

The disjunctivities of ω -languages

Yow Tzong Yeh

I.I.M.S., Massey University Albany Campus, Auckland, N.Z.

y.t.yeh@massey.ac.nz

Abstract

An ω -language over a finite alphabet X is a set of infinite sequences of letters of X . Consider congruences I_L and $P_{\omega, L}$ on X^* and a congruence O_L on X^ω introduced by an ω -language L . I_L , $P_{\omega, L}$, and O_L are called the infinitary syntactic-congruence, the principal congruence and the ω -syntactic congruence of L , respectively. If I_L ($P_{\omega, L}$, O_L) is the equality then L is called an I-disjunctive (P-disjunctive, O-disjunctive, respectively) ω -language. Properties concerning such ω -languages are explored and relations between these ω -languages are also studied.

The disjunctivity concerning the infinitary syntactic-congruence I_L

Given an ω -language L , by the infinitary syntactic-congruence I_L of L we mean the relation I_L on X^* given by $u \equiv v$ (I_L) $\Leftrightarrow \forall x, y \in X^*$, $x(uy)^\omega \in L$ iff $x(vy)^\omega \in L$. If I_L is the equality then L is called I-disjunctive. Every I-discrete I-dense ω -language is I-disjunctive. An ω -language is I-dense iff it contains an I-disjunctive language. A periodically generated ω -language L is I-dense iff L can be expressed as a disjoint union of infinitely many I-disjunctive ω -languages.

The disjunctivity concerning the principal congruence $P_{\omega, L}$

Given an ω -language L , by the principal congruence $P_{\omega, L}$ of L we mean the relation $P_{\omega, L}$ on X^* given by $u \equiv v$ ($P_{\omega, L}$) $\Leftrightarrow \forall x \in X^*$ and $\alpha \in X^\omega$, $xu\alpha \in L$ iff $xv\alpha \in L$. If $P_{\omega, L}$ is the equality then L is called P-disjunctive. Every P-discrete P-dense ω -language is P-disjunctive. A P-discrete ω -language is P-disjunctive iff the set of all its finite subwords is X^* .

The disjunctivity concerning the ω -syntactic congruence O_L

Given an ω -language L , by the ω -syntactic congruence O_L of L we mean the relation O_L on X^ω given by $\alpha \equiv \beta$ (O_L) $\Leftrightarrow \forall x \in X^*$, $x\alpha \in L$ iff $x\beta \in L$. If O_L is the equality then L is called O-disjunctive. If S is a left singular language then SL is O-disjunctive for any O-disjunctive ω -language L . If P is a finite prefix code then L is an O-disjunctive ω -language iff PL is O-disjunctive.

Families of Disjunctive ω -languages

Every I-closed I-disjunctive ω -language is P-disjunctive while not every P-disjunctive ω -language is I-disjunctive. Every O-disjunctive ω -language is P-disjunctive while not every P-disjunctive ω -language is O-disjunctive. Every O-disjunctive ω -language is I-disjunctive while not every I-disjunctive ω -language is O-disjunctive. Every P-disjunctive ω -language is P-dense

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