pipes) employ a *reed*, which vibrates in a narrow part of the instrument bore as the musician plays. Brass players use their own lips as a reed.

A number of different fluid-dynamic mechanisms may be responsible for the behaviour of a flow-induced vibration, and for many situations it is a complex interaction of several of these which determines the resultant motion (Wambsganss, 1976, 1977). Some oscillators are known to be excited by completely different mechanisms, depending upon

the incident flow velocity (Parkinson & Smith, 1962; Thang & Naudascher, 1986a, 1986b). In this section some fundamental aspects of flow-induced vibration are introduced.

<u>Movement-Induced Excitation</u>:

Consider a fluid of infinite extent moving in the x-direction past a fixed, rigid body. If the body has axial symmetry about the x-axis, then the fluid force upon the body is in the direction of fluid motion, and can be written (Massey, 1989: §8.8.3)

$$F = C_D \frac{\rho}{2} U^2 A^T$$

Here C_D is a *drag coefficient*, A^T is a *characteristic body area* transverse to the flow (the definition of which depends upon the body geometry), ρ is the fluid density and U is the *free-stream velocity*; the free-stream velocity is the velocity of the fluid at a distance so far from the body that it is unaffected by the body's presence.

Now consider that the body itself is also moving, but with velocity $\dot{x}$, in the same direction as the flow. The resulting drag force will be given by (Morse & Ingard, 1968: Equation 11.3.35)

$$F = C_D \frac{\rho}{2} (U - \dot{x})^2 A^T$$

Notice that the fluid force is now dependent upon the velocity of the body. (The reader may know that, in some cases, a velocity-dependent force upon a simple harmonic oscillator can lead to self-excited oscillations.) Any resulting flow-induced oscillation of the immersed body is said to be due to *movement-induced* excitation (Naudascher & Rockwell, 1980).

Movement-induced vibrations do not require the force upon the body to be axially symmetric as in the above example. More generally, a body moving relative to a fluid will be subject to fluid forces in three orthogonal directions, and moments about their axes. The resulting flow-induced oscillation might have up to six degrees of freedom. The term *galloping* (Blevins, 1977: ch. 4) is usually used to signify an oscillation with only one degree of freedom, while oscillations that rely upon body motions of two or more degrees of freedom for their existence are termed *flutter* (Parkinson, 1971).

Instability-Induced Excitation:

For a real fluid, the fluid velocity on the surface of a stationary immersed body will be zero. This is known as the *no-slip condition*, and it applies to any body in contact with a *viscous* fluid; all real fluids are viscous. The no-slip condition also applies to bodies in motion; it is the *relative velocity* between the body and the fluid which is then zero upon the body surface. A short distance from the surface of the body, the flow velocity is almost as great as that in the free stream. The region in between, where the fluid velocity changes appreciably (from zero at the body surface to U a short distance out), is known as a *boundary layer* (Batchelor, 1967; Schlichting, 1968).

If the fluid velocity is sufficiently large (the requisite flow speed depends upon the fluid viscosity and the geometry of the body) then the boundary layer will *separate* from the body surface, and a *wake* will be formed downstream of the body. Downstream of the body, there will continue to be a layer of fluid across which the fluid velocity changes appreciably, and this is known as a *shear layer*. Shear layers are unstable, and when appropriately perturbed may *roll up* into discrete *vortices* which are then convected away with the flow.

If a body in the fluid *sheds vortices* periodically, then the fluid force upon that body also varies periodically, since the fluid force upon the body varies throughout the vortex-shedding process. If the body is not fixed, then the time-varying force upon the body can excite oscillations at the frequency of the vortex-shedding. The resulting flow-induced vibration is said to be a result of *instability-induced excitation* (Naudascher & Rockwell, 1980).

In contrast with the movement-induced excitation described already, the fluid force upon the body does not rely upon the body's motion for its time variation. The fluid force is time-varying even when the body is fixed. Furthermore, when the body is cantilevered, there are two *resonant frequencies* to be considered - the frequency associated with the vortex-shedding from the body when held fixed and the structural resonance frequency of the body when allowed to vibrate in a vacuum. The composite flow-induced vibration is not simple when these two frequencies are close, since the vortex-shedding can *lock on* to the natural vibration frequency of the body: When a body oscillates in response to vortex shedding, the induced motion of the body periodically displaces the point of flow separation from the body, at the frequency of the body vibration. These perturbations consolidate the vortex shedding process, and the vibration amplitude can increase until

such time as the body vibrations and the oscillating flow together extract the maximum amount of energy that can be provided by the incident flow.

A flow-induced vibration caused by instability-induced excitation is sometimes referred to as a *vortex-induced vibration* (Blevins, 1977: ch. 3). However, the shedding of (discrete) vortices is not an essential feature of oscillations caused by instability-induced excitation. The essential feature is the flow instability giving rise to fluid oscillations, and thus an oscillatory force upon the body.

Fluid Oscillator Effects:

Two types of flow-induced vibration excitations have been described for a body in a moving fluid of infinite extent. There are important additions that must be considered when the motion of the oscillating body occurs within a confined region of the flow. In such cases the motion of the body can have a drastic effect upon motion of the fluid past it, and thereby, the upstream and the downstream fluid behaviour. Since it is the fluid motion which determines the subsequent forcing function for the body vibrations, the result is a coupling between the *body oscillator* and a *fluid oscillator* (Naudascher & Rockwell, 1980).

A fluid oscillator may appear in conjunction with either of the two excitation mechanisms described above. A fluid oscillator is passive; it does not add energy to the system. It contributes to the behaviour of a self-excited oscillation by modifying (the amplitude and/or phase of) the fluid forcing function upon the body oscillating in the flow.

(The vibration of the trumpeter's lips certainly occurs within a confined region of fluid. Martin (1942) observed that the lips completely obstruct the flow once during each cycle of oscillation.)

As a result of fluid oscillator coupling, the fluid forces upon a structure in a flow are altered due to the fact that an otherwise steady incident flow then has an unsteady component (Blevins, 1977: ch.6). These new effects may sometimes be even more significant than original excitation forces upon the body, even though they rely upon the latter for their existence. It is possible for these fluid oscillator forces to become the major determinants of the flow-induced vibration (Kolkman & Vrijer, 1987).

Acoustic Resonator Effects:

If a flow-induced vibration occurs in some region where there is a resonant sound field of such magnitude that the acoustic fluid motions contribute significantly to the overall fluid velocity field, then the fluid force upon the body will obviously include a significant contribution from the acoustic fluid motion. But if the flow-induced vibration is caused by instability-induced excitation, there are also other means by which acoustic fluid motion can influence the behaviour of the oscillation:

The instability at a point of flow separation, such as at the *trailing edge* of an immersed body, is known to be receptive to acoustic perturbations (Morkovin & Paranjape, 1971; Ho & Huerre, 1984). At such places, the acoustic and hydrodynamic contributions to the overall fluid motion combine to collectively satisfy a boundary condition known as the *Kutta condition* (Crighton, 1981, 1985). A hydrodynamic flow field which reponds to incident sound, in order that the combined acoustic plus hydrodynamic motion might satisfy a Kutta condition, is said to be *receptive* to acoustic perturbations. As the fluid is convected away the shear layer instability leads to amplification of such responses of the hydrodynamic flow field to the incident sound. In this manner the hydrodynamic flow field, and hence the forcing function for the oscillating body, is controlled by the sound field present.

Lock-in of vortex-shedding with the vibrations of a body in a flow has been mentioned already. The shedding of vortices from an immersed body can also lock onto the frequency of a resonant sound field (Graham & Maull, 1971). Vortex-induced vibration of a body which sheds vortices at a frequency dictated by a resonant sound field is thus constrained to motion at that same frequency.

1.3.4 On the Mechanism of Sound Production:

The science of acoustics often classifies acoustic sources as being monopole, dipole or quadrupole in nature (Morse & Ingard, 1968; Lighthill, 1952).

A *monopole* acoustic source is idealised as a point in space where new fluid is introduced, and retracted, in an oscillatory fashion. This source term has no spatial dependence; the generating motion has no preferred direction, but produces a wave which radiates spherically outwards from the centre of the source. The density fluctuation at **x**, a large distance r from the centre of the source region, will be given by (Lighthill, 1952: Equation 9).

$$\rho(x,t) - \rho_0 \approx \frac{1}{4\pi r c^2} \int\limits_{\substack{source \\ region}} \dot{Q}\left(y, t - \frac{r}{c}\right) dy$$

The function $\dot{Q}$ gives the time-rate of change of the rate of introduction of mass, per unit volume, within the source region. The density at the distance r is proportional to the rate of change of the flow introduction at a time r/c earlier. The time $t - r/c$ is known as the *retarded time*.

A dipole source is characterised by an oscillatory force upon the fluid medium. A dipole might be thought of as two monopole sources side by side, but opposite in sign, so that one expands as the other contracts. The resulting pressure field is directional. When the source region is much smaller than the wavelength of the sound (Lighthill, 1952: Equation 12),

$$\rho(x,t) - \rho_0 \approx \frac{1}{4\pi r c^3} \frac{x_i}{r} \int\limits_{\substack{source \\ region}} \dot{F}_i\left(y, t - \frac{r}{c}\right) dy$$

The far field fluctuations are greatest in the direction of the force, since $x_i = r\cos\theta$, where θ is the angle measured from the force direction.

Quadrupole sources accompany fluctuations in certain types of stresses within a fluid. The specially-formulated *Lighthill stress tensor* is usually denoted T_{ij}. A quadrupole source might be thought of as comprising four monopole sources. For a *lateral quadrupole*, the four are arranged in a tiny square, and diagonally opposite monopoles have the same sign. The monopoles which make up a *longitudinal quadrupole* lie all along the same line. For both types, the pressure field is directional. When the wavelength of the sound is much larger than the source region, the resulting far-field density fluctuations are given by (Lighthill, 1952: Equation 17)

$$\rho(x,t) - \rho_0 \approx \frac{1}{4\pi r c^4} \frac{x_i}{r} \frac{x_j}{r} \int\limits_{\substack{source \\ region}} \ddot{T}_{ij}\left(y, t - \frac{r}{c}\right) dy$$

The previous three equations are approximate solutions for the pressures far away from the source being considered. The *far field* sound pressures radiating from a dipole source are of lower order than those from a monopole source, and those from quadrupole sources are lower again; notice the increasing power of c in the denominators of the pressure equations (also of r).

Possible Sources at a Trumpeter's Lips:

Some possibilities for the sound-production mechanism at the vibrating lips of the trumpeter will now be examined. Recall that monopole sources result from a changing rate of introduction of new fluid, dipole sources can arise through fluctuating forces applied to the fluid, and quadrupole sources will be due to stresses appearing as a result of the fluid motion. Also, if there are any monopole sources present, these will be the most important, then dipoles, then quadrupoles.

A monopole source would be signalled by any changing rate of introduction of new fluid at any place within the flow. Because the aperture between the lips of the trumpet player is always changing, the rate of introduction of air into the trumpet is time-varying, and so this could be interpreted as a monopole source.

The introduction of air into the trumpet is accompanied by the release of air from the player's mouth, so that there is really a *pair* of monopoles of opposite sign. One monopole represents the changing rate of introduction of fluid into the trumpet mouthpiece; the other represents the changing output rate of fluid from the player's mouth (see Figure 1.11). The combined result of two adjacent, oppositely-signed monopoles is a dipole source.
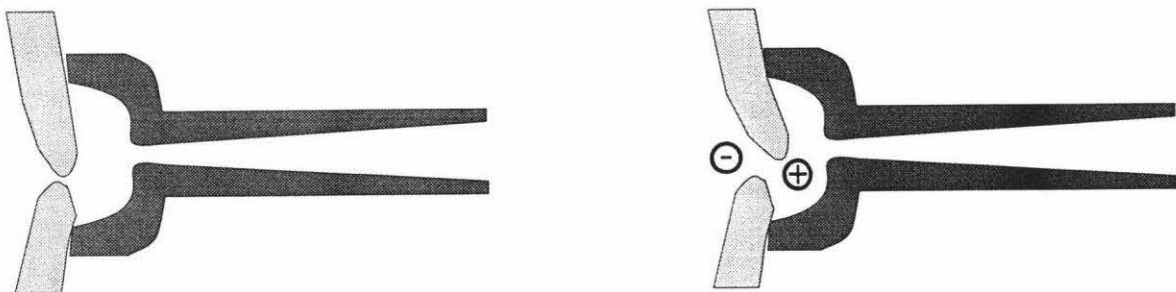

Figure 1.11: Acoustic Sources Due To Changing Flow-Rate Between The Lips

A second dipole source is evident through the motion of the trumpeter's lips. This motion indicates a fluctuating force upon the lips, and this force is necessarily provided by the flow. Since for every action there is an equal and opposite reaction (Newton's First Law), the motion of the lips also indicates the presence of a time-varying force upon the fluid, and this provides a dipole sound source. Radiation is most favoured in the direction of

motion of the lips. In Figure 1.12, the directionality of the motion, and hence the dipole sound field, is indicated by the arrow.
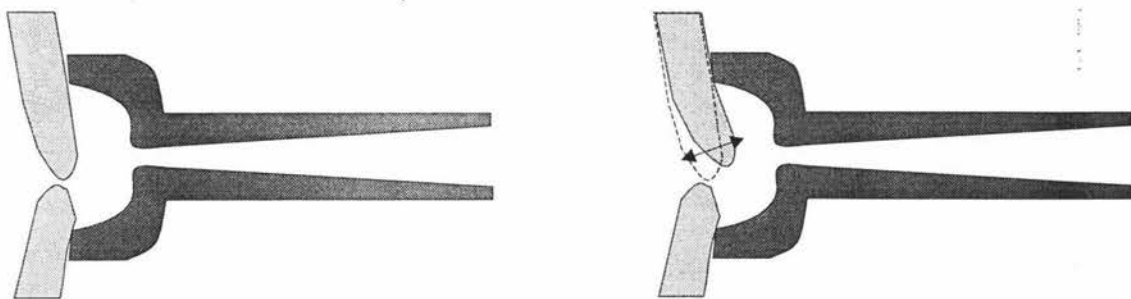

Figure 1.12:   Acoustic Source Due To Lip Vibrations

A third dipole source is possible at the back of the mouthpiece cup. Because there is a time-varying flow emerging from between the player's lips (Figure 1.11), there must be a time-varying aerodynamic force upon the inside of the mouthpiece. This time-varying force could be responsible for additional sound generation (Figure 1.13).
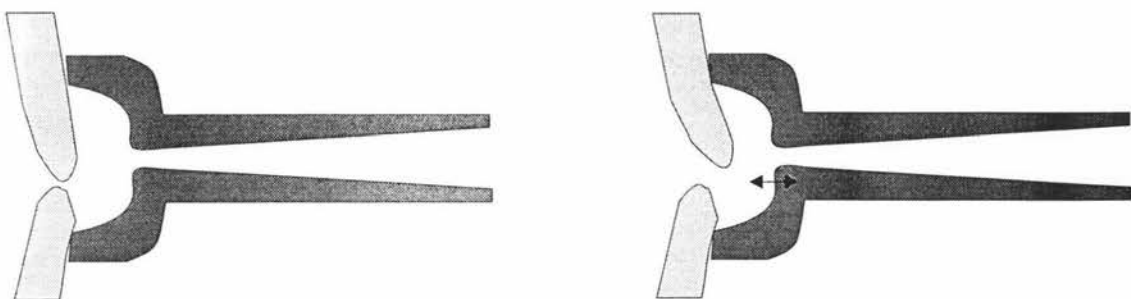

Figure 1.13:   Acoustic Source Due To Unsteady Fluid Force Upon Mouthpiece

Any unsteady introduction of fluid between the lips at a time t will produce acoustic energy at the lips at that same time t, but will produce sound at the back of the mouthpiece cup at some later time $t+T$, where T is the time taken for the flow unsteadiness to be convected downstream from the lips to the back of the mouthpiece. The mouthpiece source lags the flow-modulation source by a time interval which depends upon the depth of the mouthpiece cup and the speed at which unsteadiness is convected by the flow. Such an acoustic source is thought to be important for the production of *hole tones* (Powell, 1953; Chanaud & Powell, 1965; Wilson et. al., 1971; Ho & Nossier, 1981; Rockwell, 1983).

Three dipole sources have been identified in the vicinity of the trumpeter's lips: the unsteady influx of air from the player's mouth, the time-varying force due to the lip motion directly, and the time-varying fluid force upon the back of the mouthpiece cup.

Since dipole sound sources are of higher order than any quadrupole sources that may be present in the flow (Lighthill, 1952), the contributions of the latter to the radiated sound field are expected to be relatively insignificant.

1.3.5 Concern Over the Description of Lip Dynamics:

The function of the generator part of the complete trumpet system is to accept a steady energy influx from the player (air flow from the lungs) and to modulate this in some way to provide acoustic energy for the resonator. Many voice and wind instrument modellers have suggested that the generator subsystem should function as a *self-excited oscillator* in its own right (i.e. without relying upon feedback from the resonator subsystem). However, the generator part of the traditional trumpet model cannot operate alone as a self-excited oscillator.

Self-excited oscillation is a special type of forced oscillation. A self-excited oscillator has the ability to regulate the rate of transfer of energy from a steady (or quasi-steady) energy reserve in such a manner that oscillation is maintained. The forcing function of a self-excited oscillator is not a function of time directly, but instead depends upon the state variables of the oscillator (position, speed and acceleration).

Recall that in the traditional trumpet model the lip vibrations are represented by the motion of a simple mass-spring-damper oscillator under the influence of an applied (pressure) force. This might be written as

$$m\ddot{x} + b\dot{x} + sx = F(t) \tag{1.9}$$

The traditional trumpet model uses the following forcing function for $F(t)$:

$$F(t) = (p_1 - p_2)A^{\mathsf{T}} \tag{1.10}$$

Consider what happens to Equation (1.10) when the resonator is decoupled from the generator of the traditional trumpet model. With the trumpet absent, the pressure p_2 becomes the ambient pressure outside the mouth (atmospheric pressure). The force upon the lips then reduces to $F(t) = constant$ in the traditional trumpet model. Because the

resulting forcing function does not depend upon time (directly or indirectly), there is no scope for self-excited oscillations.

This is contrary to the experience of trumpet players, suggesting one or more of the following:

(a) p_1 or A^T are actually system variables (i.e. they are not constant);

(b)   Equation (1.10) does not accurately describe the fluid force on the lips;

(c)   the assumption of a simple harmonic oscillator is too simplistic, and maybe a modification (such as a nonlinear restoring force) is required.

The third possibility is now considered.

Suitability of a One-Mass Model:

The history of voiced-speech generator models shows that self-excited oscillations, though impossible for a single mass-spring-damper description of the vocal folds (Flanagan & Landgraf, 1968; Flanagan & Cherry, 1969), could be produced with a two-mass model (Ishikaza & Flanagan, 1972). Still more complexity has been added by later researchers - by some to give a better representation of physiological reality (Titze, 1973, 1974; Story & Titze 1995) and by others with the aim to produce a more natural-sounding speech synthesiser model (Koizumi, Tanuguchi & Hiromitsu, 1987).

While speech modellers have gone to great lengths to model as closely as possible the anatomy of the vocal folds, the buccolabial musculature has not received any mention in the trumpet-modelling literature. The situation is actually much simpler than for the vocal folds.
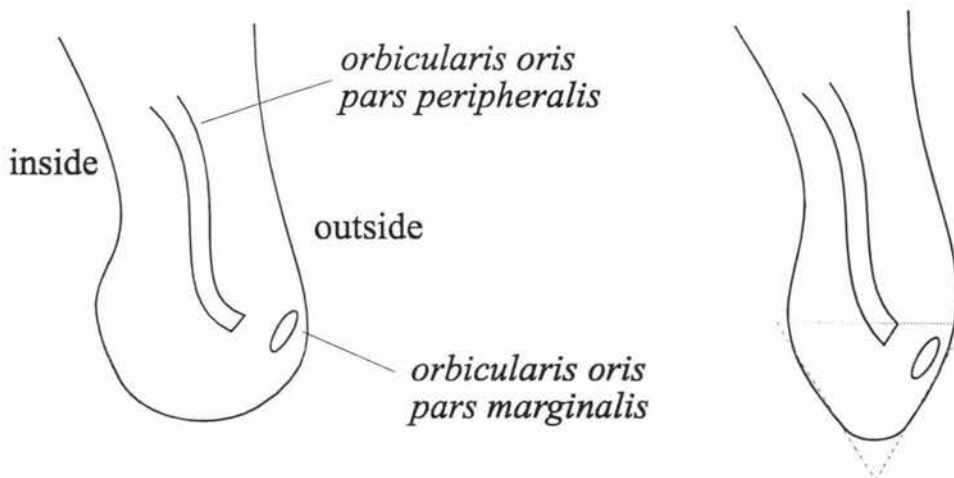

Figure 1.14:   Top-Lip Profiles For Different Tensions of *Pars Marginalis*

Contraction of the muscle *orbicularis oris pars marginalis* is considered to alter the sectional profile of the red-lip rim. The upper and lower lips both transform to a narrower shape reminiscent of a truncated isosceles triangle. Both the length and the tension, of the so-called *labial cords* that result, can be delicately controlled (Williams et al., 1989).

The vocal folds are much more difficult to model, as they may entertain oscillations of many degrees of freedom (the term *vocal folds* has gradually replaced vocal *cords* in the literature since this has become known). Titze (1973, 1974) describes a model of the vocal folds that utilises sixteen coupled SDF oscillators.

Stroboscopic and other studies of lip motion during trumpet-playing indicate that only the top lip need be modelled (Martin, 1942; Henderson, 1942; Weast, 1963).

Motional Constraint:

In the vertical direction, the distance of separation between the lips (alternatively, the area of the orifice between them) can never take a negative value. In the horizontal direction, the lip vibrations are restricted by the teeth behind them and the trumpet mouthpiece in front. One consequence of these motional constraints has been overlooked in the traditional mathematical description of trumpet operation - that the lips may already be under tension before a note begins. To illustrate this point, consider Figure 1.15 to follow:
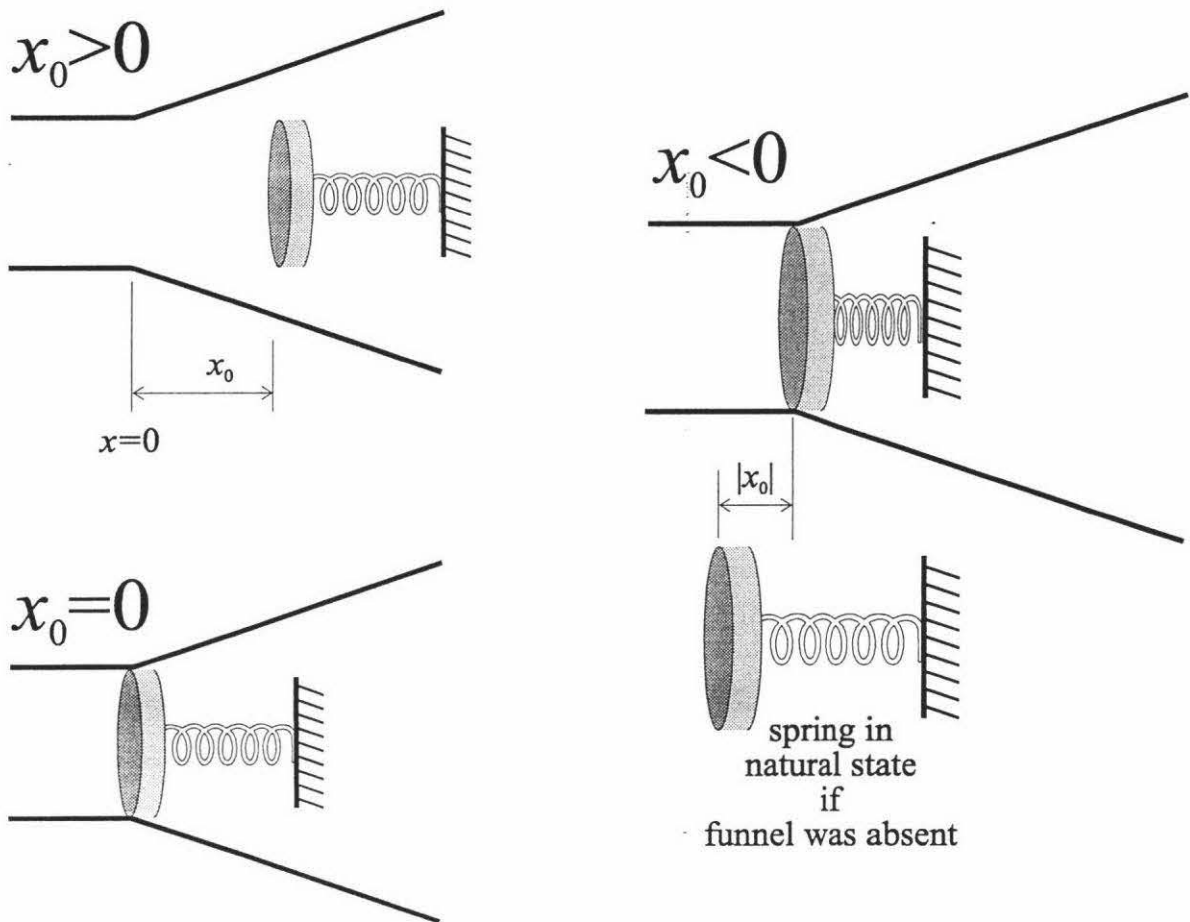
Figure 1.15: Plunger-in-Funnel Model In Absence of Fluid Forcing: "Pre-Tension"

Let x_0 denote the distance from the plunger to the funnel in the absence of fluid forces or motional constraints. Then the equation of motion for the mass, when a forcing function $F(t)$ is added, is

$$m\ddot{x} + b\dot{x} + s(x - x_0) = F(t), \quad x > 0$$
$$\Leftrightarrow \quad m\ddot{x} + b\dot{x} + sx = F(t) + sx_0, \quad x > 0$$

For the case of $x_0 < 0$, the term sx_0 indicates the amount by which the spring is "pre-tensioned" in the absence of any applied fluid forces. When the plunger is first at rest in the funnel (i.e. $x = 0$, $\dot{x} = 0$) the applied fluid force must reach a threshold value $|sx_0|$ before any motion of the plunger can commence.

Lack of consideration of motional constraints in the traditional model (and the related possibility of pre-tension forces) has led to some remarkable conclusions regarding the playing of the trumpet. For example: "If the static opening of the lips in the absence of blowing pressure is zero, as is generally the case, then there is no threshold pressure

required..." (Fletcher, 1990). This statement is contrary to the experiences of every brass instrumentalist. (The opposite of Fletcher's assertion also appears in the literature; see Worman, 1972.)

Similarly disconcerting is a statement that there is no upper limit to the static pressure provided by the musician, above which system oscillation is impossible (Fletcher, 1979a). It is interesting that Fletcher's two statements, taken in conjunction, infer that any positive pressure gradient across the musician's lips is suitable for exciting a trumpet into oscillation. It takes very little practice to refute this experimentally.

Impact

In the above plunger-in-funnel illustration, impact occurs when the plunger returns to the position $x = 0$, if it approaches with non-zero velocity.

The question of impact has received rather different treatments in models of some mildly analogous systems. Hirschberg et al. (1990a) studied the behaviour of a reed organ-pipe and modelled the reed impact as an elastic collision. The momentum instantaneously lost from the oscillation through impact was instantaneously returned in the opposite direction: ie. $\dot{x}(t) \leftarrow -\dot{x}(t)$ at the moment of collision.

Flanagan & Landgraf (1968) consider two different possibilities for the collision of the vocal folds during voiced speech. One scenario involved all momentum being instantaneously sapped from the vibration. For a completely inelastic collision $\dot{x}(t) \leftarrow 0$ at the instant of impact. The other was termed a "purely viscous contact", where the boundary constraint $x \geq 0$ was actually violated, but at such times the damping constant of the motion was substantially increased. The advantage of the latter is that *the closure times* increase with the velocity of the approach at impact.

A more comprehensive model may incorporate a second mass-spring-damper (with possibly different coefficients) to represent any yielding and/or vibration after an impact. Such an arrangement is depicted in Figure 1.16.
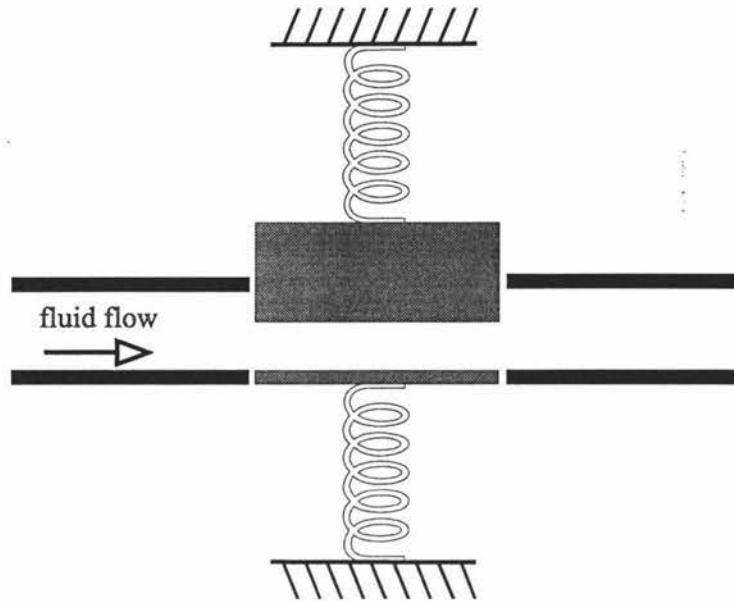
Figure 1.16:   Model of Impact of a Transversely Vibrating Piston In A Compliant Duct

The effect of impact upon the behaviour of a single degree-of-freedom oscillator can be marked (Ipanov,1993, 1994; Budd, Fox & Cliffe, 1995; Narayanan & Sekar, 1995).

1.3.6 The System is Incompletely Described:

When a model accurately represents a physical system, the model and the system both function similarly when presented with identical sets of values for the various system inputs. There is no hope for this to be achieved by the traditional trumpet model, since some of the system inputs are not even present in the model description.

Knowledge of trumpet-playing technique (described in Section 1.1) reveals the physical gestures by which the trumpeter initiates and controls the sound produced. Recognition of these gestures can be used to define the controlling variables of the musical oscillator system. Two classes of physical gestures can be distinguished (Cadoz, Luciani & Florens, 1984). These are *excitation gestures* and *modulation gestures.*

An excitation gesture is a means by which the musician effects a transfer of energy to the oscillator. When an instrumentalist has the ability to control the rate at which the energy is supplied to the system, the excitation gesture determines that rate. If this energy supply-rate determines the amount of energy that the system sheds as propagated sound, then the musician becomes part of a feedback loop that maintains the desired level of sound output. The player listens continually to the sound being produced, and makes adjustments using excitation gestures if it is too loud or too soft.

34

A modulation gesture is a control mechanism used to otherwise modify some characteristic of the oscillation. Modulation gestures may also feature within a feedback loop. For example, if a player hears a note to be out of tune then a modulation gesture may be used to correct this (see Figure 1.17).

aural feedback

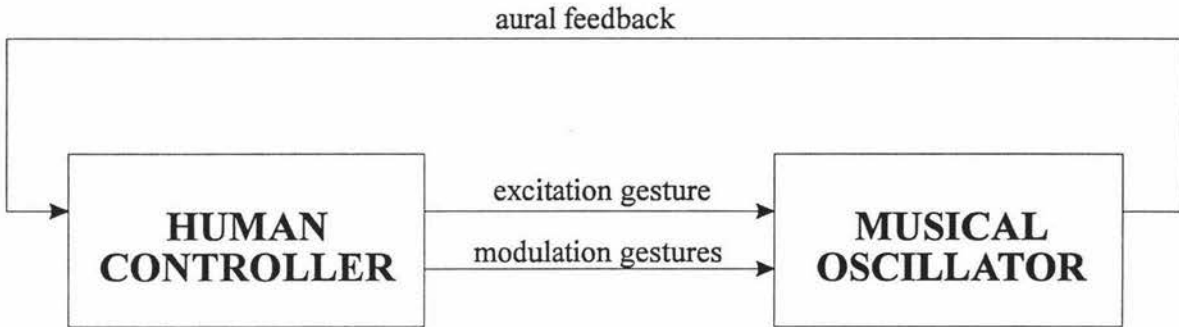HUMAN CONTROLLER → excitation gesture / modulation gestures → MUSICAL OSCILLATOR

Figure 1.17:   Human Control Of A Musical Oscillator

The excitation gesture of the brass player corresponds to the muscular control of his diaphragm, which dictates the flow of air from the lungs; in the case of wind instruments, the influx of energy into the system is coincident with the flow of air from the lungs. Adjustment of the embouchure is a modulation gesture, since such adjustment does not add energy to the system. The same applies to the variation of the vocal tract parameters, and the actions of the fingers in changing the valve positions of the trumpet. A top-level representation of the trumpet-trumpeter system is illustrated in Figure 1.18:

diaphragm force →
embouchure →
vocal tract shape →   TRUMPET OSCILLATOR SYSTEM   → acoustic output
fingering position →   → heat losses
tongue action →   → steady flow
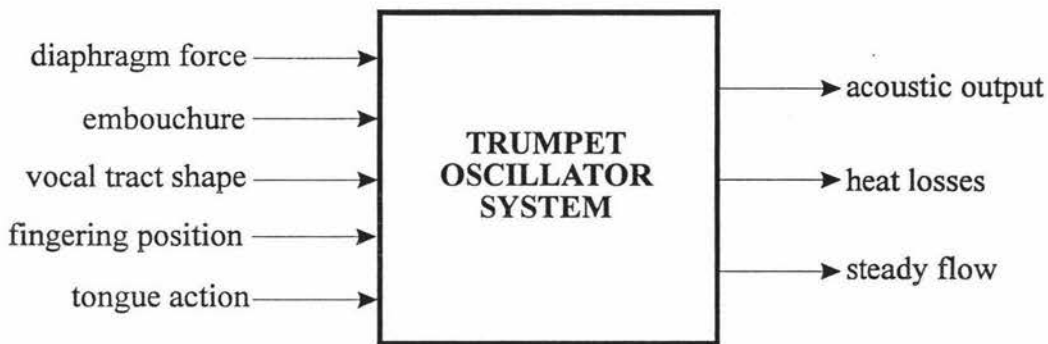
Figure 1.18:   System Inputs and Outputs

The trumpet-trumpeter system features an energy conversion mechanism. The energy provided by the excitation gesture of the musician (diaphragm force) is converted to

acoustic, heat and flow energy, and the modulation gestures of the player determine the finer details of that conversion process. The oscillating system thus comprises:

- flow within the lungs, vocal tract and oral cavity of the player,

- flow within the trumpet itself, and

- the player's vibrating lips.

In contrast to system illustrated in Figure 1.18, the system described by the traditional trumpet model begins at the mouth. The resonator subsystem includes only the inside the trumpet itself and the mouth is treated as a constant pressure source. This can give a valid representation of the trumpet oscillator only if the following two conditions are met: firstly, the modulation gestures involving variations of vocal tract characteristics must be shown to be unimportant, and secondly, the mouth pressure must be shown to be a *functional equivalent* of the diaphragm force.

The effects of vocal tract shape in the sounding of wind instruments were examined experimentally many years ago (Hall, 1955; Stauffer, 1968). Another study measured the relative tensions of the facial muscles used in blowing the instrument (Stubbins, Lillya & Frederick, 1956) and the mechanical impedance of the vocal tract walls has been measured directly (Ishizaka, French & Flanagan, 1975). More recently, x-rays of various wind instrumentalists have been used to compare the acoustic resonant frequencies of the vocal tract to the pitch of the notes being sounded (Clinch, Troup & Harris, 1982). The evidence of these experimentalists shows that the effects of the vocal tract parameters *are* important in playing the trumpet. None of these effects can be described using the traditional model.

Consider the respiratory system of the trumpet player, consisting of the lungs, vocal tract and oral cavity. At the lungs, a constant force provided by the diaphragm muscles will produce a steady flow of air from the lungs, while the vibration of the lips dictates that the flow out of the mouth is unsteady. Consequently the amount of air between the lungs and lips of the player must be continually changing. The rate of this change at each instant is determined by the difference between the instantaneous mass outflux at the lips and the (steady) mass influx from the lungs. The mass of air between the lungs and the lips must oscillate in value at the same frequency as the lip vibrations. The vocal tract thus functions as a *fluid oscillator* (Naudascher & Rockwell, 1980; see also Gupta, Wilson & Beavers, 1973).

There are two possible responses to the resulting mass oscillation. Firstly, the *density* of the air within the player's respiratory system may fluctuate in value, and secondly, the walls of the vocal tract may expand in response to the presence of additional air, altering the system's *volume*. Some variation of the density of the air is inevitable and, as a result, the mouth pressure must be oscillating at the frequency of lip vibration. Indeed, actual experimental measurements of a trombonist's mouth pressure have shown periodic fluctuations at the frequency of the lip oscillation (Elliott & Bowsher, 1982).

Since the mouth pressure of the trumpeter is oscillatory, then the force exerted by the player's diaphragm muscles and the pressure in his or her mouth *cannot* be functional equivalents. The traditional trumpet model's exclusion of the lungs and vocal tract of the player is unjustified.

The resulting model is incapable of describing any effects of the player's vocal tract parameters, nor the mass oscillation which occurs in the trumpeter's respiratory system. Some modellers have suggested that consideration of the vocal tract might lead to self-excited oscillation of the model lips even without the trumpet present (Fletcher, 1979a, 1979b; Benade & Hoekje, 1982).