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IN PURSUIT OF A SUITABLE ALTERNATIVE TO
LEAST SQUARES ESTIMATION IN NORMAL
LINEAR MODELS

A THESIS PRESENTED IN PARTIAL FULFILMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
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ABSTRACT

A search for an estimator of β in the Normal Linear Model which has better mean squared error properties than the usual least squares estimator is undertaken. The properties of some classical techniques such as restricted least squares, which includes the selection of a subset of the independent variables, are examined, along with more recent techniques such as ridge regression and Bayesian estimators. Most of these can be shown analytically to improve over least squares only when the true parameter vector β is in some subspace of the parameter space. Empirical Bayes estimators are in general difficult to handle analytically, and so several of these are studied by Monte Carlo methods. A particular modification of one of these empirical Bayes estimators is found to improve over least squares over a large region of the parameter space, and its use is demonstrated on a small data set. Some suggestions for further improvement of this estimator are given and some techniques for further study of estimators by Monte Carlo methods are recommended.

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TABLE OF CONTENTS

	PAGE
ACKNOWLEDGEMENT	
TABLE OF CONTENTS	
LIST OF TABLES	
1. INTRODUCTION	1
2. BIASED ESTIMATORS	3
2.1 Linear Estimators	3
2.2 Non-Linear Estimators	3
2.21 Restricted Least Squares or Preliminary Test Estimators	4
2.211 Biased Linear Estimators as Restricted Least Squares Estimators	4
2.212 Preliminary Test Estimators	7
2.22 Incorporation of Prior Information	8
2.221 Ridge Regression as a Bayesian Estimator	10
2.222 Another "Bayesian" Estimator	10
2.223 Numerical Example	13
2.224 Bayesian Estimator for Unscaled y	14
3. SIMULATION OF THE mse PERFORMANCE OF THE VARIOUS ESTIMATORS	17
3.1 Experimental Design for the Simulation Study	17
3.11 Method	18
3.2 Analysis of the Relative Mean Squared Errors From the Simulation Study	19
3.3 Results of Study 2	22
3.4 Testing the Size and Direction of γ	25
3.41 Testing the Direction of γ	25
3.42 Testing the Length of γ	28
3.43 Modified Estimator	30
3.44 Performance of the Modified Estimator	33
3.5 Mean Squared Error for Non-Stochastic Ridge Regression	31
3.51 Modified Estimator	33
3.52 Performance of the Modified Estimator	33
3.53 Discussion of TABLE 3	35
3.6 Recommended Estimator	35
3.61 Recommended Estimation Rule	36

	PAGE
4. NUMERICAL EXAMPLE	37
4.1 Analysis	37
4.2 Pass 1	38
4.21 Interpretation of TABLE 4	40
4.211 Variable Selection	40
4.3 Interpretation of TABLE 5	41
4.4 Interpretation of TABLE 6	42
4.5 Conclusions	43
5. SUMMARY AND DISCUSSION	45
6. APPENDIX	47
6.1 Eigenvalues and True Coefficients γ Used in the Simulation Studies	47
6.2 Random Unit-Normal Generator	49
BIBLIOGRAPHY	

LIST OF TABLES

		PAGE
TABLE 1	Relative Mean Squared Errors	20
TABLE 2a.	mse Performance of g^4 Relative to Least Squares	23
TABLE 2	Relative mse's at $\gamma \ll u$	24
TABLE 3	Relative Mean Squared Errors	34
TABLE 3a.	Percentage of Times Least Squares Was Used	34
TABLE 4	Summary Statistics From Pass 1	39
TABLE 5	Summary Statistics From Pass 2	41
TABLE 6	Summary Statistics From Pass 3	42
TABLE 7	Eigenvalues and Coefficients Used in the Simulation Studies	47

1. INTRODUCTION

This thesis is concerned with estimators of β in the Normal Linear Model,

$$y = X\beta + \xi$$

where y is an $n \times 1$ vector of observed variables, X is an $n \times p$ matrix of known constants, β is an unobservable $p \times 1$ vector of coefficients and ξ is an $n \times 1$ vector of unobservable random errors assumed to be independently and identically Normally distributed with zero mean and constant, but generally unknown variance σ^2 . This is written

$$\xi \sim N(0, \sigma^2 I).$$

The more general case where $\xi \sim N(0, \sigma^2 V)$ will not be explicitly discussed here since, if V is of full rank, the model can be reparameterised to conform to the simpler Normal Linear Model.

It is well known that among unbiased estimators of β , the least squares estimate

$$b_0 = (X^T X)^{-1} X^T y$$

has minimum variance. However this does not guarantee that the variance of the least squares estimate will be small and it is for this reason that some biased estimation techniques are considered here. The criterion adopted as a measure of the goodness of an estimator will be mean squared error, mse or MSE, where for a particular estimator b ,

$$\begin{aligned} \text{mse}(b) &= E(b - \beta)^T (b - \beta) \\ \text{and} \quad \text{MSE}(b) &= E(b - \beta)(b - \beta)^T. \end{aligned}$$

E has the usual meaning of "the expected value of". It is hoped to find an estimator which has good mean squared error properties when

compared with least squares.

Various biased estimators, including some commonly used variants of least squares, are discussed in Chapter 2. The emphasis is on their mean-squared error performance over different regions of the parameter space for β and σ^2 . Then in Chapter 3, several simulation experiments are reported. These experiments investigate the mean-squared error performance of two Stein-type estimators and several stochastic ridge estimators over a wide range of the parameters. As the experiments progress a heuristically modified stochastic ridge estimator is developed which appears to have good mean-squared error properties. A numerical example demonstrating the use of this estimator is given in Chapter 4. Finally a summary, including suggestions for further study of stochastic ridge estimators and a discussion of several aspects of simulation studies and mean-squared error performance of estimators is presented in Chapter 5.