

Copyright is owned by the Author of the thesis. Permission is given for a copy to be downloaded by an individual for the purpose of research and private study only. The thesis may not be reproduced elsewhere without the permission of the Author.

THE COOLING OF SPENT CARBON ANODES IN THE
ALUMINIUM SMELTING INDUSTRY

A THESIS PRESENTED IN PARTIAL FULFILMENT
OF THE REQUIREMENTS FOR
THE DEGREE OF MASTER OF SCIENCE IN
MATHEMATICS AT
MASSEY UNIVERSITY

Matthew Cole

March 1996

Abstract

As part of the New Zealand Aluminium Smelters (NZAS) upgrade, a hot butt cleaning system has been proposed, this would remove the bath from anodes as they are removed from the cells. It is expected that the time to cool for hot cleaned anodes would be significantly less than for current method of allowing the butts to cool before the bath is removed.

In this project a mathematical model of the cooling process of both the clean and dirty anodes is developed. This model will aid in the investigation of the hot butt cleaning system by showing the difference in cooling times between the clean and dirty anodes.

The temperature profiles within both clean and dirty anodes is calculated for one-, two- and three-dimensional models. Temperature changes in the anodes with time are also compared to experimental data.

Acknowledgments

Many people have helped me in during the writing of this thesis. In particular I would like to acknowledge the following people:

Professor Robert McKibbin for his guidance and supervision over the last year.

I thank New Zealand Aluminium Smelters (NZAS) for their support. Thank you to John Carran and Rob Wallace for your ideas and assistance.

Graeme and Rowena my parents for all their support and encouragement, for being there when I need you and letting me borrow the car when needed.

My brother Richard for all the email messages and helping out with the drawing of the diagrams. My brother Nicholas, sister Louise and brother-in-law Murray.

My flatmates who have helped me over the last year, especially Phillip and Nigel for being good mates.

Associate Professor Dean Halford and colleagues of the Mathematics Department for your encouragement and support. Professor Graeme Wake for suggesting I apply for the scholarship and do a Masters.

The other graduate assistants for your encouragement and advice, especially Mark Johnston for all help with the computers.

All my friends at Christian Community Church for your guidance and prayers, especially Hugh and Donna McGarvey and the homegroup.

Nomenclature

All constants and variables used in this thesis are defined when first used. Commonly used notation is summarized here.

| | |
|-----------------------|---|
| a, b, c, d, e, f, g | diffusivity coefficients [m^2/s] |
| A | surface area [m^2] |
| A, B, C, D | defined variables |
| A_h | horizontal downward facing surface area [m^2] |
| Bi | Biot number [—] |
| c | heat capacity [kJ/kgK] |
| c_1, c_2, c_3 | defined constant |
| D | diameter of sphere [m] |
| E_g | energy generated in a system [J] |
| E_{in} | energy transferred into a system [J] |
| E_{out} | energy transferred out of a system [J] |
| E_s | energy stored in a system [J] |
| $F(x), G(t)$ | defined function |
| Fo | Fourier number [—] |
| g | gravitational acceleration [m/s^2] |
| h | heat transfer coefficient [W/m^2K] |
| h_c | convection heat transfer coefficient [W/m^2K] |

| | |
|--------------------------------|--|
| h_r | radiation heat transfer coefficient [W/m^2K] |
| H | height [m] |
| H | characteristic length [m] |
| H | dimensionless heat transfer coefficient [-] |
| k | thermal conductivity [W/mK] |
| k_x | thermal conductivity in x-direction [W/mK] |
| k_y | thermal conductivity in y-direction [W/mK] |
| k_z | thermal conductivity in z-direction [W/mK] |
| L | length [m] |
| L_m | longest linear dimension [m] |
| L_x | x dimension [m] |
| L_y | y dimension [m] |
| L_z | z dimension [m] |
| Nu | Nusselt Number [-] |
| \bar{P} | mean horizontal perimeter [m] |
| Pr | Prandtl Number [-] |
| q | rate of heat transfer [kW] |
| q_c | rate of convection heat transfer [kW] |
| q_r | rate of radiation heat transfer [kW] |
| q'' | rate of heat transfer [kW] |
| $q_1, q_2, q_3, q_4, q_5, q_6$ | rate of heat transfer from specific direction [kW] |
| Ra_H | Rayleigh Number [-] |
| Re | Reynolds Number [-] |
| t | time [s] |

| | |
|-----------------------------|--|
| t_o | time normalisation constant [s] |
| \bar{t} | normalised time [-] |
| T | temperature [K] |
| T_i | initial temperature [K] |
| T_o | temperature normalisation constant [K] |
| T_s | surface temperature of anode [K] |
| T_∞ | ambient temperature [K] |
| \bar{T} | normalised temperature [-] |
| U_∞ | air speed [m/s^2] |
| v_o | length normalisation constant [m] |
| V | volume of body [m^3] |
| x, y, z | spatial coordinate |
| x_o | length normalisation constant [m] |
| $\bar{x}, \bar{y}, \bar{z}$ | normalised spatial coordinate [-] |
| $X(x), Y(y), Z(z)$ | defined function |
| z_f | thickness of body [m] |

Greek

| | |
|------------|--|
| α | thermal diffusivity [m^2/s] |
| β | coefficient of thermal volumetric expansion [K^{-1}] |
| δ | ratio of timesteps to grid size squared [s/m^2] |
| Δt | size of timestep [s] |
| Δx | distance between mesh points in x-direction [m] |
| Δy | distance between mesh points in y-direction [m] |
| Δz | distance between mesh points in z-direction [m] |
| ϵ | emissivity [-] |

| | |
|---------------------|---|
| λ, μ, ν | defined variables |
| μ_s | viscosity of air at surface temperature [kg/sm] |
| μ_∞ | viscosity of air at ambient temperature [kg/sm] |
| ν | kinematic viscosity [m^2/s] |
| ρ | density [kg/m^3] |
| σ | Stefan-Boltzmann Constant [W/m^2K^4] |

Subscripts

| | |
|---|-------------|
| b | bath |
| c | carbon |
| i | grid points |
| j | grid points |
| k | grid points |
| s | steel |

Superscripts

| | |
|---|-----------|
| m | timesteps |
|---|-----------|

Contents

| | |
|---|-----------|
| Abstract | ii |
| Acknowledgments | iii |
| Nomenclature | iv |
| Table of Contents | viii |
| List of Figures | xii |
| List of Tables | xiii |
| 1 Introduction | 1 |
| 1.1 Background | 1 |
| 1.2 Outline | 4 |
| 2 Modes of Heat Transfer for a Cooling Anode | 5 |
| 2.1 Convection | 5 |
| 2.1.1 Natural Convection | 6 |
| 2.1.2 Forced Convection | 7 |
| 2.2 Radiation | 8 |
| 2.3 Summary | 8 |
| 3 Lumped System Model | 10 |
| 3.1 Heat Transfer in a System | 10 |
| 3.2 Numerical Solution | 11 |
| 3.3 Summary | 11 |

| | | |
|----------|--|-----------|
| 4 | Heat Equation | 13 |
| 4.1 | Formulation of the Heat Equation | 13 |
| 4.2 | Boundary and Initial Conditions | 17 |
| 4.3 | Summary | 18 |
| 5 | Analytical Solution | 20 |
| 5.1 | One-Dimensional Solution | 20 |
| 5.1.1 | Linearisation | 20 |
| 5.1.2 | Normalisation | 21 |
| 5.1.3 | Separation of Variables | 22 |
| 5.1.4 | Orthogonality | 26 |
| 5.2 | Two-Dimensional Solution | 27 |
| 5.2.1 | Linearisation | 27 |
| 5.2.2 | Normalisation | 27 |
| 5.2.3 | Separation of Variables | 29 |
| 5.3 | Three-Dimensional Solution | 32 |
| 5.3.1 | Linearisation | 32 |
| 5.3.2 | Normalisation | 33 |
| 5.3.3 | Separation of Variables | 34 |
| 5.4 | Summary | 35 |
| 6 | One-Dimensional Numerical Solution | 37 |
| 6.1 | Constant Physical and Thermal Properties | 37 |
| 6.1.1 | Finite Difference Approximations | 37 |
| 6.1.2 | Numerical Results | 42 |
| 6.2 | Variable Physical and Thermal Properties | 44 |
| 6.2.1 | Finite Difference approximations | 44 |
| 6.2.2 | Numerical Simulation | 45 |
| 6.3 | Accuracy and Stability of Numerical Method | 47 |
| 6.4 | Summary | 50 |
| 7 | Two-Dimensional Numerical Solution | 51 |
| 7.1 | Constant Physical and Thermal Properties | 51 |
| 7.1.1 | Finite Difference Approximation | 51 |
| 7.1.2 | Anode with no bath or yoke | 59 |
| 7.2 | Variable Physical and Thermal Properties | 61 |

| | | |
|----------|--|------------|
| 7.2.1 | Finite Difference Approximation | 62 |
| 7.2.2 | Two-dimensional Simulations | 65 |
| 7.3 | Summary | 72 |
| 8 | Three-Dimensional Numerical Solution | 73 |
| 8.1 | Finite Difference Equations | 73 |
| 8.2 | Variable Physical and Thermal Properties | 90 |
| 8.3 | Numerical Simulation | 96 |
| 8.3.1 | Clean Anode | 97 |
| 8.3.2 | Dirty Anode | 98 |
| 8.4 | Summary | 98 |
| 9 | Conclusions | 105 |
| 9.1 | Effect of Hot Cleaning of Spent Anodes | 105 |
| 9.2 | The Mathematical Model | 105 |
| 9.3 | Future Work | 106 |
| | Bibliography | 107 |

List of Figures

| | | |
|-----|---|----|
| 1.1 | Schematic of smelting cell | 2 |
| 1.2 | Spent anode and new anode in cooling gallery | 2 |
| 1.3 | A new anode with assembly yoke | 3 |
| 3.1 | Cooling curve for lumped system model compared with measured data | 12 |
| 4.1 | Important energy terms for a plane-wall system | 14 |
| 4.2 | Important energy terms for a two-dimensional system | 16 |
| 4.3 | Heat flow in a one-dimensional anode | 17 |
| 5.1 | Temperature surface for one-dimensional analytic solution | 36 |
| 6.1 | One-dimensional internal node | 41 |
| 6.2 | One-dimensional surface node | 41 |
| 6.3 | Temperature surface for one-dimensional numerical solution | 44 |
| 6.4 | Temperature profiles for butts with various bath thicknesses | 46 |
| 6.5 | Cooling curve of a point in carbon compared to experimental data | 47 |
| 6.6 | Cooling curve of a point in bath compared to experimental data | 48 |
| 6.7 | Temperature of clean node | 48 |
| 6.8 | Temperature profiles of dirty anode (top) and clean anode (bottom) | 49 |
| 7.1 | Two-dimensional internal node | 52 |
| 7.2 | Two-dimensional surface node | 55 |
| 7.3 | Two-dimensional external corner node | 57 |
| 7.4 | Two-dimensional internal corner node | 58 |
| 7.5 | Temperature profile for clean anode without steel yoke | 60 |
| 7.6 | Temperature of point in carbon for clean anode without steel yoke | 61 |
| 7.7 | Two-Dimensional approximation of butt | 62 |
| 7.8 | Temperature profile for a clean anode with steel yoke | 66 |

| | | |
|------|--|-----|
| 7.9 | Temperature of point in carbon for a clean anode with a steel yoke | 67 |
| 7.10 | Temperature profile in carbon for a dirty anode without steel yoke . | 68 |
| 7.11 | Temperature of point in carbon for a dirty anode without steel yoke | 69 |
| 7.12 | Temperature of point in bath for a dirty anode without steel yoke . | 69 |
| 7.13 | Temperature profile for a dirty anode with steel yoke | 70 |
| 7.14 | Temperature of point in carbon for a dirty anode with steel yoke . . | 71 |
| 7.15 | Temperature of point in bath for a dirty anode with steel yoke . . . | 71 |
| | | |
| 8.1 | Three-dimensional internal node | 74 |
| 8.2 | Three-dimensional internal corner node | 76 |
| 8.3 | Three-dimensional internal edge node | 78 |
| 8.4 | Three-dimensional external corner/surface node | 80 |
| 8.5 | Three-dimensional surface node | 82 |
| 8.6 | Three-dimensional internal edge/surface node | 85 |
| 8.7 | Three-dimensional external edge node | 87 |
| 8.8 | Three-dimensional external corner node | 89 |
| 8.9 | Three-dimensional approximation of a quarter anode | 91 |
| 8.10 | Temperature profile for a clean anode | 99 |
| 8.11 | Cross section of temperature through clean anode | 100 |
| 8.12 | Temperature profile for a dirty anode | 101 |
| 8.13 | Cross section of temperature through dirty anode | 102 |
| 8.14 | Temperature of point in carbon for clean and dirty anode | 103 |
| 8.15 | Temperature of point in bath dirty anode | 103 |

List of Tables

| | | |
|-----|---|----|
| 2.1 | Natural Convection Heat Transfer Rates for rectangular anode and sphere (kW) | 7 |
| 2.2 | Heat Transfer Rates for sphere (kW) | 9 |
| 6.1 | Temperature at surface and centre for various physical and thermal properties | 43 |
| 6.2 | Temperature at surface and centre for various grid and timestep sizes | 50 |