# The Scattering of Waves by An Elastic Floating Body on Water of Variable Depth

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# Abstract

For many years polar scientists and offshore engineers have studied the behavior of a floating body in the presence of ocean waves. A large floating structure, such as a floating runway or an ice sheet, is sufficiently thin so that elasticity is important. The solution of the motion can found by coupling the water and elastic plate equations. However these solutions have only been calculated when the water is of constant depth.

In this thesis we shall present a solution for wave scattering by an elastic body on water of variable depth. Our solution method involves partitioning the problem domain into finite and semi-infinite regions. In the semi-infinite region the solution is obtained using an integral equation. A boundary element method together with a Green's function for the thin plate is used to solve for the finite region. The separate solution are then coupled to give the solution for the full problem.

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# Chapter 1
# Introduction

For many years the study of ocean waves has been of interest in polar and offshore engineering. The object of the study is to find out how an arbitrary large floating body on the surface of the water behaves in the presence of an ocean wave. Some researchers are interested in the motion of the floating body while others study the motion of the water itself. Our main interest is to study the motion of the water in the presence of a floating body, by calculating the scattering of the water wave.

The scattering of the wave is influenced by many factors, the most important of which are the shape and the elasticity of the floating body. For example a round-shaped body rocks in different way from a flat-shaped body. A body of significant stiffness (such as a ship) is said to have small elasticity which is not important in determining the scattered wave, but for a sufficently thin body (such as a thin ice sheet) elasticity becomes important. We only consider a floating body which is flat, rectangular, and thin, so that elasticity is important. As well as the shape and elasticity of the floating body, the shape of the seabed also influences the behaviour of the wave. A constant bed topography (flat seabed) does not scatter the wave while a non-constant bed topography (varying seabed), such that the depth of the water is variable, scatters the wave. In this thesis we seek solutions for the scattering of waves by an elastic floating body on the surface of water with variable depth. The elastic floating body and the varying seabed are referred to as the scatterers.

The main reason for our interest is that, to our knowledge, no author has solved the scattering problem for the elastic floating body with varying seabed. However many publications have presented a number of solutions for simpler problems. There are two types of simplified problem, one which considers the elastic floating body only while the seabed is constant and the other one is to have a varying seabed with the surface of the water free. The first type of the simplified problem was solved by *Meylan* (1993) for an ice floe and by *Namba and Ohkusu* (1999) for a floating artificial island. Both used linear shallow water theory (*Stoker* (1957)). The second type of the simplified problem was solved by *Staziker, Porter and Stirling* (1997) for a localized bed topography with uniform depth at its ends. Their solution method used an integral equation and a Green's function.

In solving the problem of the wave scattering by an elastic body on water of variable depth we adopt the method used by *Liu and Liggett* (1982) and *Hazard and Lenoir* (1993). Both publications used artificial boundaries dividing the water domain into a finite and two semi-infinite domains. These boundaries are located at a sufficient distance from the scatterers so that the water depth is assumed constant; this method is called the far-field method, the solution being obtained using eigenfunction expansions. Then the far-field solution in the semi-infinite domain is matched with the solution in the finite domain. *Liu and Liggett* solved this using the boundary element method (BEM) and *Hazard and Lenoir* used the finite element method (FEM).

To solve the wave scattering problem by an elastic body floating on water of variable depth we use the far-field method and the BEM. Our solution technique involves two main steps. The first is to split the water domain into finite and semi-infinite domains. The solution of the wave in the semi-infinite domain is obtained using appropriate separation of variables (leading to an eigenfunction expansion). From this solution we derive the coupling operator for the boundary condition at the artificial boundaries joining the finite and the semi-infinite domains. The solution for the wave in the finite domain cannot be solved exactly therefore we must solve it numerically. This leads us to the second step, the BEM, which involves transforming the wave problem into an integral equation together with the free-space Green's function.

The solution of the wave scattering problem is presented in terms of the wave potential, the techniques for which will be explained in detail in the first three chapters of the thesis. The first chapter explains the water domain and the governing equations for the wave scattering problem, including the derivation of the stationary model for the problem and the incident wave. The second chapter explains the reduction to finite and semi-infinite domains. In the first part of this chapter we derive the solution to the wave potential in the semi-infinite domain using separation of variables and the expression for the coupling operator for the boundary conditions. In the second part of chapter two we describe the BEM procedure for solving the wave potential in the finite domain. This procedure involves two integral equations (one is for the boundary condition underneath the elastic floating plate and one is for the solution of the wave potential) and two Green's functions (one for the elastic floating plate and the other the free space Green's function for the wave potential). Then in the third chapter we solve the integral equations from the second chapter numerically by solving an equivalent linear equation which involves matrices for the Green's function and for the boundary conditions as well as vectors for the wave potential and the constant.

The next two chapters present the results of the implementation of the BEM. In the fourth chapter we define the reflected and the transmitted waves, which are useful for error checking in the numerical solution. In the fifth chapter we present some relevant results from the numerical computation of the wave potential. At the beginning of the chapter we test for convergence. This provide us with an appropriate choice of parameters used. Then we compare the results with the previous ones of *Meylan* [5], *Staziker, Porter and Stirling* [7], and *Namba and Ohkushu* [6].