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LONG MEMORY PROPERTIES OF INTERNATIONAL *EX POST* AND *EX ANTE* REAL INTEREST RATES: ASIAN, PACIFIC AND EUROPEAN COUNTRIES

A Research Thesis submitted in fulfilment of the requirements for the degree of Master of Applied Economics at Massey University

JEREMY COUCHMAN

DEPARTMENT OF APPLIED AND INTERNATIONAL ECONOMICS

COLLEGE OF BUSINESS

MASSEY UNIVERSITY, NEW ZEALAND

2003

Acknowledgements

I would like to take this opportunity to convey my sincerest appreciation to Dr Jen-Je Su for his invaluable guidance, encouragement and support throughout this research. I would also like to thank Dr Rukmani Gounder for her significant and invaluable guidance, suggestions and corrections. Without the help of these two people, the completion of this research would never have been possible.

I would also like to thank the many friends from my time spent at Massey University who have contributed to this Thesis, in one form or another, and whom made this time in my life more enjoyable. In particular, I would like to thank: Rob Hodgson, Leanne Smith, Sam Richardson, Elizabeth Hodge, Natasha Longworth, Anne Mahon, Lauren Jewel, Hamish Grant-Fargie, Rachel Baxter, Liv Young, Elzeth van Zyl, Nathan Penny and Shayne Moar.

I would also like to thank my brother Daniel Couchman and friend Andrew Coleman for their support during this study.

Finally, I would like to acknowledge my family, who have always been there and supported me throughout my life.

Abstract

According to the Fisher equation efficient capital markets should compensate for changes in the purchasing power of money. This implies that in the long-run, the nominal interest rate and expected inflation should move together one-for-one. However, because expected inflation is unobservable, testing the Fisher relationship is problematic and an appropriate proxy for expected inflation must be employed.

Empirical results in the literature of the Fisher relationship have produced mixed findings concerning the validity of this relationship. Many recent studies have focused on the stationarity of the *ex ante* real rate in determining the acceptability of the long-run Fisher relationship. For the long-run Fisher effect to hold the *ex ante* real interest rate should display mean-reversion. Mean-reversion is characterised by the tendency of a time series to return to its mean after a shock. Most studies that have examined the stationarity of the *ex ante* real rate have concentrated on testing for restrictive integer orders of integration. This is restrictive because mean-reversion is confined to the covariance stationary I(0) process. However, an I(0) process is not the only process that displays mean-reversion. Fractional orders of integration can characterise a wider form of mean-reversion.

Many studies that observe the order of integration of the real interest rate use actual or realised inflation for expected inflation in order to generate the *ex post* real rate, which differs from the *ex ante* real rate only by a stationary forecast error. These studies have then used the *ex post* real rate to infer the dynamic behaviour of the *ex ante* real rate. However, because the difference between the *ex post* and *ex ante* real rates is unexpected, the large volatility of the forecast error can mask the more persistent behaviour of the *ex ante* real rate. The additional volatility is inherited by the *ex post* real rate and therefore estimates of the order of integration are biased downwards.

In this research the order of integration is estimated for real interest rates of nineteen European, Asian and Pacific countries. Two different econometric techniques are used in order to generate proxies for expected inflation, and it is found that these proxies exhibit a more persistent dynamic when compared to actual inflation. Employing an autoregressive fractionally integrated moving average (ARFIMA) model, the order of integration is estimated by using a maximum likelihood (ML) estimation technique. This estimation technique is applied to the two estimated *ex ante* real rates as well as the *ex post* real rate for each country studied. The empirical results show that estimated orders of integration display a distinct pattern. That is, the *ex post* real rate is found to be significantly less persistent when compared to either of the *ex ante* real rates estimated in this study. This is due to the additional volatility that is inherited within the *ex post* real rate of interest.

The Fisher relationship has also been extended to international capital and goods markets. Real interest rate parity (RIP) theory postulates that if international capital (through uncovered interest rate parity) and goods (through relative purchasing power parity) markets are efficient then the real interest rate on two perfectly comparable assets between countries and across time should equalise. Similar to the Fisher relationship, RIP has also had mixed empirical results. Early studies found limited support for RIP, on the other hand more recent studies have found evidence of real interest rate integration. In this research, a preliminary study was conducted of RIP between New Zealand and Australia. Using the same methodology mentioned above, RIP was examined for three real interest rate differentials of New Zealand and Australia. Again, these differentials differ in the method used to model expected inflation. The empirical results of RIP between New Zealand and Australia are not overwhelmingly conclusive. The order of integration of the real interest rate differentials do not differ from the order of integration of the real interest rates of New Zealand and Australia, which does not support RIP. This analysis however, does generate many possibilities for further research including data and methodological extensions.

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List of Abbreviations

ACF	Auto-Correlation Function
ADF	Augmented Dickey-Fuller
AIC	Akaike Information Criterion
AR	Auto-Regressive
ARFIMA	Auto-Regressive Fractionally Integrated Moving Average
ARIMA	Auto-Regressive Integrated Moving Average
ARMA	Auto-Regressive Moving Average
BEW	Bivariate Exact Whittle
CCAPM	Consumption-based Capital Asset Pricing Model
CER	Closer Economic Relations
CPI	Consumer Price Index
ES	Exponential Smoothing
FIGARCH	Fractionally Integrated Generalised Auto-Regressive Conditional Heteroskedastic
GARCH	Generalised Auto-Regressive Conditional Heteroskedastic
GARMA	Gegenbauer Auto-Regressive Moving Average
GNP	Gross National Product
GPH	Geweke and Porter-Hudak
HP	Hodrick-Prescott
IFS	*International Financial Statistics*
IMF	International Monetary Fund
KPSS	Kwiatkowski, Phillips, Schmidt and Shin
LMM	Long Memory Modelling
MA	Moving Average
ML	Maximum Likelihood
MMR	Money Market Rate
OECD	Organization for Economic Co-operation and Development

PP	Phillips and Perron
REH	Rational Expectations Hypothesis
RIP	Real Interest Rate Parity
RPPP	Relative Purchasing Power Parity
TBR	Treasury Bill Rate
TVP	Time Varying Parameter
UIP	Uncovered Interest Parity
VAR	Vector Auto-Regressive
WPI	Wholesale Price Index

Chapter One Introduction

1.1 Aims and Objectives

The *ex ante* real interest rate is of great importance in macroeconomics as it is used in determining all intertemporal decisions regarding savings and investment, thus influencing macroeconomic dynamics. The long-run time series properties of the *ex ante* real interest rate is linked to the Fisher (1930) equation. According to the Fisher equation, the real interest rate is the calculated difference between the nominal interest rate and expected inflation. Neutrality or superneutrality is at the core of classical economic theory; this stipulates that once-and-for-all movements in nominal variables do not have permanent effects on real variables. Therefore, in relation to the Fisher equation changes in expected inflation should not have a permanent impact on the *ex ante* real interest rate. Many economists believe that the neutrality proposition fails over business cycle horizons, although there is much debate concerning the validity of neutrality over the long-run (Atkins and Coe, 2002).

Because the *ex ante* real interest rate is the difference between nominal interest rate and expected inflation, the long-run behaviour of the *ex ante* real interest rate is linked to the behaviour of these two variables. However, since expected inflation is an unobservable variable, the *ex ante* real interest rate is also unobservable and, as a result, studying the Fisher equation is not a clear-cut case. The majority of empirical studies circumvent this problem by employing realised or actual inflation under the assumption of a stationary forecast error in order to calculate the *ex post* real interest rate and use the *ex post* real rate to infer the long-run behaviour of the *ex ante* real interest rate. Yet, as explained below, this can lead to controversial results.

Fisher (1930) originally proposed that in a world in which economic agents have perfect foresight, expected changes in the inflation rate should be immediately followed by

changes in the nominal interest rate, as borrowers and lenders adjust their behaviour keeping the real rate of interest unchanged. However, subsequent to Fisher's work a number of empirical studies, such as Nelson and Schwert (1977), Fama and Gibson (1982) and Huizinga and Mishkin (1984) argue that the *ex ante* real interest rate is in fact not constant but instead found evidence to suggest that it is time varying. Thus this evidence led studies to focus on the time series properties of the *ex ante* real interest rate in order to verify the long-run Fisher effect.

Since the work of Rose (1988) the time series properties of the *ex ante* real interest rate have been called into question. Rose found that unit root test results for the United States and seventeen other Organization for Economic Co-operation and Development (OECD) member countries suggested that the nominal interest rate contained a unit root, whereas the inflation rate did not. This result implied that a stable long-run Fisher effect could not exist because the degree of persistence of the *ex ante* real interest rate is necessarily the same as that of the dominant component, which in this case is the nominal interest rate. Rose concluded that if the *ex post* real interest rate is nonstationary (which differs from the *ex ante* real rate by a stationary forecasting error), the *ex ante* real rate must be nonstationary as well. In contrast, Mishkin (1992) found that both the nominal interest rate and inflation contained a unit root, which led to the finding that both the two variables are cointegrated. Therefore, if forecast errors are stationary this result implied that the *ex ante* real interest rate is a stationary process and is mean-reverting. Mean-reversion describes the tendency of a time series to return to a constant and finite mean after a shock. Similarly, other studies such as Evans and Lewis (1995) and Crowder and Hoffman (1996) found a cointegrating relationship between inflation and the nominal interest rate in the case of the United States.

These mixed findings mentioned above used conventional unit root test procedures, such as the augmented Dickey-Fuller (ADF) and Phillips and Perron (PP) tests. These tests assume that a particular series is integrated of the order of either I(0) or I(1). In terms of the Fisher equation for a stable long-run relationship to hold, the *ex ante* real interest rate is restricted to an I(0) stationary process. However, Lai (1997) argues that for a stable long-run Fisher

equation to exist the minimum requirement is that the *ex ante* real rate is mean-reverting; thus in the long-run the nominal interest rate and rationally expected inflation respond one-for-one to permanent shocks. Using an alternative approach Lai (1997) and Tsay (2000) found that the United States *ex ante* and *ex post* real interest rates can be properly described by a fractionally integrated I(d) process, where the order of integration d lies between zero and one. The fractional process can characterise a wide range of mean-reverting behaviour which is not accommodated by the conventional restrictive integer orders of integration. And therefore, fractional integration can verify the existence of a long-run Fisher effect.

In a recent study Sun and Phillips (2003) suggest that estimating or inferring the order of integration of the *ex ante* real rate using the *ex post* real rate can create misleading results. Sun and Phillips argue that because realised inflation contains an unexpected component (the forecast error) it tends to be more volatile than expected inflation. As a result, the extra volatility of realised inflation masks the actual slow moving persistent component buried within the volatile fluctuations. The additional short-run volatility of realised inflation is inherited within the calculated *ex post* real interest rate, which masks the more persistent *ex ante* real interest rate. Consequently, univariate long memory estimates of d based on the *ex post* real interest rate are underestimated because the additional short-run volatility create a seemingly less persistent time series. Using a new approach Sun and Phillips estimate the order of integration of the *ex post* real interest rate for the United States using a bivariate exact Whittle (BEW) estimator. This estimator controls for the additional short-run volatility and the empirical results estimated by Sun and Phillips show orders of integration that were larger compared to the estimates of the orders of integration using other estimation methods.

The main objective of this study is to determine whether the long memory behaviour found in the United States is exhibited within other countries. The long-run dynamics of the *ex post* and *ex ante* real interest rates for nineteen Asian, Pacific and European countries are studied here. However, instead of employing a particular estimation method that considers the additional short-run dynamic of realised inflation as carried out in Sun and Phillips, two econometric techniques are employed to generate a historical series of inflation forecasts in

addition to the use of realised inflation as a proxy for expected inflation. Each of the proxies for expected inflation used in this study differs in the degree of short-run volatility. This study aims to find that for each country the estimated order of integration of the real interest rate depends on the level of short-run volatility present in expected inflation. It is hypothesised that the way economic agents incorporate short-run dynamics in forming inflationary expectations may play a crucial role in determining the level of persistence of the *ex ante* real interest rate. This will have important implications for the mean-reverting dynamics of the *ex ante* real interest rate and therefore the validity of the Fisher effect. Overall, the empirical results suggest that for the majority of the nineteen countries studied here, the estimated long memory parameters are significantly smaller for the *ex post* real interest rate compared to both of the *ex ante* real interest rates.

In addition to this analysis, this study considers preliminary work of real interest rate parity (RIP) between New Zealand and Australia by examining the long-run time series properties of the real interest rate differentials. The RIP hypothesis describes the equilibrium condition between domestic and foreign real interest rates. If there is integration of world capital and goods markets, then real interest rates of identical financial assets tend to be equal across countries over time (Moosa and Bhatti, 1997). Because Australia and New Zealand have a close economic relationship, both the commodity and capital markets should have a certain degree of integration. This study therefore aims to examine the long-run properties of the real rate differentials to determine if a form of RIP exists between the two countries. Using the same methodology to examine the long memory properties of the real interest rates for various countries, the order of integration of the real rate differentials are examined to find evidence of a cointegrating relationship between the real interest rates of Australia and New Zealand.

1.2 Data and Methodology

The nineteen Asian, Pacific and European countries studied include: Australia, Belgium, Canada, France, Germany, Italy, Japan, Korea, Malaysia, the Netherlands, New Zealand, Pakistan, the Philippines, Singapore, South Africa, Spain, Sweden, the United Kingdom

and the United States. The analysis presented in this research requires a certain degree of consistency of data between each country. Thus, all the data utilised in this research is obtained from the International Monetary Fund's (IMF) *International Financial Statistics* (IFS) database, IMF (2002). The inflation data utilised for each country is calculated from quarterly consumer price index (CPI) data and the interest rate data is for quarterly three-month interest rates.

Lai (1997) explains that there are generally two approaches typically employed when studying the mean-reversion of the real interest rate. The first approach examines a cointegrating relationship between the nominal interest rate and inflation under the assumption of a stationary forecast error (for example, Mishkin (1992)). The second approach involves directly testing for a unit root in the real interest rate. In this study the latter approach is adopted by directly estimating the order of integration (d) of the real interest rate for the above-mentioned countries. Using the three proxies for expected inflation indicated above the real interest rate is calculated using the Fisher equation.

The first step in determining the order of integration of the real interest rates is to conduct unit root tests. Conventionally, the order of integration of a particular series is established using conventional unit root tests, such as the ADF and PP tests, which assume that a particular time series is integrated of the order of either I(1) or I(0). These conventional unit root tests fall into two categories; the first category tests the null hypothesis of a unit root (PP test) whereas the second type tests the null of stationarity (Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test). However, if the null hypothesis is rejected for both tests then this suggests the possibility that the time series may be fractionally integrated. Thus, in this study the PP and KPSS unit root tests are used to test for the stationarity of each real interest rate.

Following Tsay (2000), the order of integration d is estimated using a parametric approach, specifically, an autoregressive fractionally integrated moving average (ARFIMA) model is estimated for each real interest rate using a conditional time-domain maximum likelihood (ML) estimation procedure. The parametric approach allows for the estimation of the long-

run dynamic component (*d*), while controlling for any short-run dynamic behaviour (autoregressive (AR) and moving average (MA) components). Where the estimated ARFIMA models contain serial correlation in the variance of the residuals, a generalised autoregressive conditional hetroskedastic (GARCH) innovation has been estimated. The order of integration is then compared for the three real interest rates within each country in order to determine if the volatility of expected inflation has an impact on the order of integration of the real interest rate.

1.3 Chapter Outline

The rest of this study is organised as follows: Chapter two presents a review of the literature pertaining to the Fisher equation. As previously explained the Fisher equation is important as it describes the relationship between the nominal interest rate and expected inflation, thus the behaviour of the *ex ante* real interest rate is derived from this relationship. Chapter two also discusses the main concepts of fractional integration, in which basic properties, tests, estimation methodologies and macroeconomic application of the long memory models are described. Chapter three deals with the main issue faced in various studies of the Fisher equation, that is, the unobservable variable expected inflation. The methodologies utilised in this study to generate expected inflation are explained in detail. How inflationary expectations fit into the Fisher equation and the assumptions behind them are also examined. In addition, some of the methods employed in other studies to cope with this issue are discussed. Chapter four explains the data and methodologies employed in order to estimate and examine the order of integration for each real interest rate. Issues related to the unit root tests are discussed as well as the econometric methodology and the model selection strategy. Chapter five presents the empirical results for seventeen Asian, Pacific and European countries. Chapter six provides a preliminary analysis of real interest rate parity between Australia and New Zealand using the methodology outlined in chapter four. Finally, chapter seven concludes the study and provides suggested directions for future research.

Chapter Two Literature Review: Fisher Effect and Long Memory Models

2.1 Introduction

The purpose of this chapter is to present a literature review of the Fisher effect and the long memory time series models. The *ex ante* real interest rate is the calculated difference between the nominal interest rate and expected inflation. The long-run behaviour of this variable is therefore linked to the long-run relationship between the nominal interest rate and expected inflation as proposed by Fisher (1930), and known as the Fisher effect. According to the Fisher effect if economic agents are rational in the formation of inflationary expectations, then the nominal interest rate should move one-for-one with inflationary expectations in the long-run.[1] For the long-run Fisher relationship to hold the *ex ante* real rate should be a mean-reverting process with respect to time series analysis. However, the mean-reverting properties of the *ex ante* real rate have been called into question ever since the work of Rose (1988). Traditionally, the stationarity of a particular series has been described by testing for integer orders of integration - this is done by directly testing for a unit root or using cointegration analysis. The short memory I(0) process has been used to describe mean reverting of particular time series. However, if the order of integration is allowed to take a fractional value (between zero and one) then the time series process can take up richer forms of mean-reversion. If the *ex ante* real rate is fractionally integrated then the long-run Fisher equation can still be valid.

Expected inflation is an unobservable variable and therefore the *ex ante* real interest rate is also unobservable. As a result, observing the long-run behaviour of the real interest rate is a complicated process. A large proportion of the literature that studies the Fisher effect employs realised (actual) inflation. They use the assumption of rationality in order to

[1] Fisher's original proposal argued that in a world of perfect foresight, the nominal interest rate should move one-for-one with inflationary expectations immediately and therefore, leave the *ex ante* real rate unchanged.

calculate the *ex post* real interest rate, then use the *ex post* real rate to infer the long-run behaviour of the *ex ante* real rate. As explained latter in this chapter the use of realised inflation has problems associated with the implied inference of the *ex ante* real interest rate.

A large proportion of the literature concerning the Fisher effect has found a positive relationship between interest rates and inflation yet, the majority of empirical studies have not found support for the Fisher effect in its strongest form. A variety of explanations for the so-called Fisher's Paradox have been proposed in the literature, these are discussed in detail below.

Chapter two is organised as follows: Section 2.2 describes the literature pertaining to the Fisher hypothesis. The early work of Fisher (1930) is discussed, followed by a review of the work on adaptive expectations that attempted to verify the Fisher equation and Fisher's results. Subsequently, the work relating to rational expectations and efficient markets is evaluated, followed by a review of studies that incorporated the rational expectations hypothesis (REH) using advanced methodological procedures concerning time series analysis. The findings of various studies regarding the Fisher effect are then discussed. Finally, the reasons that have been proposed to explain the apparent limited empirical support for the Fisher hypothesis are examined. Section 2.3 reviews the main ideas relating to the long memory model and fractional integration. Lastly, section 2.4 presents a summary and conclusion.

2.2 The Fisher effect

The long run behaviour of the real interest rate is linked to the relationship between nominal interest rates and inflation, known as the Fisher (1930) equation. The Fisher equation states that the nominal interest rate in any period is made up of two components. The nominal interest rate is equal to the sum of the *ex ante* real interest rate and the expected rate of inflation. According to the Fisher equation, if changes in expected inflation have no permanent effect on the real interest rate then changes in expected inflation should be fully reflected in subsequent changes of the nominal interest rate. This

therefore implies that the nominal interest rate should move one-for-one with expected inflation.

> *If men had perfect foresight, they would adjust the money rate interest so as exactly to counterbalance or offset the effect the changes in the price level, thus causing the real interest rate to remain unchanged at the normal level.* (Fisher, 1930, p. 414).

The real interest rate is thought to represent real factors in the economy such as the productivity of capital and preferences of investors (Cooray, 2002). Fisher tested his theory using annual data for the United Kingdom and the United States and found that changes of inflation were not instantaneously reflected in interest rates, which was thought to be due to imperfect forethought and 'money illusion'. Fisher, however, found that a distributed lag model of inflation (inflationary expectations) was highly positively correlated to long-term interest rates for both the United Kingdom and the United States.[2] From these results Fisher concluded:

> *...price changes do, generally and perceptibly, affect the interest rate in the direction indicated by a priori theory. But since forethought is imperfect, the effects are smaller than the theory requires and lag behind price movements, in some periods, very greatly. When the effects of price changes upon interest rates are distributed over several years, we have found remarkably high coefficients of correlation, thus indicating that interest rates follow price changes closely in degree, though rather distantly in time.* (Fisher, 1930, p. 451)

A vast literature has been derived from Fisher's work that has attempted to provide evidence for or against the Fisher effect. The sub-sections below discuss various aspects of the Fisher effect in terms of early studies, the rational expectations hypothesis, time series properties, recent international studies and Fisher's Paradox.

[2] Fisher also found further evidence using short-term United States commercial paper rates, which further supported his study.

2.2.1 Fisher Effect: Early Studies

The main problem that researchers face when using the Fisher equation is that expectations of inflation are not directly observable and therefore the researcher must use an appropriate proxy. In the literature there have been various methodologies employed in order to overcome this problem. Some of these techniques are discussed in more detail in chapter three; however chapter two discusses the main ideology in the literature regarding inflation expectations. Early studies such as Gibson (1970), (1972), Yohe and Karnosky (1969), Lahiri (1976) and even Fisher himself used a distributive lag or adaptive expectations model to verify the Fisher effect. Gibson (1970) used geometrically declining weights to support the suggestion that there was a distributed lag effect in price. An important finding of Gibson's study was the presence of a significant cyclical factor in the formation of expectations. It has been suggested that economic agents use knowledge about the current state of the business cycle when forming expectations. The implication of this was that monetary policy designed to influence interest rates would eventually affect price expectations.

Studies of Yohe and Karnosky (1969), Gibson (1972) and Lahiri (1976) found a shortening of the lag length in the formation of expectations. Yohe and Karnosky's study found that the speed at which expectations were formed increased markedly in the period between 1961 and 1969. Yohe and Karnosky used the Almon lag technique to overcome the issue of multicollinearity. Gibson's study found similar results and reported that there was almost a one-to-one adjustment of interest rates to expected inflation that took place in the period after the late 1950s. Contrary to other studies, Gibson used survey data of important market participants to represent inflation expectations instead of using a distributive lag model.[3] Gibson believed that the structural break in the formation of expectations was due to lower information costs in the high inflation environment of the 1960s. Lahiri agreed with both Yohe and Karnosky, and Gibson, who also found that expectations were formed

[3] Gibson (1972) used the United States survey data known as the Livingston survey of Consumer Price inflation expectations, which is currently conducted by the Reserve Bank of Philadelphia.

more rapidly after 1960.[4] Like Gibson's (1972) study, Lahiri also found a structural break in the expectations formation equation around 1960. Subsequent studies included the assumption of rational expectations, which is discussed in the next sub-section.

2.2.2 Rational Expectations Hypothesis

With the incorporation of the REH (Muth, 1961) and the efficient market hypothesis (Fama, 1970), the subsequent studies of the Fisher effect changed their focus. The incorporation of these theories meant that it was assumed that rational economic agents would use all available information in the formation of inflation expectations. In contrast to Fisher, who thought past changes in the price level became embodied in the current interest rate, Fama (1975) argued that in an efficient market (which uses all available information) there would be a relationship between one-period nominal interest rate and one-period rate of inflation that is subsequently observed. If no such relationship exists in a market where the inflation rate is somewhat predictable and the real interest rate does not change in a way as to exactly offset changes in expected inflation, this would imply that market participants are overlooking relevant information. Fama used the United States one-month Treasury bill rates (TBR) to represent the nominal interest rate and monthly Consumer Price Index (CPI) figures to represent purchasing power for the period of 1953-1971. Autocorrelations were generated for expected change in purchasing power and the real interest rate for one-to-six month periods. Fama found significant autocorrelations for changes in purchasing power, indicating that past rates of purchasing power have information about the expected rate of change, which is consistent with the efficient market hypothesis. Sample autocorrelations of the real rate were not far from zero and supported the assumption of a constant real rate. The evidence of a constant real rate implied that changes through time of the one-to-six month nominal interest rate mirrored changes of one-to-six month expected rates of change in purchasing power, further supporting Fama's hypothesis.

[4] Lahiri (1976) used four different measures for expected inflation, which included unrestricted distributed lag, adaptive, extrapolative and Frenkel's hypothesis.

Fama's (1975) study made an important methodological contribution to the study of the Fisher effect due to the shift from distributed lag models of inflation to testing the relationship between market interest rates and subsequently observed inflation. However, Fama was consequently challenged by Hess and Bicksler (1975), Carlson (1977), Joines (1977) and Nelson and Schwert (1977). The study by Carlson (1977) utilised survey data to model inflationary expectations and rejected Fama's study.[5] Carlson included the ratio of employment to population (which was thought to have a good record of predicting inflation) into Fama's equation and found that the corresponding estimated coefficient was significant and that the relationship between expected change in purchasing power and the nominal interest rate has diminished. This indicated that the employment to population ratio had not been fully reflected in the one-month interest rate, contradicting Fama's conclusions. Joines (1977) included lagged values of the change in the wholesale prices into Fama's equation to test whether the market utilised all available information about changes in consumer price inflation contained in wholesale prices in setting the nominal interest rate. In doing this Joines found a significant seasonal pattern in the forecast residuals of consumer price inflation, which was inconsistent with market efficiency and subsequently questioned the data used by Fama.

Studies done by Nelson and Schwert (1977) and Hess and Bicksler (1975) challenged Fama'a results by generating inflation expectations using the Box-Jenkins autoregressive integrated moving average (ARIMA) methodology to construct a time series predictor. When the generated inflation expectations were incorporated in the equation of inflation, the estimated nominal interest rate coefficient was significant. This indicated that not all the information was absorbed into the nominal interest rate, or that the variation of the *ex ante* real interest rate distorted the predictability of the market interest rate or both.

Another discovery at odds with Fama's findings was studies such as Garbade and Wachtel (1978), Fama and Gibbons (1982) and Huizinga and Mishkin (1984) which found evidence that the *ex ante* real interest rate was not constant. Fama and Gibbons and Huizinga and Mishkin found evidence to suggest that the *ex ante* real rate is negatively related to both the

[5] Similar to Gibson (1972). Carlson (1977) employed United States Livingston survey data of CPI inflation.

nominal interest rate and inflation. Specifically, Fama and Gibbons attribute the finding to the 'Mundell-Tobin' effect, which is discussed in more detail below. Both Garbade and Wachtel and Fama and Gibbons found that if the *ex ante* real interest rate was modelled as a random walk process, the nominal interest rate was found to be a good predictor of inflation. However, no formal testing for a unit root was conducted by these studies. In the literature there seemed to be an overwhelming consensus that the real rate of interest was not constant, instead it is believed to be time varying (Bonham, 1991). Mishkin (1995) also presented evidence that rejected the constancy of the real interest rate, finding a negative relationship between the real rate and inflation expectations. However, this result was not found to be as strong as earlier studies. With evidence found of a time varying real interest rate, studies have focused on the time series properties of the *ex ante* real rate in order to test the validity of the Fisher effect. Thus, testing the stochastic nature of time series became crucial to studying the Fisher effect.

2.2.3 Fisher Equation: Time Series Properties

In his seminal work, Rose (1988) directly tested for a unit root contained in both inflation and interest rates using the Dickey-Fuller unit root test, which was equivalent to testing the stationarity of the *ex ante* real interest rate. Rose assumed that the expected *ex ante* real rate differed from the *ex post* real rate by a stationary forecast error.[6] This meant that if the nominal interest rate was nonstationary and the inflation rate was stationary then the *ex ante* real rate will necessarily be a nonstationary process. Using annual data of the United States for two sample periods, Rose discovered that the null hypothesis of an I(1) process was not rejected for the interest rates studied, however, the null hypothesis was rejected for the inflation rate series.[7] Rose further backed up his findings by conducting the same tests on quarterly data for eighteen Organisation for Economic Co-operation and Development (OECD) member countries. The presence of a unit root in inflation was rejected for all

[6] The REH usually requires a more stringent assumption (forecast errors must be White noise) however, since the work of Rose (1988), for the long run Fisher equation to hold the minimum requirement is that forecast errors are at least stationary.

[7] Rose used four measures of price inflation, which were the Gross National Product (GNP) deflator, the CPI, the implicit price deflator and the Wholesale Price Index (WPI). Two measures of the interest rate were used; these included the short-term commercial-paper rate and the high-grade long-term corporate rate. The two sample periods were 1892-1970 and 1901-1950.

countries at the five percent level. Rose also conducted the same tests for different samples of monthly data for the United States. For all sample periods except one, the nominal interest rate was found to contain a unit root whereas inflation did not. The implication of the discovery of a unit root in the *ex ante* real rate (as noted by Rose) was at odds with consumption-based capital asset pricing models (CCAPM), as Evans and Lewis (1995) highlighted:

> *These findings are puzzling because, as Rose (1988) points out, ...when real interest rates are subject to permanent disturbances, these models* [CCAPM] *predict that consumption growth rates should also be affected by permanent disturbances, a hypothesis easily rejected by the data.* (Evans and Lewis, 1995, p. 225)

In another influential paper Mishkin (1992) tried to address the issue regarding the strong Fisher effect that occurred in the United States during certain periods and not in others. As Mishkin explained for the United States the Fisher effect was widely accepted for the period after 1951 up until late 1979, before and after this period the short-term interest rate did not seem to have any ability at predicting future inflation. Mishkin argued that the usual methodology of regressing future inflation on the nominal interest rate in order to find a short-run Fisher effect would create spurious results if both the short-run interest rate and future inflation contained a common stochastic trend (that is, a unit root). However, if it were found that both the short-run interest rate and future inflation contained a unit root then both variables would be cointegrated if a linear combination of these variables were integrated of order zero. If a cointegrated relationship was found between future inflation and the short-run interest rate then this would imply a stable long-run relationship and hence support for a long-run Fisher effect.[8] The difference between the long-run and short-run Fisher effect is as follows: the long-run Fisher effect describes when changes in expected inflation are eventually reflected in the nominal interest rate, as opposed to the

[8] Mishkin (1992) also points out that testing for a unit root between the difference of future inflation and the short-run interest rate is the same as testing for the presence of a unit root in the *ex ante* real rate.

short-run Fisher effect in which changes in expected inflation are immediately reflected in the nominal interest rate.

Employing monthly data for the United States data of the TBR and CPI inflation for the period of 1953-1990, Mishkin (1992) conducted Dickey-Fuller and Phillips unit root tests. Mishkin found that both the nominal interest rate and inflation contained a unit root. Further analysis found there existed a cointegrating relationship between the two variables, and, as explained above, this supported a long-run Fisher effect. Mishkin did not find any evidence of the existence of a short-run Fisher effect.

Similar to Mishkin (1992), other studies have also found that inflation contained a unit root. Bonham (1991), using the augmented Dickey-Fuller (ADF) test, found that inflation contained an I(1) process for the sample period of 1955-1986. Bonham also concluded that once a proxy for the *ex ante* real rate was introduced into the model under the assumption of a unit root the null hypothesis of no cointegration was rejected. Wallace and Warner (1993) demonstrated that if one-period inflation contained a unit root then expectations of inflation would also be dominated by a unit root. Because of this Wallace and Warner suggested that one-period inflation could be cointegrated with long-term interest rates. Applying the ADF test for a sample similar to Mishkin, Wallace and Warner also found that CPI inflation contained an I(1) process and using the cointegration procedure there found evidence of a Fisher effect in the short-term and long-term.

As pointed out by Mishkin (1992), the discovery of a stationary *ex ante* real interest rate suggested that it had mean-reverting properties;[9] this is consistent with the existence of the long-run Fisher effect. As Lai (1997) explained, if both inflation and the nominal interest rate are characterised by the same permanent disturbances then for the Fisher effect to hold the *ex ante* real rate must be a stable mean-reverting process:

[9] Mean-reversion refers to the tendency of a time series to return to a constant and finite mean after a shock (Verbeck, 2000).

Unless nominal rates and rationally expected inflation respond one-for-one to permanent shocks, ex ante real rates will be affected by the same permanent shocks as expected inflation, contradicting the long-run superneutrality of money. (Lai, 1997, p. 226).[10]

A stationary I(0) process is only a special case of mean-reversion. Lai estimated a fractional order of integration for both the ex post and *ex ante* real rates of interest for the United States, which captures more subtle mean reverting dynamics.[11] Lai utilised monthly United States data, including CPI inflation, professionally forecasted inflation and various nominal interest rates. Lai found that both the *ex post* and *ex ante* real rates were fractionally integrated and therefore concluded that the real rate was mean-reverting and that the Fisher effect held in the long-run. Tsay (2000) employed monthly and quarterly data for the United States CPI and TBR and found the long memory characterised the United States *ex post* real interest rate, which implied that the United States *ex post* real rate is mean reverting in the long-run.

2.2.4 Empirical Studies

The empirical results for the United States regarding the Fisher relationship have generally been mixed. Some studies have found a positive relationship between the nominal interest rate and expected inflation, although evidence is limited with regards to a one-for-one relationship. Some studies such as Mishkin (1984a), MacDonald and Murphy (1989), Yuhn (1996) and Dutt and Ghosh (1995) have evaluated the validity of the Fisher relationship for several countries. Mishkin (1984a) examined the real interest rates for seven OECD countries over the 1967-1979 sample period, using euro deposit rates. This study rejected the constancy of the real interest rate and a negative correlation between real rates and expected inflation is found for all seven countries. Furthermore, Mishkin found that movements in nominal rates in the post-World War II period reflected changes in

[10] See also, King and Watson, 1992; Evans and Lewis, 1995.

[11] Fractional integration is also known as long memory behaviour is associated with the observation of persistent autocorrelation, which is consistent with an essentially stationary process, however, the autocorrelations take far longer to decay than the exponential rate associated with a stationary I(0) process.

expected inflation for the United States, the United Kingdom and Canada. However, the Fisher effect was weak for France, Germany, the Netherlands and Switzerland. MacDonald and Murphy (1989) examined the long-run relationship between inflation and interest rates using cointegration analysis for the United States, the United Kingdom, Belgium and Canada for the period 1955-1986. For the entire sample period the null hypothesis of no cointegration could not be rejected on the basis of the ADF test. Interestingly, MacDonald and Murphy found that once the sample period is split according to fixed and floating exchange rate regimes, cointegration was found for the United States and Canada for the fixed exchange period. No evidence of cointegration was found for the United Kingdom and Belgium.

In contrast to the results of Mishkin (1984a), Yhun (1996) used data for the post-fixed exchange rate period (that is, 1973-1993) for the United States, the United Kingdom, Japan, Germany and Canada. Yuhn found evidence of a strong Fisher effect for Germany but no evidence was found for the United Kingdom or Canada. On the other hand for Germany, Japan and the United States a long-run relationship was found which suggested that movements in the nominal interest rate reflected changes in expected inflation. Another finding of Yuhn's research was that the Fisher effect did not seem to be robust to monetary regime changes, where evidence of a Fisher effect was not found during the period of 1982-1993 in the case of the United States. Similar to Yuhn, Dutt and Ghosh (1995) found little evidence of a Fisher effect in the case of Canada. Dutt and Ghosh rejected a Fisher effect for Canada in both fixed and floating exchange rate regimes.

Few studies have analysed the Fisher effect with respect to developing countries such as Phylaktis and Blake (1993), Payne and Ewing (1997) and Berument and Jelassi (2002). Phylaktis and Blake (1993) examined the Fisher effect for three economies affected by very high inflation rated; these include Argentina, Brazil and Mexico. Using cointegration analysis Phylaktis and Blake could not reject a cointegrating relationship between nominal interest rates and inflation for all three countries. The estimated cointegrating parameters for each country could not reject a unit proportional relationship, therefore providing evidence of a strong Fisher effect. The implication of this study was that economic agents

in high inflation economies tend to invest more resources in forecasting inflation in order to incorporate fully the resulting inflationary expectations in their required returns. Employing unit root and cointegration analysis to nine developing countries Payne and Ewing (1997) evaluated the Fisher effect. Although interest and inflation rates were found to contain a unit root for each country, cointegration tests suggested that only Sri Lanka, Malaysia, Pakistan and Singapore contained a long-run Fisher effect. Moreover, a unit proportional relationship was found for Sri Lanka, Malaysia, and Pakistan, whereas, no evidence of a Fisher relationship was found for Argentina, Fiji, India, Niger and Thailand. In an extensive study Berument and Jelassi (2002) tested the Fisher hypothesis for twenty-six developed and developing countries.[12] Using the Fisher equation and controlling for the short-run dynamic of both the nominal interest rates and inflation a long-run response coefficient of the interest rate to the inflation rate was estimated. It was found that a one-to-one relationship was accepted for nine out of twelve developed countries and seven out of the fourteen developing countries, which supported the strong version of the Fisher effect. These results were also found to be robust to various empirical tests.

2.2.5 Fisher's Paradox

Although a positive relationship has emerged between interest rates and inflation in the literature, there has been very limited support for the Fisher effect in its strictest form. As Carmichael and Stebbing (1983) explain, the limited empirical support for the strict form of the Fisher effect is known as 'Fisher's Paradox'. Explanations for this paradox have been suggested, some of which are outlined below.

An early explanation of the empirical failure of the strong form of the Fisher effect was noted by Mundell (1963) and Tobin (1965). The 'Mundell-Tobin' effect explained that nominal interest rates would increase (decrease) by less than unity during times of inflation (deflation) and therefore expected inflation would have a real effect on economic activity.

[12] These countries include: Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Korea, Switzerland, the United Kingdom, the United States, Brazil, Chile, Costa Rica Egypt, Greece, India, Kuwait, Mexico, Morocco, Philippines, Turkey, Uruguay, Venezuela and Zambia, see Table 1 (Berument and Jelassi, 2002, p.1647).

It was assumed that economic agents held only two forms of wealth (that is, money and capital). When there was an increase in anticipated inflation this would lead to a reduction in real money balances. The reduction in real money balances would reduce the amount of money held as wealth and therefore economic agents would increase savings (and thus hold more capital). The resulting increase in savings would bring downward pressure on real interest rates and consequently the nominal interest rate would increase by less than unity.

Another explanation of 'Fisher's Paradox' was suggested by Darby (1975). Darby pointed out that when nominal interest rate income is taxed, the Fisher hypothesis implies that the response from nominal interest rates is greater than a unit change in expected inflation in order for rational economic agents to maintain a constant *ex ante* real interest rate.[13] If economic agents do not fully take into account the tax burden associated with a particular asset, then they are thought to suffer from 'Fiscal Illusion'. Tanzi (1980) failed to find any support for Darby's hypothesis of a greater than unitary response coefficient for the Fisher equation once the tax effect was taken into consideration. Tanzi concluded that for the United States individuals suffered from fiscal illusion. Crowder and Hoffman (1996) found support for the tax adjusted Fisher equation using quarterly data for the United States for the period 1952-1991. A fully efficient estimation method is used that separates the long-run equilibrium relationship from the short-run dynamic. It was found that a one percent increase in inflation yields a one and one third percentage point increase in the nominal interest rate. A recent study by Crowder and Wohar (1999) found support for Darby's hypothesis using monthly data for the United States for the period 1950-1995 on one-year Treasury bill and one-year municipal (tax exempt) bond rates. Crowder and Wohar used six different estimators and rejected the idea that economic agents suffered from fiscal illusion. It was discovered that the response of tax-free municipal bond rates was smaller than the response on taxable treasury bills for each estimation procedure, which suggested that other factors contributed to Fisher's Paradox.

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[13] The nominal interest rate was expected to increase by $\left(\frac{1}{1-\tau}\right)$ where τ is the average marginal tax rate, see Crowder and Wohar (1999) for details.

Alternatively, Carmichael and Stebbing (1983) proposed an explanation for Fisher's Paradox termed the 'inverted' Fisher effect. They explained that with a certain degree of regulation in the financial market (the minimum requirement proposed by Carmichael and Stebbing was the non-payment of interest on money balances) and a high substitutability between money and financial assets after tax nominal interest rates these financial assets are approximately constant over the long-term with the after tax real rate of interest moving one-for-one with inflation. This inverted Fisher hypothesis was backed up by empirical analysis for both the United States and Australia. The empirical analysis conducted by Carmichael and Stebbing (1983) was unable to reject a one-for-one relationship between the after-tax real interest rate and inflation in both countries. Other studies such as Graham (1988), Gupta (1991), Choudhry (1997) and Choi (2002) have found limited support for the inverted Fisher hypothesis. Similar to Carmichael and Stebbing, Gupta found support for the inverted Fisher hypothesis. Unlike some studies of the inverted Fisher hypothesis Gupta employs inflation expectations survey data on the GNP price deflator for the period of 1968-1985 for the United States. Also in testing the inverted Fisher hypothesis the variance, skewness and kurtosis of inflation expectations were added to the model because it is believed that inflation uncertainty has an adverse effect on the interest rate. The results of Gupta cannot reject the inverted Fisher hypothesis. Furthermore, Gupta found that uncertainty about expectations measured by the variance and higher order moments do not have any significant effect on the real interest rate. In contrast, Graham (1988) provided evidence against the findings of both Carmichael and Stebbing and Gupta. Because of the problems relating to errors-in-variables, generated repressors and omitted variables Graham rejected the inverted Fisher hypothesis and instead found strong evidence of a partial Fisher effect.

Employing cointegration analysis to determine the existence of Fisher inversion for Belgium, France and Germany for the period of 1955-1994 Choudhry (1997) showed some support for partial Fisher inversion, although little support for the inverted Fisher effect was found. Choi (2002) argued that the inverted Fisher hypothesis would hold only if inflation is stationary. This contrasted with Carmichael and Stebbing, who believed that the inverted Fisher hypothesis would not hold when inflation was high for sustained periods. This was

because high levels of inflation would be associated with an increase in the substitutability between money and capital. Therefore, high inflation would not be reflected in the real interest rate. Instead, Choi suggested that the inverted Fisher effect would not hold if inflation became persistent. If inflation was more persistent economic agents could forecast inflation using their recent experience of inflation, thus high anticipated inflation would lead to agents substituting money for capital. The increase in substitution between money and capital would lead to the validity of the normal Fisher effect. The results of Choi confirmed that for the United States the inverted Fisher hypothesis held only under a certain threshold of the persistence of the inflation process. Choi further verified these results using data from Germany, Argentina and Brazil. Overall, the evidence of Fisher inversion is variable and as explained by Cooray (2002, p. 18) "... evidence with respect to the inverted Fisher hypothesis has not been clear-cut."

Another issue regarding the validity of the Fisher effect was proposed by Evans and Lewis (1995) and is known as the 'peso problem'.[14] Evans and Lewis argue that the inflation process is characterised by infrequent shifts. If these infrequent shifts are rationally expected, forecast errors can become serially correlated if anticipated shifts do not materialise. These deviations between expected and realised inflation would be highly persistent, which would generate the appearance of a nonstationay real interest rate if realised inflation were used as a proxy for expected inflation using cointegration analysis. However, as Evans and Lewis point out, this issue is only a small sample problem because over long time periods with many shifts in the inflation process forecast error should be serially uncorrelated if economic agents are truly rational.

Addressing the 'peso problem' by controlling for a structural change in the inflation process Evans and Lewis (1995) used a Markov switching model that anticipated shifts of the inflation process for United States quarterly data. Evans and Lewis were unable to reject the idea that long-term movements in nominal interest rates reflect one-for-one long-

[14] Originally, the 'peso problem' was used to explain the situation in which the Mexican peso was expected to be devalued for a number of time periods, and which was eventually devalued in 1976. Because economic agents expected the peso to be devalued for some time before it actually was, data on forward exchange rates prior to the devaluation would be biased forecasters of future spot exchange rates, see Moosa and Bhatti (1997) chapter twelve for details.

term movements in expected inflation once inflation expectations generated by the regime switching methodology were considered. One structural break was found in the mean of the United States inflation data from the 1950s to the late 1980s. Other studies have used the argument of a common structural break in the Fisher equation in order to explain Fisher's Paradox. It has been argued that the results from studies such as Mishkin (1992) may be biased if the Fisher equation is characterised by structural breaks. If a common structural break is present in the nominal interest and inflation data then unit root tests may interpret the structural breaks as persistent stochastic disturbances and therefore be biased towards finding a unit root.[15] Garcia and Perron (1996) found that the United States inflation and real interest rate data is characterised by two structural changes for the period 1961-1986 using a Markov switching model. One of the structural changes was found to coincide with the oil price shock of 1973 and the other structural change was consistent with a federal budget deficit of 1981. The main finding of Garcia and Perron was that the *ex ante* real interest rate was found to be constant once the shifts of the relevant series were considered, supporting the earlier findings of Fama (1975). Malliaropulos (2000), using quarterly United States data from 1960 to 1995 found differing results in comparison to Evans and Lewis (1995), and Garcia and Perron (1996). Malliaropulos found that the nominal interest, inflation and the *ex ante* real rate are all trend stationary with a structural break in both conditional mean and drift rate which occurred in early 1980s. The structural break coincided with the United States Federal Reserve introducing a new operating procedure. Malliaropulos also investigated the dynamic relation between inflation and the nominal interest rate using a vector autoregressive (VAR) model utilising appropriately detrended stationary variables. The results suggested that the Fisher effect was robust in the medium and long term, and the speed of adjustment was shorter in comparison to other studies.

Some recent studies attribute the rejection of the Fisher effect to the nonstationarity of the *ex ante* real interest rate. If, for example, the *ex ante* real rate followed a random walk process then the difference between the nominal interest and inflation rate is therefore a

[15] Structural breaks are generated by exogenous events for example, oil price shocks and shifts in monetary or fiscal policy regimes.

random walk process and a stable long-term relationship cannot exist between them. As previously explained, Rose (1988) presented evidence that suggested the *ex ante* real rate is nonstationary. Peláez (1995) suggested that if the *ex ante* real interest rate is nonstationary, then for the long-run Fisher effect to hold the inflationary expectations component of the nominal interest rate should be cointegrated with actual inflation. For the period of 1959-1993 Peláez's results confirmed that there existed a cointegrating relationship between inflation and inflation expectations extracted from the nominal interest rate for the United States. Peláez attributed this finding to the nonstationary behaviour of the *ex ante* real rate.

In a more recent study, Sun and Phillips (2003) argue that univariate long memory estimates based on the *ex post* real interest rate tend to underestimate the level of persistence of the *ex ante* real interest rate. It is argued that the *ex post* Fisher equation is imbalanced because of the different timing, short-run dynamics and the small sample size of each variable. The different short-run dynamics of the variables contained in the *ex post* Fisher equation is thought to create a downward bias of the estimated order of integration (d) for the *ex post* real rate. This is because actual inflation contains an unexpected component (the forecast error) and therefore appears more volatile then actual expected inflation. This volatility is thought to mask the more slowly moving and persistent expected inflation, which is buried within the short-run fluctuations. This additional volatility is then inherited within the *ex post* real rate, and therefore masks the more persistent *ex ante* real rate. Sun and Phillips claim that the *ex ante* real rate, more or less, shares the same high level of persistence as the other components in the Fisher equation and thereby furnishing evidence against the existence of any cointegrating (or fractional cointegrating) relationship among the Fisher variables. Using both United States survey data of inflation forecasts and a new estimation method, Sun and Phillips results indicate evidence of downward bias of the level of persistence of the *ex post* real rate.[16]

This present study uses the same idea developed by Sun and Phillips (2003). Using the Fisher equation, the real interest rate is calculated using three different measures of

[16] Sun and Phillips (2003) introduce a bivariate exact Whittle estimator, that allows for the presence of short memory noise in the data.

inflation expectations, which differ in the level of volatility contained within them.[17] The three different real interest rates are calculated for nineteen countries and the order of integration is estimated for each nation using an autoregressive fractionally integrated moving average (ARFIMA) model. The estimated order of integration is compared within each country to determine whether short run volatility of expected inflation has an impact on the order of integration of the real interest rate. The way that economic agents incorporate short-run dynamics in forming inflationary expectations may play an important role in the level of persistence of the *ex ante* real rate. This in turn will have important implications for the validity of the Fisher effect, especially in terms of mean-reversion.

2.3 The long memory model

Time series data is typically classified as either an I(0) (not integrated) or I(1) (integrated of order one or unit root). It is well known that a time series process that is I(0) is referred to as being stationary, since it has a constant mean, a constant variance and the covariance between values of the process at two points in time will depend only on the distance between them, and not on time itself. On the other hand, an I(1) process is a nonstationary process. Conventionally, the terms I(0) and I(1) are synonymous with the concepts of stationarity and nonstationarity, respectively. A stationary autoregressive moving average (ARMA) process is the leading example of the I(0) process, where the autocorrelation function (ACF) decays exponentially towards zero as the lag length increases. In contrast, in a unit root process the ACF does not decay even when the lag length is large, see for example, Brooks (2002) and Tsay (2002). Some time series exhibit properties that do not characterise either an I(0) or I(1) process. Specifically, the ACF of these time series decay much slower than the exponential rate of an I(0) process, yet, unlike the I(1) process the ACF of these time series do decay. These processes are referred to as long memory time series or also known as strong dependence due to the strong association between observations widely separated in time. The original motivation for modelling a long memory process was found in hydrology, which documented the persistent nature of river

[17] The real interest rate is measured using the *ex post* rate and two forms of the *ex ante* real rate using two econometric techniques to generate inflation expectations, which are outlined in detail in chapter three.

flows known as the Hurst effect (see Hurst (1951)). The subsequent application of long memory in the physical sciences has been in areas such as climate and temperature change. Recently it has been found that many economic time series also exhibit the characteristic of long persistence. Many applications of the long memory process have been undertaken in macroeconomics and finance, some of which are discussed below; see, for example, Baillie (1996) for a recent excellent survey.

The simplest long memory model is the pure fractional I(d) unit root process, which is shown in equation (2.1) for the process y_t as follows:

$$(1-L)^d y_t = \varepsilon_t \tag{2.1}$$

Where L is the lag operator, d is the order of integration and ε_t is a white noise process. The order of integration d can be any real (fractional) and explains why this is referred to as a fractionally integrated process. Although, theoretically d can be any real number because of its time series properties the situation where $d \in [0,1]$ is of particular interest. As shown in Hosking (1981) and Granger and Joyeux (1980) the process has the following properties. For $d < 0.5$, y_t is stationary, and for $d > -0.5$, y_t is invertible. When $d = 0$, y_t is simply a white noise process. For $0 < d < 0.5$, the ACF of y_t can be shown as the following equation:

$$\rho_k = \frac{d(1+d)\ldots(k-1+d)}{(1-d)(2-d)\ldots(k-d)}, k = 1,2,\ldots \tag{2.2}$$

Equation (2.2) implies equations in the following form:

$$\rho_k = \frac{(-d)!}{(d-1)!} k^{2d-1} \text{ as } k \to \infty \tag{2.3}$$

According to equation (2.3) the ACF of a pure I(d) process decays at a hyperbolic rate as apposed to the exponential rate. On the other hand, for $d \geq 0.5$, y_t is nonstationary yet, if d

< 1, y_t is mean-reverting, that is, it has a tendency to revert back to the mean of the process in the long–run after a shock. Obviously, when $d = 1$, y_t follows a unit root process.[18]

The above process y_t can be extended to include a stationary ARMA component, when this is done the model becomes an ARFIMA model. The ARFIMA(p, d, q) model is illustrated below in equation (2.4):

$$\phi(L)(1-L)^d y_t - \mu = \theta(L)\varepsilon_t \tag{2.4}$$

Where $\phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$ and $\theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$ which represent the short run dynamic autoregressive (AR) and moving average (MA) components respectively. Both $\phi(L)$ and $\theta(L)$ are assumed have roots that lie outside the unit circle, and ε_t is a white noise process. It is easy to see that the ARFIMA(p, d, q) model encompasses the stationary ARMA process if $d = 0$, and the unit root process if $d = 1$. It is also important to note that the stationary ARMA process where $d = 0$ is often referred to as short memory as opposed to long memory for $d > 0$.[19]

As explained by Lai (1997) for the Fisher effect to hold then an equilibrium relationship should hold between the nominal interest and inflation rate in the long-run. The minimum requirement is that these two variables should be at least fractionally cointegrated. According to Baillie (1996), two time series y_t and z_t are said to be fractionally cointegrated of the order [d, b] if both y_t and z_t are I(d) and there exists β_0 and β_1 such that $y_t - \beta_0 - \beta_1 z_t \sim I(d-b)$ where $b < d$ and $(d-b)<1$, which suggest that the equilibrium relationship would mean revert in the long-run.

In practice, testing for a unit root is commonly done using two types of tests. The first type of test is to test the null of a unit root against the alternative of a stationary series. Examples of these include the ADF test (Said and Dickey, 1984) or nonparametrically

[18] See Baillie (1996) for more details.
[19] See Baillie (1996) for more details.

corrected versions, for example the Phillips and Perron (PP) test (Phillips and Perron, 1998). The second type of unit root test, considers stationarity as the null and a unit root as the alternative for example the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test (Kwiatkowski, Phillips, Schmidt and Shin, 1992). However, researches face a dilemma if both the null of a unit root and the null of stationarity are rejected for a particular time series. As a matter of fact, this may happen when the series actually follows an I(d) process. As both the test for stationarity and the test for a unit root are generally consistent with a long-memory alternative. See, Diebold (2001) and Lee and Schmidt (1996) for a detailed discussion on these issues.

Several methods have been applied to estimate the fractional integrated parameter d. One of the most popular earlier methods employed is the frequency domain (log-periodogram) method of Geweke and Porter-Hudak (1983) (GPH). The GPH method is a semi-nonparametric approach which estimates d using a two step procedure. First, d is estimated followed by the AR and MA components (for details see Maddala and Kim (1999)). On the other hand, the model can be estimated via a time-domain approach, examples are the exact maximum likelihood (ML) estimation approach of Sowell (1992) and the approximate ML approach of Baillie, Chung and Tieslau (1996) and Li and McLeod (1986). When ε_t is normally distributed the exact ML estimation method is usually preferred. However, issues can arise if ε_t is not normally distributed in high volatility data such as financial time series. Also the exact ML estimation method is computationally demanding therefore, the approximate ML estimation method is more widely used in empirical work of this nature.

Fractional integration has been applied to a variety of areas in macroeconomics. For example, Baillie and Bollerslev (1994) find evidence that the forward premium presented as the United States dollar per unit of foreign currency is fractionally integrated for Canada, Germany and the United Kingdom. The long memory model has also been applied to inflation data. Baillie, Chung and Tieslau (1996) apply the ARFIMA model to inflation data for the G7 countries as well as three high inflation countries: they are Argentina, Brazil and Israel. As well as estimating the ARFIMA model Baillie, *et al.* (1996) also include a

GARCH innovation in order to model the time variation of inflation. Strong evidence has been found for long memory in inflation rates of the G7 countries (except for Japan) and of the three high inflation economies. Baum, Barkoulas and Caglayan (1999) extend their analysis of fractional integration in the inflation rate by including a variety of countries not previously studied. Baum, *et al.* (1999) also investigate the existence of long memory in both CPI and WPI-based inflation. The CPI-based inflation is studied for twenty-seven countries while the WPI-based inflation is studied for twenty-two countries for the period of 1971-1995. Two semi-parametric estimation techniques and a frequency domain approximate ML estimation technique are employed. The overall conclusion is that long memory in inflation is a general phenomenon for both industrialised and developing countries. This result is robust for both price indexes used. Baillie, Han and Kwon (2002) find similar results for the inflation rate of eight industrialised countries. Baillie, *et al.* (2002) extend the ARFIMA model to allow for the long memory process to exist within the conditional variance using the ARFIMA fractionally integrated generalised autoregressive conditional hetroskedastic (FIGARCH) model.[20] The results suggest that inflation is unique in that it contains long-memory behaviour in both first and second conditional moments.

As explained above the studies by Lai (1997) (using GPH estimation method) and Tsay (2000) (using an approximate ML estimation method) have found long memory behaviour contained in the *ex post* and *ex ante* real interest rates. Other studies such as Tkacz (2000), found that both the nominal and the real interest rate for both the United States and Canada are characterised by a long memory process. For the United States the order of integration of nominal interest rates increases with the term to maturity and it is found that shocks to the nominal interest rates for Canada take longer to dissipate. Gil-Alana (2003), using the *ex post* real interest rate for ten European countries as well as the United States and Canada tested the null hypothesis that d was equal to various values using a Lagrange Multiplier test. Gil-Alana concluded that the real interest rate is more persistent (larger d) in some countries such as the United States, Belgium and Holland compared to others such as

[20] The ARFIMA-FIGARCH model is beyond the scope of this research, and is left as further research.

Germany and the United Kingdom. Therefore, the examination of this literature suggests further analysis that is undertaken in this study.

2.4 Conclusion

Chapter two provides a review of the literature pertaining to the Fisher effect. This is important because the long-run behaviour of the *ex ante* real interest rate is linked to the Fisher effect. Chapter two discusses the main concept of the long memory model, in which basic properties, tests, estimation methods and macroeconomic applications are discussed.

Early studies of the Fisher effect using adaptive expectations in general found support for Fisher's study. Conversely, with the incorporation of the rational expectations hypothesis in the analysis, some empirical results found mixed conclusions. Also a consensus in the literature seems to emerge that suggest that the *ex ante* real interest rate has not been constant, this is contrary to the original proposition. With the methodological advancements in time series analysis, attention has been given to examining the stationarity of the variables contained in the Fisher equation. Cointegration analysis has been employed to determine a stable long-run relationship between nominal interest and inflation rates, yet the results have been mixed. Recent studies have found evidence to suggest that the *ex ante* real rate is fractionally integrated. This result is suggestive for the validity of a stable long-run Fisher relationship. Overall, although the Fisher effect did not seem to exist in its strongest form, studies for the United States do suggest that there is a positive relationship between nominal interest and the inflation rate. On the other hand, international studies have not been so clear-cut. With the Fisher effect not being verified in its strictest form various explanations have been put forward. Many of which of these seem plausible, yet the evidence has not been consistent.

The usual method for modelling the stochastic nature of a particular time series is the use of integer orders of integration. However, in terms of mean-reversion the use of integer orders of integration (I(0)/I(1)) is unnecessarily strict because mean reverting processes do not have to be I(0) series. The long memory ARFIMA model can bridge the gap between the

knife-edge I(0)/I(1) distinction and characterise richer forms of mean-reversion. In the empirical literature support has been found for fractional integration in the *ex ante* real interest rate. The implication of this is that support for the long-run Fisher effect is maintained. Thus, evaluation for nineteen Asian, Pacific and European countries will consider the issues raised here in order to examine the Fisher effect and long memory models for these countries.

Chapter Three Inflationary Expectations: Relevant Issues

3.1 Introduction

As explained in chapter one there are generally two approaches that are adopted to test the mean-reversion of the *ex ante* real interest rate. The first approach involves testing for a cointegrated relationship between inflation and nominal interest rates using the Fisher equation (see, for example, Mishkin (1992)). Under the assumption of stationary forecast errors if both inflation and nominal interest rates are I(1) then a long-run cointegrating relationship implies that the *ex ante* real rate is a I(0) process (which is by definition mean-reverting). The second approach involves testing for a unit root in the real interest rate directly. In this study, the order of integration is directly estimated for various real interest rates using the latter approach. The first step of this approach is to calculate the real interest rate, and the real interest rate is calculated using the Fisher equation. As explained in the previous chapters the *ex ante* real interest rate is equal to the difference between the nominal interest rate and inflationary expectations, however, the main dilemma faced by researchers is that expected inflation is unobservable and therefore an appropriate proxy must be used. In chapter three the methodologies utilised to generate inflationary expectations in this study are explained. Also the issue of inflationary expectations with respect to the Fisher equation is discussed, and some of the methodologies that have been employed in the literature are briefly described.

This chapter is ordered as follows: section 3.2 describes how inflationary expectations are incorporated in the Fisher equation and techniques used in other studies are briefly explained. Section 3.3 presents the techniques employed in this study, which are used to model inflationary expectations. Finally, section 3.4 provides a conclusion.

3.2 Inflationary Expectations

According to the Fisher (1930) equation, one period nominal interest rate (i_t) at time t is constructed as the sum of the following three components shown in equation (3.1):

$$i_t = r_t + \pi_t^e + r_t\pi_t^e \tag{3.1}$$

Where r_t represents the *ex ante* real interest rate interest rate in period t, π_t^e represents inflation expectations expected at time t and $r_t\pi_t^e$ is the interaction term.[1] The interaction term $r_t\pi_t^e$ is neglected because it is dwarfed by the other components if investment horizons are short.[2] Taking this into consideration the revised Fisher equation is presented in equation (3.2):

$$i_t = r_t + \pi_t^e \tag{3.2}$$

According to equation (3.2) if changes in π_t^e have no permanent effects on r_t then those changes in π_t^e should be fully reflected in subsequent movements of i_t. Previous work on the Fisher effect involved testing for a significant relationship between interest rates and expected inflation, or alternatively using cointegration analysis to find a long-run relationship as shown in equation (3.3):

$$i_t = \alpha + \beta\pi_t^e + \varepsilon_t \tag{3.3}$$

The α term represents a constant (in early studies α was assumed to represent the constant *ex ante* real interest rate), ε_t is the error term, and β is the response coefficient. It was

[1] The tax adjusted Fisher equation of Darby (1975), implies that instead of being equal to one the Fisher effect is equal to $\left(\frac{1}{1-\tau}\right)$, however, this issue is not addressed in this research and therefore it is assumed that (τ= 0).

[2] See, for example, Gibson (1970).

expected that β was equal to one, which described the one-to-one relationship between interest rates and expected inflation (that is, the strong form of the Fisher effect). A positive value of β less than one represents the weak form of the Fisher effect, which suggests that factors such as the Mundell-Tobin effect and the Tax effect may be present.[3] With cointegration analysis, the long-run Fisher effect exists if β equals one and both i_t and π_t^e are cointegrated of order CI[1, 1], this implies that ε_t is a covariance stationary I(0) process. Equation (3.2) can be written as follows:

$$r_t = i_t - \pi_t^e \tag{3.4}$$

Equation (3.4) implies that the discovery of a cointegrating relationship in (3.3) is analogous to r_t being an I(0) process. However, for the long-run Fisher relationship to exist, the minimum requirement is that r_t is mean-reverting, hence a stable long-run relationship between i_t and π_t^e prevails in the long run. The *ex ante* real rate is mean-reverting if it follows an I(d) process with $0 < d < 1$, this implies that r_t is fractionally integrated. As explained in chapter two, fractional integration captures a much richer form of mean-reversion than an I(0) process. In this study the real interest rate is calculated using equation (3.4), from this the order of integration is estimated and examined.

The main issue that all researchers face is that π_t^e is an unobservable variable and therefore by deduction the *ex ante* real interest rate is also unobservable. Because π_t^e is unobservable an appropriate proxy must be employed in order to infer the long-run behaviour of r_t this is described in equation (3.5) in the following form:

$$\hat{\pi}_t^e = \pi_t^e + u_t \tag{3.5}$$

In equation (3.5), $\hat{\pi}_t^e$ represents some proxy of actual expected inflation (π_t^e), and u_t represents the measurement error between the actual and proxy of expected inflation. A

[3] See, Berument and Jelassi, (2002).

large proportion of the literature uses realised or actual inflation as a proxy for inflation expectations ($\hat{\pi}_t^e = \pi_t$) and then substitutes π_t into equation 3.4 in order to calculate the *ex post* real interest rate.[4] Using the assumption of a stationary forecast error, which is outlined below, the *ex post* real rate is used to infer the time series properties of the *ex ante* real rate. When realised inflation is utilised, u_t in equation 3.5 becomes the forecast error and both π_t^e and u_t are contained within realised inflation. Because u_t is contained in actual inflation some issues of the errors-in-variables problem arise, for example the 'peso problem' discussed in chapter two, which can bias the inference of r_t. As explained in chapter two it is often assumed that expectations are 'rational', that is $\pi_t^e = \mathrm{E}(\pi_t \mid I_t)$, where $\mathrm{E}(\cdot \mid \cdot)$ stands for the conditional expectations operator, and I_t is the information set available when expectations are formed at time t. However, this strict assumption is not necessary for the Fisher effect to hold. Instead, the minimum requirement is that u_t in equation (3.5) is stationary and thus the estimate of β is consistent.

Following Sun and Phillips (2003), using realised inflation in equation 3.5, it is believed that because u_t is unexpected its short-run volatility is large relative to the volatility of π_t^e. Estimates of d for the *ex post* real rate are biased downwards because the relatively large volatility of u_t masks the persistent nature of actual inflationary expectations, which in turn masks the more persistent nature of the *ex ante* real rate buried within the *ex post* real rate. Because of this issue, two econometric techniques are used here that extract the more persistent *ex ante* variable from actual inflation data and the estimated d parameters for the *ex ante* real interest rates are compared for nineteen different Asian, Pacific and European countries. The techniques employed in this study to generate inflationary expectations are outlined in detail in section 3.3.

In the literature different techniques have been employed for the purpose of adequately representing π_t^e, some of which are discussed briefly here. Earlier studies have used the

[4] This is equivalent to substituting π_t into equation 3.3 and testing the validity of the Fisher effect for example with cointegration analysis.

technique of adaptive expectations in order to generate π_t^e. Also known as 'extrapolative' or 'error-learning', adaptive expectations are defined as:

> *Faced with uncertainty about the future, an economic decision-making unit is presumed to base its predictions about future price movements on a weighted average of current and past changes in prices.* (Yohe and Karnosky, 1969, p. 353).

This approach is adaptive in the sense that π_t^e for each period is adjusted for the actual inflation; therefore, past values of inflation are extended (extrapolated) into the future. Some examples of the studies that employed adaptive expectations are Gibson (1970), Yohe and Karnosky (1969) and Lahiri (1976).

A commonly used method is the use of realised inflation as an unbiased measure of π_t^e assuming rational expectations. As explained in the literature review (chapter two) this was in contrast to early studies that employed past values of price change. Instead, the idea of the rational expectations hypothesis (REH) proposed by Muth (1961) was applied. This is defined as:

> ...[Expectations] *are essentially the same as predictors of the relevant economic theory. In particular, the* [rational expectations] *hypothesis asserts that the economy generally does not waste information, and that expectations depend specifically on the structure of the entire system.* (Muth, 1961, p. 315).

As Darin and Hetzel (1995) explain that when this is applied to realised inflation researchers make the following assumption:

> *Specifically, they assume that participants in financial markets understand the nature of the monetary regime that generates inflation.* (Darin and Hetzel, 1995, p. 26).

The assumption of the REH coupled with the assumption of market efficiency implies that with the use of all available information forecasts are unbiased and efficient. Forecasts are unbiased if the forecasted series has the same mean as actual inflation, this implies that the expected value of forecast errors is zero. Forecasts are efficient if forecasters utilise all relevant information at the time forecasts are made, this implies that forecast errors are uncorrelated to any information available at the time forecasts are made (including past values of the forecast error) (Holden, Peel and Thompson, 1985). If inflation expectations are not unbiased and efficient then the accuracy of forecasts could be improved by using publicly available information. In other words, for inflation expectations to conform to the REH then "...the long-run response of expected inflation to the actual rate of inflation should be equal to one as economic agents cannot systematically be fooled" (Paquet, 1992, p. 303). Some studies argue that the REH of Muth is far too strict and in terms of the Fisher effect the criterion of stationary forecast errors is the minimum requirement for the validity of the Fisher effect (Rose, 1988; Lai, 1997). As described in chapter two some of the studies of the Fisher effect that have utilised the REH are Fama (1975), Rose (1988) and Mishkin (1992).

A variant of the rational expectations approach is to assume that economic agents understand the time series behaviour of inflation. This approach can be used to recreate inflationary expectations using past observations of inflation. One specific method of this is to assume that the time series of inflation is characterised by a specific univariate process (for example, using the Box-Jenkins methodology). The in-sample estimates of the fitted model can then be used as a proxy for expected inflation (for example see Darin and Hetzel (1995)). However, if the inflation process is characterised by structural change (for example, changes in policy regimes) then this proxy is not rational because in-sample forecasts using univariate models assume a fixed structure (Holden, *et al.* 1985). An issue arises of persistent one-sided forecast errors that are associated with regime changes. Another separate criticism of this method is that because the regression fits the model over the full sample period, this implies that forecasters use information that is not available when forecasts are made (Dotsey, Lantz and Scholl, 2003). A method used to curb this

problem is the use of regime switching models; this issue is briefly discussed below. Another method that is employed with the purpose of mimicking the behaviour of forecasters is the use of recursive or rolling autoregressive (AR) models. Forecasts are generated using either a fixed or rolling start date. Under the fixed start date method an out-of-sample forecast is estimated for each period using an AR model for all the data up to the forecast period. As this method rolls through each forecast period, parameter estimates of the AR model are updated with the addition of a new data point and therefore the sample size used increases each period (for example see Darin and Hetzel (1995) and Fujii and Chinn (2001)).[5] A similar technique assumes that forecasters do not use data from the far past, which may be irrelevant, especially in the light of structural change. The rolling start date method follows the fixed start date method except the sample size used to make out-of -sample forecasts remains the same each period. Therefore, a fixed sample size or window of data rolls through the sample period.

Another method employed by some studies is the multivariate technique. Dotsey, *et al.* (2003) used a similar technique to the univariate methodologies described above. Using a vector autoregressive (VAR) model unemployment and nominal interest rate time series for the United States were incorporated into the formation of inflation expectations with the purpose of calculating the *ex ante* real interest rate.

One common technique employed by some studies is the use of survey data of inflation expectations in order to represent the behaviour of all market participants. Darin and Hetzel (1995) study a variety of survey data for the United States and it is found that there is a broad underlying similarity between different surveys. Because similarity is found between different inflation expectations data it is concluded that this data adequately captures the public's inflationary expectations. Sun and Phillips (2003) use the survey of professional forecasters for comparison, which is currently conducted by the Reserve Bank of Philadelphia. It is argued that because the surveyed forecasters share one thing in common, that is, they forecast as part of their current job, these forecasts represent an

[5] This recursive method requires the sacrifice of a proportion of the original sample size in order to make the initial forecasts.

overview of expectations as a whole.[6] Gibson (1972) used the United States Livingston inflation survey data, which surveyed important market participants and suggested that this data could only approximate true expectations because only a few market participants were represented. However, Gibson explains that:

> *...since the sample was chosen to include well-informed individuals. The group selected also includes those whose opinions carry much greater weight in forming market expectations than those of an ordinary citizen, for the members of the sample typically control large amounts of assets and can thus have fairly large influence on the markets in which expectations have effects.* (Gibson, 1972, p. 854).

As explained by Lai (1997) one possible criticism of using this type of proxy is that individual expectations are heterogeneous and therefore it is unclear how individual expectations of inflation aggregate.

As explained above, models that use univariate time series techniques in order to generate inflationary expectations can create persistent forecast errors if the inflation process is characterised by structural change. Some studies have incorporated techniques that model structural changes in the inflation rate. For example Evans and Lewis (1995) employed a Markov switching model and generated expected inflation with one structural break in the mean of inflation. The Markov switching model is used to capture structural changes in a certain time series by estimating the probability that the process will change from one state to another state (see Brooks (2002) for more details). The estimated probabilities are then used to generate expected inflation. Other studies that follow a similar methodology as Evans and Lewis include Garcia and Perron (1996) and Malliaropulos (2000).

Another method that has been employed to generate inflation expectations is the time varying parameter (TVP) model. This method allows for the continuous update of the forecasting model, by letting the estimated coefficients of lagged variables evolve over time

[6] Also see, Paquet (1992) and Grant and Thomas (1999) for support of the rationality of survey data.

using the Kalman filter. Dotsey, *et al.* (2003) utilised the TVP technique and found that it had good forecasting properties for the inflation rate.

One possible interesting technique that could be used to generate a proxy for inflation expectations is the use of the autoregressive fractionally integrated moving average ARFIMA model in order to capture both the short-run (AR and moving average (MA) coefficients) and long-run (*d*) dynamic of the inflation data. Also the fractionally integrated generalised autoregressive conditional hetroskedastic (FIGARCH) models could also be added to capture long memory behaviour in the second moment of inflation. Other techniques, such as the Markov switching model could also be incorporated to further improve forecasts of the ARFIMA model; however, these techniques are left for further research.

3.3 Expectations Methodology

In this study three different methods are used in order to create a proxy for π_t^e. First, following the widely used method in the literature, realised inflation is utilised as a proxy (π_t) under the assumption of a stationary forecast error to calculate the *ex post* real interest rate. The other two methods employ econometric techniques to generate a proxy for π_t^e, and are used as a comparison to realised inflation, however each differ in the level of volatility present.

The second approach that is employed in this research is the Hodrick-Prescott (HP) filter methodology (see, Hodrick and Prescott (1997)) which generates expectations from inflation data and then the generated series of expectations is used to calculate the *ex ante* real interest rate (HP-*ex ante*). The HP filter approach is proposed by Ash, Easaw and Heravi (2002) as being a good *ex post* proxy for inflation expectations when contemporaneous *ex ante* forecasts (such as survey data) are not available. The HP filter generates a smoothed series (or trend component) x_t from the original series y_t, using the following minimisation problem:

$$\sum_{t=1}^{T}(y_t - x_t)^2 + \lambda\sum_{t=2}^{T-1}((x_{t+1} - x_t) - (x_t - x_{t-1}))^2 \qquad (3.6)$$

The HP filter generates x_t by minimising the variance of y_t around x_t subject to a penalty that constrains the second difference of x_t. Applying equation (3.6) to the inflation data, the HP filter extracts a slow moving persistent component from the volatile inflation data. From the above problem, λ, is a penalty that constrains the second difference of x_t; the larger λ is the smoother the trend component x_t becomes. The value of λ, is completely arbitrary, this study uses $\lambda = 1600$ for all series which is considered in the literature to be the standard value for quarterly data (Ravn and Uhilg, 2002). Because the HP filter generates a relatively smooth series, it is assumed that economic agents only utilise a long-run component into the formation of expectation. This is a strict assumption; however the use of this method provides a means of comparison with the other two techniques.

Ash, *et al.* (2002) find that the HP series for the United States inflation data fits the criterion of 'weak rationality'. That is, the inflation forecast errors are unbiased however they are not efficient because they contain serial correlation. This form of rationality proposed by Grant and Thomas (1999) is a variant of the original definition of rational expectations,[7] because efficiency requires all the relevant information at the time forecasts are made which are then utilised for the estimation. It has been argued that the presence of serial correlation in forecast errors is not completely inconsistent with rationality. Grant and Thomas (1999) explain that first, because forecasters may be unaware of recent forecast errors due to overlapping of forecast intervals, expecting agents to readjust expectations quickly may be too stringent. And second, survey response errors may contain serial correlation if agents are unaware of the stochastic nature of the inflation-generation process. In addition, agents may continually over or under predict inflation if it assumed that it is a stationary series, when in fact it is nonstationary.

[7] See, Muth (1961).

The third method used as a proxy for π_t^e in this study is the exponential smoothing (ES) technique. The ES technique uses a linear combination of previous values of a particular series for the purpose of modelling the time series and generating forecasts. The ES technique estimates an exponentially (geometrically) weighted scheme of the lagged values of the series. Because this technique generates exponentially decaying weights, more weight is placed on recent observations, yet observations a long way in the past (that may contain relevant information) can still influence current forecasts. Equation (3.7) represents the exponentially declining series of y_t as follows:

$$\hat{y}_t = \alpha y_t + \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} \cdots, \quad 0 \le \alpha \le 1 \tag{3.7}$$

Where $\hat{y}_t$ is the current smoothed value,[8] y_t is the current realised value and α is the smoothing constant $(0 \le \alpha < 1)$. Lagging (3.7) by one period and multiplying both sides by 1-α yields equation (3.8) in the following form:

$$(1-\alpha)\hat{y}_{t-1} = \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} \cdots \tag{3.8}$$

Subtracting (3.8) from (3.7) produces equation (3.9) as follows:

$$\hat{y}_t = \alpha y_t + (1-\alpha)\hat{y}_{t-1} \tag{3.9}$$

Since α + (1-α) = 1, $\hat{y}_t$ in equation (3.9) is the weighted average of the current value of y_t and the past smoothed (predicted) value, this therefore is in a sense a form of adaptive expectations. This implies that if inflation is unexpectedly positively large, economic agents will increase their expectations of inflation for the next period.. The smoothed series $\hat{y}_t$ is calculated from y_t were α represents the smoothing factor.[9] ES is a widely used forecasting technique in practice due to its simplicity; for example, there is no decision to

[8] In terms of expected inflation $\hat{y}_t$ is conditional to the information set at t $E(y_{t+1} \mid I_t)$.

[9] The smoothing factor α is estimated by minimising the sum of squared errors. Smaller values of α mean that less weight is placed on current values of y_t therefore, $\hat{y}_t$ becomes smoother.

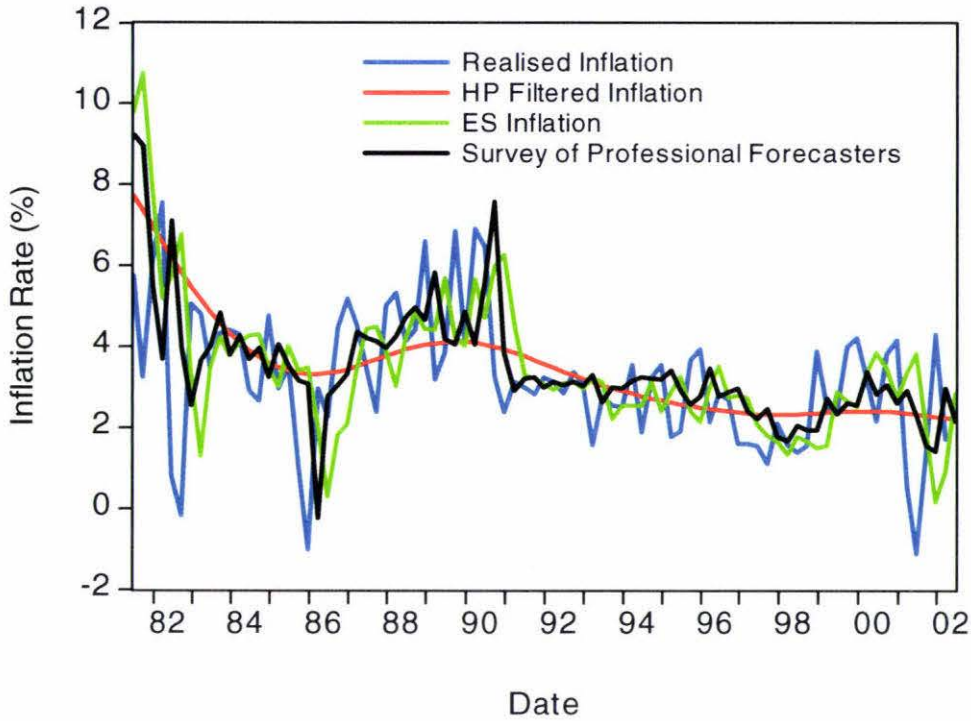
be made on how many parameters to estimate such as the autoregressive integrated moving average (ARIMA) class of models (Brooks, 2002).

As explained by Granger and Newbold (1986) exponential smoothing is also seen as a special case of a Box-Jenkins ARIMA(0,1,1) model with a MA coefficient equal to (1-α) (also see Brooks, 2002). Therefore, with the use of the ES technique it is assumed that economic agents estimate inflation considering a MA component in the difference of inflation. The exponentially smoothed series is then used to calculate the *ex ante* real interest rate (ES-*ex ante*).

Although the two econometric techniques employed in this research are not rational in the pure sense, and they do possess strict assumptions. The main purpose of these techniques is to create an insight into the univariate estimates of d when π_t^e exhibits different second moment characteristics. Thus, further research would involve the modelling of inflation utilising more complicated techniques some of which are discussed above in section 3.2, this is left as an avenue for further study.

To convey the idea about the differing short-run volatility among the different expected inflation series Figure 3.1 illustrates the quarterly inflationary expectations series for the United States over the period of 1981:3-2002:2. The length of the time series presented in Figure 3.1 is not for the entire sample period used in this study. Combined with expectations series generated in this research is the data pertaining to the Survey of Professional Forecasters, which limits the length of the series. The Reserve Bank of Philadelphia currently conducts the Survey of Professional Forecasters. The survey respondents are forecasters who forecast as part of their current job, thus it is believed that their forecasts represent expectations of the entire economy (Sun and Phillips, 2003).

Figure 3.1: Quarterly Inflation Expectations Time Series for the United States Over the Period of 1981:3-2002:3



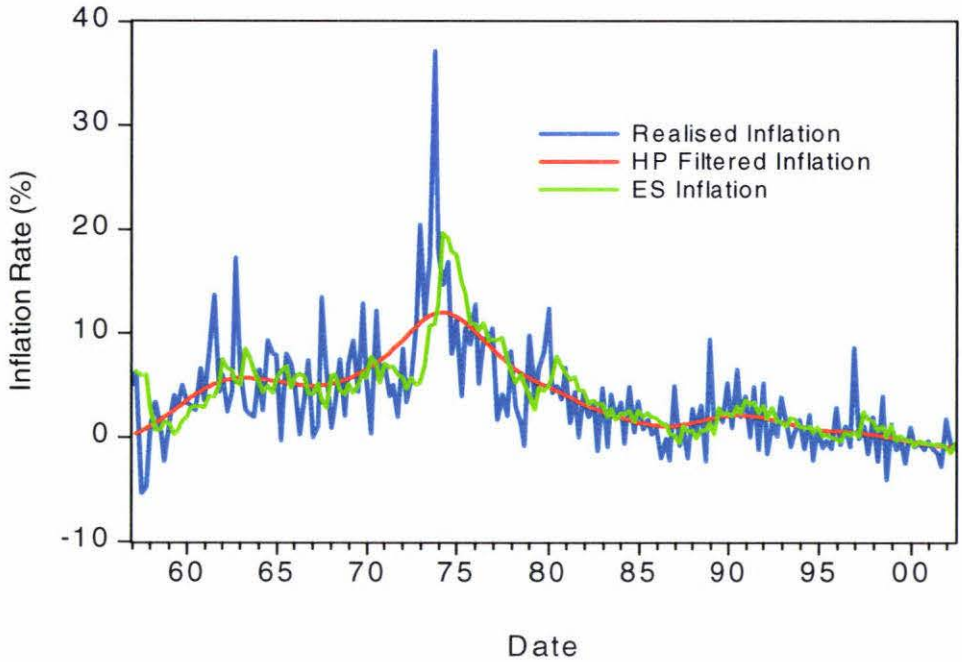
Sources: Reserve Bank of Philadelphia, see http://www.phil.frb.org/econ/spf/ for more details. International Monetary Fund (IMF) (2002).

In Figure 3.1 it can be seen that the four different series follow the same general pattern, that is, from the early 1980s inflation and inflation expectations have been decreasing. Realised inflation stands out as the most volatile series, whereas in contrast the HP filtered inflation series is extremely smooth. In comparison both the ES inflation series and the Survey of Professional Forecasters series there is a distinct similarity between them, which supports the use of the ES technique. Overall, Figure 3.1 illustrates the main difference in volatility between each expected inflation series. In order to illustrate the effect each expected inflation series has on the corresponding real interest rate Figures 3.2, 3.3, 3.4 and 3.5 that show quarterly data of three expected inflation series (panel (i)), and the calculated real interest rate series (panel (ii)) for the whole sample periods for Japan, Korea, the United Kingdom and the United States, respectively.[10]

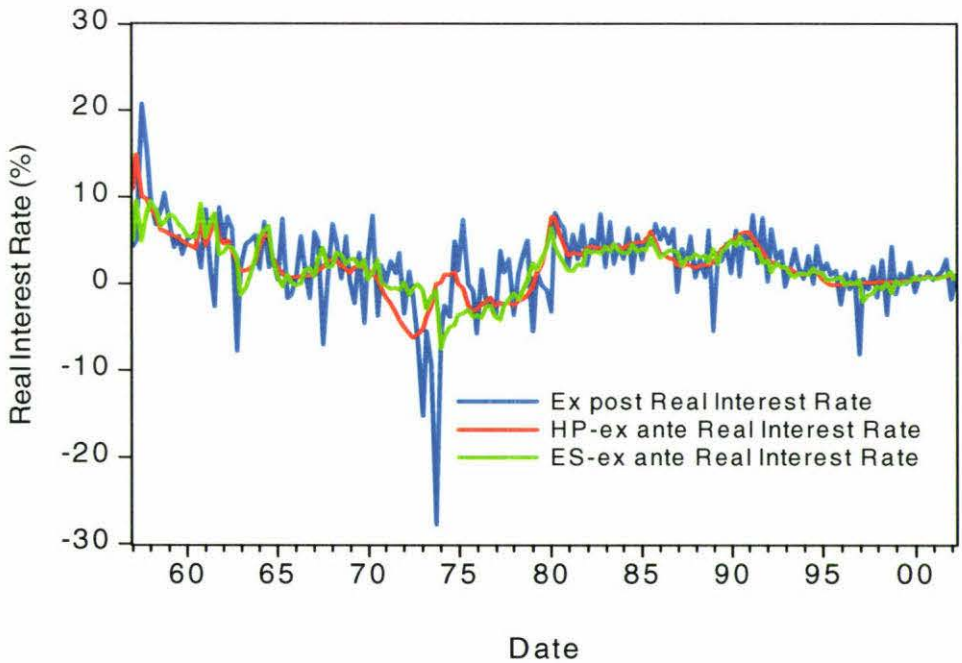
[10] For more detail on the data employed in this study refer to chapter four.

Figure 3.2: Inflation Expectations and Equivalent Real Interest Rate Time Series for Japan Over the Period of 1957:1-2002:3

Panel (i): Inflationary Expectation Series for Japan



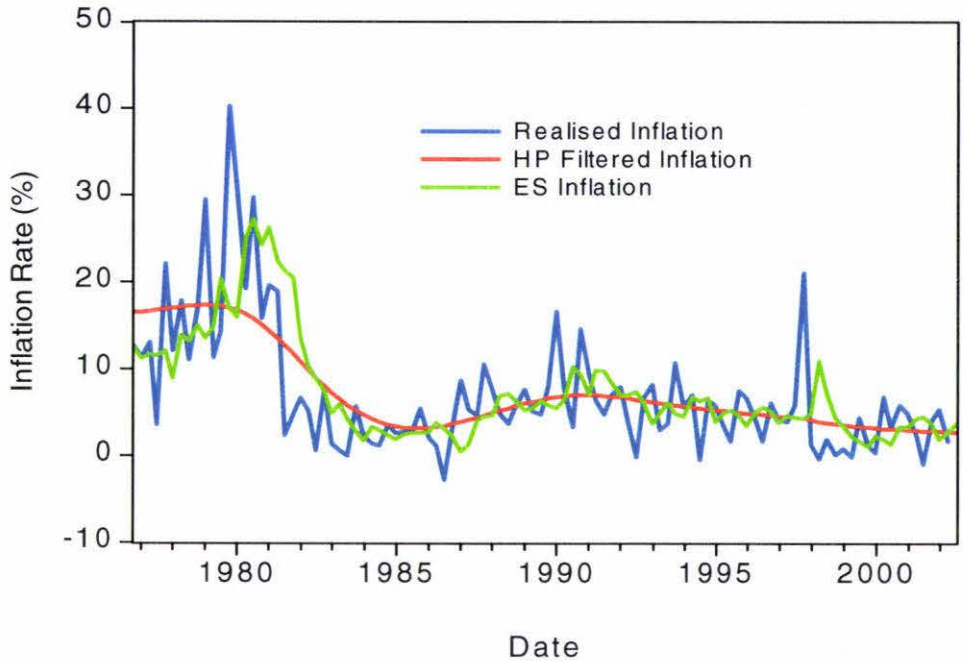
Panel (ii): Calculated *ex post* and *ex ante* Real Interest Rate Series for Japan



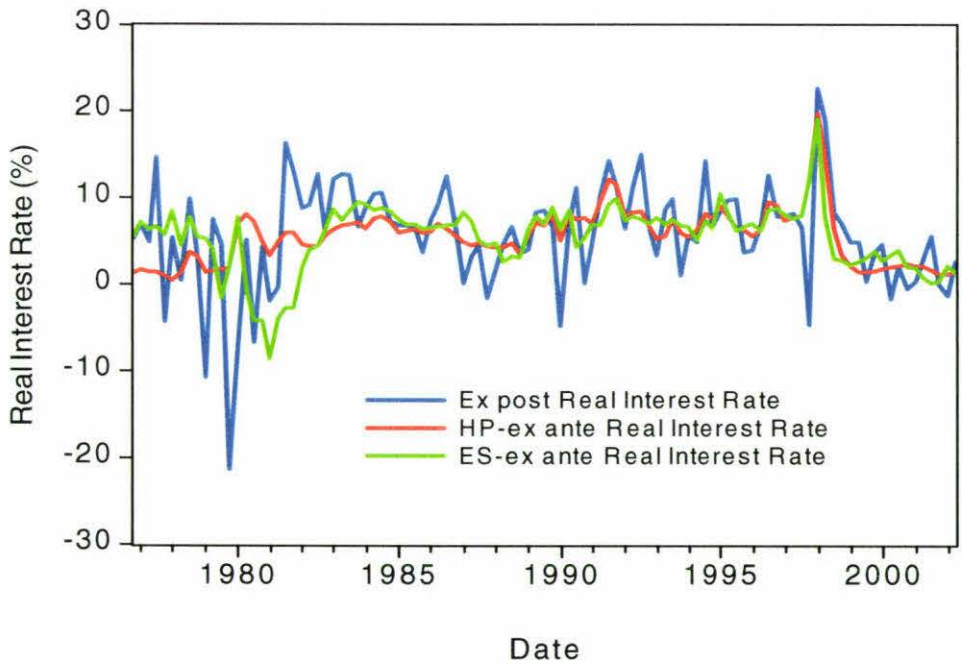
Source: IMF (2002).

Figure 3.3: Inflation Expectations and Equivalent Real Interest Rate Time Series for Korea Over the Period of 1976:4-2002:3

Panel (i): Inflationary Expectation Series for Korea



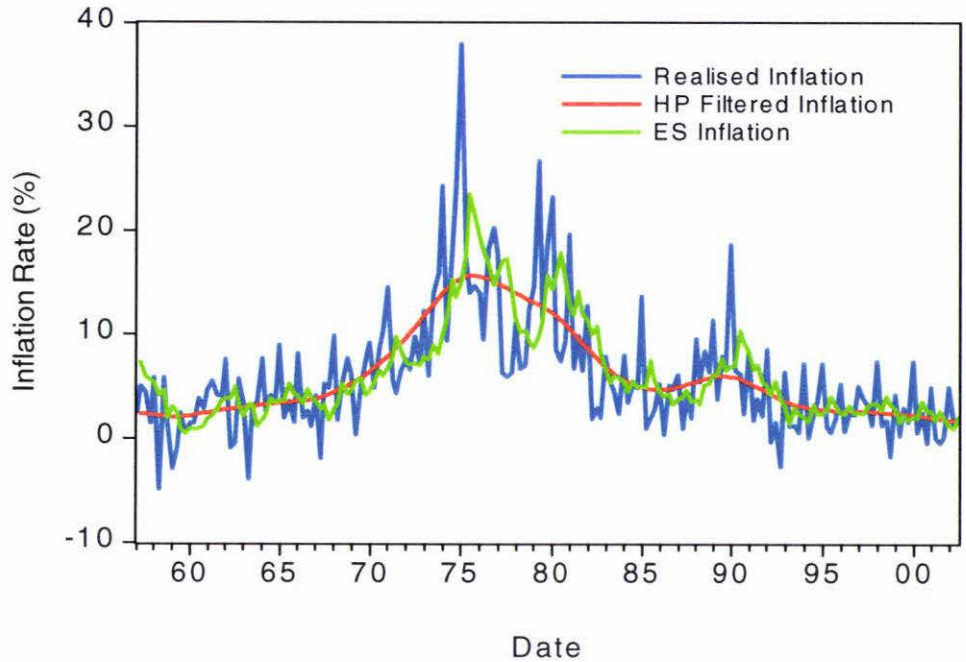
Panel (ii): Calculated *ex post* and *ex ante* Real Interest Rate Series for Korea



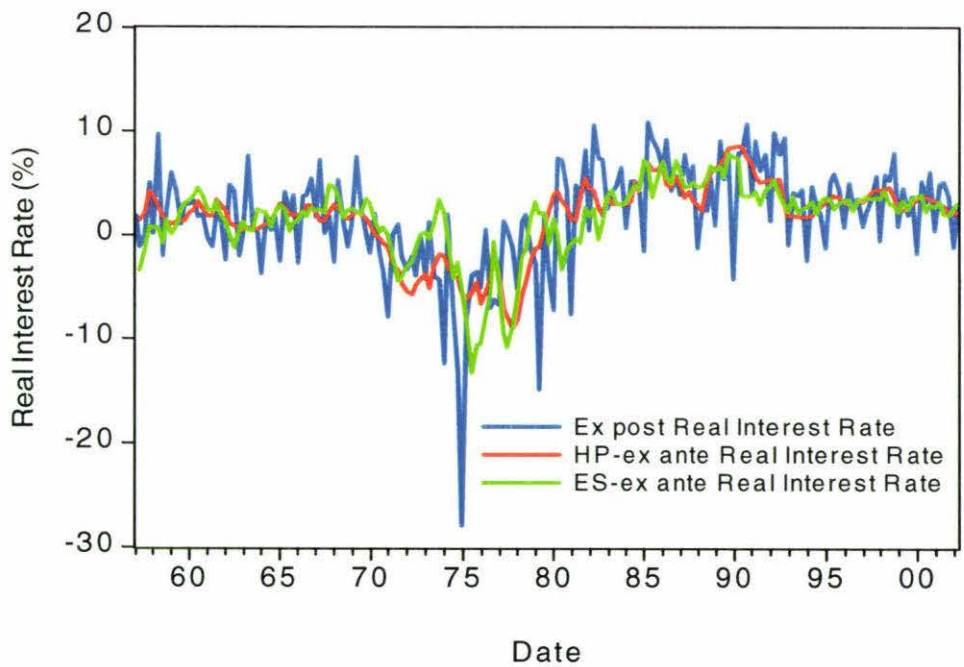
Source: IMF (2002).

Figure 3.4: Inflation Expectations and Equivalent Real Interest Rate Time Series for the United Kingdom Over the Period of 1957:1-2002:3

Panel (i): Inflationary Expectation Series for the United Kingdom



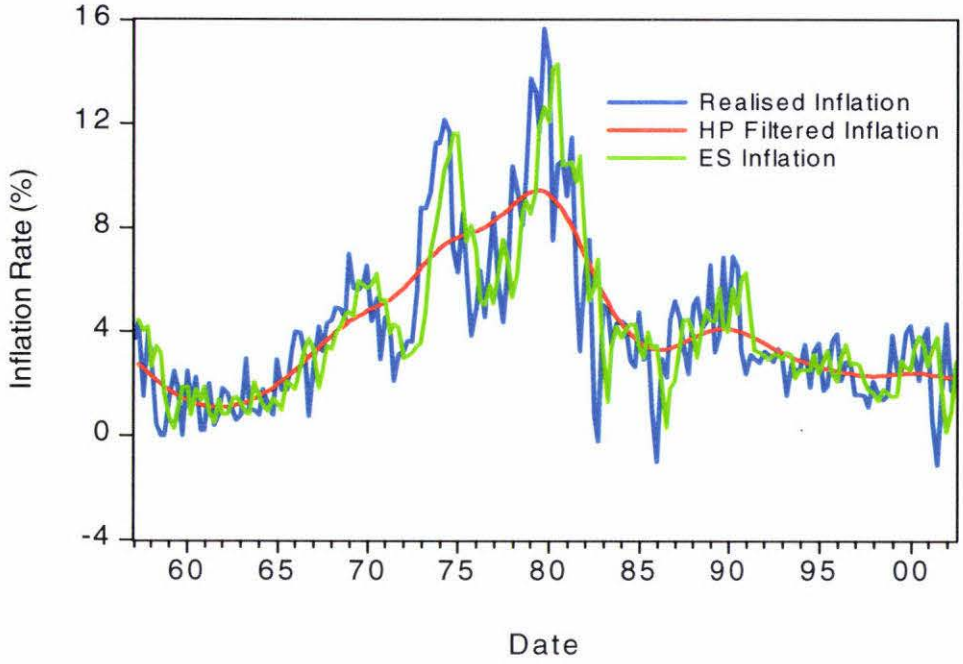
Panel (ii): Calculated *ex post* and *ex ante* Real Interest Rate Series for the United Kingdom



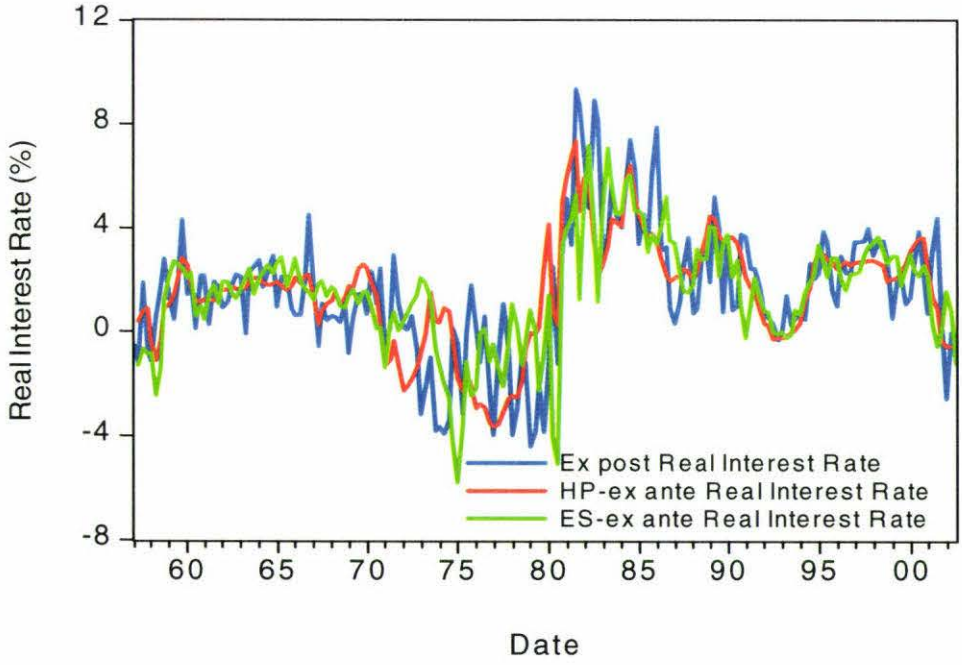
Source: IMF (2002).

Figure 3.5: Inflation Expectations and Equivalent Real Interest Rate Time Series for the United States Over the Period of 1957:1-2002:3

Panel (i): Inflationary Expectation Series for the United States



Panel (ii): Calculated *ex post* and *ex ante* Real Interest Rate Series for the United States



Source: IMF (2002).

In panel (i) of Figures 3.2, 3.3, 3.4 and 3.5 the expected inflation series for Japan, Korea, the United Kingdom and the United States are displayed. Although the sample size differs for Korea, the same general pattern emerges between each expected inflation series within each country. Over the time period for each country there are spikes in realised inflation corresponding to both positive and negative unexpected shocks. The volatile unexpected shocks generate an inflation series, which is more volatile than the other inflation expectations series. The additional volatility present in realised inflation obscures the more persistent actual expected inflation series and makes expected inflation appear more stationary. In contrast, the HP filtered inflation series is at the other end of the spectrum, that is, for each country the series contains little short-run volatility and appears extremely persistent. The ES inflation series for Japan, Korea, the United Kingdom and the United States contains a certain degree of volatility, however not to the same degree as realised inflation. In panel (ii) of Figures 3.2, 3.3, 3.4 and 3.5 there is also a general pattern in the corresponding real interest rate of Japan, Korea, the United Kingdom and the United States. In much the same way as for inflation, the *ex post* real interest rate for each country is more volatile then both *ex ante* real interest rates due to the inherited unexpected short-run volatile component. This in turn generates the *ex post* real interest rate that appears more stationary; whereas, both the HP filtered inflation and ES inflation series appear more persistent.

It is thought that because the short-run volatility differs between each real interest rate within each country studied the estimated order of integration will differ among the real interest rates. It is believed that because the *ex post* real interest rate is more volatile the estimated order of integration will be biased downward due to the extra volatility present. Whereas, the order of integration for the ES-*ex ante* real interest rate will also be biased downwards, however not to the same extent as the *ex post* real rate. Because the HP-*ex ante* real rate appears to be more persistent than the other real rates it is expected to have the highest estimated order of integration.

3.4 Conclusion

Given that inflationary expectations literature discuss the importance of the unobservable variable inflationary expectations. Chapter three deals with the issue that involves the unobservable variable inflationary expectations. The proxies of inflationary expectations employed in this study are explained. Also, how inflation expectations are incorporated into the Fisher equation is discussed, as well as methodologies that have been employed by other studies in order to solve the problems that arise in this issue.

For the long-run Fisher effects to be valid changes in expected inflation are assumed to have no permanent effect on the *ex ante* real interest rate. This assumption implies that changes in expected inflation are fully reflected in subsequent movements of the nominal interest rate. This is equivalent to the *ex ante* real interest rate being a stationary I(0) process. For the long-run Fisher equation to hold the *ex ante* real interest rate has to, at least, be mean-reverting for a stable long-run relationship between inflation expectations and the nominal interest rate to exist. However, as issue arises if the volatile realised inflation rate is used as a proxy for expected inflation. Univariate estimates of the order of integration for the real interest rate are biased downwards because the unexpected forecast error component masks the actual persistent *ex ante* real interest rate, thus many techniques have been employed to generate proxies of expected inflation.

In this study, three proxies of inflationary expectations are used with the purpose of examining the order of integration of the *ex ante* real interest rate. These proxies include realised inflation, expected inflation generated using the Hodrick-Prescott filter and expected inflation generated using the exponential smoothing technique. The important difference between each proxy of expected inflation is the degree of volatility inherent within each series. As pointed out here this volatility is transferred to the corresponding calculated real interest rate. Because of this, the *ex post* real interest rate appears less persistent than both *ex ante* real interest rates. The next step involves examining the order of integration for each real interest rate for each country. Chapter four, therefore, reviews the data and methodology utilised in this research.

Chapter Four Data, Unit Root Tests, Models and Methodology

4.1 Introduction

In chapter three, issues relevant to inflationary expectations have been considered which are crucial to the calculation of the real interest rate using the Fisher equation. As explained in chapter three this study examines the mean-reverting characteristics of the *ex ante* real interest rate by estimating the order of integration (d) directly. Thus chapter four continues the analysis of the mean-reverting behaviour of the real interest rate by explaining the data, models and methodologies employed in this work to estimate and examine the d parameter of the real interest rate.

As suggested in chapter two conventional unit root tests fall into two categories, the first type of test, tests the null hypothesis of a unit root, whereas the second type tests the null of stationarity. If the null hypothesis is rejected for both of these tests then this suggest the possibility of the process is fractionally integrated, thus unit root tests and the methodology employed in this study are discussed in this chapter. In chapter two it has been noted that there are several approaches that have been developed in order to estimate the d parameter. This study estimates the d parameter using a parametric approach, specifically, an autoregressive fractionally integrated moving average (ARFIMA) model is utilised to estimate d for each real interest rate using a conditional time-domain maximum likelihood (ML) estimation procedure. As this estimation method is a parametric approach, accordingly the sort-run autoregressive (AR) and moving average (MA) components need to be specified and from this an appropriate model must be selected.

The structure of this chapter is as follows: Section 4.2 describes the data that is obtained for this study. Section 4.3 illustrates the unit root tests that are employed to test the stationarity of each real interest rate. Section 4.4 briefly describes the ARFIMA model, the GARCH model and the estimation procedure. Section 4.5 then explains the method used to

determine the short-run AR and MA components of the ARFIMA model, flowed by the criteria utilised to select a parsimonious model. The final section summarises and concludes this chapter.

4.2 Data

In analysing the real interest rates for a variety of countries the data employed in this research needs to be, to some degree, consistent between countries. Therefore, all the data employed in this study is obtained from the International Monetary Fund's (IMF) *International Financial Statistics* (IFS) database (IMF, 2002). For each country quarterly three-month interest rate data and quarterly Consumer Price Index (CPI) data is employed. In general, examining the *ex ante* real interest rate requires that the nominal interest rates have low or no default risk. Therefore, the interest rate series examined is chosen from either the Treasury bill rate (TBR) or the Money Market rate (MMR). IMF (2002) described the CPI as follows "...[The CPI] reflects changes in the cost of acquiring a fixed basket of goods and services by the average consumer." (IMF, 2002, p. xxii). The TBR is "the rate at which short-term securities are issued or traded in the market." (IMF, 2002, p. xxiii) and the MMR is "the rate on short-term lending between financial institutions." (IMF, 2002, p. xxii). The IMF calculates the quarterly interest rates used in this research, as arithmetic averages of monthly interest rates. The specific rate that is used for each country depends essentially on the availability of quality time series data. However, because TBR is widely used in the literature the TBR data is preferred to the MMR. Unfortunately, TBR data is not available for all the countries so MMR data has been utilised in those cases, as it is thought to be a relatively risk free measure of the interest rate. The chosen sample size is the largest available period reported in the IFS database and sample sizes differ among countries due to data availability. Appendix Table A4.1 illustrates the sample size, interest rate employed and the series code of the data for each country. The data are for the following nineteen countries: Australia, Belgium, Canada, France, Germany, Italy, Japan, Korea, Malaysia, the Netherlands, New Zealand, Pakistan, the Philippines, Singapore, South Africa, Spain, Sweden, the United Kingdom and the United States.

The inflation rate for each country is the calculated percentage change of the CPI; which is multiplied by 400 to obtain annualised inflation rates. The use of quarterly interest rate data avoids the issue of overlapping data, which can create spurious serial correlation and generate additional dynamic. This issue arises as a result of data having a holding period larger then the observation period, for example monthly series of three-month TBR data. In terms of the Fisher equation, the inflation forecast error would be serially correlated when using realised inflation as a proxy for expected inflation. The serially correlated forecast error is inherited into the *ex post* real interest rate possibly creating spurious serial correlation.

As explained in chapter three the proxies of inflationary expectations are generated using the inflation data obtained from the IMF IFS database. From this each real interest rate is calculated as the difference between the current period's nominal interest rate and expected inflation. The calculated real interest rate is then employed to examine the order of integration between each real interest rate within each country. The first step of this analysis is to conduct unit root tests with the purpose of examining the stochastic nature of each real interest rate, the procedures utilised in this study for unit root tests are outlined in section 4.3 below.

4.3 Unit Root Test Procedures

As explained in chapter two there are two different types of unit root tests that are used to determine the stochastic nature of a time series. Most of the standard tests for stationarity test the null hypothesis that the particular series contains a unit root against the alternative being a stationary test (for example, the Augmented Dickey Fuller (ADF) test). Alternatively, certain unit root tests consider stationarity as the null hypothesis and a unit root as the alternative (for example, the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test). However, both types of unit root test have power to suggest a I(d) process, therefore if both types of tests are utilised and both reject the null hypothesis, the researcher faces a dilemma as the test results may indicate that the time series process is not well

characterised by either an I(0) or I(1) process. One possible explanation of this result is that the series is actually characterised by an order of integration which lies between the two null hypotheses, in other words the series may be follow a fractional I(*d*) process. Following other empirical studies,[1] this research employs two unit root tests in order to test the stationarity of the real interest rate data of each of the nineteen countries. First, the Phillips and Perron (PP) test is used which follows the usual unit root tests with a null hypothesis of a unit root. Secondly, each real interest rate series is tested for a unit root using the KPSS test, which has an opposite null hypothesis.

The Phillips and Perron (1998) test was proposed as an alternative to the ADF unit root test. The PP test uses a nonparametric method to control for serial correlation in the residuals of the unit root test equation.

$$y_t = \rho y_{t-1} + x_t' \beta + \varepsilon_t \tag{4.1}$$

Equation (4.1) presents a simple AR(1) process (y_t), where x_t represents optional exogenous variables which consist of a either a constant, trend or both, and ε_t is the models residual component. In this study a constant is included in the test equation. Testing for a unit root in equation (4.1) is equivalent to testing if $|\rho| = 1$. Subtracting y_{t-1} from both sides of equation (4.1) with a constant term gives equation (4.2):

$$\Delta y_t = \mu + \alpha y_{t-1} + \varepsilon_t \tag{4.2}$$

Where μ is the constant term, $\alpha = (\rho - 1)$, therefore testing for a unit root is equivalent to testing if $\alpha = 0$. However, if the models residual component ε_t contains additional short-run dynamics then the asymptotic properties of the test statistic will be altered. The PP test modifies equation (4.2), however instead of including extra lagged terms to control for autocorrelation in the residual (which is done in the ADF test), the PP test adjusts the t-statistic of α to take into account the potential autocorrelation. This adjustment modifies

[1] See for example, Baillie, Chung and Tieslau (1996), Lai (1997) and Tsay (2000).

the t-statistic of α so that serial correlation does not affect the asymptotic distribution of the test statistic. The null and alternate hypothesis of the PP test are presented below:

$$\begin{aligned} H_0 &: \alpha = 0 \\ H_1 &: \alpha < 0 \end{aligned} \tag{4.3}$$

The PP test is based on the following Z_t statistic presented in equation (4.4):

$$Z_t = \frac{s_e}{s} t_{\hat{\alpha}} - \frac{1}{2} \frac{\left(s^2 - s_e^2\right)}{s\left[T^{-2} \sum_1^T (y_{t-1} - \bar{y}_{-1})^2\right]^{1/2}} \tag{4.4}$$

Where $t_{\hat{\alpha}}$ is the conventional t-ratio for the significance of α in equation (4.2), s_e^2 is the short-run variance of the residual and s^2 is a consistent estimator of the long-run variance or nuisance parameter, these are shown below in equations (4.5) and (4.6):

$$s_e^2 = T^{-1} \sum_{t=1}^{T} \hat{\varepsilon}_t^2 \tag{4.5}$$

$$s^2 = T^{-1} \sum_{t=1}^{T} \hat{\varepsilon}_t^2 + 2T^{-1} \sum_{\tau=1}^{l} \omega_{\tau,l} \sum_{t=\tau+1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_{t-\tau} \tag{4.6}$$

Where $\omega_{\tau,l}$ represents the Bartlett weights which is a modification to the variance estimator (s^2) to ensures nonnegative values, this is shown below in equation (4.7):

$$\omega_{\tau,l} = 1 - \frac{\tau}{l+1} \tag{4.7}$$

Where l is known as the bandwidth or truncation lag and must be chosen by the researcher. There is no simple rule for the selection of l, generally l should increase as the sample size T increases (but not as fast as T) see for example Mills (1999). It is widely known that low

lag lengths can create size distortion of the test statistic, yet large lag lengths decrease the power of the test. The choice of l in this research is discussed below.

As explained above the Kwiatkowski, Phillips, Schmidt and Shin (1992) test is a unit root test with stationarity as the null hypothesis. Similar to the PP test the KPSS test uses a nonparametric method to control for serial correlation in the test equation. The KPSS test assumes that a series is composed of the sums of a deterministic trend, random walk and a stationary error term (not necessarily white noise), this is shown bellow in equation (4.8).

$$y_t = \delta t + \zeta_t + \varepsilon_t \tag{4.8}$$

Where ε_t is the stationary residual and ζ_t is a random walk process given by equation (4.9):

$$\zeta_t = \zeta_{t-1} + v_t \tag{4.9}$$

Where v_t is short memory with a zero mean and constant variance (say, σ_v^2). The null hypothesis for the KPSS test is the following:

$$H_0 : \sigma_v^2 = 0, \text{in other words } \zeta_t \text{ is constant} \tag{4.10}$$

In this study, similar to the PP test, only a constant term is included in equation (4.8) (that is, $\delta = 0$). The KPSS test statistic is given by equation (4.11), which is shown below:

$$\eta = \frac{\sum_{t=1}^{T} s_t^2}{\hat{\sigma}_\varepsilon^2} \tag{4.11}$$

Where $\hat{\sigma}_\varepsilon^2$ is the estimate of the residual variance and s_t is the partial sum of errors defined bellow in equation (4.12):

$$s_t = \sum_{j=1}^{t} \hat{\varepsilon}_j, \quad t = 1,2,\ldots,T \tag{4.12}$$

Similar to the PP test the asymptotic distribution of the test statistic is only valid if the residuals are *iid*. If the errors are *iid* then the denominator of equation (4.11) will converge to the true residual variance σ^2, however if the residuals are not *iid* then a modification must be made to control for the nuisance parameter. An appropriate estimator of the denominator in equation (4.11) must be employed, which in this case is equation (4.6). This implies that for both the PP and KPSS test a truncation lag length (l) must be set. The methodology employed in this work is noted below. For more details on either the PP or KPSS test refer to chapters 3 and 4 of Maddala and Kim (1999).

As Baillie (1996) explains, it is well known that conventional unit root tests under the null hypothesis are consistent against I(d) alternatives. Hassler and Wolters (1994, cited in Baillie 1996, p. 29) find evidence to suggest that the PP unit root test has power to distinguish an I(d) process from a unit root:

> *...with a nonstationary value of d = 0.75 generating the fractional white noise, the rejection frequencies of the unit root hypothesis are about 50% when T = 100 and about 70% with T = 250.* (Baillie, 1996, p. 29).

Another study Lee and Schmidt (1996, cited in Baillie 1996, p. 29) examined the performance of the KPSS unit root test to distinguish short memory from long memory stationary processes and it is found that the KPSS test is consistent against an I(d) alternative. Therefore, this suggests that if both these tests are applied to a particular series, and both unit root tests reject the null hypothesis this suggests the possibility that the series is characterised by a I(d) process.

The tests for stationarity in this study are conducted using the econometrics software package Eviews 4.1. For the selection of l the following strategy utilised was applied to each real interest rate for the nineteen countries studied. As explained above there is no set

rule for choosing an optimal lag length, therefore, in this study two specifications are used. Firstly, for both the PP and KPSS tests a fixed lag of $l = 4$ is tested for the three real interest rate series. Secondly, as the Eviews 4.1 econometrics software package is used in this analysis, l is selected utilising the Newey and West (1994) data-driven automatic bandwidth selection methodology. The next section presents the models employed here.

4.4 The Long Memory ARFIMA and ARFIMA-GARCH Model

As explained in chapter two the ARFIMA model can characterise a wide range of long-run behaviour. In this research the order of integration is estimated using an ARFIMA model. The ARFIMA (p, d, q) model is presented below in equation (4.13):

$$\phi(L)(1-L)^d(y_t-\mu)=\theta(L)\varepsilon_t \tag{4.13}$$

Where $\phi(L)=1-\phi_1 L-\ldots-\phi_p L^p$ and $\theta(L)=1+\theta_1 L+\ldots+\theta_q L^q$ which represent the short run dynamic AR and MA components respectively. Both $\phi(L)$ and $\theta(L)$ are assumed to have roots that lie outside the unit circle, and ε_t is a white noise process. The order of integration is represented by d, theoretically d can be any real number because of its mean-reverting behaviour the situation where $d \in [0,1]$ is of particular interest when the real interest rate is examined. Model (4.13) encompasses both the stationary autoregressive moving average (ARMA) class of models (d = 0) and the unit root process (d = 1). For more details on the properties of the I(d) process refer to chapter two of this study.

The ARFIMA model is employed to model the conditional mean of the real rate series, various models can be used to estimate the conditional variance. In this research the generalised autoregressive conditional hetroskedastic (GARCH) model is utilised to characterise any time dependent hetroskedasticity present within the second moment of each real rate series. The use of a GARCH model is used to model the characteristic of the variance of certain time series known as 'volatility clustering', which describes the presence of large changes in volatility being followed by further large changes. The

GARCH model was introduced by Bollerslev (1986) and allowed the conditional variance (σ_t^2) to be modelled on its own lags and the lags of the squared error (ε_t^2), the GARCH(p, q) model is shown below in equation (4.14):

$$\beta(L)\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 \tag{4.14}$$

Where ω is the mean, $\beta(L) = (1 - \beta_1 L - \ldots - \beta_p L^p)$, $\alpha(L) = (1 + \alpha_1 L + \ldots + \alpha_q L^q)$ and the roots of $\beta(L)$ and $\alpha(L)$ lie outside the unit circle. Because model (4.14) models the conditional variance, its value must always be strictly positive, thus this implies that $\beta(L)$ and $\alpha(L)$ must be non-negative. In general the parsimonious GARCH(1, 1) model is employed, which is shown below in equation (4.15):

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{4.15}$$

Where σ_t^2 is the conditional variance, which is conditional on past information. The lag of the squared residual ε_{t-1}^2 is know as the autoregressive conditional hetroskedastic (ARCH(1)) term and represents news about the volatility from the previous period. The lag of the conditional variance σ_{t-1}^2 is known as the GARCH(1) term and represents last period forecast variance. The GARCH(1, 1) model is parsimonious because only three parameters need to be estimated (ω, α, β) and the model can be presented in the following manner. If equation (4.15) is recursively lagged, and the lagged equations are substituted back into equation (4.15), the following equation is generated (equation (4.16)):

$$\sigma_t^2 = \frac{\omega}{(1-\beta)} + \alpha\sum_{j=1}^{\infty}\beta^{j-1}\varepsilon_{t-j}^2 \tag{4.16}$$

Equation (4.16) shows that the current variance is influenced by an infinite number of past squared errors with geometrically declining weights (see Brooks (2002) for details).

It can also be shown that the GARCH(1, 1) model is equivalent to an ARMA(1, 1) process for the squared residual. To illustrate this v_t in equation (4.17) equals the error of squared residuals:

$$v_t = \varepsilon_t^2 - \sigma_t^2 \tag{4.17}$$

Rearranging (4.17) for σ_t^2 and substituting into equation (4.15) yields equation (4.18):

$$\varepsilon_t^2 = \omega + (\alpha + \beta)\varepsilon_{t-1}^2 + v_t - \beta v_{t-1} \tag{4.18}$$

Where the sum of α and β govern the persistence of volatility, and therefore they are expected to sum to less than unity.

In order to estimate the models presented in section 4.4, this study employs the Ox 3.10 econometrics software package. The ARFIMA and ARFIMA-GARCH models are estimated using Long Memory Modelling (LMM) 2.1 by Davidson (2002). Several estimation procedures are available in LMM 2.1, in this research the conditional time domain ML estimation procedure is utilised, as it is the only estimation procedure that allows for the inclusion of a GARCH innovation. The main disadvantage of the time domain ML method is that the standard error of estimated parameters are slightly larger than other methods available.

Using the real interest rate data the *d* parameter is estimated using ARFIMA and ARFIMA-GARCH models, which are estimated employing the estimation procedure explained above. However, the short-run dynamic AR and MA parameters must be specified when the models are estimated, and an appropriate model must be selected that adequately represents the particular real rate series. Section 4.5 therefore outlines the procedure adopted in this study to specify the short–run dynamic components and the criteria used to select an appropriate model.

4.5 Model Selection Methodology

Given that estimation procedure described above is a parametric approach the short-run dynamic components (that is, the AR and MA components) must be specified in order to estimate the ARFIMA model. Because an ARFIMA model must be estimated for three real interest rates for nineteen different countries, an appropriate estimation strategy must be created. The strategy employed here is outlined below. The key to this strategy is to find a parsimonious model that adequately describes the data generation procedure of a particular real interest rate. This requires that the lag structure not be complicated, for example if lag structures are of high orders then the estimate of the long-run parameter (d) may be subject to contamination. The model selection strategy is described below, which details how p and q were chosen in the ARFIMA models for each real rate.

The modelling specification strategy creates the possible combination of eleven different ARFIMA models for each real interest rate. Initially, an ARFIMA(0, d, 0) model is estimated, if seasonality was present within the real rate data it was controlled using an AR (ϕ_4) and MA (θ_4) seasonal component.[2] After controlling for the seasonal component, lower order AR and MA components are estimated to improve the representation of the real interest rate process. When an AR component is used to control for seasonality, low order MA terms are controlled for, where q = 0, 1, 2 equation (4.19) described the possible estimated models:

$$\left(1-\phi_4 L^4\right)\left(1-L\right)^d\left(y_t-\mu\right)=\left(1+\theta_1 L+\theta_2 L^2\right)\varepsilon_t \qquad (4.19)$$

When a seasonal MA component is used, low order AR components are controlled for, where $p = 0, 1, 2$ equation (4.20) describes the possible combination of estimated models:

$$\left(1-\phi_1 L-\phi_2 L^2\right)\left(1-L\right)^d\left(y_t-\mu\right)=\left(1+\theta_4 L^4\right)\varepsilon_t \qquad (4.20)$$

[2] Seasonality was identified by examining the ACF and partial ACF of each real interest rate series. Because of the additional short-run volatility present in the *ex post* real interest rate seasonality was found to be an issue manly for the *ex post* real rate.

If seasonality is not an issue then the ARFIMA model shown in equations (4.19) and (4.20) are estimated without the seasonal AR and MA components (that is, $\phi_4 = \theta_4 = 0$). For low order AR components there are three different combinations of the model, where $p = 0, 1, 2$. The selection of MA components is analogous to the AR component selection, where $q = 0, 1, 2$. Overall, Given the possibility of eleven different models the model selection criterion is outlined below.

In order to select an adequate representation of the real interest rate series the model selection criterion is outlined here. For each of the estimated ARFIMA model the estimated standard errors of the estimated ARMA coefficients are examined to determine the significance of the estimates. Models with significant ARMA coefficients are selected over models with insignificant coefficients. A portmanteau test for the residuals is employed to determine whether each estimated model has captured the short-run dynamic characteristic of the real rate series. Specifically, the Box-Pierce test (developed by Box and Pierce (1970)) is employed to jointly test that a set of autocorrelation coefficients are equal to zero.[3] The Box-Pierce Q-statistic is shown below in equation (4.21):

$$Q = T\sum_{k=1}^{m} \hat{\tau}_k^2 \sim \chi_m^2 \tag{4.21}$$

Where T is the sample size, $\hat{\tau}_k$ is the kth autocorrelation coefficient and m is the maximum lag being considered, in this study $m = 10$ and therefore the Q-statistic is referred to as the $Q(10)$ statistic. The $Q(10)$ statistic is asymptotically distributed as a χ_{10}^2 under the null that all the 10 autocorrelation coefficients are zero, therefore, the critical value at the five percent level of significance is $\chi_{10}^2 = 18.31$. Models with the estimated Q(10) statistics less than the critical value are considered for selection, as they are thought to characterise the short-run dynamic of the real rate series. For models that are indistinguishable using the

[3] The Box-Pierce test is employed as it is the only portmanteau test available in Long Memory Modelling 2.1.

above criterion, the Akaike Information Criterion (AIC) is employed to determine the selected model the is outlined below in equation (4.22):

$$AIC = \log\left(\frac{\sum \hat{\varepsilon}_t^2}{T}\right) + \frac{2k}{T} \tag{4.22}$$

Where $\Sigma\hat{\varepsilon}_t^2$ is the sum of squared residuals, k is the number of independent variables and T is the sample size. A model is chosen that essentially minimises the sum of squared residuals subject to a penalty (the second part on the right hand side of equation (4.22)) due to the loss of degrees of freedom by including additional parameters, therefore the model minimises the AIC.

For each of the selected model the Box-Pierce Q-statistic is examined for squared residuals (the is, the $Q(10)^2$ statistic) in order to identify non-linear time series, see McLeod and Li (1983). This is done in order to test for serial correlation in the second moment of the estimated models residuals in the same way as the portmanteau test is conducted for the first moment of the residuals. The critical value is examined ($\chi_{10}^2 = 18.31$) and if the calculated test statistic is larger than the critical test value then this indicates the presence of serial correlation in the conditional variance of the estimated models residuals. As explained in section 4.5 a GARCH(1, 1) innovation is applied, and the selected model is re-estimated to control for any affect that serial correlation might have on the estimated order of integration. Serial correlation in the second moment is not the primary concern of this study, therefore only a GARCH(1, 1) innovation is considered, no other second moment innovations are examined and are left for further research.[4] Using the data, unit root tests, the ML estimation technique and model selection criteria discussed above the empirical results of this study are presented in chapter five.

[4] One possible extension is to model any long memory behaviour of the conditional variance using a fractionally integrated generalised autoregressive hetroskedastic (FIGARCH) model however, this is beyond the scope of the present study.

4.6 Conclusion

This chapter discusses the data, unit root tests, methodology and the model specification and selection strategy. The data utilised in this study is obtained from the International Monetary Fund's *International Financial Statistics* database, which is used to calculate the three different real interest rates for the nineteen countries studied.

In order to examine the mean-reverting dynamics of each real interest rate for each real interest rate, first both the PP and KPSS unit root tests are employed to examine the stationarity of each real rate; secondly the ARFIMA model is used to directly estimate the order of integration (d). A GARCH(1, 1) innovation is also added to the selected models in which non-linearity in the particular real rate is present. A conditional time-domain maximum likelihood estimation method is employed to estimate the ARFIMA model. Because this procedure is a parametric approach the specification of the short-run dynamic components (AR and MA parameters) is important to characterise a particular real rate. The model specification strategy creates a variety of possible models, thus the model selection criteria is used to select the most parsimonious model.

Given the methodology outlined in this chapter the time series characteristics of the real interest rate for nineteen Asian Pacific and European courtiers are examined in chapters five and six, using the unit root testing procedures and the ARFIMA modelling methodology.

Appendix 4.1

Table A4.1: List of the Chosen Interest Rate Used, Sample Size of the Data and the IMF's Series Code of Each Time Series Studied

Country	Interest Rate	Sample period	Series code	CPI sample period	CPI series code
Australia	TBR	1969Q3-2002Q2	19360C..ZF	1957Q1-2002Q3	19364...ZF
Belgium	TBR	1957Q1-2002Q3	12460C..ZF	1957Q1-2002Q3	12464...ZF
Canada	TBR	1957Q1-2002Q3	15660C..ZF	1957Q1-2002Q3	15664...ZF
France	TBR	1970Q1-2002Q3	13260C..ZF	1957Q1-2002Q3	13264...ZF
Germany	MMR	1957Q1-2002Q3	13460B..ZF	1957Q1-2002Q3	13464...ZF
Italy	MMR	1971Q1-2002Q3	13660B..ZF	1957Q1-2002Q3	13664...ZF
Japan	MMR	1957Q1-2002Q3	15860B..ZF	1957Q1-2002Q3	15864...ZF
Korea	MMR	1976Q1-2002Q3	54260B..ZF	1970Q1-2002Q3	54264...ZF
Malaysia	MMR	1968Q1-2002Q3	54860B..ZF	1957Q1-2002Q1	54864...ZF
Netherlands	MMR	1960Q1-1998Q4	13860B..ZF	1957Q1-2002Q3	13864...ZF
New Zealand	TBR	1978Q1-2002Q3	19660C..ZF	1957Q1-2002Q3	19664...ZF
Pakistan	MMR	1957Q1-2002Q3	56460B..ZF	1957Q1-2002Q1	56464...ZF
Philippines	TBR	1976Q1-2002Q3	56660C..ZF	1957Q1-2002Q3	56664...ZF
Singapore	TBR	1973Q1-2002Q2	57660C..ZF	1961Q1-2002Q3	57664...ZF
South Africa	TBR	1957Q1-2002Q3	19960C..ZF	1957Q1-2002Q1	19964...ZF
Spain	MMR	1974Q1-2002Q3	18460B..ZF	1957Q1-2002Q3	18464...ZF
Sweden	TBR	1963Q1-2001Q3	14460C..ZF	1957Q1-2002Q3	14464...ZF
UK	TBR	1957Q1-2002Q2	11260C..ZF	1957Q1-2002Q3	11264...ZF
USA	TBR	1957Q1-2002Q3	11160C..ZF	1957Q1-2002Q3	11164...ZF

Source: IMF (2002).
Notes: Treasury Bill Rate (TBR), Money Market Rate (MMR), Consumer Price Index (CPI).

Chapter Five Empirical Results for Asia, Pacific and Europe

5.1 Introduction

Given the importance of the real interest rate as a crucial variable that influences the macroeconomic dynamics of all countries, this chapter presents the econometric results of the long memory properties of the real interest rates studied. Based on the Fisher equation the real interest rates are examined using the models presented in chapter four. The empirical results generated using the calculated real interest rates for seventeen Asian, Pacific and European countries are presented in chapter five. Utilising the data, model and the model selection criteria discussed in chapter four the order of integration (d) is examined for the three real interest rate series of each country here. The estimates of d are compared within each country to investigate whether the short-run volatility of inflationary expectations impact the level of persistence of the real interest rate.

The chapter is organised as follows: Section 5.2 presents the preliminary results of the three real interest rates for seventeen Asian, Pacific and European countries. The preliminary results show the results of unit root tests as outlined in chapter four. Section 5.3 presents the estimated long memory models for the three real rates of each country. The final Section summarises the empirical findings and presents the conclusion.

5.2 Unit root test results

For the empirical investigation undertaken here the first step involves the unit root test. The unit root test results are for seventeen countries (excluding Australia and New Zealand). As discussed in chapter four, the Phillips and Perron (PP) and the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) tests are employed in order to indicate the long run stochastic nature of each of the real interest rate series. The results of the PP and KPSS unit root tests are illustrated in Table 5.1. The different truncation lag selections are

denoted as follows: (l = 4) represents the truncation lag set equal to four, (l = auto) represents the tests where the truncation lag is automatically selected using automatic lag selection in Eviews 4.1 econometric software package under the Newey and West (1994) methodology.[1] There are three panels in Table 5.1 each presenting the results for one of the three real rates. Panel (i) presents the results for the *ex post* real rate, Panel (ii) presents the results for the Hodrick-Prescott-*ex ante* (HP-*ex ante*) real rate and Panel (iii) presents the results for the exponentially smoothed-*ex ante* (ES-*ex ante*) real rate.

For the *ex post* real rate all countries fall into two groups with respect to the test results. Under the fixed truncation lag $l = 4$ the PP test rejects the null hypothesis of a unit root for all the countries at the one percent level of significance. Under the automatic truncation lag the PP test presents the same results, that is, the null hypothesis of a unit root is rejected at the one percent level of significance. For the KPSS test with the truncation lag ($l = 4$) the computed test statistics reject the null hypothesis of no unit root for majority of the countries at least at the five percent level of significance. However, there are some exceptions where the computed test statistics do not reject the null hypothesis for some countries, these include Japan, Korea, Pakistan, the Philippines and Singapore. For the KPSS unit root test, using the automatic truncation method, the results are similar to the fixed truncation lag selection. However, for some countries the power of the test is reduced and the test statistic fails to reject the null hypothesis of no unit root at the five percent level of significance. This applies to three countries, that is Belgium, Malaysia and the United States. The result is not unexpected as the automatic truncation lag increases the lag length and this can reduce the power of the KPSS test (Tsay, 2000). He points out that "...we realise that the increase of the lag length to reduce the size distortion of the test statistic will decrease the power of the KPSS tests at the same time" (Tsay, 2000, p.327). Overall, for the *ex post* real rate the results fall into two groups, for the first group both the PP and KPSS tests are rejected at least at the five percent level of significance. These countries include Belgium, Canada, France, Germany, Italy, Malaysia, the Netherlands, South Africa, Spain, Sweden, the United Kingdom and the United States.

[1] The automatic truncation lags for each country is presented in Table A5.1 of Appendix 5.1.

Table 5.1: The PP and KPSS Unit Root Test Results for European, Asian and Pacific Countries

Test Statistic	Bl	Ca	Fr	Ge	It	Ja	Ko	Ml	Nt	Pk	Ph	Si	SA	Sp	Sw	UK	US
Panel (i) *ex post*																	
PP (l = 4)	-6.497‡	-5.772‡	-3.576‡	-9.517‡	-4.165‡	-9.360‡	-7.688‡	-5.962‡	-8.993‡	-9.628‡	-5.836‡	-8.931‡	-5.730‡	-5.675‡	-7.915‡	-8.905‡	-5.064‡
PP (l = auto)	-7.466‡	-6.437‡	-3.750‡	-10.056‡	-4.327‡	-10.157‡	-7.988‡	-6.310‡	-10.129‡	-9.907‡	-5.731‡	-9.124‡	-6.387‡	-5.968‡	-9.079‡	-10.410‡	-5.557‡
KPSS (l = 4)	0.468†	0.993‡	1.015‡	0.926‡	1.071‡	0.340	0.160	0.495†	1.214‡	0.209	0.220	0.285	0.908‡	0.789‡	1.275‡	0.871‡	0.485†
KPSS (l = auto)	0.250	0.530†	0.559†	0.667†	0.613†	0.229	0.127	0.355*	0.745‡	0.169	0.217	0.267	0.494†	0.519†	0.742‡	0.472†	0.273
Panel (ii) HP-*ex ante*																	
PP (l = 4)	-1.759	-2.610*	-2.190	-3.499‡	-1.936	-3.236†	-3.024†	-2.722*	-2.992†	-3.130†	-3.149†	-3.135†	-2.236	-5.260‡	-2.456	-2.113	-2.745*
PP (l = auto)	-1.777	-2.633*	-2.080	-3.466‡	-1.665	-3.227†	-2.828*	-2.673*	-2.965†	-3.410†	-3.149†	-3.142†	-2.271	-5.260‡	-2.439	-2.127	-2.819*
KPSS (l = 4)	0.554†	0.940‡	1.017‡	0.767‡	1.196‡	0.453*	0.331	1.027‡	1.267‡	0.530†	0.574†	0.783‡	0.984‡	0.365*	1.424‡	0.967‡	0.478†
KPSS (l = auto)	0.274	0.508†	0.574†	0.496†	0.659†	0.251	0.252	0.572†	0.744‡	0.277	0.445*	0.459*	0.521†	0.283	0.733†	0.488†	0.266
Panel (iii) ES-*ex ante*																	
PP (l = 4)	-2.843*	-3.383†	-2.342	-3.540‡	-2.252	-2.900†	-3.364†	-3.354†	-3.293†	-4.304‡	-4.625‡	-3.238†	-2.636*	-3.768‡	-2.995†	-3.213†	-4.220‡
PP (l = auto)	-2.865*	-3.401†	-2.230	-3.540‡	-2.335	-2.900†	-3.405†	-3.310†	-3.293†	-4.411‡	-4.571‡	-3.148†	-2.722*	-4.047‡	-2.847*	-3.213†	-4.382‡
KPSS (l = 4)	0.401*	1.039‡	1.070‡	0.630†	1.231‡	0.537†	0.216	0.568†	0.978‡	0.274	0.416*	0.404*	0.766‡	0.908‡	1.321‡	0.810‡	0.467†
KPSS (l = auto)	0.211	0.581†	0.599†	0.404*	0.664†	0.278	0.155	0.364*	0.598†	0.165	0.371*	0.332	0.434*	0.578†	0.759‡	0.435*	0.264

Notes: Bl: Belgium, Ca: Canada, Fr: France, Ge: Germany, It: Italy, Ja: Japan, Ko: Korea, Ml: Malaysia, Nt: Netherlands, Pk: Pakistan, Ph: Philippines, Si: Singapore, SA: South Africa, Sp: Spain, UK: United Kingdom, US: United States. *, significant at the 10% level; †, significant at the 5% level; ‡, significant at the 1% level. (l = 4) indicates that the truncation lag is equal to 4, (l = auto) indicates that the truncation lag is automatically chosen in EViews 4.1 following the methodology of Newey and West (1994).

This suggests that neither an I(1) or I(0) process can explain the time series, the results are discussed in more detail later in this section. For the second group the PP test rejects the null hypothesis at the one percent level of significance, however the null hypothesis of the KPSS test is not rejected these countries include Japan, Korea, Pakistan, the Philippines and Singapore. This suggests that a covariance stationary I(0) process cannot be rejected.

For the HP-*ex ante* real rate (Panel (ii) Table 5.1) the results are somewhat mixed. Under the fixed truncation lag the PP test statistics show that nine of the seventeen countries cannot reject the null hypothesis of a unit root at the five percent level of significance, these include Belgium, Canada, France, Italy, Malaysia, South Africa, Sweden the United Kingdom and the United States. For eight of the seventeen countries the null hypothesis of a unit root is rejected at the five percent level of significance, these include Germany, Japan, Korea, the Netherlands, Pakistan, the Philippines, Singapore, and Spain. The PP test results are identical when the truncation lag is selected automatically except for Korea, which does not reject the null hypothesis at the five percent level. For the KPSS unit root test under a fixed truncation lag, the computed statistics for majority of the countries reject the null hypothesis of a stationary I(0) process at the five percent level of significance. These countries include Belgium, Canada, France, Germany, Italy, Malaysia, the Netherlands, Pakistan, the Philippines, Singapore, South Africa, Sweden, the United Kingdom and the United States. For Japan, Korea, and Spain the null hypothesis for the KPSS test could not be rejected under the fixed truncation lag. Again, for the KPSS unit root test the truncation lag length is increased when selected automatically, therefore for some countries the power of the KPSS test is reduced. For Belgium, Pakistan, the Philippines, Singapore and the United States the result changes and the null hypothesis for the KPSS test cannot be rejected at the five percent level. Overall, for nine of the countries: Belgium, Canada, France, Italy, Malaysia, South Africa, Sweden the United Kingdom and the United States the PP test is not rejected whereas the KPSS test indicates that the HP-*ex ante* real rate contains a unit root (I(1)). For Japan, Korea, and Spain the opposite result is found, that is the PP test is rejected whereas the KPSS is not which indicates that the HP-*ex ante* real rate for these countries is covariance stationary (I(0)). For the remainder of the countries Germany, the Netherlands, Pakistan, the Philippines, and Singapore both unit root

tests are rejected indicating that neither an I(1) or I(0) process can explain the HP-*ex ante* real rate.

Finally, for the ES-*ex ante* real rate (see Panel (iii) Table 5.1) the PP and KPSS unit root test results are again mixed. Under the fixed truncation lag the null hypothesis is rejected for the PP test for the majority of countries. For the remainder Belgium, France, Italy and South Africa, the null hypothesis cannot be rejected at the five percent level of significance. Under the automatically selected truncation lag the PP test has identical results, except for Sweden, which now fails to reject the null hypothesis at the five percent level. For the fixed truncation lag the KPSS test is rejected for majority of the countries. For Belgium, Korea, Pakistan, the Philippines, and Singapore the KPSS test under fixed lag length is not rejected. As for the *ex post* and HP-*ex ante* real rates the automatic truncation lag length selection method has decreased the power of the KPSS test for the ES-*ex ante* real rate. Therefore, Germany, Japan, Malaysia, South Africa, the United Kingdom and the United States now fail to reject the null hypothesis at the five percent level. For ES-*ex ante* real rate overall, nine of the seventeen countries reject both the PP and KPSS test these include Canada, Germany, Japan, Malaysia, the Netherlands, Spain, Sweden, the United Kingdom and the United States. For France, Italy and South Africa the PP test is not rejected, whereas the KPSS test is. For Korea, Pakistan, the Philippines and Singapore the PP test is rejected whereas the KPSS test is not. And finally, for Belgium neither the PP nor the KPSS test is rejected suggesting perhaps that the time series fails to illustrate a long run process.

For the results described above, there is no certainty regarding the long run characteristics of any particular time series. There is always a probability that the null hypothesis is falsely accepted or rejected. However, the results do indicate that for the *ex post* real rate we should expect that the estimated difference parameter is either covariance stationary or fractionally integrated. For both *ex ante* real rates there is a proportion of countries that cannot reject the hypothesis of an I(1) process. Therefore, the estimated difference parameter may in fact be larger for the *ex ante* real rates. For all the real rates there is a proportion of countries that rejected both the PP and KPSS unit root tests as explained in

chapter four, thus real rates may not be represented by either an I(1) or I(0) process. The double rejection of the unit root tests offer evidence that a fractionally integrated I(*d*) process may represent real interest rate.

Similar results can be found for the United States in the literature for example, Tsay (2000) used both the PP and KPSS test to test for a unit root in the United States *ex post* real rate. Tsay rejected both the unit root tests and concluded that his results suggest that the *ex post* real rate follows an I(*d*) process. Lai (1997) employed the Augmented Dickey-Fuller (ADF) and the KPSS tests to test for a unit root in the United States *ex ante* real rate.[2] Lai marginally rejected the ADF test and rejected the KPSS test and argued that the *ex ante* real rate contained an I(*d*) process.[3] The unit root tests only give an indication to the stationarity (or non-stationarity) of a particular time series. The actual order of integration is examined more closely in section 5.3 using the long memory autoregressive fractionally integrated moving average (ARFIMA) model.

5.3 Long memory model results

Following other studies that use fractional integration the section 5.3.1 presents the autocorrelation functions for certain countries. Next, section 5.3.2 presents the selected ARFIMA models for each real interest rate for the seventeen European, Asian and Pacific countries. And finally, section 5.3.3 presents the results of the estimated ARFIMA models that included a GARCH (1, 1) innovation.

5.3.1 Autocorrelation Functions

As Baillie (1996) explains "One of the characteristics of long memory series is that the autocorrelations of the original series frequently have the appearance of being nonstationary, while the differenced series can appear over differenced." (Baillie, 1996, p.10). That is, the autocorrelations for the level of a process decay at a slow rate, and the

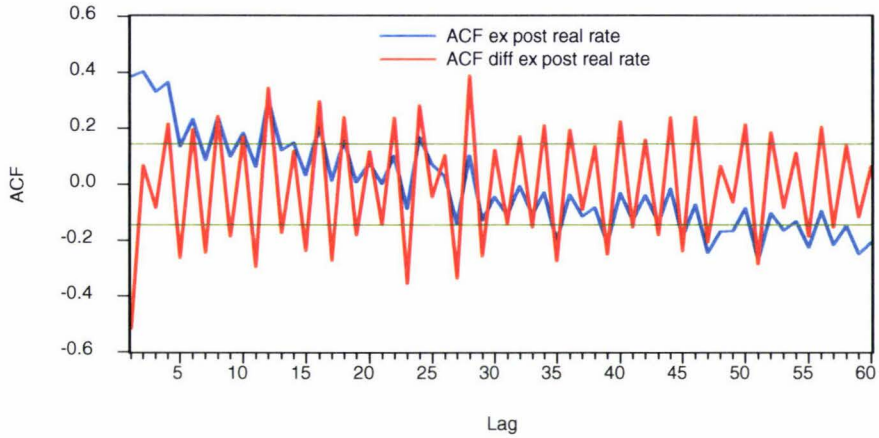
[2] Lai used professionally forecasted CPI inflation as a proxy for expected inflation.
[3] Lai pointed out that the ADF test is known to have low power against fractionally integrated alternatives.

autocorrelations of the differenced process display negative values at low lags (Ballie, Han and Kwon, 2002). Figures 5.1, 5.2, 5.3 and 5.4 depict the autocorrelation functions (ACF) for the levels and first-difference of each real interest rate for four of the countries studied here; these countries are Japan, Korea, the United Kingdom and the United States respectively. In Figures 5.1, 5.2, 5.3 and 5.4 Panel (i) represents the ACF for *ex post* real rate, Panel (ii) represents the ACF for HP-*ex ante* real rate and Panel (iii) represents the ACF for ES-*ex ante* real rate. The two horizontal lines in each panel of Figures 5.1, 5.2, 5.3 and 5.4 represent the critical values of the standard errors for a white noise process (equal to $0 \pm 1.96\sqrt{T}$ where T is the sample size).

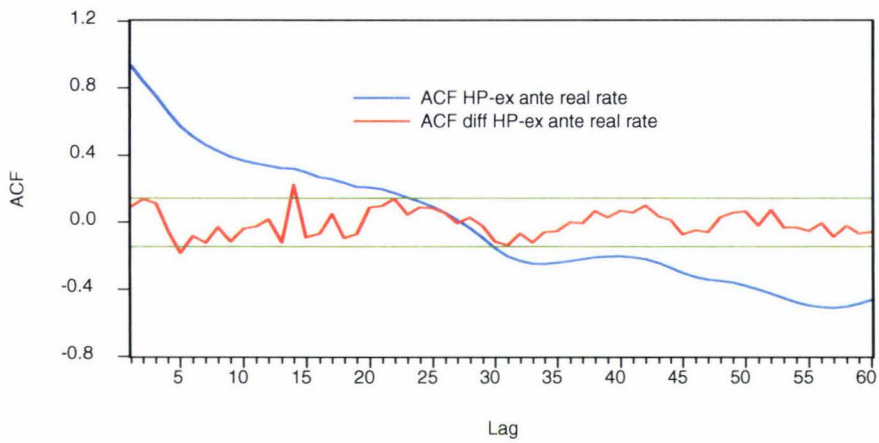
There is a similar pattern among the three real interest rates in Figures 5.1, 5.2, 5.3 and 5.4. Compared to the *ex ante* real rates the ACF for the *ex post* real rate at the level decays relatively quickly, whereas the ACF for the first-difference is extremely volatile which may indicate that the series is over differenced. For Japan in Figure 5.1 Panel (i) the ACF for the level decays relatively quickly and the ACF for the first-difference is very volatile, this indicates that this series may in fact be a stationary process. In Figure 5.2 Panel (i) the *ex post* real rate for Korea follows a similar pattern to Japan, however the ACF decays much more quickly. In Figure 5.3 Panel (i) the ACF of the *ex post* real rate for the United Kingdom decays relatively slower compared to both *ex post* real rates of Japan and Korea. The ACF for the *ex post* real rate of the United Kingdom contains a seasonal pattern, which is shown as the decaying spike every fourth consecutive lag. The slow decay of the ACF of the levels, and the volatile ACF of the first difference indicates that the series may be fractionally integrated. For the United States the ACF in Figure 5.4 Panel (i) for the *ex post* real rate decays relatively slowly in levels and has a somewhat less volatile ACF for the first-difference compared to Japan and the United Kingdom. For each country, both *ex ante* real rates the ACF shown in Panels (ii and iii) of Figure 5.1, 5.2, 5.3 and 5.4 show a similar pattern. The ACF at the level have a more persistent decaying pattern. For Japan, the United Kingdom and the United States autocorrelations for *ex ante* real rates are still significant after at least fifteen lags. For Korea autocorrelations of both *ex ante* real rates at the level decay quicker when compared to Japan, the United Kingdom and the United States.

Figure 5.1: Autocorrelation Functions of Real Interest Rates for Japan at the Level and First-Difference

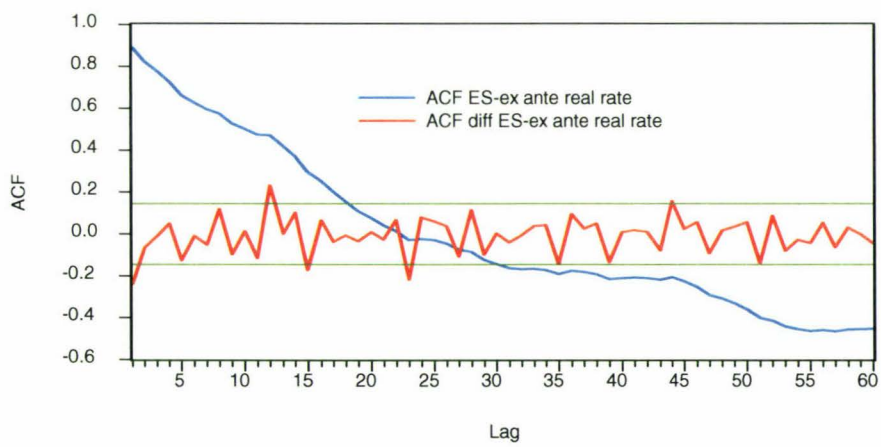
Panel (i) ACF of the level and first difference for the *ex post* real rate



Panel (ii) ACF of the level and first difference for the HP-*ex ante* real rate



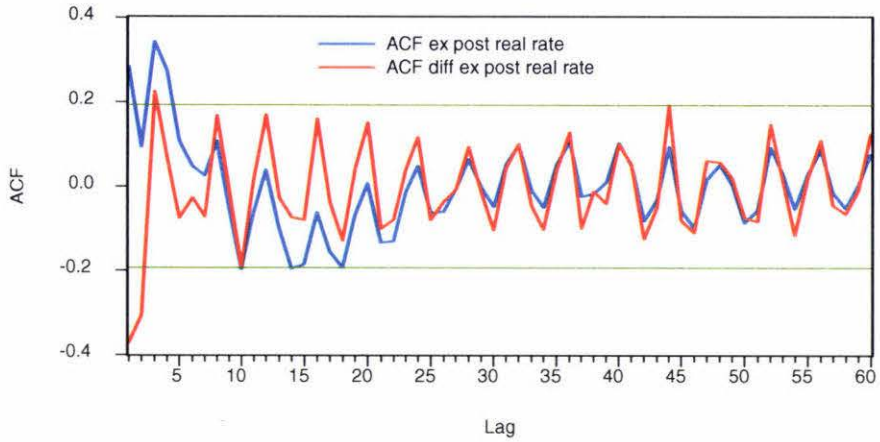
Panel (iii) ACF of the level and first difference for the ES-*ex ante* real rate



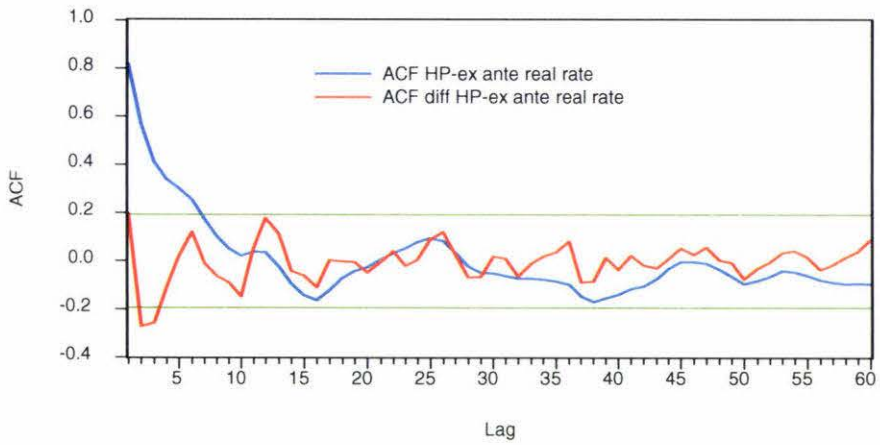
Notes: ACF: Autocorrelation function. The two horizontal lines in each panel represent the critical values of the standard errors for a white noise process ($0 \pm 1.96\sqrt{T}$ where T is the sample size).

Figure 5.2: Autocorrelation Functions of Real Interest Rates for Korea at the Level and First-Difference

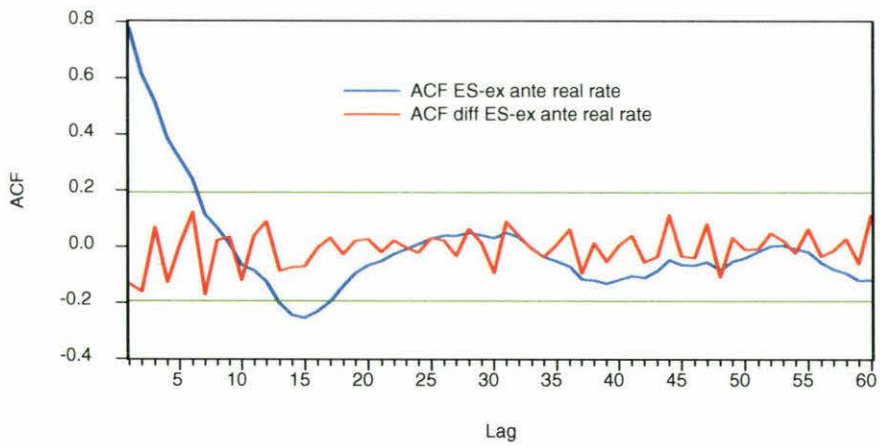
Panel (i) ACF of the level and first difference for the *ex post* real rate



Panel (ii) ACF of the level and first difference for the HP-*ex ante* real rate



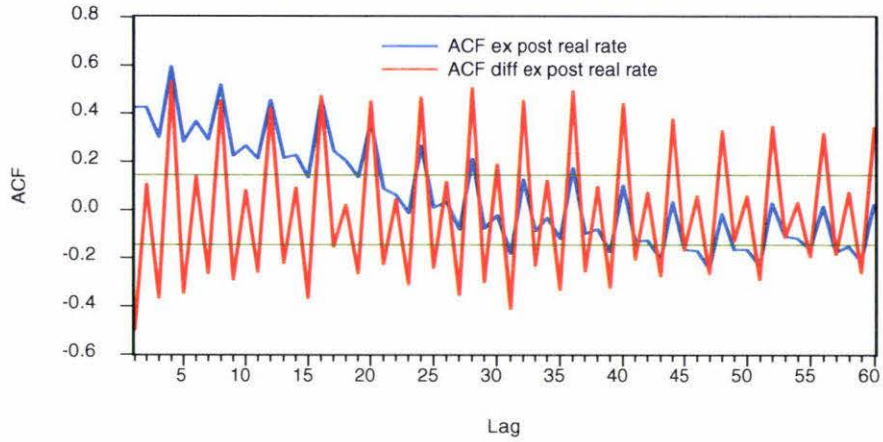
Panel (iii) ACF of the level and first difference for the ES-*ex ante* real rate



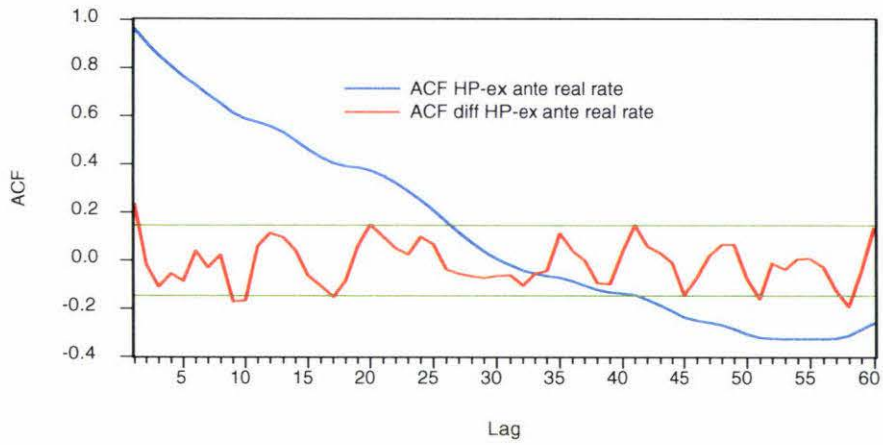
Notes: ACF: Autocorrelation function. The two horizontal lines in each panel represent the critical values of the standard errors for a white noise process ($0 \pm 1.96\sqrt{T}$ where T is the sample size).

Figure 5.3: Autocorrelation Functions of Real Interest Rates for the United Kingdom at the Level and First-Difference

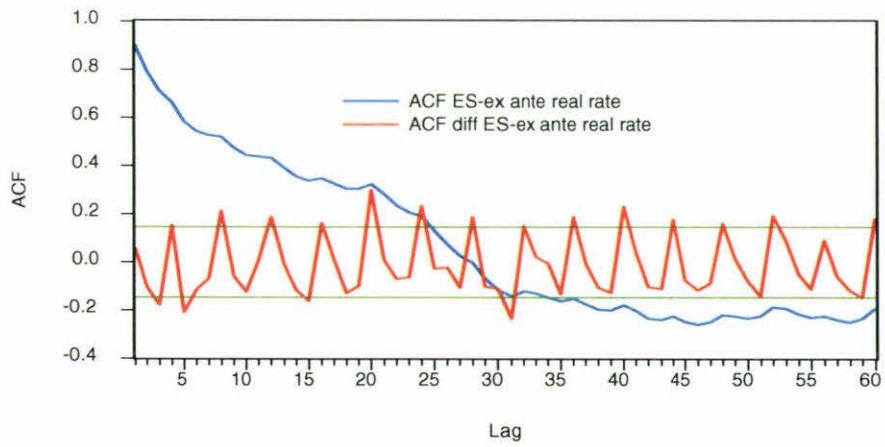
Panel (i) ACF of the level and first difference for the *ex post* real rate



Panel (ii) ACF of the level and first difference for the HP-*ex ante* real rate



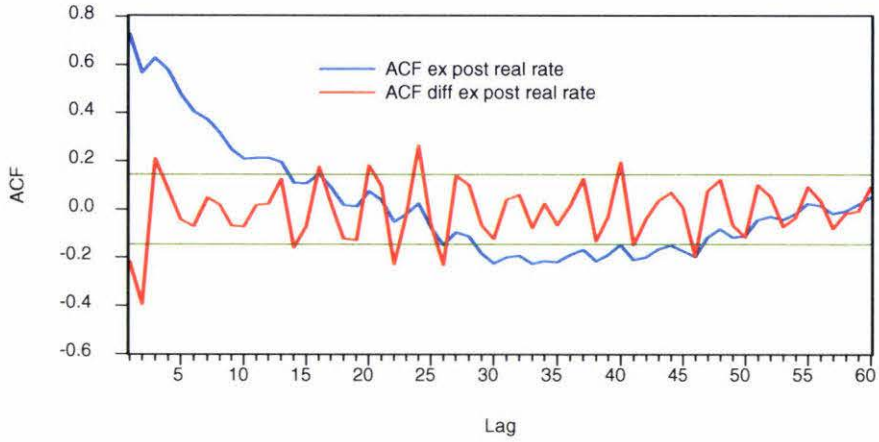
Panel (iii) ACF of the level and first difference for the ES-*ex ante* real rate



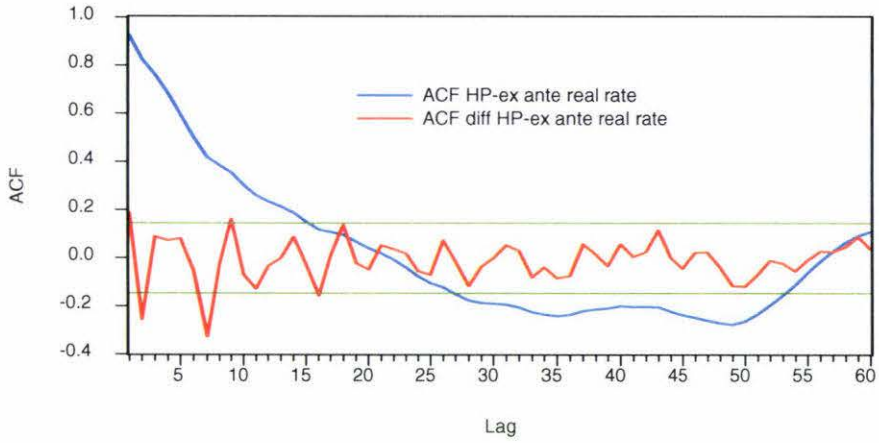
Notes: ACF: Autocorrelation function. The two horizontal lines in each panel represent the critical values of the standard errors for a white noise process ($0 \pm 1.96\sqrt{T}$ where T is the sample size).

Figure 5.4: Autocorrelation Functions of Real Interest Rates for the United States at the Level and First-Difference

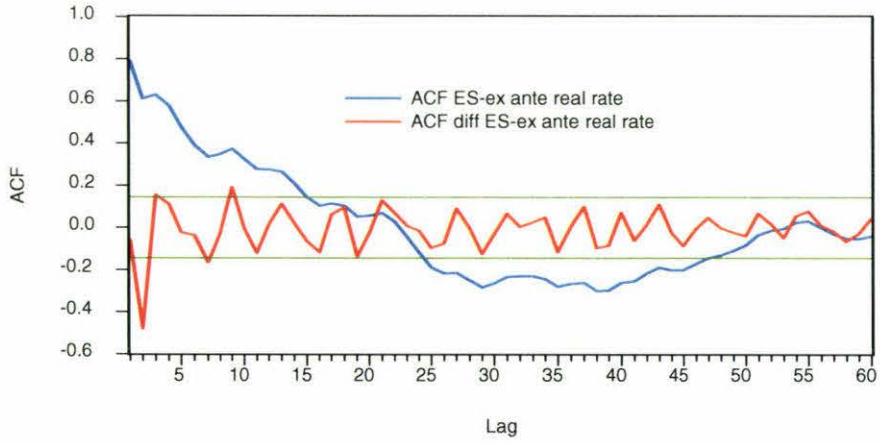
Panel (i) ACF of the level and first difference for the *ex post* real rate



Panel (ii) ACF of the level and first difference for the HP-*ex ante* real rate



Panel (iii) ACF of the level and first difference for the ES-*ex ante* real rate



Notes: ACF: Autocorrelation function. The two horizontal lines in each panel represent the critical values of the standard error for a white noise process ($0 \pm 1.96\sqrt{T}$ where T is the sample size).

The ACF for the first difference does not seem to be significantly volatile which, may indicate a stationary time series. It is also important to note that for some of the ACF of the *ex ante* real rates at the levels illustrate a significant cyclical pattern at high lag lengths. Smallwood and Norrbin (2001) deal with the issue of long cyclical decay using the Gegenbauer Autoregressive Moving Average (GARMA) model. Smallwood and Norrbin argue that the GARMA model can capture the slow non-monotonic decay through the cosine function. However, this issue is beyond the scope of this study and is for further research.

5.3.2 Empirical Results of the ARFIMA Model

Following the model selection methodology outlined in chapter four, section 5.3.2 presents the estimated ARFIMA models for each real interest rate for the seventeen countries studied here. As explained in section 5.2 and subsection 5.3.1 the unit root tests and ACFs present evidence that some real rates may in fact be fractionally integrated. Therefore, in section 5.3.2 the top three competing estimated ARFIMA models for each real interest rate are presented for each country. Each country has a table reporting *ex post*, HP-*ex ante* and ES-*ex ante* estimated values. Each table is outlined as follows: the second column of each table illustrates the estimated difference parameter (d), the third column shows the estimated mean (μ) of the real rate series, the fourth column (where necessary) presents the estimated autoregressive (AR(p)) (ϕ_p represents the equivalent AR component) or moving average (MA(q)) (θ_q represents the equivalent MA component) components where $p = 0, 1, 2$ and $q = 0, 1, 2$. The fifth column presents (where necessary) the estimated seasonal AR or MA component. The sixth and seventh columns contain the Box-Pierce statistic for the first and second moment of the real interest rate respectively. And finally, the eighth column presents the Akaike Information Criterion (AIC) statistic. The statistics in parenthesis represent the estimated standard errors. A dash indicates AR or MA components that were not specified in the corresponding estimated model. The shaded rows in each country Table indicate the overall selected ARFIMA model for each real interest rate. For these overall selected models the estimated d parameters that have (*)

indicates that an I(0) process cannot be rejected at the five percent significance level, and d parameters that have (†) cannot reject an I(1) process at the five percent significance level.

Belgium

Table 5.2 contains three chosen competing estimated ARFIMA models for each real interest rate of Belgium.[4] Looking at Table 5.2 it is important to note that for each real interest rate seasonality is an issue. This is shown by significant seasonal MA components for each shaded selected model. The presence of seasonality in each of the real rates distinguishes Belgium from other countries. This is because the inflation expectations generated in this study are thought to remove seasonality from the inflation data, and therefore remove seasonality from the corresponding *ex ante* real rates. The three models displayed in Panel (i) of Table 5.2 for *ex post* real rate of Belgium show that the difference parameter (d) is relatively stable between models 1, 2 and 3.[5] Models 2 and 3 in Panel (i) are very similar, however, model 3 is selected due to the fact that it has a lower AIC. The Box-Pierce $Q(10)$ statistic for model 3 is substantially larger than the critical test value. However, given the model selection methodology outlined in chapter four, model 3 seems to be the best representation of Belgium's *ex post* real rate. The Box-Pierce $Q(10)^2$ statistic is less than the critical value and therefore, serial correlation in the second moment is not present. The estimated d in model 3 shows that fractional integration is contained in the 95 percent confidence interval, and therefore not rejected.

The chosen models for the HP-*ex ante* real rate of Belgium are shown in Panel (ii) in Table 5.2. The estimated d between these models are relatively less stable and is more sensitive to model specification. Models 1 and 3 in Panel (ii) are very similar due to the significant seasonal component and significant low order AR or MA component. Model 3 is however chosen, and the $Q(10)$ statistic shows that model 3 provides a good description of the real rate data. However, the high $Q(10)^2$ statistic indicates that serial correlation in the second moment of the residuals is present this is dealt with in the next subsection. It is also

[4] In each table, estimated models that are not presented are omitted due to space.
[5] Stability or sensitivity of the long memory component with respect to ARFIMA model specification is found to be an issue for various countries and real interest rates, this is explained in more detail below.

important to note that the estimated d of model 3 indicates that the 95 percent confidence interval contains an I(1) process.

Panel (iii) of Table 5.2 contains the chosen models of ES-*ex ante* real rate of Belgium. The estimated d is relatively stable between the different estimated models. In Panel (iii) model 3 is chosen over model 2 due to the lower AIC. The $Q(10)$ statistic for model 3 is marginally larger than the critical test statistic, however as for the *ex post* real rate model 3 was chosen given the model selection criterion. Model 3's $Q(10)^2$ statistic suggests that there is no serial correlation in the second moment of the residual. Similar to the HP-*ex ante* real rate the I(1) process is also contained within the 95 percent confidence interval for d.

Table 5.2: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of Belgium

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.445 (0.065)	3.117 (0.907)	-	-	41.27	13.17	-608.46
2	0.408 (0.086)	2.949 (1.647)	-	ϕ_4=0.195 (0.087)	29.14	18.55	-588.32
3	0.415 (0.079)	3.050 (0.936)	-	θ_4=-0.171 (0.074)	29.02	17.83	-595.29
HP-ex ante				**Panel (ii)**			
1	0.840 (0.120)	3.436 (0.961)	θ_1=-0.330 (0.146)	ϕ_4=0.288 (0.109)	15.35	32.95	-193.21
2	1.066 (0.102)	3.840 (0.031)	-	θ_4=-0.156 (0.065)	20.58	32.35	-194.87
3	0.713† (0.192)	2.364 (1.027)	ϕ_1=0.447 (0.213)	θ_4=-0.220 (0.070)	16.83	32.17	-193.64
ES-ex ante				**Panel (iii)**			
1	0.861 (0.078)	1.011 (0.410)	-	-	28.82	9.91	-255.46
2	0.866 (0.077)	-4.992 (4.000)	-	ϕ_4=0.175 (0.063)	20.25	8.29	-251.99
3	0.854† (0.086)	0.861 (0.345)	-	θ_4=-0.134 (0.059)	20.04	9.12	-253.66

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

Overall, the selected models for Belgium show that the *d* is larger for both *ex ante* real rates compared to the *ex post* real rate, thus both *ex ante* series may contain a unit root. The selected model for the HP-*ex ante* real rate indicates that serial correlation in the variance of the residuals is present, which may impact on the estimate of *d*.

Canada

The estimated ARFIMA results for Canada are presented in Table 5.3. Panel (i) indicates that seasonality is an issue for the *ex post* real rate. Model 2 is selected which contains a significant seasonal MA component. Both the $Q(10)$ and $Q(10)^2$ statistics are below the critical test statistic value, this indicates that selected model is adequate and no GARCH innovation is needed to improve the model. Fractional integration cannot be rejected as the standard error of the estimated *d* is significantly different from zero.

In Panel (ii) model 2 is chosen, and seasonality is not present in the generated HP-*ex ante* real rate. Model 2 is preferred over model 1 due to an issue with the estimation procedure that seems to find difficulty distinguishing between the long run component (*d*) and the short run dynamic component (AR(1)).[6] Model 3 shows that inclusion of more MA components does not improve the model as these components are not significantly different form zero. The $Q(10)$ statistic value shows that model 2 is sufficient, however, the high $Q(10)^2$ statistic indicates evidence for use of a GARCH innovation. For the ES-*ex ante* real rate model 2 is chosen over model 1 as shown in Panel (iii), this is for the same reason as the HP-*ex ante* real rate that was chosen in Panel (ii). Overall, the estimated *d* is again shown to be larger for the *ex ante* real rate compared to the *ex post* real rate. For the HP-*ex ante* real rate an I(1) process is not rejected at the five percent significance level. For the 3 selected models the estimated *d* is larger for the HP-*ex ante* real rate, and the estimated *d* for the ES-*ex ante* real rate being between the other real interest rates.

[6] This estimation issue was found in several countries and is explained in more detail below.

Table 5.3: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of Canada

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.436 (0.074)	2.588 (2.190)	-	ϕ_4=0.239 (0.083)	7.94	13.32	-644.09
2	0.448 (0.069)	1.772 (1.107)	-	θ_4=-0.178 (0.064)	10.88	12.76	-654.65
3	0.383 (0.097)	1.923 (1.463)	ϕ_1=0.107 (0.130)	θ_4=-0.185 (0.062)	10.22	12.37	-652.62
HP-ex ante				**Panel (ii)**			
1	0.563 (0.217)	1.673 (2.168)	ϕ_1=0.593 (0.139)	-	15.92	92.85	-245.03
2	0.761† (0.159)	1.269 (0.152)	θ_1=-0.505 (0.106)	-	9.35	85.14	-242.86
3	0.740 (0.205)	1.270 (0.156)	θ_1=-0.529 (0.172) θ_2=-0.025 (0.119)	-	9.63	84.91	-242.83
ES-ex ante				**Panel (iii)**			
1	0.457 (0.209)	2.518 (1.578)	ϕ_1=0.481 (0.217)	-	6.24	35.77	-291.66
2	0.633 (0.093)	-0.397 (0.662)	θ_1=-0.369 (0.086)	-	3.68	28.81	-291.25
3	0.637 (0.142)	-0.409 (0.704)	θ_1=-0.364 (0.153) θ_2=0.005 (0.110)	-	3.71	28.55	-291.25

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

France

Displayed in Table 5.4 are the estimated ARFIMA models for France. It is important to note that for France the $Q(10)^2$ statistic is lower than the critical value for each real interest rate. Absence of serial correlation in the second moment of the residuals for each real rate distinguishes France from most countries. For the *ex post* real rate (see Panel (i)) the estimated d parameters are relatively stable, and model 1 adequately describes the real interest rate series, as shown by the low $Q(10)$ statistic value. In Panel (ii) of Table 5.4 model 3 is chosen based on the statistically significant MA component. The estimated d for model 3 is relatively large and cannot reject the possibility of being an I(1) process at the five percent level of significance. The ES-*ex ante* real rate is adequately described by

model 3 in Panel (iii). Overall, similar to Canada, the estimated d parameters for France are larger for both *ex ante* real rates compared to the *ex post* real rate. Also, the estimated d for the ES-*ex ante* real rate lies in between the other real interest rates.

Table 5.4: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of France

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.575 (0.072)	3.205 (1.340)	-	-	14.31	3.28	-373.37
2	0.546 (0.076)	0.853 (3.404)	-	ϕ_4=0.130 (0.093)	13.87	2.89	-363.71
3	0.564 (0.077)	3.443 (1.278)	-	θ_4=-0.101 (0.058)	12.18	3.00	-369.89
HP-ex ante				**Panel (ii)**			
1	1.237 (0.151)	4.752 (0.373)	-	-	17.46	7.65	-176.71
2	0.717 (0.161)	6.140 (2.111)	ϕ_1=0.581 (0.106)	-	8.07	6.18	-171.03
3	0.909† (0.123)	4.613 (0.353)	θ_1=-0.432 (0.085)	-	8.72	6.10	-171.19
ES-ex ante				**Panel (iii)**			
1	0.967 (0.141)	4.272 (0.340)	-	-	13.59	8.41	-194.32
2	0.567 (0.138)	1.624 (3.109)	ϕ_1=0.453 (0.172)	-	5.03	9.62	-189.17
3	0.714 (0.114)	4.272 (0.450)	θ_1=-0.394 (0.096)	-	4.13	6.05	-188.73

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

Germany

Panel (i) of Table 5.5 presents the estimated ARFIMA results for *ex post* real interest rate of Germany. Model 3 is selected as it contains a significant seasonal MA component. The estimated d has relatively large standard errors and cannot reject the possibility that the series is an I(0) process. Both the $Q(10)$ and $Q(10)^2$ statistic value indicate that model 3 represents the *ex post* real rate series. For the HP-*ex ante* real rate the estimated d parameters are sensitive to different short-run dynamic specification. Given this issue model 1 in Panel (ii) is selected due to the significance of the AR component compared to

model 2. Another issue with the German HP-*ex ante* real rate is the large $Q(10)$ and $Q(10)^2$ statistics that indicate an inadequacy with modelling this series. However, given the model selection methodology model 1 is chosen, and the inclusion of a GARCH innovation may improve model 1. In Panel (iii) model 1 is selected for the same reason as model 1 is selected in Panel (ii). Again there is an issue with the large $Q(10)$ and $Q(10)^2$ statistic values. Overall, it is found for Germany that the magnitude of the estimated d coefficient between the three real rates is the same as both Canada and France. However, the estimated models for both *ex ante* real rates do not adequately represent the data, an issue that is addressed in the subsection 5.3.3 below.

Table 5.5: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of Germany

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.234 (0.082)	2.339 (0.918)	-	ϕ_4=0.322 (0.073)	11.52	12.24	-714.32
2	0.062 (0.088)	2.295 (0.495)	θ_1=-0.294 (0.102)	ϕ_4=0.348 (0.075)	4.00	9.67	-697.06
3	0.232* (0.125)	2.041 (0.705)	-	θ_4=-0.295 (0.074)	17.18	14.43	-728.128
HP-ex ante				**Panel (ii)**			
1	0.397 (0.110)	2.216 (1.563)	ϕ_1=0.751 (0.056)	-	33.85	77.83	-221.60
2	0.992 (0.192)	2.368 (0.091)	θ_1=-0.214 (0.183)	-	45.19	71.20	-225.93
3	0.392 (0.122)	2.381 (1.454)	ϕ_1=0.779 (0.066)	θ_4=0.183 (0.126)	26.72	90.80	-220.06
ES-ex ante				**Panel (iii)**			
1	0.287 (0.131)	2.274 (1.206)	ϕ_1=0.770 (0.068)	-	20.50	73.27	-236.20
2	0.840 (0.137)	0.837 (0.141)	θ_1=-0.284 (0.143)	-	26.15	73.71	-239.91
3	0.701 (0.172)	0.972 (0.290)	θ_1=-0.413 (0.172) θ_2=-0.154 (0.165)	-	25.10	69.55	-239.12

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

Italy

Table 5.6 shows the estimated ARFIMA results for Italy. Model 1 is chosen in Panel (i), as the short-run dynamic components are insignificant for models 2 and 3. For model 1 the $Q(10)$ statistic is marginally greater that the critical test statistic. However, Model 1 is chosen due to the fact that the estimated d is stable between different models and the AR and MA components are insignificant for models 2 and 3. Model 2 is selected in Panel (ii) because the single MA component captures the short-run dynamic, whereas the second MA component in model 3 is insignificant. Panel (iii) shows that model 1 is selected because both the AR component in model 2 and the MA component in model 3 are insignificant. The order of magnitude of the estimated long-run dynamic (d) between the 3 real interest rates of Italy is similar to Germany. However, the a high $Q(10)^2$ statistic is present for all the selected estimated models of each real interest rate.

Table 5.6: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of Italy

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.526 (0.112)	0.603 (2.002)	-	-	20.86	39.44	-910.62
2	0.512 (0.139)	0.292 (6.577)	-	ϕ_4=0.107 (0.135)	17.68	44.18	-901.52
3	0.516 (0.124)	0.729 (2.013)	-	θ_4=-0.074 (0.096)	18.23	43.09	-904.65
HP-ex ante				**Panel (ii)**			
1	0.668 (0.134)	3.849 (3.679)	ϕ_1=0.559 (0.111)	-	14.53	25.28	-212.08
2	0.816† (0.130)	0.699 (0.470)	θ_1=-0.460 (0.126)	-	10.84	26.20	-212.71
3	0.706 (0.135)	0.677 (0.509)	θ_1=-0.594 (0.212) θ_2=-0.152 (0.120)	-	7.83	24.68	-210.32
ES-ex ante				**Panel (iii)**			
1	0.745 (0.101)	0.537 (0.416)	-	-	18.81	40.12	-288.12
2	0.670 (0.106)	0.299 (2.946)	ϕ_1=0.120 (0.192)	-	16.05	40.42	-287.97
3	0.680 (0.094)	0.530 (0.529)	θ_1=-0.109 (0.169)	-	16.16	40.89	-287.99

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

Japan

For Japan, the results of the estimated ARFIMA models are presented in Table 5.7. Model 2 in Panel (i) contains a marginally significant seasonal MA component and is selected due to the lowest AIC. In both Panels (ii) and (iii) model 1 is selected as the $Q(10)$ statistic shows that these are adequate parsimonious models. For the HP-*ex ante* real interest rate the selected model cannot reject a unit root process contained in the estimated d. The magnitude of Japan's estimated d parameters are ordered in a similar way as countries above (for example, Italy).

Table 5.7: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of Japan

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.241 (0.082)	1.227 (1.564)	-	ϕ_4=0.215 (0.114)	16.62	23.30	-1577.88
2	0.296 (0.088)	3.349 (1.906)	-	θ_4=-0.173 (0.092)	16.19	12.53	-1782.45
3	0.448 (0.161)	5.012 (4.755)	ϕ_1=-0.218 (0.137)	θ_4=-0.142 (0.101)	11.44	13.96	-1756.03
HP-ex ante				**Panel (ii)**			
1	1.084† (0.135)	10.882 (0.487)	-	-	16.34	32.98	-250.30
2	1.054 (0.165)	10.748 (1.030)	θ_1=-0.039 (0.207)	-	16.04	32.86	-250.26
3	1.009 (0.178)	11.675 (3.260)	θ_1=-0.014 (0.405) θ_2=-0.165 (0.255)	-	14.31	36.73	-248.91
ES-ex ante				**Panel (iii)**			
1	0.742 (0.056)	5.498 (1.245)	-	-	5.86	6.75	-333.95
2	0.979 (0.181)	22.122 (15.984)	ϕ_1=-0.231 (0.107)	-	11.18	10.37	-328.48
3	0.801 (0.117)	5.761 (1.528)	θ_1=0.096 (0.178)	-	5.28	8.49	-333.47

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

Korea

In Panel (i) of Table 5.8 model 3 is selected to represent the *ex post* real rate for Korea. The estimates of d are insensitive to model specification, and like Japan the Korean *ex post* real rate contains a marginally significant seasonal MA component. For the HP-*ex ante* real rate model 1 is selected, which is shown in Panel (ii). Although not emphasised in Panel (ii), the estimates of d are unstable to model specification. The estimated standard error of d for model 1 is relatively large compared to the estimated value of d. This indicates that a covariance stationary process cannot be rejected at the five percent level of significance. In Panel (iii) model 1 is chosen and as indicated by (†) the estimated d parameter cannot reject an I(1) process. The result in Panel (iii) illustrates the problem that was encountered with some countries (for example, Malaysia and the Philippines) where

difficulty existed in distinguishing between the long-run component and the short-run AR(1) coefficient.[7] Overall, even though the estimates of d for each selected model are not ordered in the same way as for example Japan, it is important to note that both *ex ante* real interest rates have larger estimates of d than the *ex post* real rate.

Table 5.8: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of Korea

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.242 (0.086)	5.363 (1.688)	-	-	18.36	8.23	-1813.72
2	0.200 (0.091)	4.987 (2.056)	-	ϕ_4=0.184 (0.092)	17.02	10.96	-1715.66
3	0.202 (0.089)	5.535 (1.574)	-	θ_4=-0.155 (0.078)	14.76	10.12	-1769.88
HP-ex ante				**Panel (ii)**			
1	0.420* (0.277)	5.533 (2.262)	ϕ_1=0.558 (0.084)	-	14.86	33.68	-244.52
2	0.516 (0.341)	4.371 (2.923)	ϕ_1=0.635 (0.268) ϕ_2=-0.284 (0.166)	-	3.56	30.23	-232.71
3	0.607 (0.188)	1.774 (0.947)	θ_1=-0.520 (0.122)	-	6.54	35.44	-237.47
ES-ex ante				**Panel (iii)**			
1	0.739† (0.182)	5.520 (0.760)	-	-	6.96	47.80	-395.93
2	-0.128 (0.227)	5.355 (0.915)	ϕ_1=0.862 (0.080)	-	7.02	41.87	-381.33
3	0.564 (0.227)	5.150 (1.182)	θ_1=-0.248 (0.258)	-	7.47	33.00	-390.61

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

Malaysia

Panel (i) in Table 5.9 shows that seasonality is present in the Malaysian *ex post* real rate, this is indicated by the significant seasonal AR component present in the selected model (see Model 1). The low $Q(10)$ statistic value confirms that model 1 characterises the *ex post* real rate. The estimated models for the Malaysian HP-*ex ante* real rate have stable

[7] See model two which is presented in panel (iii) of Table 5.8.

estimated d parameters, which are shown in Panel (ii). Model 1 is preferred compared to other models due to insignificant short-run dynamic coefficients presented in models 2 and 3. In Panel (iii) model 1 is chosen because of similar issues faced in the Korean ES-*ex ante* real rate. An I(1) process cannot be rejected at the five percent level of significance for the estimated order of integration of model 1 in Panel (iii). Overall, the distribution of d parameters for the Malaysian real interest rates is similar to that of Korea. That is, d parameters of both *ex ante* real rates are larger than the *ex post* real rate. However, the estimated d is larger for the ES-*ex ante* real rate than for the HP-*ex ante* real rate.

Table 5.9: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of Malaysia

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.477 (0.155)	-0.897 (6.384)	-	ϕ_4=0.274 (0.099)	13.92	52.78	-1072.33
2	0.367 (0.263)	-0.053 (4.778)	θ_1=-0.142 (0.214)	ϕ_4=0.308 (0.082)	13.92	37.66	-1186.71
3	0.494 (0.159)	4.518 (3.604)	-	θ_4=-0.236 (0.085)	16.88	50.34	-1117.50
HP-ex ante				**Panel (ii)**			
1	0.726 (0.114)	2.728 (0.925)	-	-	12.63	30.12	-271.63
2	0.708 (0.152)	-0.735 (1.720)	ϕ_1=0.012 (0.207)	-	12.63	30.67	-269.91
3	0.739 (0.313)	2.752 (0.988)	θ_1=0.020 (0.514)	-	12.31	30.59	-271.62
ES-ex ante				**Panel (iii)**			
1	0.811† (0.126)	0.700 (0.730)	-	-	15.62	10.61	-411.51
2	-0.073 (0.146)	1.547 (1.053)	ϕ_1=0.886 (0.079)	-	13.58	12.97	-399.01
3	0.721 (0.199)	0.760 (0.885)	θ_1=-0.115 (0.174)	-	16.92	11.44	-410.18

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

The Netherlands

The low $Q(10)$ statistic and the significant seasonal AR component confirms that model 1 is the preferred model for the Netherlands *ex post* real interest rate which is shown in Panel

(i) of Table 5.10. Model 2 in Panel (ii) is preferred because of the significant MA component. Given the marginally high $Q(10)$ statistic, model 2 is still preferred given the model selection methodology employed here. Model 2 in Panel (iii) is selected again based on statistically significant MA coefficient. Overall, each selected real interest rate is fractionally integrated. Both *ex ante* real rates are larger than the *ex post* real rate. It is also important to note that the estimated d coefficient is slightly larger for the ES-*ex ante* real rate compared to the selected HP-*ex ante* real rate. However, each real rate has a large $Q(10)^2$ statistic, which indicates that serial correlation may impact on the estimated d parameters.

Table 5.10: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of the Netherlands

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.253 (0.086)	1.571 (2.121)	-	ϕ_4=0.455 (0.099)	11.26	33.47	-1102.90
2	0.152 (0.146)	1.591 (1.494)	θ_1=-0.150 (0.190)	ϕ_4=0.486 (0.113)	11.78	32.57	-1095.29
3	0.248 (0.075)	1.358 (1.657)	-	θ_4=-0.303 (0.062)	30.62	46.27	-1262.69
HP-ex ante				**Panel (ii)**			
1	0.484 (0.266)	0.932 (2.731)	ϕ_1=0.475 (0.307)	-	23.21	42.88	-305.36
2	0.615 (0.128)	0.711 (0.405)	θ_1=-0.390 (0.161)	-	19.35	46.07	-303.27
3	0.590 (0.170)	0.726 (0.474)	θ_1=-0.422 (0.265) θ_2=-0.030 (0.179)	-	19.56	46.48	-303.19
ES-ex ante				**Panel (iii)**			
1	0.869 (0.126)	-0.425 (0.166)	-	-	24.19	47.99	-325.61
2	0.643 (0.133)	-0.328 (0.572)	θ_1=-0.330 (0.149)	-	16.18	54.98	-316.78
3	0.584 (0.183)	-0.191 (0.809)	θ_1=-0.402 (0.274) θ_2=-0.064 (0.165)	-	16.40	55.18	-316.40

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

Pakistan

Pakistan is similar to Belgium, as each real interest rate has a significant seasonal component. In Panel (i) of Table 5.11 model 2 is selected as both the seasonal AR coefficient and the MA(1) coefficient are significant. The low $Q(10)$ and $Q(10)^2$ statistic indicates how well model 2 characterises the *ex post* real rate. The estimated d parameter for model 2 is close to zero and cannot be an I(0) process. In Panel (ii) model 3 is preferred over model 2 due to the lower AIC value. However, the large $Q(10)^2$ statistic indicates justification for a GARCH innovation. Panel (iii) shows that model 3 is selected over model 2 based on the lower AIC value. Similar to other countries, the estimated d parameters are larger for the selected *ex ante* real rates compared to the selected *ex post* real rate model.

Table 5.11: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of Pakistan

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.238 (0.141)	0.976 (2.726)	-	ϕ_4=0.245 (0.098)	14.51	7.11	-5879.08
2	0.010* (0.142)	0.326 (1.064)	θ_1=-0.325 (0.118)	ϕ_4=0.299 (0.099)	5.88	2.05	-5633.10
3	0.245 (0.135)	-0.381 (2.890)	-	θ_4=-0.184 (0.071)	20.69	6.15	-6045.70
HP-ex ante				**Panel (ii)**			
1	0.709 (0.083)	-1.942 (0.656)	-	-	30.77	52.68	-392.09
2	0.634 (0.099)	2.659 (3.278)	-	ϕ_4=0.344 (0.118)	5.03	67.32	-363.93
3	0.667 (0.089)	-1.872 (0.654)	-	θ_4=-0.267 (0.089)	9.98	63.72	-373.25
ES-ex ante				**Panel (iii)**			
1	0.726 (0.086)	-4.315 (1.013)	-	-	21.92	3.75	-583.73
2	0.672 (0.120)	7.711 (8.697)	-	ϕ_4=0.210 (0.072)	11.60	5.81	-553.92
3	0.713 (0.099)	-4.480 (0.839)	-	θ_4=-0.185 (0.060)	12.75	4.75	-566.40

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

The Philippines

Table 5.12 presents the estimated ARFIMA models for the Philippines. In Panel (i) none of the selected models have estimated orders of integration significantly different from the I(0) process. Model 2 is selected because of the significant MA coefficients. The estimated *d* parameters for the HP-*ex ante* real interest rate are not stable with different specified models, as shown in Panel (ii). Model 1 is selected because there seems to be an inadequacy for the estimation procedure to distinguish between the long-run and short-run dynamic components as shown in model 2. In Panel (iii) model 1 is chosen for the same reason as the HP-*ex ante* real interest rate.

Table 5.12: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of the Philippines

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	-0.084 (0.710)	4.280 (1.685)	ϕ_1=0.513 (0.720) ϕ_2=0.130 (0.442)	-	14.70	82.94	-4708.97
2	-0.005* (0.222)	4.047 (1.860)	θ_1=-0.445 (0.201) θ_2=-0.499 (0.210)	-	12.48	83.09	-4474.31
3	-0.002 (0.217)	4.190 (1.733)	θ_1=-0.454 (0.216) θ_2=-0.483 (0.230)	ϕ_4=-0.043 (0.256)	11.82	79.02	-4460.88
HP-ex ante				**Panel (ii)**			
1	0.970† (0.337)	-1.950 (0.277)	-	-	11.69	62.10	-525.08
2	0.244 (0.210)	4.240 (2.936)	ϕ_1=0.713 (0.120)	-	8.60	55.92	-484.63
3	0.615 (0.276)	-1.166 (1.466)	θ_1=-0.486 (0.313)	-	11.67	58.29	-494.89
ES-ex ante				**Panel (iii)**			
1	0.612 (0.185)	3.330 (2.197)	-	-	19.11	31.63	-1125.57
2	-0.042 (0.374)	3.945 (1.311)	ϕ_1=0.696 (0.270)	-	11.66	34.27	-1051.41
3	0.363 (0.197)	3.622 (1.922)	θ_1=-0.318 (0.195)	-	17.69	30.68	-1081.67

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

Singapore

Although not shown in Panel (i) of Table 5.13, the estimated d parameter for the *ex post* real rate of Singapore are very sensitive to model specification. Like the Philippines, the 3 chosen competing models for the *ex post* real rate have d parameters that cannot reject the I(0) process. In spite of this, model 2 is selected as it has significant short-run dynamic components including a seasonal AR coefficient. Model 2 is preferred for the HP-*ex ante* real rate shown in Panel (ii). A significant seasonal AR coefficient represents the HP-*ex ante* real rate well as shown by the low $Q(10)$ and $Q(10)^2$ statistics. For the ES-*ex ante* real rate the estimated d parameters are again sensitive to the model specification shown in

Panel (iii). Model 3 is selected, but the standard error of the estimated d is relatively large and an I(1) process cannot be rejected.

Table 5.13: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of the Singapore

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	-0.078 (0.107)	0.563 (0.342)	θ_1=-0.281 (0.139)	ϕ_4=0.192 (0.116)	24.83	35.43	-646.37
2	-0.142* (0.086)	0.405 (0.309)	θ_1=-0.426 (0.167) θ_2=-0.274 (0.096)	ϕ_4=0.228 (0.096)	10.34	40.57	-608.23
3	-0.078 (0.277)	0.505 (0.705)	ϕ_1=0.299 (0.230) ϕ_2=0.252 (0.126)	θ_4=-0.114 (0.116)	17.55	28.14	-788.99
HP-ex ante				**Panel (ii)**			
1	1.097 (0.178)	-5.654 (0.209)	θ_1=0.469 (0.182)	-	19.83	4.45	-149.63
2	0.571 (0.085)	2.789 (1.824)	-	ϕ_4=0.183 (0.074)	12.14	5.33	-145.67
3	0.753 (0.068)	-5.247 (0.335)	-	θ_4=-0.225 (0.128)	13.90	7.45	-149.64
ES-ex ante				**Panel (iii)**			
1	0.927 (0.180)	-9.439 (1.197)	-	-	18.21	41.01	-531.76
2	0.787 (0.183)	-9.032 (1.364)	θ_1=-0.179 (0.105)	-	23.16	45.45	-523.93
3	0.535† (0.247)	-5.645 (4.567)	θ_1=-0.483 (0.235) θ_2=-0.322 (0.191)	-	14.10	62.65	-499.40

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

South Africa

Table 5.14 presents the estimated ARFIMA models for South Africa. In Panel (i) model 1 is selected as it adequately represents the *ex post* real rate, which is indicated by the portmanteau tests of both the first and second moments. In Panel (ii) the estimated d parameters are relatively unstable with different model specification. Model 2 is chosen and the I(1) process is not rejected at the five percent level of significance. This is due to

large estimated standard errors. For the ES-*ex ante* real rate (shown in Panel (iii)) the estimation procedure does not distinguish between the long-run and short-run dynamic. For model 2 the estimated d has a very large standard error relative to the estimated value, which shows that the estimate d parameter is somewhat meaningless. Given this issue model 3 is chosen which does not reject an I(1) process for the estimated d.

Table 5.14: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of South Africa

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.474 (0.061)	0.728 (1.796)	-	-	15.44	18.54	-1486.59
2	0.440 (0.074)	1.344 (2.433)	-	ϕ_4=0.172 (0.113)	10.81	16.56	-1419.65
3	0.445 (0.066)	0.666 (1.829)	-	θ_4=-0.131 (0.080)	11.41	15.65	-1460.48
HP-ex ante				**Panel (ii)**			
1	1.375 (0.201)	-0.110 (0.083)	-	-	10.32	36.70	-250.37
2	0.632† (0.274)	-0.284 (0.510)	ϕ_1=0.737 (0.117)	-	4.05	24.93	-243.27
3	1.138 (0.182)	-0.098 (0.079)	θ_1=-0.283 (0.150)	-	7.39	32.40	-247.39
ES-ex ante				**Panel (iii)**			
1	1.229 (0.164)	-2.644 (0.317)	-	-	8.98	40.60	-304.78
2	0.426 (0.301)	-0.877 (7.014)	ϕ_1=0.795 (0.108)	-	5.82	31.77	-295.87
3	1.032 (0.162)	-2.812 (0.476)	θ_1=-0.248 (0.154)	-	8.41	35.21	-301.68

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

Spain

In Table 5.15 model 1 is preferred for all the real interest rates of Spain because there is a lack of significance in the estimated short-run dynamic components. In Panels (i), (ii) and (iii) each selected model has a large $Q(10)^2$ statistic which justifies the inclusion of a GARCH innovation. For both *ex ante* real rates, the standard errors for the estimated d parameters are relatively large and further justify the inclusion of a GARCH model.

Table 5.15: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of Spain

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.440 (0.119)	-2.278 (5.633)	-	-	11.21	31.92	-1767.08
2	0.361 (0.122)	3.591 (4.137)	-	ϕ_4=0.166 (0.148)	8.02	60.83	-1606.15
3	0.422 (0.128)	-2.427 (5.858)	-	θ_4=-0.124 (0.114)	9.80	46.28	-1738.79
HP-ex ante				**Panel (ii)**			
1	0.500 (0.242)	2.425 (3.055)	-	-	9.54	30.92	-777.17
2	0.249 (0.267)	5.045 (2.195)	ϕ_1=0.327 (0.371)	-	6.81	36.16	-745.80
3	0.292 (0.214)	3.901 (2.717)	θ_1=-0.359 (0.240)	-	4.32	36.32	-742.86
ES-ex ante				**Panel (iii)**			
1	0.609 (0.177)	-3.076 (3.094)	-	-	14.61	30.62	-829.86
2	0.422 (0.229)	0.884 (4.947)	ϕ_1=0.275 (0.308)	-	10.29	42.53	-801.34
3	0.447 (0.166)	-2.148 (4.025)	θ_1=-0.343 (0.237)	-	7.46	43.75	-792.18

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

Sweden

Model 1 is selected for the *ex post* real rate of Sweden as shown in Panel (i) of Table 5.16. Model 1 shows a significant seasonal AR component and the portmanteau tests of the first and second moments confirm that model 1 characterises the data well. For the HP-*ex ante* real rate model 1 is selected because of the lack of significant AR and MA coefficients in models 2 and 3, as shown in Panel (ii). In Panel (iii) model 3 is chosen due to the significant MA coefficient and lower AIC compared to model 2.

Table 5.16: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of Sweden

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.259 (0.073)	2.219 (1.595)	-	ϕ_4=0.316 (0.104)	14.74	14.71	-1208.68
2	0.464 (0.172)	1.988 (3.018)	θ_1=0.294 (0.174)	ϕ_4=0.265 (0.108)	11.30	12.82	-1184.17
3	0.290 (0.067)	1.748 (1.291)	-	θ_4=-0.214 (0.068)	18.12	14.08	-1250.11
HP-ex ante				**Panel (ii)**			
1	0.948† (0.109)	-1.089 (0.277)	-	-	18.05	19.42	-264.43
2	0.591 (0.306)	1.845 (2.622)	ϕ_1=0.419 (0.334)	-	12.30	22.09	-260.26
3	0.784 (0.142)	-1.079 (0.351)	θ_1=-0.229 (0.150)	-	14.90	20.57	-261.70
ES-ex ante				**Panel (iii)**			
1	0.863 (0.108)	-0.480 (0.486)	-	-	18.38	26.10	-282.78
2	0.480 (0.165)	2.319 (1.625)	ϕ_1=0.454 (0.178)	-	8.51	27.22	-275.26
3	0.681 (0.114)	-0.320 (0.649)	θ_1=-0.253 (0.104)	-	11.93	26.62	-278.16

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

The United Kingdom

The *ex post* real rate of the United Kingdom is characterised well by a seasonal AR coefficient, this is shown by the selection of model 1 in Panel (i) of Table 5.17. In Panel (ii) models 1 and 2 are indistinguishable, however, model 2 is preferred over model 1 because of the lower AIC. Seasonality is an issue for the United Kingdom ES-*ex ante* real rate. This is indicated by the significant seasonal AR coefficient present in the selected model, model 2 of Panel (iii). For all of the selected ARFIMA models in Table 5.17 the high $Q(10)^2$ statistic value indicates an issue with serial correlation in the second moment, this is dealt with in the subsection 5.3.3.

Table 5.17: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of the United Kingdom

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.286 (0.098)	1.820 (2.342)	-	ϕ_4=0.462 (0.115)	14.77	30.59	-1499.22
2	0.186 (0.099)	1.853 (1.723)	θ_1=-0.146 (0.101)	ϕ_4=0.507 (0.127)	12.10	31.00	-1487.70
3	0.301 (0.082)	1.807 (1.311)	-	θ_4=-0.317 (0.067)	33.80	16.96	-1634.18
HP-ex ante				**Panel (ii)**			
1	0.659 (0.263)	-0.331 (2.220)	ϕ_1=0.557 (0.270)	-	11.94	27.88	-245.19
2	0.909† (0.100)	1.580 (0.274)	θ_1=-0.300 (0.099)	-	14.25	30.49	-246.48
3	0.773 (0.100)	1.395 (0.288)	θ_1=-0.462 (0.142) θ_2=-0.172 (0.093)	-	10.59	29.33	-245.12
ES-ex ante				**Panel (iii)**			
1	0.713 (0.202)	-3.927 (21.199)	θ_1=-0.327 (0.082)	ϕ_4=0.233 (0.179)	20.19	187.78	-365.97
2	0.473 (0.121)	2.126 (3.167)	θ_1=-0.578 (0.142) θ_2=-0.262 (0.098)	ϕ_4=0.228 (0.089)	10.13	137.59	-357.34
3	0.358 (0.289)	2.047 (1.651)	ϕ_1=0.711 (0.323) ϕ_2=-0.126 (0.118)	θ_4=-0.235 (0.083)	16.54	128.71	-361.07

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

The United States

Model 2 is selected for the *ex post* real rate of the United States, which is shown in Panel (i) of Table 5.18. The estimated seasonal MA component of model 2 is marginally significant at the five percent level of significance and is selected because of the lower AIC value. In Panel (ii) model 2 is selected to represent the HP-*ex ante* real rate of the United States. Both the $Q(10)$ and the $Q(10)^2$ statistics for the United States HP-*ex ante* real rate are larger than the critical test values, this presents an inadequacy of the model to characterise the temporal dependence of the interest rate series. However, given the model selection methodology, model 2 is preferred. For the ES-*ex ante* real rate model 3 is selected, as both

MA coefficients are significant for model 3. Similar to the United Kingdom, each selected real interest rate model of the United States has a large $Q(10)^2$ statistic, this may impact on the estimation of d, again this is dealt with in the next subsection.

Table 5.18: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of the United States

Estimates	d	μ	AR/MA	Seasonal AR/MA	$Q(10)$	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.534 (0.081)	2.322 (2.180)	-	ϕ_4=0.172 (0.096)	18.00	27.70	-412.84
2	0.534 (0.080)	0.355 (0.952)	-	θ_4=-0.173 (0.097)	18.26	25.21	-419.23
3	0.471 (0.133)	1.278 (1.398)	ϕ_1=0.093 (0.152)	θ_4=-0.166 (0.099)	18.39	24.07	-417.89
HP-ex ante				**Panel (ii)**			
1	0.587 (1.099)	0.729 (5.558)	ϕ_1=0.550 (1.016)	-	49.73	57.93	-220.64
2	0.694 (0.098)	0.572 (0.263)	θ_1=-0.714 (0.083)	-	25.76	131.89	-213.20
3	0.825 (0.310)	0.581 (0.280)	θ_1=-0.546 (0.325) θ_2=0.147 (0.257)	-	23.47	134.74	-212.90
ES-ex ante				**Panel (iii)**			
1	0.993 (0.182)	-1.593 (0.903)	ϕ_1=-0.086 (0.157) ϕ_2=-0.492 (0.183)	-	18.61	124.02	-289.05
2	0.405 (0.091)	-0.173 (1.183)	θ_1=-0.715 (0.119)	-	19.49	82.55	-292.85
3	0.755† (0.126)	-1.346 (0.596)	θ_1=-0.286 (0.130) θ_2=0.424 (0.111)	-	11.66	86.88	-284.98

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model.

Significance of the Results

The results obtained above are put in the context of other recent studies. Lai's (1997) study on long-term persistence in the real interest rate estimated the d parameter of both the *ex post* and *ex ante* real interest rate of the United States. Lai used monthly data including monthly CPI inflation, professionally forecasted inflation and various nominal interest

rates. Lai's results show that for monthly six-month treasury bills and monthly six-month commercial papers estimates of d are slightly larger for *ex ante* real rates.[8] Tsay (2000) finds that both monthly and quarterly *ex post* real interest rates of the United States are fractionally integrated. Gil-Alana (2003), utilising *ex post* real rate data, tested the null hypothesis of various levels of d using a Lagrange Multiplier test for ten European countries as well as the United States and Canada. Gil-Alana finds that real interest rates are more persistent for some countries (showed larger d) such as the United States, Belgium and Holland compared to other countries such as Germany and the United Kingdom. As explained in chapter two Sun and Phillips (2003) find, using quarterly survey data of inflation for the United States, that the estimated d is larger for the *ex ante* real rate due to the downward bias of estimates of d using *ex post* real rate data.

In summary, the results of this study presented above in subsection 5.3.2 indicate that for each country the point estimate of the d parameters follow the same pattern. That is, estimates of d for both *ex ante* real rates for the selected models are larger than the estimated d for the selected *ex post* real rates. For nine of the seventeen countries the estimated d parameter is largest for the HP-*ex ante* real rate with the ES-*ex ante* real rate having an estimated d between the HP-*ex ante* real rate and *ex post* real rates. These countries include Canada, France, Germany, Italy, Japan, the Philippines, Singapore, Sweden and the United Kingdom. This was the expected outcome for all countries due to the perceived persistent nature of the real interest rates. In the above estimated models there are issues with estimates of the long run dynamic for some of the real rates. For some countries the estimated d parameter is sensitive to the inclusion of different short-run dynamic components, also in some cases the estimation procedure had difficulty distinguishing between the long-run and short-run (AR(1)) dynamic. However, when using small samples we may expect these issues given the fact that the order of integration represents a long-run characteristic of time series data. One reason for these issues may be due to serial correlation in the second moment of the residual, which was present in some real interest rates shown by the large $Q(10)^2$ statistic values. Subsection 5.3.3 presents the results that deal with this issue.

[8] See Tables 3 and 5 (Lai, 1997, p.232-233).

5.3.3 Estimated ARFIMA-GARCH Models

As explained in chapter four, the selected ARFIMA models that have larger $Q(10)^2$ statistic values than the critical value $(\chi_c^2 = 18.31)$ are re-estimated with a GARCH(1, 1) innovation. The inclusion of the GARCH(1, 1) component enables hetroskedastic residuals to be modelled. No other model specifications were used to model the variance of the residuals because the time series properties of the second moment are not the primary concern of this research.[9] The purpose of including a GARCH(1, 1) innovation is to control any effect that serial correlation in the variance of residuals has on the estimation of *d*. Table 5.19 contains the estimated ARFIMA-GARCH models for *ex post* real rates for various countries. All columns in Table 5.19 are the same as Tables 5.2-5.18 except the following: Column six of Table 5.19 presents the estimated mean of the GARCH innovation (ω). In column seven of Table 5.19 (α) represents the estimated coefficient for the lagged unconditional variance (ARCH(1) component ε_{t-1}^2). Column eight (β) of Table 5.19 contains the estimated coefficient for the lagged conditional variance (GARCH(1) component σ_{t-1}^2). Tables 5.20 and 5.21 are organised identically to Table 5.19 and present the estimated ARFIMA-GARCH models for the HP-*ex ante* and ES-*ex ante* real rates respectively.

The $Q(10)^2$ statistic values for all countries in Table 5.19 indicate that the GARCH(1, 1) addition has captured the serial correlation in the variance of the residuals. For Italy, the Philippines, Singapore, South Africa, Spain, the United Kingdom and the United States both GARCH parameters are significant. The estimated $Q(10)$ statistic for Italy, the Netherlands and Spain is now larger than the critical value. Comparing the *d* estimates to the selected *ex post* real rate models shown in subsection 5.3.2 the estimated *d* parameters have not changed much with the inclusion of the GARCH(1, 1) component. One exception

[9] One possible extension of the GARCH model is to model the long memory behaviour (if present) of both the conditional mean and conditional variance using the ARFIMA-FIGARCH (Fractionally Integrated Generalised Autoregressive Conditional Heteroskedasticity) model. For an empirical application using inflation data, see Baillie, Han and Kwon (2002).

is Malaysia, the results now cannot reject the *ex post* real rate being a covariance stationary I(0) process. For the Philippines and Singapore the estimated d parameter still cannot reject an I(0) process. For Malaysia, the Netherlands, the United Kingdom and the United States the estimated seasonal coefficients are significant. Given these results there are some issues concerning the estimates shown in Table 5.19, these are discussed below.

Table 5.19: Estimated ARFIMA(p,d,q)-GARCH (1,1) Model for Selected *ex post* Real Rates of European Asian and Pacific Countries

Country	d	μ	AR/MA	Seasonal AR/MA	ω	α	β	$Q(10)$	$Q(10)^2$
Italy	0.651	1.090	-	-	0.942	0.288	0.746	24.53	9.41
	(0.085)	(0.183)			(0.749)	(0.098)	(0.072)		
Malaysia	0.353*	0.274	-	ϕ_4=0.332	5.116	0.296	0.546	9.68	4.36
	(0.320)	(3.183)		(0.116)	(7.764)	(0.693)	(1.112)		
Netherlands	0.520	11.148	-	ϕ_4=0.662	0.298	0.444	0.693	19.77	5.20
	(0.075)	(3.546)		(0.082)	(1.105)	(0.251)	(0.140)		
Philippines	0.222*	1.838	θ_1=-0.266	-	8.173	0.324	0.682	11.73	8.29
	(0.140)	(1.768)	(0.216)		(4.599)	(0.091)	(0.167)		
			θ_2=-0.274						
			(0.157)						
Singapore	0.124*	0.474	θ_1=-0.251	-	1.505	0.108	0.793	11.70	12.30
	(0.208)	(0.644)	(0.256)		(0.728)	(0.035)	(0.111)		
			θ_2=-0.155						
			(0.164)						
South Africa	0.476	-0.426	-	-	1.053	0.141	0.873	12.42	4.46
	(0.075)	(0.980)			(1.226)	(0.033)	(0.058)		
Spain	0.540	-5.788	-	-	1.525	0.330	0.702	19.70	8.27
	(0.057)	(1.798)			(1.521)	(0.081)	(0.061)		
UK	0.235	2.589	-	ϕ_4=0.524	5.340	0.098	0.849	9.28	13.87
	(0.096)	(1.791)		(0.075)	(2.805)	(0.064)	(0.103)		
US	0.487	0.646	-	θ_4=-0.120	0.776	0.142	0.810	18.02	3.21
	(0.067)	(0.640)		(0.073)	(0.377)	(0.044)	(0.071)		

Notes: A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root.

Table 5.20: Estimated ARFIMA(p,d,q)-GARCH (1,1) Model for Selected HP-*ex ante* Real Rates of European Asian and Pacific Countries

Country	d	μ	AR/MA	Seasonal AR/MA	ω	α	β	$Q(10)$	$Q(10)^2$
Belgium	0.379	3.125	ϕ_1=0.912	θ_4=-0.034	0.026	0.460	0.614	11.92	4.65
	(0.141)	(0.513)	(0.053)	(0.078)	(0.016)	(0.132)	(0.147)		
Canada	0.825†	1.301	θ_1=-0.442	-	0.191	0.291	0.661	7.08	14.93
	(0.136)	(0.039)	(0.097)		(0.104)	(0.096)	(0.102)		
Germany	0.578	3.123	ϕ_1=0.791	-	0.062	0.247	0.740	13.29	8.44
	(0.103)	(0.700)	(0.059)		(0.029)	(0.058)	(0.115)		
Italy	1.105†	0.961	θ_1=-0.308	-	0.549	0.752	0.078	9.59	3.55
	(0.115)	(0.584)	(0.136)		(0.256)	(0.344)	(0.264)		
Japan	1.304†	3.687	-	-	0.256	0.452	0.266	21.58	0.54
	(0.226)	(15.977)			(0.144)	(0.179)	(0.127)		
Korea	0.729†	1.354	ϕ_1=0.484	-	1.487	0.446	-0.067	16.46	4.91
	(0.212)	(4.404)	(0.184)		(0.548)	(0.286)	(0.060)		
Malaysia	0.700	2.994	-	-	0.174	0.614	0.531	18.48	3.12
	(0.109)	(0.364)			(0.256)	(0.242)	(0.069)		
Netherlands	1.004†	0.838	θ_1=-0.313	-	0.035	0.316	0.734	13.73	5.55
	(0.103)	(0.050)	(0.109)		(0.024)	(0.058)	(0.045)		
Pakistan	0.962†	-2.962	-	θ_4=-0.175	0.125	0.417	0.664	19.66	11.67
	(0.134)	(0.674)		(0.083)	(0.174)	(0.129)	(0.086)		
Philippines	0.996†	-1.973	-	-	0.017	0.773	0.642	21.53	6.73
	(0.126)	(0.002)			(0.036)	(0.334)	(0.092)		
South Africa	0.965†	-0.234	ϕ_1=0.830	-	0.011	1.176	0.478	2.33	1.40
	(0.152)	(0.034)	(0.059)		(0.008)	(0.607)	(0.142)		
Spain	1.325	3.231	-	-	0.144	0.725	0.421	14.75	13.81
	(0.150)	(1.674)			(0.097)	(0.233)	(0.256)		
Sweden	0.799†	-0.410	-	-	0.049	0.232	0.810	16.96	8.95
	(0.152)	(0.522)			(0.611)	(0.183)	(0.081)		
UK	0.911†	2.351	θ_1=-0.327	-	0.055	0.114	0.881	13.85	1.45
	(0.120)	(0.275)	(0.074)		(0.085)	(0.027)	(0.038)		
US	0.826†	1.025	θ_1=-0.637	-	0.059	0.366	0.636	19.95	7.97
	(0.094)	(0.382)	(0.056)		(0.033)	(0.105)	(0.061)		

Notes: A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root.

Presented in Table 5.20 are the estimated ARFIMA-GARCH results for the selected HP-*ex ante* real rates. Similar to the *ex post* real rate estimates the $Q(10)^2$ statistic indicates that serial correlation in the variance of the residuals has been captured with the inclusion of the GARCH(1, 1) component. For countries such as Belgium, Italy, Korea, the Netherlands, Pakistan, South Africa, Spain and the United States the estimate of d has changed considerably. For some countries the estimate of d has changed to a point that an I(1) process cannot be rejected, these countries include Italy, Korea, Pakistan, South Africa and the United States. All estimates of the short-run dynamic components are significantly different from zero (except the seasonal MA component for Belgium). It is important to note that the estimate of β for Korea violates the non-negativity constraint on the conditional variance, however, this estimate is relatively small and is not significantly different from zero. Similar to the results for the *ex post* real rate the $Q(10)$ statistic values have increased for a few countries and now are larger than the critical value, these countries include Japan, Malaysia, Pakistan and the Philippines.

The estimated ARFIMA-GARCH models for selected ES-*ex ante* real rates are shown in Table 5.21. Again, the $Q(10)^2$ statistics indicate that the GARCH(1, 1) improvement has captured the dynamics of the residuals variance. Singapore is the only exception where the estimated $Q(10)^2$ statistic is marginally larger than the critical value, however, this is still a large improvement from the results reported in Table 5.13. Similar to the HP-*ex ante* real rates, estimates of d for the ES-*ex ante* real rate have changed dramatically for certain countries. For Italy, the Netherlands, the Philippines and Spain the estimated of d now cannot reject an I(1) process. However, for Singapore and the United States the estimated d parameter has decreased as well as the estimated standard errors, thus the results cannot reject the ES-*ex ante* real rate containing a unit root. For the Netherlands, Singapore, South Africa and the United Kingdom the estimated AR and MA coefficients are now no longer significant for the GARCH model.

Table 5.21: Estimated ARFIMA(p,d,q)-GARCH (1,1) Model for Selected ES-*ex ante* Real Rates of European Asian and Pacific Countries

Country	d	μ	AR/MA	Seasonal AR/MA	ω	α	β	$Q(10)$	$Q(10)^2$
Canada	0.650	-0.670	θ_1=-0.347	-	0.756	0.117	0.742	4.68	6.20
	(0.088)	(0.438)	(0.087)		(0.253)	(0.076)	(0.101)		
Germany	0.278	1.876	ϕ_1=0.806	-	0.164	0.153	0.809	9.80	12.71
	(0.098)	(1.217)	(0.054)		(0.067)	(0.050)	(0.053)		
Italy	0.974†	0.598	-	-	0.122	0.317	0.741	22.31	1.68
	(0.104)	(0.019)			(0.076)	(0.070)	(0.074)		
Korea	0.832†	5.699	-	-	1.636	0.449	0.406	4.18	6.17
	(0.106)	(0.437)			(0.795)	(0.205)	(0.209)		
Netherlands	1.032†	-1.190	θ_1=-0.110	-	0.000	0.298	0.753	15.98	7.40
	(0.142)	(0.167)	(0.167)		(0.000)	(0.050)	(0.050)		
Philippines	0.765†	3.305	-	-	3.562	0.392	0.620	12.93	6.23
	(0.244)	(1.334)			(2.436)	(0.187)	(0.333)		
Singapore	0.325	1.402	θ_1=-0.406	-	0.840	0.212	0.674	7.41	18.98
	(0.158)	(1.117)	(0.220)		(0.480)	(0.080)	(0.258)		
			θ_2=-0.176						
			(0.146)						
South Africa	1.048†	-2.846	θ_1=-0.188	-	0.0745	0.047	0.970	4.48	12.10
	(0.121)	(0.314)	(0.138)		(0.205)	(0.014)	(0.041)		
Spain	0.964†	-1.123	-	-	0.326	0.893	0.305	16.53	7.73
	(0.092)	(1.465)			(0.162)	(0.244)	(0.192)		
Sweden	0.673	-0.101	θ_1=-0.253	-	0.337	0.240	0.736	9.60	8.43
	(0.148)	(0.477)	(0.097)		(0.271)	(0.077)	(0.104)		
UK	0.574	-4.583	θ_1=-0.423	ϕ_4=0.425	0.315	0.177	0.795	11.71	6.64
	(0.082)	(4.714)	(0.101)	(0.083)	(0.170)	(0.035)	(0.075)		
			θ_2=-0.141						
			(0.085)						
US	0.705	-1.392	θ_1=-0.276	-	0.280	0.356	0.575	9.24	12.24
	(0.078)	(0.307)	(0.112)		(0.096)	(0.088)	(0.069)		
			θ_2=0.269						
			(0.083)						

Notes: A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root.

As stated above there are some issues concerning the estimation of the ARFIMA-GARCH models for the three real interest rates. Firstly, for some countries the sum of the estimated GARCH coefficients (α and β) is greater than one. As Brooks (2002) explains this issue has highly undesirable properties, since it implies that the real interest rates contain a unit root in variance, and forecasts of GARCH(1, 1) models with this property will not converge to the long-term average value of the variance. Baillie, Chung and Tieslau (1996) study the estimates of the ARFIMA-GARCH models for various countries' inflation rates contained the same issue, however, this was not discussed.[10] Secondly, a more important issue concerns the variability of the estimated d once the GARCH(1, 1) innovation is added, this issue is more prominent for both *ex ante* real rates. (?)

5.4 Summary and Conclusion

This chapter presents the empirical results for seventeen Asian, Pacific and European countries. The estimation of the ARFIMA models is employed in order to capture the long run dynamic component or order of integration (d) for the *ex post* and two *ex ante* real interest rates for each country. Overall, a distinct pattern is found; firstly, for all countries the point estimated value of d is larger for both *ex ante* real interest rates compared to the *ex post* real interest rate. Secondly, and more importantly, once a GARCH(1, 1) innovation is added to the estimated ARFIMA model it is found that for twelve of the seventeen countries the estimate of d for the ES-*ex ante* real rate lies between the estimates of d for the other two real interest rates. These countries include Canada, France, Germany, Italy, Japan, Pakistan, the Philippines, Singapore, Spain, Sweden, the United Kingdom and the United States. The reason for this result is due to the volatility that is inherited within each real interest rate from the corresponding expectations of inflation. Thus, because the *ex post* real rate inherits the relatively large volatility of actual inflation, the estimate of d is biased downwards.

[10] See Table VII (Baillie, Chung and Tieslau, 1996, p.33).

Appendix 5.1

Table A5.1: Automatic Bandwidth selections for PP and KPSS tests

Test	Bl	Ca	Fr	Ge	It	Ja	Ko	Ml	Nt	Pk	Ph	Si	SA	Sp	Sw	UK	US
Ex post																	
PP	9	8	7	9	6	9	7	8	10	8	0	6	8	7	9	10	8
KPSS	10	10	9	9	9	9	7	8	9	8	5	6	10	8	9	10	10
HP-*ex ante*																	
PP	5	3	1	3	10	5	12	2	3	8	4	7	5	4	5	2	5
KPSS	10	10	9	10	9	10	7	9	9	10	7	9	10	7	10	10	10
ES-*ex ante*																	
PP	6	0	6	4	2	4	0	3	4	6	0	5	6	1	6	4	6
KPSS	10	10	9	10	9	10	8	9	9	10	6	7	10	8	9	10	10

Note: Bl: Belgium, Ca: Canada, Fr: France, Ge: Germany, It: Italy, Ja: Japan, Ko: Korea, Ml: Malaysia, Nt: Netherlands, Ph: Philippines, Si: Singapore, SA: South Africa, Sp: Spain, UK: United Kingdom, US: United States. Eviews 4.1 automatic Bandwidth selection is based on Bartlett Kernal estimator using the Newey and West (1994) procedure.

Chapter Six The Real Interest Rates and Real Interest Rate Differential of Australia and New Zealand

6.1 Introduction

Given the empirical results presented in chapter five for seventeen Asian, European and Pacific countries, chapter six extends this work by testing the validity of real interest rate parity (RIP) between New Zealand and Australia by way of examining the long-run time series properties of the real rate differential. Due to similar institutional structures, geographical location and closer economic relations (CER) between Australia and New Zealand, it is relevant to undertake a comparative study of the real interest rate differential. In recent times, policy changes have been implemented to increase the integration between both these countries through the CER agreement (which came into force in January 1983), thus it is adequate to examine RIP due to trade relations while other economic and financial sector functions proceeded rapidly during the sample period studied further justifying the comparative analysis. The RIP hypothesis describes an equilibrium condition between domestic and foreign real interest rates. Specifically, if the world capital and commodity markets are integrated, then real rates on perfectly comparable financial instruments should equalise across countries and across time (this is explained in more detail below). This chapter provides a preliminary analysis of RIP between the real rates for Australia and New Zealand. The analysis presented in this chapter is an extension of the work presented in chapter five, however due to data shortcomings different avenues for further detailed research are suggested.

Over the sample period studied (that is, 1978:1-2002:2) New Zealand went from being one of the most interventionist economies amongst the Organisation for Economic Co-operation and Development (OECD) member countries, to one of the most open market-

oriented economies.[1] During the mid and late 1980s New Zealand undertook many major economic reforms. A part of the early reform period included the deregulation of the financial system; Bowden and O'Donovan (1996, p. 281) explain the progression of these financial reforms:

> *Dominant features of the financial reform program include removal of interest rate controls in July 1984, removal of international capital controls (December 1984), abolition of all compulsory investment requirements on the major groups of financial institutions (February 1985), free floating of the currency, with no official foreign intervention (March 1985) and progressive reduction in entry/exit barriers to participation in financial markets (November 1985).*

With the lifting of the financial system controls mentioned above and implementation of other economic reforms, has meant that both goods and capital are now able to flow freer within New Zealand and abroad. Similar to New Zealand, Australia also had significant changes in the financial system in the early eighties. For example the Australian dollar was floated in December 1983 and the removal of interest rate ceilings started in the late 1980s (Lewis, Drake, Jüttner, Norris and Treadgold, 1994; Macfarlane, 1998). This would suggest that any form of parity condition between Australia and New Zealand is likely to be more significant for data after the reform period of the 1980s.

Another important feature of these two countries is the similar shifts in monetary policy during the sample period. In the late 1980s and early 1990s both Australia and New Zealand shifted their monetary policy frameworks towards inflation targeting. New Zealand formulated this change with the introduction of the Reserve Bank of New Zealand Act 1989, which stipulated central bank independence and a single goal of achieving and maintaining price stability with an initial inflation target of between 0-2 percent per annum

[1] For commentary on the New Zealand reform period see Silverstone, Bollard and Lattimore (1996) and Dalziel and Lattimore (2001).

(Dalziel and Lattimore, 2001).[2] In 1999 a new Policy Targets Agreement (PTA) was created which broadened the focus of the Reserve Bank of New Zealand (RBNZ). The primary objective was still to maintain price stability; however in addition, the RBNZ was also required to avoid unnecessary instability in output, interest rates and the exchange rate (Dalziel and Lattimore, 2001). Recently, in 2002 with the introduction of a new RBNZ Governor (Dr Alan E Bollard) a new PTA was designed to give the RBNZ more flexibility in the operation of monetary policy. The main objective of the RBNZ is now to maintain price stability, however the RBNZ is required to take a more forward-looking, medium term approach by keeping the level of inflation between 1-3 percent on average in the medium term (RBNZ, 2002). In Australia inflation targeting was adopted officially in 1993, which was laid out in the Reserve Bank Act 1959. The Reserve Bank of Australia (RBA) is required to best contribute to the following objectives: Stability of the currency, the maintenance of full employment and economic prosperity and welfare of the people of Australia. In order to contribute to these objectives the RBA is required to keep the level of inflation between 2-3 percent per annum over the medium term (Crosby and Milbourne, 1999).

In the literature recent studies have tried to verify RIP using cointegration analysis. Within this analysis a cointegrating relationship is assumed to be found if both the domestic and foreign real interest rates share the same unit root, overall the results have been mixed and in many cases not very supportive. Yet, if both the real interest rates of a domestic and foreign country are fractionally integrated a long-term stable relationship can still exist if two series are fractionally cointegrated. As explained in chapter two, two time series are fractionally cointegrated if they are integrated of the same order and the order of integration of a linear combination of the series is less than the order of integration of the original series. In this chapter using the same methodology outlined in chapter four the real interest rates of both Australia and New Zealand are examined, in addition the real interest rate differential is also examined between the three real interest rates calculated here. It is expected that because both Australia and New Zealand have a close economic relationship,

[2] Subsequent changes of the inflation target took place in 1996 (0-3 percent per annum) and 2002 (1-3 percent on average over the medium term).

both the commodity and capital markets could have some degree of an interrelationship; therefore, some form of RIP could exist. The order of integration is examined for each real rate differential with the aim of finding evidence of a cointegrating relationship between the real rates of Australia and New Zealand.

Chapter six is organised as follows: Section 6.2 briefly explains the theory of RIP followed by a brief description of some of this. Section 6.3 explains the data employed in this study as well as certain relevant data issues related to RIP. Section 6.4 presents the empirical findings, and finally section 6.5 concludes chapter six.

6.2 Real Interest Rate Parity Theory

This section highlights the RIP literature and postulates the hypothesis briefly as Australia and New Zealand have close linkages under the CER. Moosa and Bhatti (1997) point out that if markets for goods, capital and foreign exchange are efficient, real interest rates on financial assets comparable in all respects tend to be equalised across countries. If RIP holds then there is an important implication for monetary policy. The ability of monetary authorities to influence the domestic *ex ante* real interest rate is limited to the extent to which they can influence the world real rate under RIP (Mark, 1985; Smallwood and Norrbin, 2001; Moosa and Bhatti 1997). Several studies have been undertaken to examine the RIP hypothesis, particularly since globalisation of the world markets and increasing integration of many economies. It is important to highlight the key assumptions and present the empirical results for Australia and New Zealand.

The RIP condition is derived from the implication of two other parity conditions that describe the equilibrium condition of the international asset (capital) and commodity markets. These are known as uncovered interest parity (UIP) and relative purchasing power parity (RPPP), these conditions are presented below in equations (6.1) and (6.2):

$$\left(i^{d}_{t+k} - i^{f}_{t+k}\right) = \left(s^{e}_{t+k} - s_{t}\right) \tag{6.1}$$

$$\left(\pi_{t+k}^{ed} - \pi_{t+k}^{ef}\right) = \left(s_{t+k}^{e} - s_{t}\right) \tag{6.2}$$

Equation (6.1) presents the UIP condition where i_{t+k}^{d} is the nominal interest rate for a domestic debt instrument of maturity k and i_{t+k}^{f} is the foreign equivalent, s_t is the spot exchange rate at time t and s_{t+1}^{e} is the expected spot exchange rate expected for period $t+k$. UIP describes the equilibrium condition between the expected change in the exchange rate and the interest rate differential between perfectly substitutable foreign and domestic debt instruments. Equation (6.1) suggests that if the interest rate differential diverges from the expected change of the exchange rate then economic agents will transfer funds across financial markets which re-establishes equilibrium. Equation (6.2) presents the *ex ante* RPPP, where π_{t+k}^{ed} is the domestic inflation rate expected during $t+k$, and π_{t+k}^{ef} is the foreign equivalent of the inflation rate. The RPPP condition stipulates that the expected rate of depreciation of the exchange rate is equal to the difference in the rate of inflation of commodity prices between the domestic and foreign countries. Setting equations (6.1) and (6.2) equal to each other and rearranging yields equation (6.3):

$$\left(i_{t+k}^{d} - \pi_{t+k}^{ed}\right) = \left(i_{t+k}^{f} - \pi_{t+k}^{ef}\right) \tag{6.3}$$

From the Fisher equation, equation (6.3) states that *ex ante* real interest rates of the domestic and foreign countries should be equal, or alternatively the difference between the *ex ante* real interest rates should be zero, which implies the existence of RIP. The equalisation of the domestic and foreign real interest rates have been considered with the following assumptions: investors are assumed to be risk neutral, no barriers to movements of goods and capital exist, and assets are assumed to be perfect substitutes. For a detailed theoretical description of international parity conditions see, Moosa and Bhatti (1997). Testing for RIP is equivalent to testing the following equation (equation (6.4)):

$$r_{t+k}^{d} = \mu + \lambda r_{t+k}^{f} + \zeta_{t+k} \tag{6.4}$$

Where, r_{t+k}^{d} represents the domestic *ex ante* real interest rate, r_{t+k}^{f} is the foreign counterpart and ζ_{t+k} are the economic agents forecast errors of the real rate differential and is assumed to be orthogonal. For RIP to hold the restriction of $(\mu, \lambda) = (0,1)$ must be satisfied. In the recent literature empirical studies have applied cointegration analysis in order to verify RIP, under the hypothesis that the *ex ante* real interests of both countries contain a unit root (see, for example, Chinn and Frankel (1995)). For example, in a simple bivariate cointegrating framework for RIP to hold an implied cointegrating vector of (1 –1) should be found, this suggests that in the long-run a change in the foreign real interest rate is matched by a unit change in the domestic real interest rate.

There is a substantial literature pertaining to RIP, and the general consensus of studies using 1970s and 1980s data have rejected the hypothesis (Mishkin, 1984b; Mark, 1985; Merrick and Saunders, 1986). The empirical failure of the RIP in early studies was thought to be due to the failure of the UIP caused by stickiness of commodity prices or the failure of the RPPP caused by the presence of a risk premium or both (Mark, 1985). On the other hand, more recent studies have found some support for RIP and international capital market integration (Chinn and Frankel, 1995; Fujii and Chinn, 2001). However, as explained by Flemingham, Qing and Healy (2000) it is unclear whether recent support is derived from increasing globalisation or due to the utilisation of superior diagnostic methodologies. Flemingham, *et al.* (2000) studies specifically the interdependence of the Australian real interest rate with some major trading partners (New Zealand included). It has been noted that the real interest rates of the major trading partners are characterised by structural change, for example, New Zealand's real interest rate is characterised by a structural break following the floating of the New Zealand dollar in March 1985. Flemingham, *et al.* find that once a common structural break is controlled for a cointegrating relationship is found between the Australian and Foreign real rates, however there is little support for RIP. Fuji and Chinn (2001) analyse RIP for the G-7 countries using a variety of interest rates of differing maturity. Using both the consumer price index (CPI) and the wholesale price index (WPI) Fuji and Chinn find support for RIP when debt instruments with long-run maturities are employed. Support is also found for RIP at longer horizons with the use of alternative methods for modelling inflationary expectations.

6.3 Data and Data Issues

The data for Australia and New Zealand employed in this chapter is similar to the data explained in chapter four. That is, quarterly three-month interest rate and quarterly CPI data is employed. Examination of RIP requires the use of the same debt instrument; therefore, the Treasury bill rate (TBR) is obtained for Australia and New Zealand. From the International Monetary Fund (IMF) *International Financial Statistics* (IFS) database (IMF, 2002). The sample periods for the TBR differs between Australia and New Zealand, which means that the real interest rate of Australia is examined for two different periods. The first period for Australia covers the whole sample size (that is, 1969:3-2002:2) available for the TBR data, whereas the second sample period is for a shorter sample period (that is 1978:1-2002:2). Because of lack of larger time series TBR data for New Zealand, the estimation period for both Australia and New Zealand is for 1978:1-2002:2 for comparative purposes.[3] However, results for Australia's larger time series data is also reported to show the consistency of the results reported in chapter five.

For the New Zealand TBR obtained from the IFS database, there were missing observations for the period of 1985:1-1986:1 (inclusive). Unfortunately, no adequate substitute data was found in Statistics New Zealand's database or the Reserve Bank of New Zealand; therefore the missing observations were generated using an autoregressive (AR) model. It is assumed in this study that the New Zealand TBR is characterised by an AR model, therefore in order to capture the time series characteristics of the TBR an AR(4) model has been estimated for the sample period excluding the missing observations. The missing observations were then generated using the estimated AR(4) model. Future research could avoid this issue by either including a complete data set, or employing a more rigorous technique in order to estimated the missing observations of the IFS database.

[3] For size of the sample periods and IMF (2002) series codes, refer to Table A4.1 in the appendix of chapter four.

It is worth noting several issues pertaining to the data of RIP analysis as this chapter presents a tentative analysis. First, the analysis of real interest rates between countries requires serious consideration in terms of the particular nominal interest rate used. The use of national interest rates (such as the TBR) may not be appropriate in determining the equality of international interest rates. It can be argued that the TBR, which are denominated in different currencies, are not comparable across countries due to the varying degree political and default risk. Eurocurrencies (offshore rates), on the other hand, are suggested as an alternative because they are issued by the same financial institution in the same political jurisdiction, and therefore, involve the same political and default risk (Moosa and Bhatti, 1997). Another issue regarding the TBR is that for some countries the TBR is not market determined over entire data sample periods, and therefore, hinders empirical analysis of international market integration (Flemingham, *et al.* 2000).[4] It is also important to note that long-term interest rate (for example, maturities of five and ten years) data may be considered for RIP analysis. Fujii and Chinn (2001) explain that RIP is better represented by long-term interest rate data because firms usually make investment decisions on the basis of long-term yields. However, long-term debt instruments are more heterogeneous then short-term rates and thus may not be comparable across countries.

It is also important to consider an appropriate price index to be used. Mishkin (1984b) explains that the price index employed should depend on what economic decision is being studied. For example, the CPI is seen as being appropriate if savings and consumption decisions are being examined. Alternatively, if decisions concerning trade among countries is of interest then a price index with a large proportion of tradable goods in its consumption bundle is more appropriate, for example, the WPI. Another issue concerning inflation data is the timing of the data, for both New Zealand and Australia CPI data is only calculated quarterly. In contrast, as pointed out by Flemingham, *et al.* (2000) countries such as the United Kingdom, the United States, Canada, Germany and Japan CPI figures are calculated monthly.[5]

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[4] Interest rate controls were present in the TBR market of New Zealand in the 1970s and early 1980s.
[5] Also, the timing of price observations vary as well, some are calculated based on a single day, while others use prices observed throughout the calculated period.

With the data described in section 6.3, the same methodology (that is, the application of the autoregressive fractionally integrated moving average (ARFIMA) model) explained in chapter four is applied to the real interest rates for Australia (for both sample periods) and New Zealand as well as the real interest rate differential between the two countries. The estimated order of integration (*d*) of the real interest rate is examined within each country to see if the same pattern that was observed in chapter five is consistent with Australia and New Zealand. The difference parameter is also estimated for the real interest rate differential between New Zealand and Australia, with the purpose of discovering if a stable long-run relationship can hold between the real interest rates of both countries. The empirical results are presented in section 6.4.

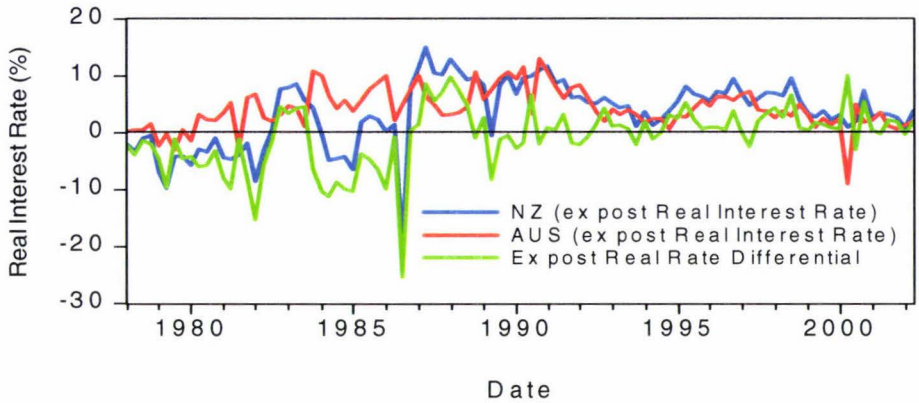
6.4 Empirical Results

This section presents the empirical results for the calculated real interest rates and real interest rate differentials for Australia and New Zealand. In order to get an understanding of the nature real interest rate series Figure 6.1 presents the three real interest rates of Australia and New Zealand, as well as the corresponding real interest rate differentials. Comparing the three real interest rates in panels (i), (ii) and (iii) of Figure 6.1 the same volatility characteristics are present within the three real rate series for Australia and New Zealand as was found in the real rate series presented in chapter three. That is, because the *ex post* real rate contains the unexpected volatile forecast error from realised inflation it appears more volatile than both *ex ante* real interest rates.[6] The second moment characteristic present in each of the three real interest rates is inherited within the corresponding real interest rate differential. Panel (i) of Figure 6.1 clearly shows that the *ex post* real rate differential contains more short-run volatility than both the HP-*ex ante* and ES-*ex ante* real rate differentials displayed in panels (ii) and (iii) of Figure 6.1 respectively. Comparing the three real rate differentials in Figure 6.1 indicates that each of them follow a similar pattern over the sample period shown. That is, in the late 1970s and through the 1980s the real rate differential fluctuated substantially from zero, which suggests a lack of continuity between the real interest rates for Australia and New Zealand.

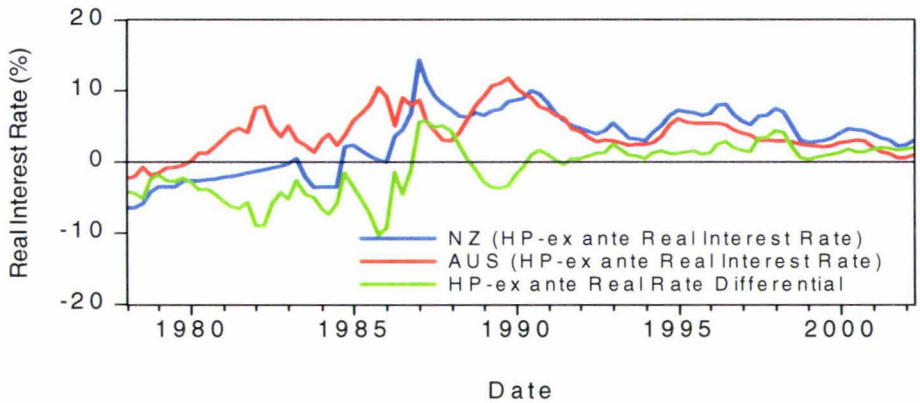
[6] Refer to chapter three for more details.

Figure 6.1 Real Interest Rate and Real Interest Rate Differential for Australia and New Zealand Over the Period of 1978:1-2002:2

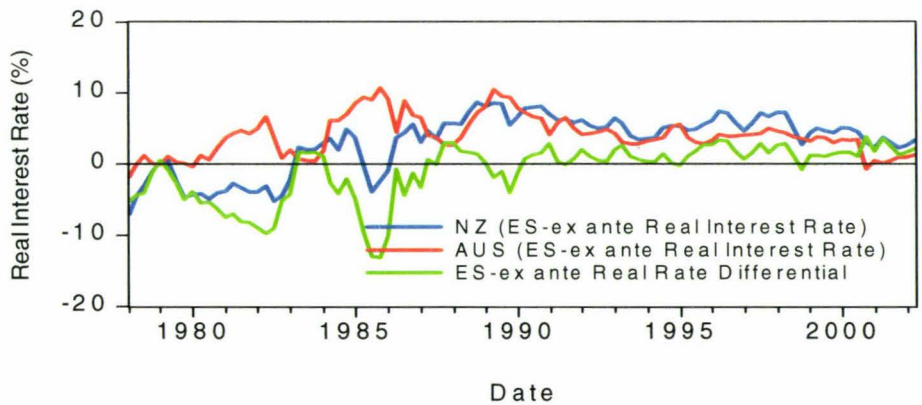
Panel (i): *Ex post* Real Interest Rate series and *ex post* Real Interest Rate Differential



Panel (ii): HP-*ex ante* Real Interest Rate series and HP-*ex ante* Real Interest Rate Differential



Panel (iii): ES-*ex ante* Real Interest Rate series and ES-*ex ante* Real Interest Rate Differential



Source: IMF (2002).

Yet, from the early 1990s the real rate differentials seem more stationary around a constant close to zero.[7] This is a function of increased continuity between the real interest rates of Australia and New Zealand around this period.

The unit root test results of Phillips and Perron (PP) and Kwiatkowski, Phillips, Schmidt and Shin (KPSS) described in chapter four are presented in Table 6.1.[8] The second column of Table 6.1 presents the unit root test results for each real interest rate of Australia for the entire sample period available. Double rejection of both unit root tests indicates that the Australian *ex post* real rate maybe fractionally integrated. However, for both *ex ante* real rates the KPSS test is rejected at least at the five percent level of significance, which suggests that the series is nonstationary. In contrast, the third and fourth column of Table 6.1, which presents the unit root test results for Australia and New Zealand over the reduced sample period, does not support the hypothesis that both New Zealand and Australian real interest rates contain the same order of integration.

Looking at Australia, for all three real interest rates, the PP test is rejected at least at the ten percent level of significance for the truncation lag length equal to four ($l = 4$), that suggests that each real rate for the reduced sample period is stationary. For New Zealand, the results differ substantially from Australia for the same sample period. For both the *ex post* and ES-*ex ante* real rates the PP and KPSS unit root tests are rejected at least at the ten percent level of significance, this indicates that these real interest rates maybe fractionally integrated. For the HP-*ex ante* real rate the KPSS test is rejected for both lag specifications at least at the five percent level of significance, whereas the PP test is not rejected, this suggests that this real rate is nonstationary. The unit root tests for Australia and New Zealand over the same sample period are not very supportive of any form of a cointegrating relationship, since the unit root tests seem to indicate different orders of integration. This is confirmed by the estimated unit root test values of the real interest rate differentials presented in column five of Table 6.1. The results are identical to the test results for New

[7] The real rate differential may not equal zero because of differing default characteristics of the debt instrument.

[8] For the truncation lag length automatically selected (l = auto) using Eviews 4.1 econometrics software package, refer to Table A6.1 in Appendix 6.1.

Zealand, which indicates that the real interest rate differentials have inherited the same stochastic nature of New Zealand's real interest rates. However, as shown below the estimated ARFIMA models do not support this view.

Table 6.1: The PP and KPSS Unit Root Test Results for Australia, New Zealand and the Real Interest Rate Differentials

Test Statistic	Australia (1969:3-2002:2)	Australia (1978:1-2002:2)	New Zealand (1978:1-2002:2)	$(r_{t,NZ} - r_{t,Aus})$
Ex post				
PP test ($l = 4$)	-5.375‡	-5.639‡	-4.261‡	-6.017‡
PP test (l = auto)	-6.289‡	-5.639‡	-4.140‡	-6.017‡
KPSS test ($l = 4$)	1.189‡	0.346	0.777‡	0.885‡
KPSS test (l = auto)	0.654†	0.270	0.624†	0.767‡
HP-*ex ante*				
PP test ($l = 4$)	-1.772	-2.633*	-2.409	-2.469
PP test (l = auto)	-1.773	-2.566	-2.427	-2.463
KPSS test ($l = 4$)	1.260‡	0.327	1.045‡	1.280‡
KPSS test (l = auto)	0.689†	0.240	0.709†	0.918‡
ES-*ex ante*				
PP test ($l = 4$)	-2.083	-3.110†	-2.741*	-2.776*
PP test (l = auto)	-2.102	-3.110†	-2.693*	-2.655*
KPSS test ($l = 4$)	1.113‡	0.313	1.211‡	1.211‡
KPSS test (l = auto)	0.617†	0.237	0.810‡	0.967‡

Notes: *, significant at the 10% level; †, significant at the 5% level; ‡, significant at the 1% level. ($l = 4$) indicates that the truncation lag is equal to 4, (l = auto) indicates that the truncation lag is automatically chosen in EViews 4.1 following the methodology of Newey and West (1994).

Following the methodology and model selection strategy outlined in chapter four the ARFIMA models were estimated for the real interest rates and real rate differentials. Table 6.2 presents the estimated ARFIMA models for Australia over the entire available sample period (1969:3-2002:2). The point estimates of d follow the same pattern as the results of the majority of countries shown in chapter five. In panel (i) of Table 6.2 model 1 is chosen to represent the *ex post* real rate of Australia due to the significant seasonal AR component. The portmanteau test statistic for the mean of the *ex post* real rate indicates that model 1 represents the series adequately. In panel (ii) model 1 is selected even though the Q(10) statistic is larger than the critical value, which indicates that serial correlation may still be present in the residuals of model 1. However, given the model selection criteria the competing models (models 2 and 3) have large estimated standard errors for the AR and

moving average (MA) coefficients, justifying the choice for model 1. In panel (iii) model 1 is again selected due to the insignificant short-run dynamic components present in models 2 and 3. The estimated d parameters in panel (iii) are relatively stable and the Q(10) statistic indicates that model 1 characterises the time series properties of the data. For each selected model in Table 6.2 the $Q(10)^2$ statistic indicates that serial correlation is present in the second moment, this issue is addressed below.

Table 6.2: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of Australia for the Sample Period 1969:3-2002:2

Estimates	d	μ	AR/MA	Seasonal AR/MA	Q(10)	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.363 (0.078)	1.504 (2.715)	-	ϕ_4=0.274 (0.130)	10.11	27.51	-917.57
2	0.645 (0.201)	-0.349 (7.908)	θ_1=0.398 (0.264)	ϕ_4=0.180 (0.137)	4.27	30.35	-888.66
3	0.383 (0.067)	1.346 (1.498)	-	θ_4=-0.225 (0.110)	12.49	25.74	-931.63
HP-ex ante				**Panel (ii)**			
1	1.070† (0.116)	0.779 (0.072)	-	-	19.47	50.92	-201.16
2	0.837 (0.940)	-0.123 (6.099)	ϕ_1=0.287 (1.130)	-	15.28	45.92	-200.38
3	0.965 (0.207)	0.751 (0.097)	θ_1=-0.141 (0.255)	-	17.38	51.94	-200.63
ES-ex ante				**Panel (iii)**			
1	0.854† (0.076)	1.759 (0.178)	-	-	8.01	21.90	-266.84
2	0.862 (0.209)	0.064 (5.017)	ϕ_1=-0.014 (0.283)	-	8.24	21.88	-266.76
3	0.870 (0.184)	1.775 (0.217)	θ_1=0.023 (0.237)	-	8.13	21.57	-266.82

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model. Akaike Information Criterion (AIC).

Tables 6.3 and 6.4 present the estimated ARFIMA models for both Australia and New Zealand for the equivalent sample period. Table 6.3 shows the estimated ARFIMA models for Australia over the reduced sample period (1978:1-2002:2). In panels (i), (ii) and (iii) the estimated d parameters are sensitive to the specification of the AR and MA components. In panel (i) model 1 is selected for the *ex post* real rate due to the insignificant AR and MA components displayed in models 2 and 3. In panel (ii) and (iii) model 1 is also

chosen for both the HP-*ex ante* and ES-*ex ante* real rates; yet, in both these cases shown in model 2 the results do not clearly distinguish between the long-run dynamic component (d) and the short-run AR(1) component. This is important to consider since if model 1 is selected it suggests that the series contains a unit root, in contrast if model 2 is selected then this model can not reject a short memory I(0) process. However, in spites of this, the estimated AR(1) component of model 2 panels (ii) and (iii) can not reject an estimated coefficient equal to one, therefore indicating that the series contains a unit root, this problem seems to be an issue generated by the reduced sample size. Overall, in comparison with Table 6.2 the estimated d parameters of each selected model in Table 6.3 are similar in the reduced sample period as they are in the entire sample period in the case of Australia.

Table 6.3: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of Australia for the Sample Period 1978:1-2002:2

Estimates	d	μ	AR/MA	Seasonal AR/MA	Q(10)	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.410 (0.096)	2.691 (1.546)	-	-	8.83	9.33	-490.89
2	0.538 (0.112)	2.828 (1.830)	ϕ_1=-0.230 (0.162)	-	6.66	8.09	-477.47
3	0.770 (0.287)	0.821 (1.185)	θ_1=0.481 (0.277)	-	5.85	8.65	-475.12
HP-ex ante				**Panel (ii)**			
1	1.045† (0.128)	-2.310 (0.137)	-	-	18.92	36.32	-159.83
2	0.161 (0.170)	4.750 (1.590)	ϕ_1=0.834 (0.104)	-	15.73	33.55	-154.87
3	0.945 (0.235)	-2.197 (0.346)	θ_1=-0.136 (0.289)	-	17.31	36.77	-159.40
ES-ex ante				**Panel (iii)**			
1	0.904† (0.109)	-1.568 (0.494)	-	-	13.90	19.79	-182.25
2	-0.015 (0.163)	4.259 (0.920)	ϕ_1=0.862 (0.101)	-	12.50	15.30	-175.47
3	0.864 (0.206)	-1.526 (0.559)	θ_1=-0.053 (0.247)	-	13.49	20.20	-182.15

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model. Akaike Information Criterion (AIC).

Table 6.4 illustrates the estimated ARFIMA models for New Zealand over the sample period of 1978:1-2002:2. Panel (i) reveals the relatively stable estimates of d for the *ex*

post real interest rate of New Zealand. Model 1 is selected due to the insignificant AR and MA components of models 2 and 3. In panel (ii) model 2 is selected for the HP-*ex ante* real rate given the significant MA component. In panel (iii) the estimated *d* parameter is again sensitive to the model specification, yet model 2 is selected for the ES-*ex ante* real rate. Even though the MA component is not significant from zero in model 2 the Q(10) statistic is significantly smaller than the Q(10) statistics of models 1 and 3, therefore justifying the selection of model 2. As for Australia the $Q(10)^2$ statistic for the selected model for each real rate is larger than the critical value which justifies the inclusion of a model for the second moment. In comparison, the estimated *d* parameters between each *ex post* real interest rate for New Zealand and Australia suggest that the 95 percent confidence intervals overlap each other. This does not rule out the possibility that both real rates are integrated of the same order. This is seen for both *ex ante* real interest rates of Australia and New Zealand. The finding that each real interest rate of New Zealand is integrated of the same order as the Australian counterpart suggests that a form of cointegration may exist between each real rate that may verify the existence of RIP. Although given this comparison the results presented in Table 6.5 are less supportive of RIP.

Table 6.4: Three Chosen Competing Estimated ARFIMA Models for Each Real Interest Rate of New Zealand

Estimates	d	μ	AR/MA	Seasonal AR/MA	Q(10)	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.541 (0.173)	-0.673 (3.033)	-	-	2.55	32.00	-948.25
2	0.498 (0.222)	1.201 (4.687)	ϕ_1= 0.047 (0.412)	-	2.61	29.80	-945.36
3	0.536 (0.134)	-0.629 (2.729)	θ_1= -0.007 (0.276)	-	2.54	31.83	-948.23
HP-ex ante				**Panel (ii)**			
1	0.424 (1.819)	4.669 (1.910)	ϕ_1=0.653 (1.748)	-	19.41	14.40	-171.10
2	0.860† (0.159)	-6.231 (0.428)	θ_1=-0.348 (0.120)	-	19.26	20.44	-173.86
3	0.873 (0.194)	-6.255 (0.454)	θ_1=-0.333 (0.101) θ_2=0.022 (0.142)	-	19.07	20.63	-173.83
ES-ex ante				**Panel (iii)**			
1	1.015 (0.136)	-7.153 (0.525)	-	-	20.84	22.27	-194.50
2	0.581 (0.161)	4.439 (3.424)	ϕ_1=0.372 (0.209)	-	14.11	33.02	-184.70
3	0.842 (0.103)	-7.082 (0.537)	θ_1=-0.281 (0.098)	-	21.99	34.66	-188.91

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root for a selected ARFIMA model. Akaike Information Criterion (AIC).

Table 6.5 displays the estimated ARFIMA models for the real rate differentials. The first noticeable part of Table 6.5 is that the estimates of d for all the real rates vary substantially with differing model specification. This paints a very uncertain picture of the validity of RIP between Australia and New Zealand. In panel (i), model 1 seems to be an adequate representation of the *ex post* real rate differential data given the low Q(10) and $Q(10)^2$ statistics. However, in comparison to the selected models for the *ex post* real interest rate of New Zealand and Australia, there is no significant difference in the estimate of d for the real rate differential. In panel (ii) model 2 is chosen given the significant AR component and the low estimated Q(10) statistic. In comparison with the selected models of the HP-*ex ante* real rates of Australia and New Zealand in Tables 6.3 and 6.4 respectively, the estimated d parameter of the HP-*ex ante* real rate differential is not very suggestive in terms

of RIP. Even though the point estimate of the *d* parameter for the HP-*ex ante* real rate differential is small in comparison to the *d* estimates of the HP-*ex ante* real rates, the standard error of this estimate is relatively large and encompasses a large range of long-run dynamic behaviour. In panel (iii) it can be seen that model 3 adequately describes the ES-*ex ante* real rate differential given the significant MA component as well as the low Q(10) statistic.

Table 6.5: Three Chosen Competing Estimated ARFIMA Models for the Real Interest Rate Differentials of New Zealand and Australia $(r_{t,NZ} - r_{t,Aus})$

Estimates	d	μ	AR/MA	Seasonal AR/MA	Q(10)	$Q(10)^2$	AIC
Ex Post				**Panel (i)**			
1	0.380 (0.116)	-1.720 (1.973)	-	-	9.77	14.61	-1177.76
2	0.516 (0.261)	-2.306 (3.621)	ϕ_1=-0.183 (0.327)	-	10.36	19.90	-1167.88
3	0.450 (0.176)	-2.026 (2.306)	θ_1=0.091 (0.180)	-	10.14	17.21	-1173.46
HP-ex ante				**Panel (ii)**			
1	0.947 (0.159)	-4.172 (0.096)	-	-	18.76	31.81	-222.83
2	0.367* (0.304)	-0.409 (2.591)	ϕ_1=0.639 (0.282)	-	15.14	39.90	-213.33
3	0.658 (0.128)	-3.986 (0.578)	θ_1=-0.429 (0.274)	-	15.93	50.50	-213.87
ES-ex ante				**Panel (iii)**			
1	0.174 (0.303)	-0.490 (1.723)	ϕ_1=0.752 (0.248)	-	14.67	34.02	-269.35
2	0.782 (0.197)	-4.877 (0.750)	θ_1=-0.130 (0.165)	-	24.26	20.23	-279.98
3	0.567 (0.122)	-4.123 (1.553)	θ_1=-0.337 (0.147) θ_2=-0.342 (0.136)	-	15.10	24.08	-264.78

Notes: Shaded row represents selected ARFIMA model for the corresponding real interest rate. A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated *d*, while (†) indicates that *d* is within two standard errors of a unit root for a selected ARFIMA model. Akaike Information Criterion (AIC).

In spite of this, the estimate of *d* is again not significantly different from the *d* estimates of the ES-*ex ante* real rates for Australia and New Zealand. The results presented here do not indicate a straightforward conclusive finding. Because there is no statistically significant change in the estimated *d* parameter for the real rate differential there is little support for

the hypothesis that the real interest rates between Australia and New Zealand share a significant similarity, or in other words share a cointegrating relationship. Although the results are inconclusive this is not surprising given the very mixed findings in the literature and also for RIP to hold two other strict equilibrium conditions must also hold, that is both RPPP and UIP are requirement to hold concurrently.

In the last step, Table 6.6 presents the re-estimated models which include a GARCH(1, 1) innovation for selected models that have $Q(10)^2$ statistic values larger than the critical value. Overall, the estimated *d* parameters have not changed significantly in comparison with the selected models presented in Tables 6.2-6.5. One exception is the estimated *d* of the HP-*ex ante* real rate differential presented in panel (iv). Although this estimate is now significantly below the estimates of *d* for the HP-*ex ante* real interest rates, the estimate of *d* for the HP-*ex ante* real rates presented in panels (ii) and (iii) in Table 6.6 are now significantly different, therefore offering no support for a cointegrating relationship.

Although the results presented in section 6.4 are not conclusive this preliminary analysis leaves the study of RIP between New Zealand and Australia open ended. Further research of RIP between Australia and New Zealand can take many different directions. The first major concern is related to the data employed in this study, as several data issues were discussed in section 6.3. As well as the data issues another concern is that of structural change present in the real interest rate series for both Australia and New Zealand. As explained above Flemingham, *et al.* (2000) found the presence of structural breaks in both the real interest rate series of Australia and New Zealand. Due to large institutional changes present in New Zealand during the 1980s and early 1990s studies on RIP may require the inclusion of a structural break either in the levels of the real interest rates or the real interest differential itself. To further extend this tentative analysis, the inclusion of other major trading and financial partners could be introduced such as Japan, Korea, the United Kingdom and the United States.

Table 6.6: Estimated ARFIMA(p, d, q)-GARCH (1, 1) Model for Selected Real Rates of Australia and New Zealand

Model	d	μ	AR/MA	Seasonal AR/MA	ω	α	β	$Q(10)$	$Q(10)^2$
Australia (1969:3-2002:2)				**Panel (i)**					
Ex post	0.415	1.882	-	ϕ_4=0.239	6.793	0.226	0.487	7.68	9.41
	(0.087)	(3.454)		(0.098)	(2.781)	(0.283)	(0.338)		
HP-*ex ante*	1.349	0.789	-	-	0.125	0.455	0.604	10.98	5.19
	(0.104)	(0.038)			(0.123)	(0.141)	(0.285)		
ES-*ex ante*	0.890†	1.832	-	-	0.885	0.148	0.768	4.13	2.61
	(0.126)	(0.023)			(0.579)	(0.086)	(0.115)		
Australia (1978:1-2002:2)				**Panel (ii)**					
HP-*ex ante*	1.337	-3.081	-	-	0 (0)	0.585	0.631	11.10	4.73
	(0.099)	(0.310)				(0.279)	(0.171)		
ES-*ex ante*	0.899†	-1.568	-	-	1.029	0.079	0.828	9.42	6.95
	(0.157)	(0.896)			(0.614)	(0.067)	(0.056)		
New Zealand (1978:1-2002:2)				**Panel (iii)**					
Ex post	0.557	-3.194	-	-	3.667	0.209	0.793	7.51	1.28
	(0.056)	(2.564)			(2.743)	(0.154)	(0.065)		
HP-*ex ante*	0.884†	-6.447	θ_1=-0.529	-	0.387	0.294	0.717	13.81	11.82
	(0.145)	(0.129)	(0.141)		(0.397)	(0.245)	(0.049)		
ES-*ex ante*	0.640	3.013	ϕ_1=0.245	-	0.730	0.184	0.720	13.25	11.43
	(0.116)	(5.118)	(0.165)		(0.280)	(0.069)	(0.135)		
$(r_{t,NZ} - r_{t,Aus})$				**Panel (iv)**					
HP-*ex ante*	0.144*	2.625	ϕ_1=0.792	-	0.202	0.411	0.613	5.55	3.20
	(0.143)	(0.940)	(0.113)		(0.114)	(0.096)	(0.062)		
ES-*ex ante*	0.648†	-4.237	θ_1=-0.242	-	0.967	0.198	0.741	11.98	8.86
	(0.183)	(3.087)	(0.155)		(1.109)	(0.139)	(0.219)		
			θ_2=-0.207						
			(0.110)						

Notes: A (-) indicates models where AR or MA components were not estimated. Numbers in parenthesis are the standard errors. A (*) indicates that a I(0) process is within two standard errors of the estimated d, while (†) indicates that d is within two standard errors of a unit root.

6.5 Conclusion

This chapter has extended the empirical analysis of chapter five by looking at the real interest rates and also the real rate differentials of Australia and New Zealand. Given the close economic relationship between these two countries, it was expected that real interest rates would share a high degree of continuity and some form of real interest rate parity would hold. The order of integration is estimated using the ARFIMA model. However, because the estimated d parameters of the ARFIMA models of the real rate differentials vary substantially between different model specifications, and there is no significant change in the estimated order of integration of the real rate differentials, the results are not very conclusive.

Although the empirical results reported in this chapter are not overwhelmingly conclusive, this analysis suggests further research of real interest rate parity between New Zealand and other countries. Because the Treasury Bill Rate data used for New Zealand in this study had missing observations, future research could look at obtaining a complete data set and/or utilising a more sophisticated extrapolation technique. A variety of different interest rates could be employed for differing assets and maturities. As well as data issues, further study could incorporate structural change in the model for the real interest rate series in order to control for major structural changes that occurred during the main economic reform period of the 1980s

Appendix 6.1

Table A6.1: Automatic Bandwidth selections for PP and KPSS tests of Australia and New Zealand

Test	Australia (1969:3-2002:2)	Australia (1978:1-2002:2)	New Zealand (1978:1-2002:2)	$(r_{t,NZ} - r_{t,Aus})$
Ex post				
PP	9	4	1	4
KPSS	9	6	6	6
HP-*ex ante*				
PP	6	3	2	3
KPSS	9	7	7	7
ES-*ex ante*				
PP	5	4	6	1
KPSS	9	7	7	6

Notes: Where $(r_{t,NZ} - r_{t,Aus})$ represents the real interest rate differential. Eviews 4.1 automatic Bandwidth selection is based on Bartlett Kernal estimator using the Newey and West (1994) procedure.

Chapter Seven Summary, Conclusion and Further Research

7.1 Introduction

According to the Fisher (1930) equation, the nominal interest rate should move one-for-one with expected inflation in the long-run. Although a positive relationship between the nominal interest and the inflation rate seems to exist, there is limited support in the literature for the Fisher effect in its strictest form. Even though there have been various reasons suggested for the so called 'Fisher's Paradox', no single explanation has conclusively resolved this issue. An important issue regarding the validity of the Fisher effect is the stationarity of the *ex ante* real interest rate. If the *ex ante* real interest rate does, in actual fact, contain a unit root then any shock to either the nominal interest rate or expected inflation would have a permanent impact on the *ex ante* real rate, and therefore contradict the classical long-run neutrality proposition. However, for the long-run Fisher effect to hold the *ex ante* real interest rate needs to be, at least, mean-reverting.

In recent studies, the mean-reverting properties of the real interest rate have been examined in order to test the validity of the Fisher effect (Lai, 1997; Tsay, 2000). These studies have examined whether or not the real interest rate contains a fractional order of integration (d). A fractional I(d) process offers a wider range of mean-reversion behaviour compared to the restrictive integer orders of integration. In a specific study, Sun and Phillips (2003) found evidence to suggest that univariate estimates of d for the *ex post* real interest rate is biased downwards for the United States. It is thought that because realised inflation contains a volatile unexpected component, it follows that the *ex ante* real interest rate appears to be more stationary because the inherited volatility masks the actual persistent movement of the *ex ante* real interest rate. This study extended this idea by examining d for both the *ex post* and *ex ante* real interest rate for various Asian, Pacific and European countries. The major conclusions and findings are presented in section 7.2, section 7.3 provides some suggestions for further research.

7.2 Chapter Summary and Conclusion

The primary focus of this study has been to examine the degree of persistence of three different real interest rates for nineteen Asian, Pacific and European countries. The secondary focus of this study has been the examination of the real interest rate parity (RIP) between New Zealand and Australia. Chapter one provides a brief overview of the main ideas and issues faced by relevant past and recent studies. More importantly chapter one explains the main aims and objectives of this study in relation to recent empirical studies.

Chapter two provides a review of the relevant literature associated with the Fisher effect as it is used to calculate the real interest rate for this research. Also discussed in chapter two are the main theory and concepts of the long memory model.

Overall, the vast empirical studies related to the Fisher effect have produced mixed results. Early studies of the Fisher effect (including those undertaken by Fisher himself) found a certain degree of support for the Fisher effect using adaptive expectations. In contrast, latter empirical studies in which the rational expectations hypothesis (REH) was integrated found mixed results. Also, at this stage there seemed to be a consensus that emerged that suggested that the *ex ante* real interest is not constant, which contrasted Fisher's original proposition of a constant *ex ante* real rate. With advances in time series analysis attention was given to examining the stationarity of the variables within the Fisher equation. Many of these studies used cointegration analysis in order to determine the validity of the Fisher equation, and although some support was found, overall the results of these studies were mixed. Recent empirical studies found results which suggest that the *ex ante* real interest rate is fractionally integrated. The implication of this result is that it offers support for the long-run Fisher effect because fractional integration characterises mean-reverting characteristics. Overall, the literature associated with the Fisher equation does not find conclusive evidence of the Fisher effect in its strictest form, however studies regarding the United States do suggest that there is a positive relationship between the nominal interest and the inflation rate. The results of international studies have not been so straightforward.

Various reasons have been suggested to explain the inadequate empirical support for the Fisher effect in its strictest form, many of which seem plausible, yet the empirical evidence is not consistent.

Chapter two also explains that traditionally integer orders of integration have been used to model the stochastic nature of time series. However, this is restrictive in the sense that mean-reversion is characterised only by the short memory I(0) process. However, the long memory autoregressive fractionally integrated moving average (ARFIMA) model can be used which models a wider range of mean-reverting behaviour. Therefore, the ARFIMA model is employed to examine the validity of the Fisher effect.

The technique employed in order to deal with the issue of expected inflation is essential to the study of the real interest rate. Therefore, chapter three examines the main issues in regards to expected inflation, as well as the methods employed in this research to proxy expected inflation. As explained in chapter three many studies have used realised inflation to model inflation expectations, assuming that forecast errors follow the REH. However, the use of realised inflation can cause problems for the issue of errors invariable. As explained in recent literature this assumption is not necessary and the minimum requirement is that forecast errors are stationary. In chapter three various methods of expected inflation that have been employed in the literature are discussed.

The three methods employed in this study with the purpose of modelling expected inflation are described in chapter three. Different methods are used in order to compare the effect this has on the estimate order of integration of the real interest rate for the nineteen countries studied. First, realised inflation is used to represent expected inflation, from this the *ex post* real interest rate is generated. Second, the Hodrick-Prescott (HP) filter technique is applied in order to generate a series of expected inflation, from this the *ex ante* real rate is calculated. And finally, the exponential smoothing (ES) technique is used, which also generates a series of expected inflation, again from this the *ex ante* real interest rate is calculated. As shown in chapter three each of the generated expected inflation series differ in the amount of volatility contained in the series. It is also shown that this volatility

is inherited within the corresponding real interest rate. From this it is concluded that the volatility inherited into each real interest is expected to impact the estimated order of integration.

Chapter four discusses the data, unit root tests, methodology and model selection strategy employed in this study. Because of consistency, the data used in this study was obtained from the International Monetary Fund's (IMF) *International Financial Statistics* (IFS) database. The data consisted of the relatively risk free interest rate data (money market rate (MMR), the Treasury bill rate (TBR)) and price index data (consumer price index (CPI)) in order to calculate the inflation rate. As explained in chapter four the two unit root tests employed in this study have the opposite null hypothesis, it is explained that if both of the null hypotheses are rejected this provides evidence that suggests that the particular time series may be fictionally integrated. The order of integration is estimated directly using the ARFIMA model, in addition a generalised autoregressive conditional hetroskedastic (GARCH) innovation is also estimated for selected models in which non-linearity is present. The estimation procedure utilised is a conditional tome-domain maximum likelihood (ML) estimation procedure, which is parametric. Because the method used to estimate the order of integration is a parametric approach, chapter four outlines the strategy used to select an appropriate ARFIMA model. The strategy outlined is constructed with the purpose of estimating a parsimonious model that adequately represents the particular real interest rate.

For the real interest rates of seventeen of the nineteen countries the empirical results are presented in chapter five. The unit root test results are initially presented and for the *ex post* and both *ex ante* real interest rates a pattern emerges. Firstly, for the *ex post* real interest rate the majority of countries reject both unit root tests indicating evidence of an I(d) process. For the remaining countries the unit root tests suggest that the series are a stationary I(0) process. For both the *ex ante* real interest rates the unit root tests suggest that there is a significant proportion of countries that can not reject the possibility that they contain a unit root. Overall, the results indicate that the estimated d for the *ex ante* real interest rates are larger than the *ex post* real interest rate. The estimated ARFIMA models

are then presented and a distinct pattern appears; firstly, for all countries the estimated value of *d* for both *ex ante* real interest rates is larger than the *ex post* real rate. Secondly, and more significantly, once the GARCH(1, 1) innovation is added to the ARFIMA model it is found that for twelve of the seventeen countries the estimate of *d* for the ES-*ex ante* real rate lies between the estimate of *d* for the other real interest rates. This result is in-line with the expected result suggested in earlier chapters. This result implies that because there is more short-run volatility present in the *ex post* real rate the actual persistent nature of the *ex ante* real interest rate is buried within the fluctuations. Therefore this generates a downward bias in the estimate of *d* for *ex post* real interest rates, a pattern which is found among different countries. This has a significant implication in terms of the validity of the Fisher equation; that is, the degree of persistence of the *ex ante* real interest rate will depend on how actual expected inflation is generated by economic agents. For example, if economic agents are long-run focused in generating expected inflation, expected inflation will be more persistent and hence will generate an *ex ante* real interest rate that is also more persistent.

An extension of the empirical results presented in chapter five is contained in chapter six. Specifically, chapter six provides preliminary analysis of real interest rate parity (RIP) between New Zealand and Australia. RIP represents the equilibrium condition between domestic and foreign real interest rates. If both world commodity and capital markets are integrated then the real interest rates on comparable assets should equilibrate across countries and time. As explained in chapter six early empirical studies on RIP failed to find a great deal of evidence of real interest rate equalization. In contrast, recent empirical studies found some support for RIP between certain countries.

The initial empirical results presented in chapter six, follow the estimate of the ARFIMA model for the real interest rates of both New Zealand and Australia. The same broad pattern presented in chapter five is also displayed for both New Zealand and Australia in chapter six. Although it was expected that real interest rates would share a high degree of continuity and some evidence of RIP would be found, the actual empirical results were not so clear-cut. Using the same methodology described in chapter four the *d* parameter was

estimated for the real interest rate differentials. However, the estimated d parameter differed substantially between different model specifications, and there was no substantial change in the estimated d of the real interest rate differentials. The results do not conclusively support the hypothesis that real interest rates of New Zealand and Australia share a significant cointegrating relationship.

7.3 Further Research

Throughout this study there have been several suggestions regarding certain aspect of the current research which could be extended. Because the unobservable variable expected inflation is crucial to the study of the *ex ante* real interest rate, focus of further research could examine alternative methodologies of modelling this variable. In chapter three it is suggested that more complicated methodologies could be employed in order to generate the expected inflation variable. Specifically, the ARFIMA model could be applied to inflation data and in-sample inflation forecasts could be generated in order to represent expected inflation. As pointed out in chapter three this method would have the benefit of modelling the short memory autoregressive (AR) and moving average (MA) components, as well as the long memory order of integration. In addition other models could be applied to the second moment of inflation, such as the fractionally integrated generalised autoregressive conditional hetroskedastic (FIGARCH) model, with the purpose of capturing both short and long memory behaviour of conditional variance. It would also be possible to incorporate regime switching models such as the Markov switching model with the purpose of improving the forecasts of the ARFIMA model.

Another possible extension of this research which is mentioned in chapters two, four and five is the application of ARFIMA-FIGARCH model to the real interest rate. As well as implementing this model for expected inflation, the short and long memory behaviour of the conditional mean and variance could also be modelled for the real interest rate.

In chapter six the preliminary study of RIP between New Zealand and Australia created some areas for further research. Firstly, the TBR data employed for New Zealand was

incomplete, therefore a complete data set could be employed or otherwise a more rigorous extrapolation method could be utilised to generate the missing observations. Another consideration regarding the data that could be addressed in future studies is the choice of the inflation rates and nominal interest rates to be modelled. As explained in chapter six, the nominal interest rate that is used is important, for example some studies suggest using Eurocurrency rates as they have the same political and default risk. Also, the price index employed is also important depending on what economic decision is being studied, and therefore different price indexes could be utilised. In addition to the data, possible structural change could be controlled for within the real interest rate series of both New Zealand and Australia. Two approaches could be employed; first, structural change could be controlled within each of the real interest rate series of New Zealand and Australia. Second, structural change could also be controlled within the real interest rate differential itself. By controlling for possible structural change, the integration of New Zealand and Australian real interest rates could become less ambiguous.

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