Minimising Weighted Mean Distortion

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ABSTRACT

There has been considerable recent interest in geometric function theory, nonlinear partial differential equations, harmonic mappings, and the connection of these to minimal energy phenomena. This work explores Nitsche's 1962 conjecture concerning the nonexistence of harmonic mappings between planar annuli, cast in terms of distortion functionals. The connection between the Nitsche problem and the famous Grötzsch problem is established by means of a weight function. Traditionally, these kinds of problems are investigated in the class of quasiconformal mappings, and the assumption is usually made *a priori* that solutions preserve various symmetries. Here the conjecture is solved in the much wider class of mappings of finite distortion, symmetry-preservation is proved, and ellipticity of the variational equations concerning these sorts of general problems is established. Furthermore, various alternative interpretations of the weight function introduced herein lead to an interesting analysis of a much wider variety of critical phenomena — when the weight function is interpreted as a thickness, density or metric, the results lead to a possible model for tearing or breaking phenomena in material science. These physically relevant critical phenomena arise, surprisingly, out of purely theoretical considerations.

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Dedicated to the memory of Jan Vermeulen, my Opa.
24 September 1932 – 21 September 2009

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