ADVANCES IN CLASSICAL AND QUANTUM
WAVE DYNAMICS ON QUASIPERIODIC
LATTICES

A dissertation submitted for the degree of
Doctor of Philosophy
in
Physics

Center for Theoretical Chemistry and Physics
New Zealand Institute for Advanced Study
Massey University, Albany
New Zealand

Carlo Danieli

June 2016
This dissertation is a presentation of the research conducted between February 2013 and June 2016 at the Center for Theoretical Chemistry Physics within the New Zealand Institute for Advanced Study at Massey University (Albany campus, Auckland, New Zealand) while enrolled in the Doctor of Philosophy degree.

The degree was carried out under the supervision of Prof. Sergej Flach, Dr. Joshua D. Bodyfelt and Dist. Prof. Gaven J. Martin.

The material reported is, at the best of my knowledge, original (except where acknowledged), and it has not been submitted in whole or in part for a degree in any university.

Carlo Danieli
June 2016
The most merciful thing in the world, I think, is the inability of the human mind to correlate all its contents. We live on a placid island of ignorance in the midst of black seas of infinity, and it was not meant that we should voyage far. The sciences, each straining in its own direction, have hitherto harmed us little; but some day the piecing together of dissociated knowledge will open up such terrifying vistas of reality, and of our frightful position therein, that we shall either go mad from the revelation or flee from the deadly light into the peace and safety of a new dark age.

H.P. Lovecraft
The Horror in Clay
Abstract

Lattices and discrete networks are cornerstones of a number of scientific subjects. In condensed matter, optical lattices allowed the experimental realization of several theoretically predicted phenomena. Indeed, these structures constitute ideal benchmarks for light and wave propagation experiments involving interacting particles, such as clouds of ultra-cold atoms that Bose-Einstein condensate. Moreover, they allow experimental design of particular lattice topologies, as well as the implementation of several classes of spatial perturbations. For example, Anderson localization being observed for the first time in atomic Bose-Einstein condensate experiments and Aubry-André localization discovered with light propagating through networks of optical waveguide.

This thesis considers different types of lattices in the presence of quasiperiodic modulations, mainly the celebrated Aubry-André potential. Particular attention will be given to spectral properties of models, localization features of eigenmodes and the transition from delocalized (metallic) eigenstates to localized (insulating) ones within the energy spectrum. We additionally discuss the relation between the model’s properties and the dynamics of particles hopping along the lattice.

After introducing the linear discrete Schrödinger equation, we first discuss the spectral properties of the Aubry-André model. We then study the transition between metallic and insulating regimes of a class of quasiperiodic potentials constructed as an iterative superposition of periodic potentials with increasing spatial period. Next, we discuss the Aubry-André perturbation of flat-band topologies, their energy-dependent transition (mobility edge), which can be expressed in analytical forms in case of specific onsite energy correlations, highlighting existence of zeroes, singularities and divergences. We then discuss two cases of driven one-dimensional lattices, namely an Aubry-André chain with a weak time-space periodic driving and an Anderson chain with a quasiperiodic multi-frequency driving. We show analytically and numerically how drivings can lift the respective localization and generate delocalization by design. Finally we discuss the problem of the possible generation of correlated metallic states of two interacting particles problem in one-dimensional Aubry-André chains, under a coherent drive of the interaction.
The thesis is based on the following publications:

Published:

- H. Hatami, **C. Danieli**, J. D. Bodyfelt and S. Flach, "Quasiperiodic driving of Anderson localized waves in one dimension", Phys. Rev. E **93**, 062205 (2016);


Selected Conference Presentations:

- **C. Danieli**, J. D. Bodyfelt and S. Flach, "Flatband engineering of mobility edges ", Australian and New Zealand School in Ultracold Physics, Otago University, Dunedin, New Zealand, December 2016;

- **C. Danieli**, J. D. Bodyfelt and S. Flach, "Flatband engineering of mobility edges ", IONS, KOALA, University of Auckland, Auckland, New Zealand, November 2015;

- **C. Danieli**, J. D. Bodyfelt and S. Flach, "Flatband engineering of mobility edges ", Conference on Frontiers of Nanoscience, ICTP, Triest, Italy, August 2015;


• C. Danieli, J. D. Bodyfelt and S. Flach, "Flatband models with correlated onsite perturbation", INMS Postgraduate Conference, Massey University, Albany, New Zealand, September 2014;

• L. Morales-Molina, E. Doerner, C. Danieli and S. Flach, "Delocalization in driven quasiperiodic lattices", III Dynamic Days South America, Vina de Mar, Chile, November 2014;

• S. Flach, C. Danieli, J. D. Bodyfelt, D. Leykam, P. Matthies and A. S. Desyatnikov, "Perturbing flat-band", Let’s Face Chaos Through Nonlinear Dynamics, University of Maribor, Maribor, Slovenia, July 2014;

• C. Danieli, K. Rayanov and S. Flach, "Approximating metal-insulator transitions", Dodd-Wall Symposium, Otago University, Dunedin, New Zealand, October 2013;

• C. Danieli, K. Rayanov and S. Flach, "Approximating metal-insulator transitions", INMS Postgraduate Conference, Massey University, Albany, New Zealand, October 2013;

A PhD dissertation is the culmination of several years of someone’s life. It means much more than just the hundred and odd pages, and between its lines one can find joy, sadness, emotions and people. People. Countless people that have been an invaluable part of this experience, which can hardly be captured in this section.

Prof. Sergej Flach has not been only an extraordinary supervisor, he has been a friend and a guide. I am so grateful for the time we have spent together during these years. I thank him for the possibility to join his research within condensed matter physics, complete new subject for me, and for the freedom he gave me to approach the topics I was more interested in while minding that my projects were not losing focus from forming a coherent plot. Moreover, I want to thank him for sharing with me his love and passion for life and science. Next, I want to thank Dr. Joshua Bodyfelt. As a friend and co-supervisor, he has always been there supporting me with ideas and motivations, in particular when I was demoralized. Moreover, I want to thank all the people I had the luck and the pleasure to collaborate during these years: my co-supervisor Prof. Gaven Martin, Prof. Boris Pavlov (RIP), Dr. Xiaoquan Yu, Dr. Daniel Leykam, Dr. Luis Morales-Molina, Dr. Edoardo Doerner, Dr. Hani Hatami and Kristan Rayanov.

I found the CTCP an ideal place to grow and do science. There I have been surrounded by amazing people and phenomenal scientists, ready to help and support each other for scientific but also personal issues. The various Lukas, Krista, Jayson, Lauri, Peter, Xiaoquan, Alberto, Sasha, Sophie, Paul, Odile, Andy and everybody, thank you so much because I truly felt within a community. In particular, I want to thank Prof. Peter Schwerdtfeger for running the place with so much passion and his friendly attitude to everyone of us (you have been more that a boss, you are our godfather); and Vesna Davidovic-Alexander, I am so grateful for your constant and invaluable help and support. I want to extend these acknowledgements to those people that have hosted me as a visitor during this project, from Erica at the ICTP in Italy to Ma-Young, Yoorin, Sol, Dominika, Hwa-Sung, Jaehee, Hee-Chul, Ivan and everyone at PCS in South Korea.

Outside of science, my first thoughts go to my family back in Italy. This experience has been challenging not only for me, but also for everyone of us. During these years there have been losses, disease and grief, but also lots of joy, the arrival of new young family members, weddings and so on. Mum, dad, Luciano, Anna, Simone, all the aunts, uncles, cousins and so on, I am so grateful to you all for this. Love you all.
During this time spent in New Zealand, many friends departed for long travels or begun new experiences somewhere, each one following dreams and aspirations. Enrico, Andrea, Nevenka, Claudio, Francesco, Federica, Felix, Sabrina, Tjafa, Jakob, Matteo, Giacomo, Lorena, Tommaso, Celeste, Luca; this list can go ahead endlessly. Even if keeping in touch with everyone is not easy, you all have been with me during this time. But from friends left, new friends were expecting me along my journey. After the positive experiences of Padua and Berlin, I was hoping to again meet good people to spend time with and share interests starting from the domestic environment. This happened from the very first day I arrived in New Zealand. Russell, Shannon, Kelsie, Caitlin, Michael, Jack, Dave, Cloud, Jenna, Steven, young Manu, when I stepped for the first time in that flat and met you all, I immediately felt home. During these years you have been more than a family for me; therefore I thank you so much for welcoming me and for this awesome time together.

New Zealand didn’t only gave me the opportunity to move my first steps within the scientific research, but it also gave me the opportunity to grow some personal passions. Heaps of DIY projects, house concerts organized and attended, the amazing experiences of the Off The Radar and the Chronophonium festival; moments where the enthusiasm for the projects gave rise to overwhelming atmosphere and strong connections between the people involved. Among these experiences, I would like to particularly highlight the Worn Out collective, as an undeniable evidence that even the most disorganized group of lazy people can create something great. Benny, Rory, Junior, Fluffy, Anhand, Grant, Ritchie, Glenn, Cyprian, we had such a great time and amazing experiences together, thank you so much!

Not every single moment of this project have been easy and pleasant. A bunch of people taught me to do not be afraid of pain and stress, both mental and physical, but instead to try to accept them, since they can uncover energies and motivations that you were not even thinking to have. I am so deeply grateful to Eden and Kyrsten, two wonderful human beings that have welcomed me and guided through this magical adventure. With them, I would like to thank the Skindependent family. Josh, Christopher, Fran, Betty, Travis, Scott, Martin, you guys are incredible. So are the people met during this three years in various locations and conventions. Too many to be mentioned, but none forgotten, from practitioners to enthusiasm like myself. Alice, Snow, Kat and Clay, Alan, June, Marita, Havve, Bruno. I love you my friends.

Music is, has been and will be a fundamental part of my life, and even the smallest detail of this dissertation is laced with the music I was listening while it was conceived, developed and formalized. At last then, I would like to thank all those bands, and in a broader way, all those musicians, performers and independent artists I have met and enjoyed their music and performances between Europe, New Zealand and South Korea. Also, to all the artists I haven’t met but that I have deeply relished and enjoyed their art, even though we do not know each other I am so grateful to you all for all the emotions and experiences you let me have in these years, because they made this dissertation the way it is and myself the way I am.
Contents

1 Motivations and Outline 1

2 Introduction 5
   2.1 Wave Propagation in Continuous and Discrete Systems 5
   2.2 Periodic Structures 7
   2.3 Periodicity-Breaking and Localizing Potentials 8
   2.4 The Quasiperiodic Class 11
   2.5 Summary and Discussions 15

3 Wave Localization in Quasiperiodic Potentials 17
   3.1 Introduction 17
   3.2 Aubry-André model 18
   3.3 Approximating Metal-Insulator Transition - A Cantor-like Construction of a Class of Quasiperiodic Potentials 21
   3.4 Summary and Discussions 27

4 Quasiperiodicity and Mobility Edges in Flat-Band Topologies 29
   4.1 Introduction 29
   4.2 Flat-Band Topologies 30
   4.3 Rotation of Flat-Band lattices into Lattices of Fano Defects 35
   4.4 Flat-Band Engineering of a Mobility Edge 38
   4.5 Summary and Discussions 45

5 Driven Lattices with Quasiperiodic and Random Potentials 47
   5.1 Introduction 47
   5.2 Resonant Metallic States in Driven Quasiperiodic Lattices 48
   5.3 Quasiperiodic Driving of Anderson Localized Waves in One Dimension 54
   5.4 Summary and Discussions 66

6 The Two Interacting Particles Problem in Quasiperiodic Chains 67
   6.1 Introduction 67
   6.2 The Two Particles Problem 68
   6.3 Two Interacting Particles in an Aubry-André Chain 71
   6.4 Summary and Discussions 78

7 Conclusions and Outlook 81

A Numerical Methods 85
   A.1 Numerical Diagonalization 85
A.2 Wave Packet Dynamics ............................................. 85
A.3 Mode’s Characterization .......................................... 87

B Quasiperiodic Driving of Disordered Chains 89
B.1 Multi-Frequency Perturbation ..................................... 89
B.2 Floquet Analysis .................................................... 90

C The Two Interacting Particles Problem - Technical Issues 97
C.1 Mapping ............................................................. 97
C.2 Overlap Integral .................................................... 99
C.3 Floquet Analysis .................................................... 100

Bibliography 105

D Statements of Contribution to Doctoral Thesis Containing Publications 121
List of Figures

2.1 Optical and Photonic Lattices ................................... 8
2.2 Extended and Localized Waves ................................... 9
2.3 Anderson States. Participation, Volume and Localization Length ...... 10
2.4 Anderson Localization in Photonic and Optical Lattices ............... 11
2.5 Aubry-André. Metal-Insulator Transition .......................... 14
2.6 Exponential Localization in quasiperiodic BEC ........................ 14
3.1 Energy spectrum of the Aubry-André model, golden ratio ............. 19
3.2 Aubry-André model, time evolution ................................ 20
3.3 Aubry-André model, group velocity scaling .......................... 21
3.4 Schematic picture of the potential $E_1$ ............................ 22
3.5 Schematic picture of the potential $E_2$ ............................ 22
3.6 Schematic picture of the potential $E_k$ ............................. 22
3.7 Schematic picture of the potential $E_1 + E_2$ ...................... 23
3.8 Quasiperiodic chain, energy spectrum ................................ 24
3.9 Quasiperiodic chain, inverse of highest participation ratio ........... 25
3.10 Quasiperiodic chain, phase diagrams ................................ 25
3.11 Quasiperiodic chain, inverse of highest participation ratio - subgaps .. 26
3.12 Quasiperiodic chain, mobility edge ............................... 26
3.13 Quasiperiodic chain, time evolution ................................ 27
4.1 Flat band tolerogies ................................................. 31
4.2 Two-dimensional Lieb lattice ....................................... 32
4.3 Diamond chain with magnetic field .................................. 34
4.4 Fano-Anderson model ............................................. 35
4.5 Cross-Stitch Lattice ............................................... 37
4.6 Diamond Chain ................................................... 37
4.7 Energy spectrum of the cross-stitch lattice, symmetric case .......... 40
4.8 Energy spectrum of the cross-stitch lattice, asymmetric case ....... 41
4.9 Energy spectrum of the cross-stitch lattice, anti-symmetric case ... 42
4.10 Energy spectrum of the cross-stitch lattice, antisymmetric case .... 43
4.11 Diamond chain, extended state .................................... 43
4.12 Energy spectrum of the diamond chain, symmetric case ............ 44
4.13 Energy spectrum of the diamond chain, antisymmetric case ....... 45
5.1 Spectrum of the Aubry-André model ................................ 50
5.2 AA model: highest participation number with weak driving .......... 52
5.3 AA model: participation number and second moment .................. 52
5.4 AA model: highest participation number vs. the system size $N$ ........ 53
5.5 AA model: highest participation number with strong driving .......... 53
5.6 AA model: phase dependence of the second moment ...................... 54
5.7 AA model: time evolution of the second moment .......................... 55
5.8 Anderson chain: Bessel function representation .......................... 58
5.9 Anderson chain: single color driving, weak driving regime ............ 63
5.10 Anderson chain: single color driving, strong driving regime ........... 63
5.11 Anderson chain: driving strength dependence of the second moment .... 64
5.12 Anderson chain: weak driving regime, one color versus two colors .... 65
5.13 Anderson chain: two colors driving, strong driving regime ............ 65
6.1 Two Interacting Particles - Lattice model ................................. 70
6.2 Two Particles States in Quasiperiodic Chains ............................. 71
6.3 Spreading of Two Interacting Particles in Quasiperiodic Chains ........ 72
6.4 Phase Diagram - Region A and B ........................................... 74
6.5 Region A: frequency dependence of the second moment .................. 75
6.6 Region A: wave packet dynamic ............................................. 76
6.7 Region A: regime of small frequencies ..................................... 77
6.8 Region B: frequency dependence of the second moment .................. 77
6.9 Region B: wave packet dynamic ............................................. 78