THE RELATIONSHIP BETWEEN SPATIAL ABILITY AND MATHEMATICAL ABILITY

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Abstract

The purpose of this study was to examine the relationship between spatial ability and general mathematical ability. Many researchers have assumed that a positive correlational relationship exists between mathematics and spatial ability. However, a review of the literature shows that the relationship is not as simple as thought, partly because there is disagreement among researchers on a definition of spatial ability. In the present study general mathematical ability was indexed by the Progressive Achievement Test: Mathematics. A group of 50 high ability and a group of 50 low ability children completed five tests relating to spatial ability from the Kit of factor Related Cognitive Tests. Results from a discriminant function analysis supported the hypothesis that a positive correlational relationship exists between spatial ability and general mathematical ability. This result is important because it provides new evidence to support the argument that there is a relationship between spatial ability and general mathematical ability. The potential for spatial ability tasks to aid in the understanding of mathematics is discussed. However, it is argued that there is a need for greater refinement of the spatial ability construct before more research using it as a factor is conducted.

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Chapter 1

Spatial Ability

Human Intellectual Ability

From the earliest beginnings of psychology, human intellectual ability has been a topic of interest. In 1869, Galton (cited in Guilford, 1967) began to investigate the nature of individual differences in human abilities. Galton believed that physical attributes, such as motor abilities, were markers of intellectual ability. Thus the type of sensory information he tested was confined to features such as reaction time and hand grip strength of a person. Other researchers such as Kraeplin, Ebbinghaus, and Binet were not convinced by Galton's notion of "good senses, good intellect", and developed tests that were more "mentally" oriented. These mental tests were concentrated on more cognitive functions such as reasoning involving relations, recognition of forms, and specific memory tests.

Binet was vocal in his criticism of Galton's sensory tests in that they were too simple. In 1896 Binet together with Henri, proposed ten more complex mental functions to be tested: memory, imagery, imagination, attention, suggestibility, comprehension, aesthetic appreciation, moral sentiment, motor skill and judgement of visual space (Guilford, 1967). By 1905 Binet had formalised his theories into the first modern individual intelligence test. While quite crude in some areas, the test had many subtests still used in today's revision such as repeating digits, defining simple words, and visual memory. Other tests with the same basic objective of measuring complex mental activities were developed around the same time, notably the Wechsler scale (see

Anastasi, 1990, for details of the Wechsler scale). However, it is not the purpose here to discuss in depth the development of individual intelligence tests. The development of the tests is related closely to the development of the mental factors themselves. That is, with increased measurement of complex mental functions by early individual difference psychologists, cognitive factors became more noticeable in their own right. Researchers began to study constructs such as memory, imagery, verbal comprehension, and reasoning as a constructs removed from the general intellectual function. Included in these constructs was spatial ability, which forms the focus of this chapter.

Early Studies: Finding a Separate Spatial Factor

Identification of a spatial-type factor began during the 1920's and 1930's with researchers using factor analysis to separate a sub-factor of intellectual ability labelled Spatial Ability (e.g. Brown & Stephenson, 1933; El-Khoussy, 1935, cited in Clements, 1983; Smith, 1938). At that time many of the studies were looking at spatial-type factors as secondary factors from intellectual ability. Earlier studies had been conducted and hypotheses posited as to the nature of human spatial ability, the best example being Galton's work on human abilities (Bishop, 1980; Bootzin, Bower, Zajonc & Hall, 1986), but the more exact identification of such constructs was not seen until the advent of strong statistical methods such as factor analysis.

Murphy (1936), while investigating mechanical ability, indicated the existence of several sub-factors, one of which he labelled, "Mental manipulation of spatial relations" (p 359). El-Khoussy (1935; cited in Lohman, Pelligrino, Alderton, & Regian, 1987) administered both figural tests, which involved reasoning with abstract figures, and reference tests, such as paper folding and embedded figures, to a sample of primary school children. He found a group of correlations which he called K, or creating, maintaining and using spatial imagery, above a general intelligence factor *g*.

British and European researchers concentrated on the Spearman (1927; cited in McGee, 1979) hierarchical model of factors in which there is a single global factor, *g*, at the top of a hierarchy of group factors, and multiple specific factors, *s*. This model suggests that all intellectual abilities (e.g., verbal, mechanical reasoning, spatial relations) correlate with a single factor. Thus any study of sub-factors, such as space relations, was conducted not to examine the factor itself but to show the factor was a correlate of a central factor *g*. However, it was the work of American theorists and researchers, who followed the multiple-factor theory proposed by Thurston (1938), that led to the identification of different spatial factors (Lohman et al., 1987).

The Primary Mental Abilities model (PMA) described by Thurston was a collection of basic abilities which he (and other associates) argued were the foundation or primary modules of higher order abilities and of intelligence itself. Using the then newly developed mathematical procedure of factor analysis, Thurstone examined the patterns within scores from many different types of tests including perceptual tasks, word fluency, arithmetic accuracy, general reasoning, and spatial and geometric relations. From the results of these studies Thurstone developed the first multiple factor test. The multiple factor test was designed to measure the separate mental abilities which Thurstone had identified from factor analytic studies.

Although Thurston's (1938) PMA study offered a new perspective for those studying the spatial factor, it also brought with it limitations. The new idea of Thurston's was to rotate the factor axis of a factor matrix to produce oblique factors. The classical method of rotation was to plot test scores on a set of axes, then rotate the axes keeping them perpendicular to each other. By keeping the axes perpendicular, the grouping of test scores could be interpreted without the aid of any complicated mathematical transformations of the data. This method did have the restriction of not always being powerful enough to produce factors that were not linear, or in some other way not well differentiated from each other. (See Tabachnick & Fidell, 1989, for more detail on factor analytic procedures.) Thurstone rotated the axes to the best fit of the groupings without keeping the axes perpendicular. In doing so it seemed possible, theoretically, to extrapolate to a *g* factor similar to that described by Spearman. However the difficulty lay in trying to interpret such a rotated factor. Groupings of factors that had been rotated obliquely (that is, the factors that correlate with each other, as was the case in Thurston's model) did not have a clear interpretation as a psychological construct (Lohman et al., 1987).

The failure to show that Spearman's model of intellectual ability could be derived from Thurston's model discouraged research into higher order factors, but it did encourage further research into the multiple factor theory of ability. It is this step to more direct analysis of spatial abilities that will be covered next.

The Differentiation of Spatial Ability

The change that came about from the study of multiple factors of abilities was exemplified by researchers such as Thurston (1951, 1967; cited in Clements, 1981), French, Ekstrom, and Price (1963) and Guilford (1967). By looking at the myriad of cognitive factors reported in different studies, these researchers mapped the central constructs reportedly being measured but being called by different names. In doing so a smaller but still vague group of factors, including spatial factors, emerged. At the same time, the work of British researchers was progressing but with the emphasis on an hierarchical interpretation of the spatial factor. The result is that much of the early British work ignores any differentiation within the spatial factor and hence was incompatible with the American approach of multiple factor theory. However, an increase in the amount of studies by British researchers led to a widening of the definition used in the hierarchical model originally proposed by Spearman and so went some way towards uniting the two models, as described below.

Vernon (1961) proposed a new hierarchy for mental abilities that, while still true to the Spearman ideal, incorporated certain aspects of the multiple factor model. At the top of the hierarchy was the *g* factor. Below that were the two major group factors: Verbal-Educational (v:ed) and Practical-Mechanical (k:m). These major group factors branched out to minor group factors (such as verbal, number, mechanical, spatial, and psychomotor), which in turn branched out to specific factors (see Figure 1). The importance of this model is the emphasis Vernon placed on the intercorrelations and contributions of factors at the minor group level.

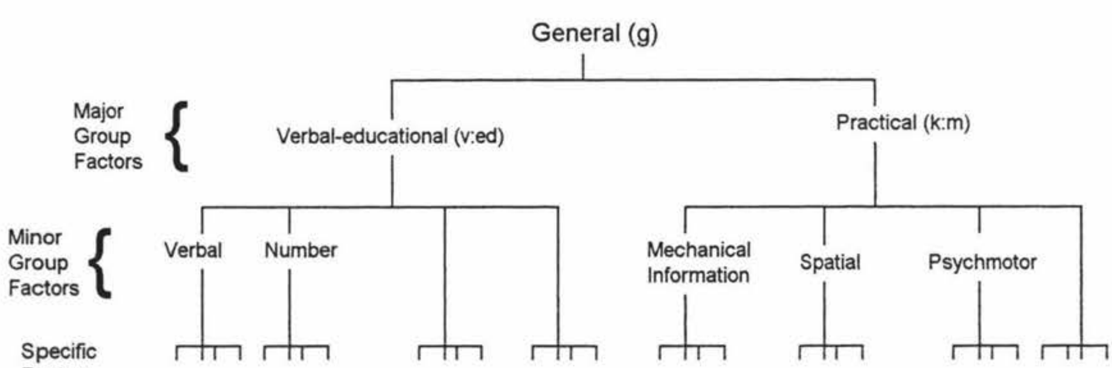

Figure 1. Model of a hierarchical organisation of abilities. (Adapted from Anastasi, 1990, p 387)

So what spatial factors emerged from the abundance of new work? The answer is both many and few. Many due to the nomenclature differences in the various studies, and few based on the robust nature of the factors themselves. Table 1, showing some of the factors found by factor analytic researchers, offers some explanation of this. Thurston's (1950) and Guilford and Lacey's (1947; cited in Guttman, Epstein, Amir, & Guttman, 1990) definitions are good examples of a first attempt to encapsulate and define separate spatial factors (see Table 1). Guilford and Lacey, while producing names of two spatial factors which are now well known to us, Spatial Relations and Visualisation, gave rather convoluted definitions of the two factors.

Table 1. Spatial factors from major factor analytic studies and the tests most commonly used to index them.

Investigator	Factor Label	Factor Name	Factor description	Tests
Guilford & Lacey (1947)	Vz	Visualisation	Ability to imagine the rotation of depicted objects, the folding or unfolding of flat patterns, the relative changes of position of objects in space	Vz: pattern comprehension, mechanical comprehension.
	SR	Spatial Relations	Relating different stimuli to different responses, arranged in spatial order	SR:Instrument comprehension, aerial orientation.
Thurston (1950)	S1		An ability to recognise the identity of an object when it is seen at different angles	S1: Flags, Card Rotation.
	S2		An ability to visualise a configuration in which there is movement or displacement among the internal parts of the configuration.	S2: Surface Development paper puzzles.
	S3		An ability to think about those spatial relations in which the body orientation of the observer is an essential part of the problem.	S3: Cube Comparisons

Table 1 continued

Investigator	Factor label	Factor name	Factor desciption	Tests
French (1951)	S	Space	Ability to perceive spatial patterns accurately and to compare them with each other.	
	SO	Spatial Orientation	Ability to remain unconfused by the varying orientations in which a spatial pattern may be presented.	
	V1		Ability to comprehend imaginary movement in three-dimensional space or the ability to manipulate objects in the imagination.	
Michael, Zimmerman, & Guilford (1957)	SR-O	Spatial Relation and Orientation	Ability to comprehend the nature of the arrangement of elements within a visual stimulus pattern with respect to the examinee's body as a frame of reference.	SR-O: Cubes, Flags, paper puzzles, aerial orientation.
	Vz	Visualisation	Mental manipulation of a highly complex stimulus pattern	Vz: Form Board, directional plotting, pattern comprehension.
French, Ekstrom,& Price (1963)	Vz	Visualisation	Ability to manipulate the stimulus and alter its image	Vz: Form Board, paperfolding, Surface Development
	S	Spatial Orientation	Perception of the position and configuration of objects in space with the observer as reference point.	S: Card Rotations, Cube Comparisons.

Adapted from Guttman, Epstein, Amir, & Guttman, (1990, p 218) and McGee (1979, p 891)

Lohman et al. (1987) point out that the definition of Spatial Relations, 'relating different stimuli to different responses arranged in spatial order', given in Table 1, is another way of saying, 'the ability to compare and contrast rotated or reflected images'. That is acceptable, but as part of the definition of Visualisation Guilford and Lacey (1947) include the 'ability to imagine the rotation of depicted objects', which according to Lohman is the same or extremely similar to the Spatial Relation description. Thurston's (1951) three factors S1, S2 and S3 are in turn different from Guilford and Lacey's (1947, cited in Guttman, Epstein, Amir, & Guttman, 1990) definitions, although a combination of S1 and S2 make up the Vz of Guilford and Lacey (Lohman et al., 1987). The Spatial Relation and Orientation factor of Michael, Zimmerman and Guilford (1957) appeared to be a synthesis of two apparently separate abilities: discrimination with respect to the object and discrimination with respect to the examinee.

The result of all this was mild confusion within multiple factor model research. It seems the latitude afforded by the American model of studying multiple factors had turned back on itself and had begun to diverge rather than converge on a set of useful spatial factors. Different tests were used to index seemingly identical factors (as seen in Table 1) and conversely one test was used to index seemingly different factors. Smith (1964, cited in Guttman, Epstein, Amir, & Guttman, 1990) noted that "*a remarkable feature of the American research on spatial ability is the difficulty the American psychologists are finding in clarifying the distinction between the different spatial factors*" (p. 95). It was not until the late 1970s that a comprehensive review of the spatial ability field, and in particular the multiple factor model, was carried out by McGee (1979). This offered some hope of clarification, and is discussed in the next section.

More Recent Studies

McGee (1979) reviewed spatial ability literature published between 1928 and 1979. In particular, McGee cited evidence from the literature supporting the existence of two spatial factors: Spatial Orientation and Spatial Visualisation. He argued that while there appeared to be much confusion and divergence among the literature, there were underlying patterns of sub-abilities of the spatial factor. The first spatial factor, Spatial Visualisation, was described by McGee as "*the ability to mentally rotate, manipulate, and twist two- and three-dimensional stimulus objects*" (p. 909). The second spatial factor was Spatial Orientation, which McGee described as *"the comprehension of the arrangement of elements within a visual stimulus pattern, the aptitude to remain unconfused by the changing orientations in which a spatial configuration may be presented, and an ability to determine spatial orientation with respect to one's body"* (p. 909). Indeed, a table in McGee's (1979) paper presents much the same information as can be seen in Table 1, but in an interesting fashion. The definitions and names of the factors from four major factor analytic studies are grouped in two columns representing the spatial visualisation factor and the spatial orientation factor. Although only four studies appeared in McGee's study (Ekstrom, French, Harman, & Dermen 1976; French, 1951; Guilford & Lacey, 1947; and Thurston 1950), they were the four most recognised studies in the area at that time.

Although McGee's definitions have survived to the present day, they have taken their share of criticism. Clements (1981) suggested that McGee's definitions were vague and overlap with each other. In particular, Clements argued that the Ekstrom et al. (1976) definition of Spatial Orientation, *"An ability to perceive spatial pattern or to maintain orientation with respect to objects in space; requires that a figure be perceived as a whole"*, (p. 11) which McGee supported and used to develop his own definition of Spatial Orientation, is contradictory. Clements argued that the use of the word 'pattern' suggests there is a second order operation of combining parts of an array involved. However, the second part of the definition states the figure "...must be perceived as a whole" which contradicts the idea of combining parts into a pattern. It was suggested by Clements that this could remedied by the inclusion of perceiving both part and the whole of an object in the definition, although he conceded that this would blur the boundary of the Orientation factor (Clements, 1981).

Lohman's (1979) reanalysis of factor analytic studies presented another view of spatial ability factors. As McGee had, Lohman pointed out the inconsistencies within the literature regarding the spatial factor. He also reported that while there was little agreement among researchers in general, there was definitely a consistency in factor structures since Thurston's work in 1938. Lohman concluded that the evidence indicated there were three separate spatial factors: Spatial Relations, Spatial Orientation, and Visualisation. Spatial Relations was described as the performance on tasks requiring subjects to rotate figures or objects mentally. It represents both the ability to solve the problems as well as the amount of time a person takes to carry out the necessary mental rotations. Spatial Orientation was described as the ability to imagine how a stimulus array would appear from a different perspective. Visualisation was not defined as such by Lohman but described as the loading on tests which had difficult rotation, reflection and folding of complex figures.

Lohman appeared to be vague in his descriptions of the spatial factors. He did not even give a formal definition for the Visualisation factor, just a description of the type of tests it weighted on (Lohman et al., 1987). Lohman's reason for his vagueness was that he believed there were no hard boundaries between spatial factors. Because there is some mental manipulation in all spatial dimensions, there will always be difficulty in trying to produce 'perfect spatial factors' (Lohman, 1979). Lohman defined two kinds of mental manipulation: mental movement and mental construction. Mental operations involving rotation, folding and reflection were regarded as mental movement; tasks requiring reproduction or construction of objects were regarded as mental construction. According to Lohman any spatial task could use both mental construction and mental movement, although it may be difficult to separate out the operations or to discover which operation is being used.

Lohman's (1979) suggestion of mental movement and mental construction appears to take a step back from the work of traditional factorial researchers. Rather than continuing with factorisation to produce increasingly 'pure' factors such as those discussed by McGee, Lohman advocated a halt to that type of research. Lohman and his colleagues (Lohman et al., 1987) went on to deconstruct the idea of producing a global score when assessing spatial ability. The suggestion was to study smaller mental ability factors such as encoding speed, rotation speed and encoding accuracy. The result may be to produce a battery of tasks and measures which have been already been grounded in cognitive theory, thus allowing the final goal of indexing the factor Spatial Ability while bypassing the much studied but still complex spatial factors. Clements (1981) echoes these sentiments by stating, "*... factor analysts may have been continually basing their analyses on tests which, although called 'spatial', in fact measure some unknown combination of spatial and non-spatial traits*" (p. 12)

The deconstruction of the spatial ability construct (as Lohman and his colleagues see it) leads to an interesting observation. The deconstruction takes the multiple factor approach much closer to the pure hierarchical model proposed by Vernon (1961). As outlined earlier in this chapter, Vernon's model had many small specific factors at the bottom of the hierarchy, narrowing to major group factors then finally to *g*. This is similar to Lohman's concept of many small factors (Vernon's specific factors) being the indices of the larger spatial factor (a minor group factor in Vernon's model). This resemblance is important because it represents a shift in theoretical debate, albeit a small one, to a closer and perhaps unified theory of spatial ability.

Chapter 2

The Spatial - Mathematical Link

The main aim of this section is to review the literature on the relationship between mathematical ability and spatial ability. In doing this there is a need to examine the separate parts of the relationship. The previous section highlighted the research and theory related to spatial ability. The following section will discuss the literature on mathematical ability, which is primarily factorial in nature, before considering the link between mathematical ability and spatial ability.

The first step is to examine the nature of mathematics. In simple terms mathematics may be defined as the study of abstract concepts of number and space. Although this gives some very general clues as to the nature of mathematics, it is still too vague. It is similar to saying "To drive a car you need to know clutch, brake, accelerator, and steering wheel", it is *almost* true but missing one important point: interaction. To drive a car you need to know how the clutch and accelerator interact. To do mathematics you need to know how basic concepts like number, addition, multiplication, circle, and cosine interact. The ability to do mathematics, then, requires an individual to not only understand basic concepts like addition and circle, but to understand how they interact with each other to produce useful algorithms.

Perhaps the best way to view mathematics is as a set of tasks. Just as spatial ability requires some form of spatial task to measure it, mathematical ability is measured by mathematical tasks. Thus the set of possible tasks on which an individual could be said to show mathematical ability could be labelled 'mathematics'. Lean and Clements (1981) describe mathematics as the "...*course content, teaching and learning associated* *with the subject 'mathematics', as it is studied in schools and tertiary institutions throughout the world.*" (p. 268). While in the definition of other constructs such as spatial ability or visual perception it would be circular to say that a construct is defined by the tasks which are used to measure it, this is not so with mathematics. Mathematics is a concrete and observable subject. Thus the tasks which are mathematical are also concrete. When a student completes a series of multiplication problems they have completed a mathematical task. They have demonstrated their mathematical ability. That is because certain tasks such as arithmetic, geometry, calculus, trigonometry and algebra are uniquely mathematical. There is no ambiguity as to what kind of tasks they are. This contrasts with spatial tasks, in which there is often uncertainty as to whether the task has any other underlying constructs involved. Of course this argument applies only to the basics of mathematics, those parts of the curriculum such as basic arithmetic, geometry, algebra and calculus (such subjects as used in the present study). At a more abstract level mathematics melds into problems we face in everyday life and the tasks become confounded with extraneous factors.

Early Studies

In the 1950s researchers such as Barakat (1951) and Wrigley (1958) began to investigate mathematical ability as a separate factor from general intellectual ability. Wrigley, in particular, was concerned with the question of whether mathematical ability could be regarded as an integrated whole. The question he addressed was whether there is some existing cognitive factor on which the many aspects of mathematics are dependent. The primary factors Wrigley referred to are those such as verbal ability and numerical ability. This approach is similar to that of Lohman et al. (1987), who suggested that the best way to study the spatial factor was to examine smaller sub-factors which (may) make up the factor.

Wrigley (1958) investigated the structure of mathematical ability by administering a battery of tests to adolescents (average age 14) in England and Ireland. Four types of ability were tested: verbal, spatial, number, and mathematical. Verbal ability was indexed by tests of vocabulary, reading comprehension, and grammar. Spatial ability was indexed by the Moary Advanced Space test. Number ability was indexed by basic tests of addition, multiplication, subtraction and division. Mathematical ability was indexed by tests of algebra, arithmetic and geometry. Wrigley concluded that there was a significant mathematical factor separate from a general ability factor. Wrigley made the further observation that, although there appeared to be a mathematical factor, it was clear that two sub-factors existed. When the mathematical factor and general ability factor were held constant, a numerical factor and a geometrical factor were identified. Wrigley suggested this reflected the idea that mathematicians were of two types: geometrically oriented and analytically oriented. Geometrically oriented mathematicians use primarily visual means to interpret and solve mathematical problems, whereas analytically oriented mathematicians primarily use the rigour of equation and algorithm to interpret and solve mathematical problems.

Around the time of Wrigley's (1958) study, other researchers studying mathematical ability were finding similar results. Barakat (1951), using similar tests to those used by Wrigley, found a mathematical factor with mathematics, numerical and symbolic tests weighting on it. The evidence from these and other studies was that a mathematical ability factor separate from a general ability factor could be distinguished, and that it was characterised by weightings on numerical and geometrical tests. When a piece of research is undertaken which involves mathematical ability as a factor, it will almost always be indexed with basic tests of numerical ability (e.g., arithmetic, algebra, perhaps even calculus at an advanced level) and geometric ability (e.g., trigonometry, graph problems, and mechanics) .

This, then, will be part of the description of mathematical ability used in the present study. That is, mathematical ability is a robust factor statistically different from, but closely related to, general ability. One further aspect to notice is that all of these early factorial (and other) studies isolated a factor in mathematical ability correlating to a spatial factor, namely the geometrical ability component. Although studies such as Barakat (1951) and Wrigley (1958) only looked at the possibility of relationship between the geometrical factor and spatial ability, these findings offer a first step towards the goal of looking for a relationship between mathematical ability and spatial ability.

Recent Studies

No large scale studies of the mathematical ability concept were forthcoming until the 1970s. This coincides with the proliferation of work focusing on the spatial factor. With more sophisticated methodology and a richer theoretical background, researchers began to investigate the possible link between spatial ability and mathematical ability. An important researcher at this time was Richard Skemp. Skemp was an educationalist, mathematician and psychologist, offering him an unique perspective on the role that psychological constructs such as spatial ability and imagery could play in the understanding of mathematical ability. The 'fresh' perspective of this researcher allowed discussion of how spatial ability could be related to mathematical ability from an information processing approach.

Skemp introduced his ideas in a book called *Intelligence, Learning, and Action* (1979). He suggests that when confronted with a mathematical problem people recall a particular 'cognitive framework'. In a well developed cognitive framework there will be a present state, p, in which is placed all the available information about the problem facing us; a goal state, g, the solution of the problem; and a director system, a set of links between the p and g states. The links between the p and g states can be interconnected, thus allowing different paths to the solution. This framework, while not a mental representation of anything physical, is still a 'map' of our mathematical knowledge. In this way, Skemp suggests, we need the ability to store and manipulate mental images in order to carry out basic mathematical tasks.

A second important discussion Skemp (1986) develops is that of imagery within mathematical language. According to Skemp one of the most distinctive features of mathematics is the 'dual language'. By this Skemp means the use of symbols which can "*...be read aloud, or communicated without ever taking visual form.*" (1986, p 88). By looking through mathematics text books, one can see that the use of symbols is an integral part of mathematics, from the graphical representation of equations to the more abstract symbols used to denote actions such as addition and integration. To the non-mathematician there are only certain parts of mathematics which appear to have a clear spatial link, such as geometry and perhaps trigonometry. These have been shown to be correlated to spatial ability as far back as Barakat's study (1951). However, careful analysis of the content of mathematics will disclose the vast amount of symbol use right across the mathematical spectrum.

A good example of imagery in mathematics, and one that Skemp (1986) employs, is calculus. In calculus almost all of the important information is presented in a shorthand version using symbols. The symbols themselves can be manipulated to produce an answer or they can be translated to a more understandable form for the novice mathematician. Even the simple symbolic description of a set of numbers such as $\{x: x \geq 0\}$, evokes both a verbal description and the image of a graph of the set. One can mentally build up the graph by putting numbers into the set or by translating it into an equation. Within statistics, too, there is shorthand imagery: a z-score statistic can be thought of graphically as the area beyond or within certain limits on the normal curve.

Word problems are yet another example of an application of spatial ability within mathematics. Skemp (1986) believes that being able to successfully translate a word problem into a mathematical equation can be facilitated greatly by the ability to form a mental diagram of the problem (assuming the problem is one able to be visualised). Even though a solution may not come directly from the production of an image of the problem, the image helps in, " *representing the overall structure of the problem. It gives the context from which the particular calculation needed is abstracted* " (Skemp, 1986, p 91).

The formation of mental images and the manipulation of them, as outlined by Skemp, is very similar to the definitions of spatial orientation and spatial visualisation described in the previous section. One important difference is that in mathematics the action performed on the images is less than in the spatial test situation, but the image itself is (usually) much more complex than those images used in spatial tests. It is important that the actions and images be very clear cut in the testing situation so that the construct being measured will be as unconfounded as possible. However, in real mathematical situations the image produced is loaded with numerical information, relations with other images, and linkages to verbal information.

The implication here is that if Skemp is correct about the use of imagery in *general* mathematical problem solving, then there is possibly a relationship between spatial ability and general mathematical ability. The key factor Skemp provides us with is that the complexity of the real life situation is an important variable in the relationship. The underlying factors involved may be the number factor, the geometry factor, any of Thurston's Primary Mental Ability factors, or the k:m factor of Vernon, but these will tend to correlate only with very specific parts of mathematical ability.

Another researcher in the mathematical education area who discusses the use of mental images to facilitate mathematical problem solving is Greeno (1991). Greeno proposed that we construct and operate on mental models of real world situations within conceptual domains. In Skemp's (1986) model of a cognitive framework there was no direct relation between the mental model and the physical actuality of the mathematical problem. This is quite different from Greeno's (1991) model, which is very much analogous to the physical environment. According to Greeno, in this conceptual domain we recreate the immediate environment, be it our neighbourhood map or a mathematical problem. In the case of mathematics, we use the conceptual domain to not only place physical representations of objects but to operate on them using some given rule.

Greeno's conceptual domain model allows us to operate on the mental representation alone, as in rotating, flipping or stretching an object, or to compare two or more mental models. It is the action of mentally operating on images within the domain that provides us with evidence that there is a spatial role in mathematical ability. In this model a procedure such as addition could be represented by the manipulation of imagined groups of objects. The problem $5 + 8 = 13$ may be represented by picturing a set of five blocks and a set of eight blocks, then mentally moving them together and counting the total. As such calculations become formalised and replaced by more advanced techniques the domain can house groups of rules and properties, such as equivalence and subset, for solving more complex problems. The inclusion of symbolism and proposition as part of a conceptual domain makes it a useful tool. Just as in Skemp's model, the opportunity to create complex images allows the full range of spatial factors to be related to mathematical thinking.

Battista (1994) elaborates on Greeno's model and suggests a link between mathematical ability and spatial ability. Although Greeno's model is not explicitly about spatial aspects of mathematics, the discussion of imagery and mental operation on images suggests that there is some basis for believing that spatial ability is involved. Battista makes this point by likening the cognitive domain model to cognitive maps (e.g., Garling, Book, & Lindberg, 1986) and to the constructs of Spatial Orientation and Spatial Visualisation as described by French et al. (1976). Greeno himself stated *"...I propose a conceptual environment with spatial properties"* (1991, p 179). It is this point that Battista elaborates, arguing that not only is the conceptual domain inherently spatial but the operations that are performed within this environment are comparable to those used in spatial domains (Battista, 1994).

Battista (1994) echoes Skemp's concept of using symbols in mathematics to replace more complex language. He argues that, " *with repeated experiences, words for* *important operations on the models become symbols* " (1994, p 93). This idea of the symbolic language of mathematics is perhaps the more important part of this discussion. Many researchers are willing to concede that there is a relationship between spatial ability and certain aspects of mathematical ability, as it is clear that manipulation of symbols in mathematics such as graphs, shorthand notations and geometric figures has a spatial component to it. However, it is more than the basic factors of spatial orientation or spatial visualisation that are involved in the ability to do mathematics. Mathematical symbols, as stated above, can be manipulated in more than one way. They can be translated into written English, or into graphical form, or left as symbols. If Skemp's and Greeno's models hold, then we can say that acting on these symbols will be a spatial task. The symbol *represents* several different modes of communication, but the underlying structure that allows the individual to solve the mathematical problem is spatial. Indicators of spatial components in mathematics include the production of the conceptual framework which is associated with the problem (e.g., subtraction), the translation of the symbols into another form, the manipulation of the resulting translation, and the generation of results from propositional statements using groups of rules governed by spatial relations (e.g. the use of logic symbols such as If, Then, If and Only If, to prove a simple proposition). All these point to a strong spatial component in general mathematics.

Chapter 3

Present Research

The review of the literature on spatial ability demonstrates that researchers have found it difficult to produce a universally agreed definition. Although from a theoretical perspective it is acceptable to disagree on the definition of spatial ability, it does leave researchers with the awkward practical question: what definition should be used? For the present study there is a need for compromise between theoretical debate and practicality. That is not to say pragmatism will dictate the theoretical path of the rest of this work. What it does mean is that in order to accomplish an experimental result in a piece of research of this nature, certain points will be assumed tacitly. Hence a definition that may be less than perfect must be used for the sake of argument.

Of the two main constructs under investigation in the present study, spatial ability has been the most challenging to define. The literature review has identified various sub-factors, with spatial orientation, spatial visualisation and spatial relations being the most robust (Lohman et al., 1987), but researchers seemed to have removed themselves from the task of identifying spatial ability in the singular. This is not an argument for or against any one theoretical model of spatial ability, but rather an observation of the diverging paths of the theorist and the experimental researcher.

There are essentially two types of research done in the spatial ability area. The first type is theoretical research which concentrates on defining and refining the construct itself, usually through large meta-analytic factorial studies. There has been very little done in this fashion since McGee's (1979) study. The second kind is empirical research in which the spatial ability factor is used as a construct in the study. In such studies the spatial ability factor is usually secondary to the main research topic; often spatial ability is simply a factor to be correlated with another factor. One problem to note is the continuing assumption of experimental researchers that when they use a spatial factor in a study they have a pure and isolated factor. From the discussion above relating to the nature of spatial thought in mathematics, it seems questionable to assume that one can isolate and measure (say) spatial visualisation, without in some way accounting for its intercorrelation with spatial orientation.

The difficulty inherent with this is to marry the two broad models of general ability (general factor and multiple factor) into one cohesive model, or at least one model with a minimum of conflicting postulates. Lohman and his colleagues (Lohman et al., 1987) went some way towards bringing the models together when they introduced their idea of deconstructing the separate spatial factors to produce more pure and more measurable sub-factors. Whether this is a practical solution is yet to be shown, but it does offer some insight to a theoretical solution. Measurement of spatial factors at present is still based on the notion that there is a well defined construct or constructs. Yet the majority of studies using a spatial factor begin with statements of how difficult it is to define spatial ability. I would contend that until production of reliable and statistically pure sub-factors of spatial ability occurs, measurement of the current factors in isolation is non-productive. The factor analysts, in their parsimony, have led us astray. Not because they have produced spurious results, but because of the way the results have been interpreted: interpreted by researchers using the spatial ability factor to be more precise and of higher construct validity than the factor analysts should claim. The fault is on both sides and thus is compounded.

The present study will use the definitions of the two sub-factors Spatial Orientation and Spatial Visualisation as described by Ekstrom et al. (1976). Spatial Visualisation will be defined as:

> An ability to manipulate or transform the image of spatial patterns into other arrangements; requires either the mental reconstruction of a figure into components for manipulation or the mental rotation of a spatial configuration in short term memory, and it requires performance on serial operations, perhaps involving an analytic strategy. (p. 173)

and Spatial Orientation will be defined as:

> An ability to perceive spatial patterns or to maintain orientation with respect to objects in space; requires that figures be perceived as a whole. (p. 149)

More important is the way in which the present study will address the issue of measurement. Because at present there are no strong theoretically driven measures of smaller factors as proposed by Lohman (1987; and others such as Guilford, 1967), it is necessary that the most reliable measure available is used. To that end, McGee's concept of Spatial Ability appears to offer the best theoretical approach available. Thus, it is the most useful theory that decides the choice of measurement tools in this research. McGee's (1979) theoretical view is derived from Ekstrom et al. (1976) discussion of Spatial Ability. It follows then that the tests produced by Ekstrom et al. (1976) are the best choice of indices for the Spatial Ability factor in the present study.

There will be five tests used in the present study (discussed in detail in the Method section) associated with the spatial orientation and spatial visualisation factors from the Kit of Factor Referenced Cognitive Tests (Ekstrom et al. 1976). Note, however, that only a global 'Spatial' score will be indexed and not separate visualisation and orientation scores. This researcher suggests that the most efficient method of measuring spatial ability with current tests is to use a wide angle approach. That is, to use many smaller tests designed to measure smaller factors but discuss a *global* spatial score only, to try to minimise intercorrelations within the Spatial Orientation and Spatial Visualisation factors.

With the above definitions of spatial abilities in place, hypotheses can be developed. The discussion of mathematical ability revolved around the idea that at some point most mathematical thinking involves the production and manipulation of a mental image or reference to a cognitive framework. It may be that this reference is obvious and observable for basic mathematical concepts and that with practice these concepts meld into a unconscious connection of complex propositions and rules. But the notion of Skemp's cognitive map and Greeno's conceptual domain still suggest there may be a spatial factor involved in general mathematical ability.

The problem with mathematical ability is not finding a reliable measurement tool, but the philosophical debate concerning the nature of mathematics, which affects how we measure mathematical ability. In the present study mathematical ability is defined as an individual's performance on a set of mathematical tasks which are derived from those concepts taught from the general mathematics syllabus in New Zealand schools. The Progressive Achievement Test: Mathematics (Reid, 1991), designed in New Zealand to test general mathematical achievement, was chosen to index mathematical ability in the present study.

The hypothesis to be tested in the present study is that there is a significant positive correlational relationship between global spatial ability and general mathematical ability. The hypothesis will be testsed through a discriminant function analysis of each participant's PAT Mathematics score and their spatial ability score.

Chapter 4

Method

Participants

Participants were chosen from three different schools in the central North Island area: Flaxmere Primary School in Hawkes Bay, Hawera Intermediate School in Taranaki, and Palmerston North Intermediate Normal School in Manawatu. Each school's Principal was contacted by letter asking permission to carry out the study at their school.

On the researcher's behalf each school sent out an information sheet and consent form supplied by the researcher (see Appendix A) to all students whose scores fell above the 75^{th} percentile or below the 25^{th} percentile of the Progressive Achievement Test Mathematics (PAT Mathematics) (Reid, 1991). Once permission had been gained from the students and parents the researcher was allowed access to the PAT Mathematics scores of those students. From the pool of 200 returned consent forms, students were invited to take part through a second information sheet and consent form (see Appendix A). A total of 105 students across the three schools consented to take part in the study. At the time of testing five students were unable to attend leaving 100 participants. The participants were 49 Form 1 students and 51 Form 2 students, with a mean age of 11.90 years (*SD* = .697). The sample consisted of 60 males and 40 females.

Participants were assigned to High or Low mathematical ability groups based on their PAT Mathematics score. Students above the 75^{th} percentile were grouped as High mathematical ability, and those below the 25^{th} percentile were grouped as Low mathematical ability. There were 50 participants in each group. The High mathematical ability group consisted of 20 females and 30 males with a mean age of 11.90 years (*SD* = .647). The Low mathematical ability group consisted of 20 females and 30 males with a mean age of 11.88 years (*SD* = .746). Participants were not told what group they were in. This was done to reduce performance anxiety in the Low mathematical ability participants, as feelings of real or perceived inability to complete a given task have been shown to significantly affect participants' performance (Anastasi, 1990).

Materials

Mathematical ability was measured using PAT Mathematics scores for all children. The PAT Mathematics was first published in 1972 after extensive norm and item checks using ten specialist committees and smaller teacher review panels (Reid, 1991). A new revision was commissioned in 1987 which included many item changes to ensure the content of the test remained current. Norms were obtained through administration of the PAT Mathematics to 1000 students across New Zealand from Standard 2 to Form 4. The three separate reliability coefficients[1] derived from the standardisation sample were: Equivalent Forms = 0.88; Split-half = 0.90 and KR20 = 0.91. All these coefficients show high consistency of the items within the PAT Mathematics (Reid, 1991).

Validity checks of the PAT Mathematics included content-type and concurrent validity checks. Subjective examination of the content by a representative group of New Zealand teachers was carried out during and after test construction. This examination found that the PAT Mathematics assesses a wide variety of skills and abilities important to mathematics learning (Reid, 1991). Furthermore, concurrent validity checks have shown high correlations with tests of general mathematical ability such as the ACER

[1] Mean coefficients from Form A and Form B from Part Two to Part Eight of the PAT-Mathematics (Reid, 1991)

Review and Progress Tests ($r = 0.75$), ITBS Mathematical Skills L9 ($r = 0.76$) and the SAT Modern Mathematics Concepts Advanced ($r = 0.81$), (Reid, 1991).

Spatial ability was measured through five separate scales taken from the Kit of Factor-Referenced Cognitive Tests (Ekstrom et al., 1976), hereafter referred to as the KIT. Spatial visualisation, by the definition given earlier in the introduction, required a measure which involved transforming a spatial pattern into other arrangements. The tests chosen to assess spatial visualisation were the Form Board Test (VZ1), the Paper Folding Test (VZ2) and the Surface Development Test (VZ3). Spatial orientation was indexed by the Card Rotation Test (S1) and the Cube Comparisons test (S2).

Spatial Visualisation Tests

The first Spatial Visualisation test was the Form Board Test (Ekstrom et al., 1976). In this test participants are presented with an outline of a geometric figure at the top of the test page. Below the figure are the test problems. For each of the six problems below each figure there are five shapes. The task is for the participants to indicate which of the shapes, when fitted together, would form the outline of the figure (see Figure 1). There are 24 questions in each of the two parts of the test. Participants are given eight minutes for each part with a one minute rest between each part.

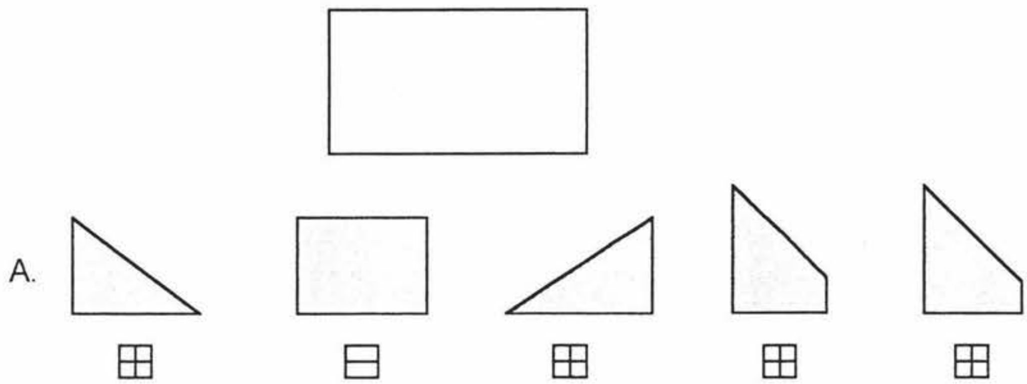

Figure 2. Example of an item from the Form Board Test (Ekstrom et al., 1976)
A cross indicates the piece should be used.

The Paper Folding Test, which was developed from Thurston's Punched Holes (Ekstrom et al., 1976), involves the simulated folding of a piece of paper. Figure 2 provides an illustrative example of an item from this test. Two, three or four illustrations each representing a new fold in the paper are presented. A hole is shown punched through the last of the illustrations. The task is to mentally unfold the paper and match the resulting punctured paper with one in a series of five target pieces of paper. There are ten items in each of the two parts of this test. Participants are given three minutes given for each part and a one minute rest between each part.

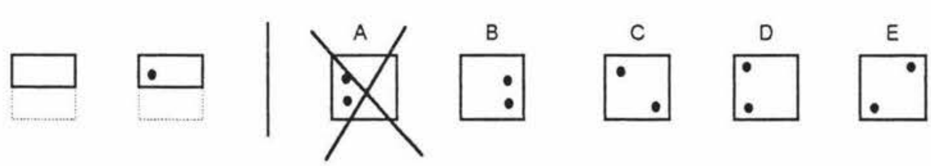

Figure 3. Example of item from Paper Folding Test (Ekstrom et al., 1976).
The correct choice is indicated by a cross through the figure.

The version of the Surface Development Test found in the KIT (Ekstrom et al., 1976) was adapted from Shepard and Feng (1972). The Surface Development Test consists of 12 multiple choice questions in which flat surfaces must be mentally folded to produce a three-dimensional structure, as illustrated in Figure 3. Participants were required to match unfolded surface sides to folded structure edges. There are six items in each of the two parts. Participants are allowed six minutes for each part with a one minute rest between each part.

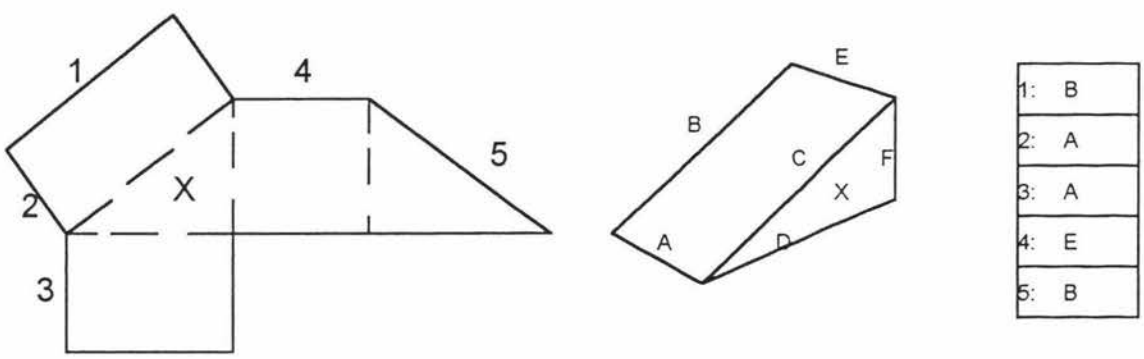

Figure 4. Example of item from Surface Development Test (Ekstrom et al., 1976)

Spatial Orientation Tests

Spatial orientation was indexed by the Card Rotation Test (S1) and the Cube Comparisons Test (S2) from the KIT (Ekstrom et al., 1976). The Card Rotation Test requires the participant to compare an irregular shaped drawing of a card with eight other drawings of the same card. The eight other drawings may be rotated (in the two-dimensional paper plane) or reflected (in the three-dimensional plane) as illustrated in Figure 4. The task is for the participant to indicate whether or not each of the eight drawings has been rotated (same) or reflected (different). There are ten items in each of the two parts of this test. Participants are given three minutes for each part with a one minute rest between each part.

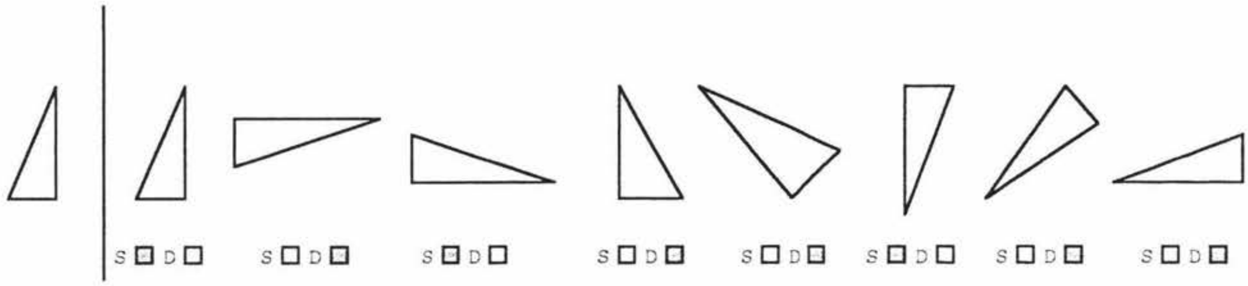

Figure 5. Example of an item from the Card Rotations Test (Ekstrom et al., 1976)

The Cube Comparisons Test consists of 42 items, each of which contain drawings of two cubes. Each drawing shows three faces of the cube and there is a different design, number or letter on each face of a given cube. The task is for the participant to decide whether the two cubes are the same or different (see Figure 5). There are 21 items in each of the two parts of the test. Participants are allowed three minutes for each of the two parts with a one minute rest between each part.

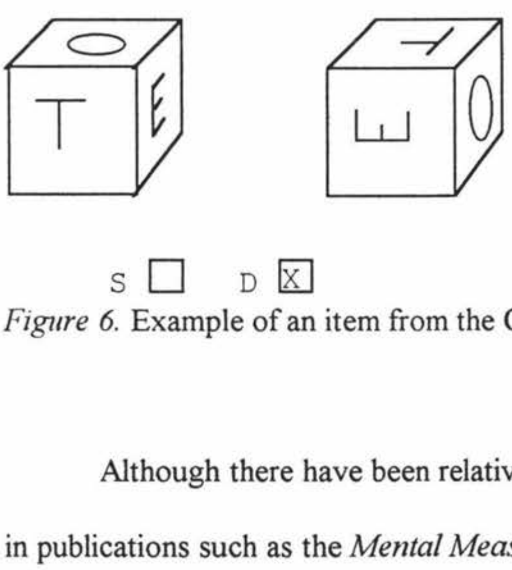

Figure 6. Example of an item from the Cube Comparisons Test (Ekstrom et al., 1976)

Although there have been relatively few psychometric data produced for the KIT in publications such as the *Mental Measurements Yearbook* and *Tests in Print*, there has been much written in the literature about the use and function of the KIT. Guttman et al. (1990) carried out a meta-analysis of major factor analytic studies of spatial ability from 1928 to 1979. Approximately 90% of those studies which specified the spatial factors spatial orientation and spatial visualisation (the factors used in the present study) used one or more of the tests from the KIT or tests from which the KIT was developed.

For the present study the construct validity of the measures chosen to index spatial ability is built into the rationale of the project. That is to say, the definition of spatial ability used in the present study was synthesised and developed by the author. Once the sub-factors had been defined, the content descriptions of many different spatial-type tests were screened. Those that approximated the definitions were selected.

Procedure

Participants were tested in small groups of ten or fewer. All five spatial ability tests were administered together as a battery[2]. The three spatial visualisation tests were presented first, in the order Form Board, Paper Folding and Surface Development. These were followed by the two spatial orientation tests, Cube Comparisons and Card Rotation. The same order of administration was followed for all participants. Participants were seated at separate desks in a well lit and quiet room at their school. A brief outline of the study was read to the participants from the information sheet on the front of the test. Before each test there was a short rest period of two minutes followed by the instructions for the next test being read aloud to the participants. These instructions were also printed on the participants' test papers. Once the instructions had been read, any questions arising were dealt with, and then the test period began. Each time period was measured with a stop-watch by the researcher. The total time period for each session including rests was approximately 58 minutes.

[2] See Appendix B for complete cover pages, including examples and instructions of each test described

All procedures in this study were approved by the Massey University Human Ethics Committee prior to the commencement of the study. Participants (and their parents) were informed of their right to withdraw (or parents to withdraw their children) from the study at any time. The three schools involved were informed that they would be sent a summary of the findings from the research when the thesis had been completed.

Chapter 5

Results

Statistical Analysis

The two statistical analyses chosen for the present study were a Discriminant Function Analysis (DFA) and a t-test. Both analyses were carried out using the SPSS/PC statistical package.

DFA can be used for two reasons. The first is to predict group membership through classification functions. The second is to supply a means of interpreting group differences. The present study utilises the group membership prediction ability of DFA. While similar to a MANOVA, a DFA differs in that the research question is "does difference in a set of DV scores, or *predictors,* allow us to predict group membership?" While this is very much the same as a reversed MANOVA, DFA has the advantage of actually classifying cases into groups and testing the accuracy of the classification.

In simple terms, DFA takes a set of cases for which group membership is known and assigns weightings to each of the predictor variables in that set. Weightings are based on, among other things, maximising the difference between group means. The resulting collection of weightings form a function, a Discriminant Function. This function is an equation which takes the scores from a set of predictor variables for some case and predicts, with some margin of error, the group membership of that case. In essence, DFA uses the relationship between a group of descriptive scores to produced an algorithm, and then applies the algorithm to place an ungrouped case into a group. Furthermore, DFA derives a measure of adequacy of the classification made (see Appendix C) which provides information about what cases are misclassified and with what frequency this occurs.

DFA was chosen for the predictive component of its output and for its compatibility with the type of question posed in this research. Primarily DFA is used to produce classification functions which help to predict group membership (Tabachnick & Fidell, 1989). This is the exact focus of the present research: to determine whether a spatial ability score helps predict mathematical ability. Although other techniques could have given us the same *statistical* answer, the DFA is more suitable because the way in which it produces an answer is similar to the way the research question in the present study has been posed.

The t-test was chosen to provide an answer for the hypothesis question of whether spatial ability differs significantly across mathematical ability groups. Using a DFA with one predictor is a trivial case and can be very susceptible to noise within the data. Moreover, including the Total score in a DFA with the other predictors would cause singularity as the Total variable is the sum of the other five predictors.

Scoring of Spatial Tests

Each of the spatial tests produced a score based on accuracy. In the case of the Surface Development Test and the Paper Folding Test the score was calculated as the total items answered correctly minus a fraction of those items answered incorrectly. As the fraction to be subtracted was not specified in the manual, it was set as 0.2 of the number of incorrect answers. This was deemed appropriate as there were five choices (multiple-choice) per item. The Orientation tests, S1, S2 and the third Visualisation test

VZ1 scores were simply the total items answered correctly minus the total answered incorrectly (as per test instructions). Six separate scores were derived, VZ1, VZ2, VZ3, S2, S1 and Total, to be used as dependent measures.

Descriptive Statistics

Table 2 shows the mean, standard deviation, and range of the six dependent variables for the complete sample.

Table 2. Descriptive statistics of predictor variables.

Test Name	Variable	*M*	*SD*	Min	Max possible	Actual Max
Form Board	VZ1	76.97	38.82	0	240	204
Paper Folding	VZ2	8.48	4.57	0	20	19
Surface Development	VZ3	7.82	14.92	0	60	57.6
Cube Comparisons	S2	8.98	8.32	0	42	38
Card Rotation	S1	79.55	32.97	0	160	146
	TOTAL	180.91	87.98	13.4	522	395.4

Analyses

The t-test results supported the hypothesis that spatial ability is related to mathematical ability. The Total spatial score for the High mathematical ability group (11150.90) was significantly greater than the Total spatial score for the low mathematical ability group (6940.30), t (86.3) = 5.43, $p < .01$.

Logically the tests in each of the spatial categories, spatial visualisation and spatial orientation, will measure the same construct. Therefore tests within each variable would be expected to correlate positively. This is reason for concern in variables in a DFA as redundant (correlating) predictors may cause multicollinearity. In the present study the predictors did not correlate highly (the highest being VZ3 with VZ1 $r = .43$, see Table 3) ensuring multicollinearity was not an issue.

All data were screened for linearity and multivariate outliers and no problem cases were identified. Checks on normality showed variable S2 had a slightly skewed negative kurtosis. However, transformation (square root and natural log) of the variable did not significantly improve the distribution so it was left unchanged. All of the 100 original cases were included in the analyses.

One discriminant function was produced with $\chi^2(5) = 71.59$, $p < 0.01$. Table 3 shows the loading matrix for the predictor variables and the discriminant function. All of the five predictors were significant in the discriminant function with VZ3 and VZ2 presenting the highest correlations of 0.89 and 0.72 respectively.

Table 3. Results of discriminant analysis of spatial ability variables.

Predictor Variable	Correlations of predictor variables with discriminant function	Univariate F F(1,98)	Pooled within group correlations among predictors				
			VZ1	VZ2	VZ3	S1	S2
VZ1	.43	20.58*	1.0				
VZ2	.72	56.64*	.26	1.0			
VZ3	.89	86.99*	.43	.38	1.0		
S1	.44	20.96*	.40	.31	.31	1.0	
S2	.31	10.34†	.17	.17	.09	.13	1.0

*p < .01
†p < .05

Classification results show that 38 (76%) of cases were correctly classified into the High mathematical ability group, and 43 (86%) into the Low mathematical ability group. Overall the discriminant function correctly classified 81% of the cases into the two groups. Of those classified incorrectly 12 (63%) were classified into the Low mathematical ability group and 7 (37%) into the High mathematical ability group.

Discussion

The primary aim of the research described in this thesis was to examine the relationship between general mathematical ability and spatial ability. The emphasis was on the production of a general or global spatial ability score from several spatial tests and examination of the correlation between that score and a general mathematical ability score. It was hypothesised that there would be a positive correlational relationship between spatial ability and mathematical ability. Although such a relationship seems apparent due to the pictorial nature of some parts of mathematics, there has been little research carried out on this exact question. The question has been asked of the relationship between mathematic ability and other spatial skills such as spatial visualisation and spatial orientation but not of mathematical ability and *spatial ability* as a whole.

A PAT Mathematics score for each participant was obtained from the school of the participant. Participants completed a battery of five spatial tests from the Kit of Factor Referenced Cognitive Tests. The raw scores from the tests were used to produce a general spatial ability score for each participant. Statistical analysis using a t-test showed that the High mathematic ability group spatial score was significantly higher than the Low mathematics ability group spatial score. A discriminant function analysis was carried out using each of the separate scales in the spatial ability test battery (3 spatial visualisation tests and 2 spatial orientation tests). This was in order to examine the relative importance (or weighting) each sub-scale had in the relationship of general spatial ability to mathematical ability. The discriminant function showed all five of the spatial ability scales were significant predictors of mathematical ability.

The results from the present study support the hypothesis that spatial ability is positively correlated to mathematical ability. While this was the main aim of the research there were some new observations made about the nature of the spatial ability construct that are very important. The discussion of the results will initially focus on the spatial scores and the relation they have separately and jointly to the mathematics factor. Although the results show that there is a relationship between spatial ability and mathematical ability, the implications from the research are not quite as simple. In particular, the need for greater clarification of the spatial ability construct has shown itself to be very closely linked to any project of this type. It would be sensible to discuss this at length because the central factor being employed is spatial ability and the theoretical structure of spatial ability has been shown to be questionable.

Weighting of Variables

Although a rank ordering was found for the five separate scales of spatial visualisation and spatial orientation this will not be discussed in terms of one particular spatial score being a *better* predictor than another. For example, the score from the VZ3 test had the highest ranking in the DFA structure matrix and the score from the S2 test the lowest. The traditional interpretation of a single function DFA such as this is to describe each of the separate weights individually, but not as if each were mutually exclusive. This is the power of the DFA in the present research: it supplies a means of looking at individual predictive weightings while accounting for intercorrelations between variables.

The argument made here is that to concentrate on the relationship between separate factors of spatial ability, such as spatial orientation or spatial visualisation, and mathematical ability is erroneous. Most major reviews such as McGee (1977) and Lohman et al. (1987) have highlighted the overlap between the commonly used spatial constructs. This point is reiterated because it is of such great importance and must be considered carefully when asking whether there is a relationship between spatial ability and mathematical ability.

This research has supplied evidence that a relationship between mathematical ability and general spatial ability does exist. A significant result was found for both a global spatial ability score using a t-test and for separate spatial visualisation and spatial orientation scores using a DFA. While it cannot be said that one type of spatial score *independently* will better predict mathematical ability, from the results of the present research it is possible to talk about the relative importance of each of the scores.

Spatial visualisation scores provided more predictive power in the function equation than spatial orientation scores (see Table 2). This is similar to results found in the literature (see e.g. Lohman et al., 1987). One interpretation of this occurrence is that the spatial visualisation factor is relatively robust. Even though it correlates with other spatial factors (and possibly with other non-spatial factors), it is sufficiently central to the construct of spatial ability to be evident. An alternative, and perhaps more likely, reason for the relative predictive power of the spatial visualisation factor is the difficulty in defining the boundaries of the spatial orientation factor. That is to say, the spatial orientation factor is troubled with statistical noise and theoretical uncertainty.

Many sources, while acknowledging the 'literature standard' factors of spatial visualisation and spatial orientation, are weary of the present definition of spatial orientation. The definition provided earlier mentions the, *"...ability to maintain orientation [of the body] with respect to objects in space..."*. The problem here is that the subject may not be keeping his or her body as the reference point. Clements (1981) makes the very important point that it is possible for a person to complete a spatial orientation task by rotating the figure (a visualisation action) rather than reorienting the position of the imagined self to the imagined object. For this reason a superior performance by participants on visualisation tasks may have occurred in the present study.

As a whole, then, the positive relationship between spatial ability and general mathematical ability that was hypothesised has been supported by the present research. However a problem lies with the current tests: it is not possible to be completely positive that the participants are using spatial ability to complete the tests. Or if they are, whether they are using the methods which the test purports to measure. This is an example of problematic construct validity, a problem not confined to this particular set of tests. To go inside the tests used here and examine their construct validity is beyond the scope of this piece of research.

Adequacy of Instrumentation

Most mathematical problems can be solved in more than one way using more than one technique. The area under a graph can be found using the Calculus or it can be approximated using analytical techniques. Whichever way it is done it is almost always possible for an examiner to know which method was chosen simply by looking at the test script. The same cannot be said of spatial tasks. The problem of construct validity in a test is considerable in the spatial ability field and therefore must be considered in the present study.

Because the ultimate aim of studies such as this is to make some assumptions about the possible use of spatial tasks to improve skills in mathematics, it is therefore vital to ensure the type of spatial task is well defined and understood. This does *not* appear to be happening. The type of test used in the present study is widely used in the spatial ability field. However, not all studies use the tests for the same purpose. For example, the Card Rotation test (Ekstrom et al., 1976) is sometimes used as an orientation task, and at other times it is used as a visualisation task. The question is what part of this large (and fuzzy) construct we call spatial ability is involved with the understanding of mathematics? In a way the present study has used spatial tests tacitly to prove a point. While on one hand it can be argued that most spatial tests are inadequate and have less than optimum construct validity, I then went on to derive a score from the tests and argue that there is some statistical relationship between this score and mathematical ability. That in itself is correct, there is a relationship. But what is it between? It is between mathematical ability and *some part* of the whole spatial ability construct. There is statistical noise which is yet to be uncovered in the construct. This problem will hinder any further examination of the spatial ability-mathematical ability link. The importance of discussing the fuzziness of the spatial ability construct and the tests themselves is to inform the reader that while the best path had been chosen, there are still problematic areas to be considered.

Lohman et al. (1987) proposed the best solution to this problem thus far. Their suggestion was to disregard the usual differentiation of spatial visualisation and spatial orientation and to talk about Mental Movement and Mental Construction. The advantage of this type of classification (see introduction for more detail) is that it appears to encompass the most important parts of spatial ability without the cumbersome and ambiguous titles of visualisation and orientation. Further, within these classifications Lohman et al. (1987) suggest that it may be possible to use smaller well defined factors, such as Rotation speed and Encoding speed, to try to quantify spatial ability. In this way it may be possible to discover which parts of spatial ability are involved in mathematical ability. However, as these new types of classifications (or constructs) have yet to be discussed at length in the literature, let alone used as the bases for spatial ability tests, it is still too early to make any statements about their effectiveness.

To an extent the implications of the present research are borne of the difficulties inherent in this area of research. Clearly the need for a more precise concept of spatial ability is required to allow a deeper understanding of the relationship between mathematical ability and spatial ability. The result found in this study showed evidence for a relationship between general mathematical ability and spatial ability. However, in looking at this relationship it was found that one of the constructs involved, spatial ability, had less than a universally accepted definition. Nevertheless, a relationship was found using one set of definitions from the literature. This is an important statement to make. As discussed earlier, there are few studies in the literature which actually present empirical evidence to support claims of there being a relationship between these two constructs. The assumption is made but rarely tested.

Perhaps more importantly the notion of looking at spatial ability as a whole rather than as separate spatial abilities was made. This reiterates Clements' (1981) words of caution about the difficulty in testing spatial ability and the need to be flexible when using spatial type tasks in mathematics education. This leads to a further and very important question: Assuming a more robust and indeed causal relationship is found between spatial ability and general mathematical ability, will it be possible to actually change a student's spatial ability? This question has not been discussed at length in the literature.

Some research has been carried out on spatial orientation and spatial visualisation and how they are involved with mathematical ability (Conner & Serbin, 1985; Fennema & Sherman, 1977, 1978; Fennema & Tartre, 1985; Tartre, 1990). Moreover, most of the studies used only geometry related questions only in the mathematical problem solving and so were not studying the relationship between general mathematics and spatial ability per se. However some of these studies have shown encouraging results. Ben-Chaim, Lappan and Houang (1988) found that after instructional intervention (e.g., drawing pictures of problems, moving paper representations of problems around) on spatial visualisation tasks with intermediate age children (11-13 years), those students who received training performed significantly better in mathematical tasks than those who did not receive training. On the whole there is not much in the literature discussing whether or not spatial ability can be effectively increased in students. This may be a direct result from the problem discussed above of not having available a definite and universal definition of spatial ability.

Limitations

Perhaps the most obvious limitation of the present study is that it is correlational in nature. Although the statistical analysis using a DFA was discussed in terms of *predictors* in the relationship, this was referring only to the ability of one factor to predict the most likely statistical grouping of a subject based a set of variables of that subject. What it does not say is that high ability in spatial tasks will lead to increased ability in mathematical understanding. So the first of the limitations is the possibility of reverse causation.

It is entirely possible that high mathematical ability will result in an increased aptitude for spatial tasks. The pursuit of mathematics itself may involve certain tasks that encourage and extend abilities we would call spatial. For example, the transfer of a quadratic equation to a graphical representation can be done entirely using coordinates and simple differentiation; that is, without using any spatial abilities. However, once recognition of the relationships between equation and graph occur, then the graph may be imagined before it is physically drawn. Doing so may develop spatial skills.

Another limitation is the possibility of other factors in the relationship. Confounding factors are very difficult to control for in simple correlational experiments such as the present study. Two confounds that may be involved in the spatial ability-mathematical ability relationship are general intelligence and reading ability. As most of the major intelligence tests such as the Stanford-Binet 4 and the WISC-R have both mathematical, reading, and spatial components it would appear that there is indeed need for concern about such confounding variables. In the present study there was no feasible way for the researcher to test the WISC-R score for all one hundred participants and then examine the relationships between these variables and mathematical ability. However, it is important that the reader is aware of the possibility of general intelligence as a third variable in the relationship between spatial ability and mathematical ability in the present study. To try to include this factor in the present research and control its influence would be an enormous task, and one too large for the scope of a master's thesis.

One issue still topical in the spatial ability and mathematical ability fields is that of gender differences. Much research has been done and much controversy has arisen from it. Sherman (1967) proposed an explanation of gender differences in mathematical ability based on cognitive factors such as spatial ability. Many other studies have followed this line of reasoning (Fennema & Tartre, 1985; Linn & Petersen, 1985; Tartre 1990) and found that there are indeed gender differences in both mathematical ability and spatial ability, but that they are decreasing (Friedman, 1994). However, the present study did not include any discussion of the gender difference issue. This was due simply to the nature of the gender issue itself. It is such a large field, with many implications, that to bring it into a study of this size would greatly unbalance it. The gender problem is one that requires an entire thesis of its own rather than perfunctory inclusion. However, it is a factor that would need to be included as another possible confounding factor in further research into this topic.

Conclusions

This study has provided evidence that a relationship between spatial ability and general mathematical ability exists. This is important. As noted earlier, the relationship between spatial ability (not just spatial visualisation or spatial orientation) and mathematical ability has been assumed but rarely verified through research. The result from the present study, although a simple one, is a step towards examining the role of spatial ability in mathematics learning.

From the preceding discussion on the nature of spatial ability a secondary conclusion can be drawn: that the construct itself needs to be refined further before a general agreement on it will occur. While it was not the purpose of this study to redefine the spatial ability construct, this researcher cannot stress enough the importance this issue plays in this area. It is also possible that a new tack needs to be taken altogether by breaking down the incumbent factors, spatial visualisation and spatial orientation, and taking the more axiomatic approach proposed by Lohman et al. (1987) of using mental movement and mental construction as descriptors of spatial ability. It is possible that the construct is sound but the measures in use are not reliable.

The conclusion drawn from the research described in this thesis is that a relationship between one definition of spatial ability and mathematical ability has been shown to exist. However, this does not conclude the new questions raised within the thesis. The question of the definition of spatial ability and the use of spatial tasks to increase understanding of mathematics are now ready to be addressed.

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Appendix A

This appendix contains:
Initial information form
Initial consent form
Student information form
Parent/Guardian information form

Research Project on

The Relationship Between Mathematical Ability and Spatial Ability

Information Sheet

Dear Student and Parents/Guardians,

My name is Peter Flynn. I am completing a Master of Science degree at Massey University. As part of my degree, I am conducting a research project involving Spatial Ability and Mathematical Ability in intermediate school aged children. Spatial Ability is to do with the way we mentally move images of objects around in our mind. I am studying this because it may be important to the way mathematics is taught at schools in New Zealand. The research project is supervised by Dr. Julie Bunnell, a lecturer in the Department of Psychology at Massey University.

As part of the research I need to measure Mathematical Ability. To do this I will use the scores from the Progressive Achievement Tests for Mathematics, (PAT) completed by students at most New Zealand schools.

Before I can look at the PAT scores I need to have both the parent's/guardian's and child's permission. Your school is helping out with this by sending this letter to you on my behalf. This is to make sure that I am following the Privacy Act regulations. No other person will see the PAT scores or will be given any information about the PAT scores.

I intend to use this information to help me select those students who will be invited to take part in the project. Students who are invited to be in the project will soon receive an information sheet outlining the details of the project.

Again, at this stage I only wish to get your permission to have access to the PAT scores. This does not mean that any student is already part of a study, or that any student is obligated to become part of the research project.

Once you have made your decision, would you please complete the consent form and return this to the school. Thank you very much.

Sincerely yours,

Peter Flynn
Psychology Department
Massey University
Palmerston North
06 350 4117 (work)
06 355 4880 (home)

Julie Bunnell
Psychology Department
Massey University
Palmerston North
06 350 4122

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Research Project on

The Relationship Between Mathematical Ability and Spatial Ability

Consent Form

I DO/DO NOT give permission for Peter Flynn to have access to my PAT scores. No one else will see or be given information about the PAT scores. The information gained will be used only for the purposes stated above.

Signed (student): ______________________________

Name (student): ______________________________

I DO/DO NOT give permission for Peter Flynn to have access to my child's PAT scores. No one else will see or be given information about the PAT scores. The information gained will be used only for the purposes stated above.

Signed parent(s)/guardian(s): ______________________________

Name(s) parent(s)/guardian(s): ______________________________

Research Project on :

The Relationship Between Spatial Ability

and Mathematical Ability.

<u>Information Sheet</u>

Dear Student,

My name is Peter Flynn. I am studying for a Masters degree at Massey University. As part of the degree, I am conducting a research project. The project is being supervised by Dr. Julie Bunnell, a lecturer at Massey University. The project is concerned with the relationship between Spatial Ability and Mathematical Ability. Spatial Ability is our ability to make images of our surroundings and move those images around in our minds.

Can you imagine a yellow block with the letter Y on it? Can you imagine what the block would look like if you turned it upside down? This is what I mean when I talk about moving images or pictures in your mind. I am studying this because it may be important to the way mathematics is taught at schools in New Zealand. As the project is about your ability to move images around in your mind it does not matter whether you enjoy mathematics, or whether you do well in mathematics.

Not every student at your school has been invited to be in this research project. Before the project started, I obtained permission from the Massey University Human Ethics Committee and your Principal to carry out the study. I also contacted you and your parents/guardians earlier to get permission to look at your PAT Mathematics score so that I could choose students with a wide range of ability in mathematics.

If you would like to take part in this project it will take about one hour of your time during normal school hours. You will be asked to do a pencil and paper task in which you decide if one shape is the same as another shape. One of the shapes will have been turned around, turned over, or folded up. All you will have to do is look at one picture and choose which of several shapes you think is the same. You will do the task in a group with about ten other students, and I will be present.

If you take part in the project, you have the right to:

- refuse to answer any particular question
- pull out of the project at any time
- ask questions at any time about the study
- be given a report of the results of the project when it is finished.

In addition you will have complete confidentiality of anything you write to or tell the researcher. All information you write will be coded so that it cannot be identified as your work.

Your parents/guardians also have an information sheet like this one. Please discuss this with them, as you will need their permission if you wish to be in the project. If you or your parents/guardians have any questions about the project please contact one of us through your school or through the address below. If you wish to participate in the project, please return the consent form (attached to this letter), signed by you and your parents/guardians, to your teacher.

Sincerely yours

Peter Flynn
Psychology Department
Massey University
Palmerston North
06 350 4117 (work)
06 355 4880 (home)

Julie Bunnell
Psychology Department
Massey University
Palmerston North
06 350 4122

Research Project on :

The Relationship Between Spatial Ability

and Mathematical Ability.

Information Sheet

Dear Parent/Guardian,

My name is Peter Flynn. I am studying for a Masters degree at Massey University. As part of the degree, I am conducting a research project. The project is being supervised by Dr. Julie Bunnell, a lecturer at Massey University. The project is concerned with the relationship between Spatial Ability and Mathematical Ability. Spatial Ability is our ability to make images of our surroundings and move those images around in our minds.

Can you imagine a yellow block with the letter Y on it? Can you imagine what the block would look like if you turned it upside down? This is what I mean when I talk about moving images or pictures in your mind. I am studying this because it may be important to the way mathematics is taught at schools in New Zealand. As I have said, the project is about children's Spatial Ability so it does not matter if they enjoy mathematics, or whether they do well in mathematics.

Not every student at your child's school has been invited to be part of this research project. Before the project started, I obtained permission from the Massey University Human Ethics Committee and your child's Principal to conduct the study. I also contacted you and your child earlier to obtain permission to look at your child's PAT Mathematics score so that I could choose students with a wide range of ability in mathematics.

If your child volunteers, and you give your permission for your child to take part in this project, it will take about one hour of his or her time during normal school hours. The children will be asked to do a pencil and paper task in which they decide if one shape is the same as another shape. One of the shapes will have been turned around, turned over, or folded up. All they will have to do is look at one picture and choose which of several shapes they think is the same. Your child will do the task in group of about ten other students, and I will be present.

Children who take part in the project, have the right to:

- refuse to answer any particular question
- pull out of the project at any time
- ask questions at any time about the study
- be given a report on the results of the project when it is finished.

In addition to this there will be complete confidentiality of what ever the children write or tell to the researcher. All information he or she gives to the researcher will be coded so that it can not be identified as the child's work.

Your child also has an information sheet like this one. Please discuss this with your child and if you or your child have any questions about the project please contact one of us through your child's school or through the address below. If you are willing for your child to participate in the project, please return the consent form (attached to this letter), signed by yourselves and your child, to your child's teacher.

Sincerely yours

Peter Flynn
Psychology Department
Massey University
Palmerston North
06 350 4117 (work)
06 355 4880 (home)

Julie Bunnell
Psychology Department
Massey University
Palmerston North
06 350 4122

The Relationship between Spatial Ability and Mathematical Ability.

Consent Form

I have read the Information Sheet for this study and have had the details of the study explained to me in full. My questions about the study have been answered to my satisfaction, and I understand that I may ask further questions at anytime.

I also understand that I am free to withdraw from the study at any time, or decline to answer any particular question in the study. I agree to provide information to the researcher on the understanding that it is completely confidential.

I DO/DO NOT wish to participate in this study under the conditions set out on the information sheet:

Signed (Student): ______________________________

Name (Student): ______________________________

We have read the Information Sheet for this study and have had the details of the study explained to us in full. Our questions about the study have been answered to our satisfaction, and we understand that we may ask further questions at anytime. We understand that we may withdraw our child from the project at any time and that any information our child provides will be kept confidential.

I DO/DO NOT give permission for my child to participate in this study under the conditions set out in the information sheet:

Signed Parent(s)/Guardian(s): ______________________________

Name(s) Parent(s)/Guardian(s): ______________________________

Appendix B

This appendix contains:

The spatial ability test booklet
and information sheet

Hello, Kia Ora, Talofa and thank you for coming along.

The tasks you are about to do will help me to measure your Spatial Ability. Can you imagine a yellow block with the letter Y on it? Can you imagine what the block would look like if you turned it upside down? Spatial Ability is our ability to imagine how objects and images change when we see them from different positions. I am studying this because it may be important to the way mathematics is taught at schools in New Zealand. As the project is about your ability to move images around in your mind it does not matter whether you enjoy mathematics, or whether you do well in mathematics.

There are instructions on each of the tests inside this booklet which you should read very carefully. If you have any questions about the instructions, please let me know as soon as possible so I can help you. It is very important that you understand what you have to do so you are able to do your best.

When I ask you to, turn this page over and read the instructions for the first test.
DO NOT START THE TEST UNTIL ASKED TO DO SO.

At the end of each test put your pencil down and wait for further instructions.
DO NOT GO ON TO THE NEXT TEST UNTIL ASKED TO DO SO.

There is a blank piece of paper between each test, so when you come to a blank page you have finished the test. Stop, put your pencil down, and wait for further instructions.

Please fill out the spaces below now:

Code: .... Age: .... Sex: .... Form: ....

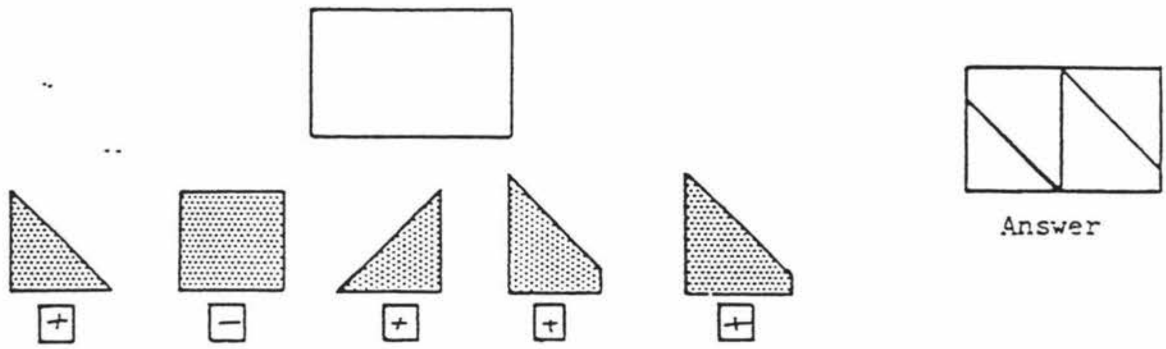
Name: .... School: ....

FORM BOARD TEST — VZ-1

This is a test of your ability to tell what pieces can be put together to make a certain figure. Each test page is divided into two columns. At the top of each column is a geometrical figure. Beneath each figure are several problems. Each problem consists of a row of five shaded pieces. Your task is to decide which of the five shaded pieces will make the complete figure when put together. Any number of shaded pieces, from two to five, may be used to make the complete figure. Each piece may be turned around to any position but it cannot be turned over. It may help you to sketch the way the pieces fit together. You may use any blank space for doing this. When you know which pieces make the complete figure, mark a plus (+) in the box under ones that are used and a minus (-) in the box under ones that are not used.

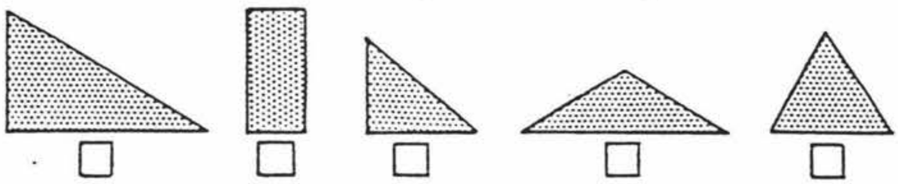
In Example A, below, the rectangle can be made from the first, third, fourth, and fifth pieces. A plus has been marked in the box under these places. The second piece is not needed to make the rectangle. A minus has been marked in the box under it. The rectangle drawn to the right of the problem shows one way in which the four pieces could be put together.

A.

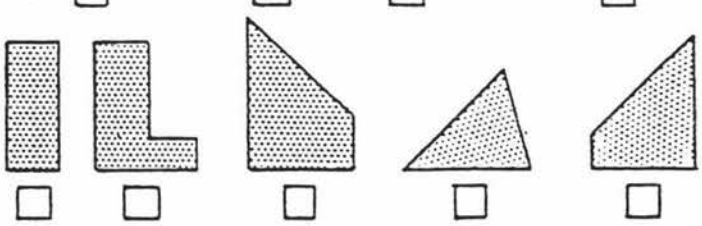


Now try to decide which pieces in Examples B and C will make the rectangle.

B.



C.



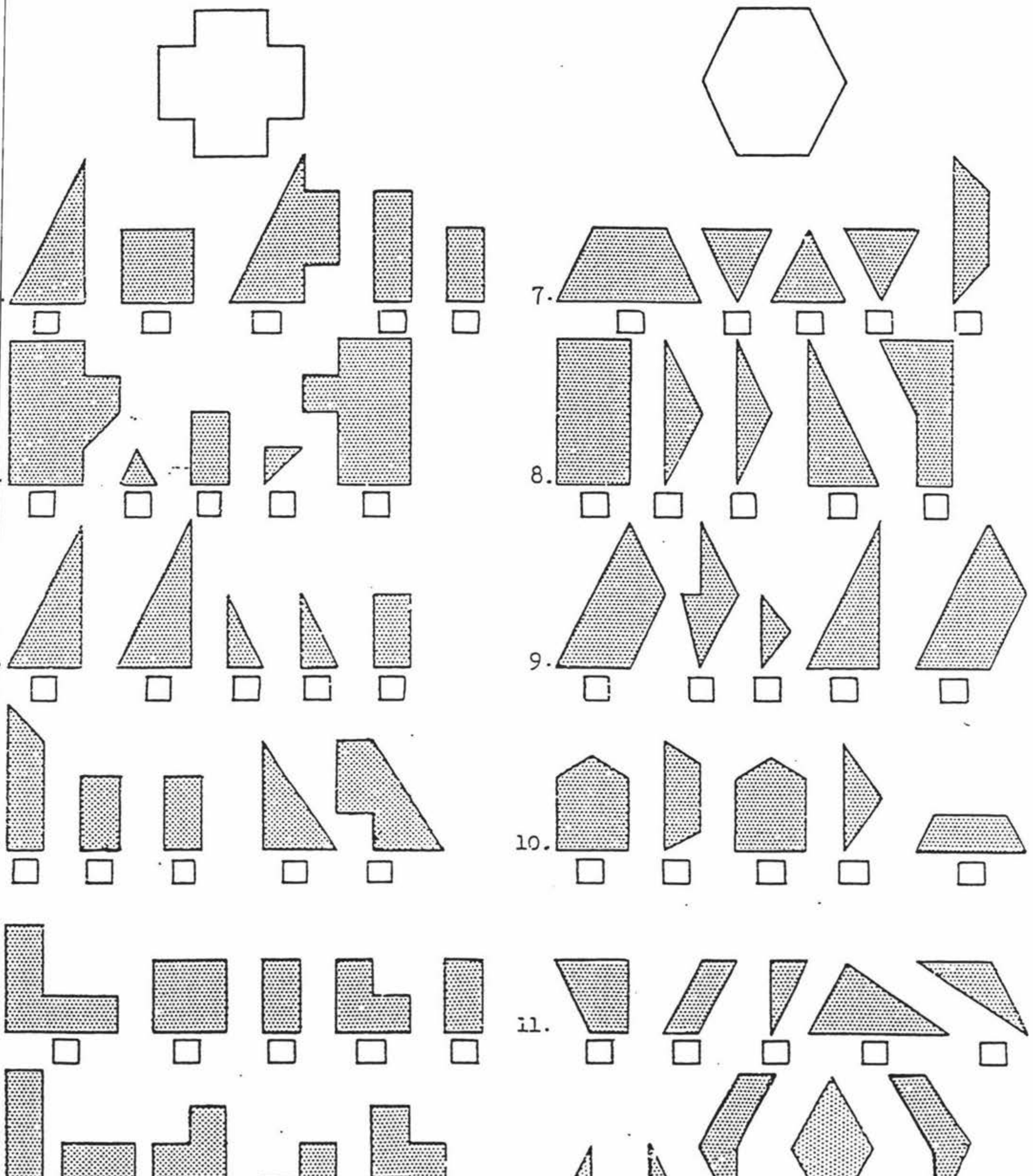
In Example B, the first, fourth, and fifth pieces are needed. You should have marked a plus under these three pieces and a minus under the other two pieces. In Example C, the second, third, and fifth pieces should be marked with a plus and the first and fourth with a minus.

Your score on this test will be the number marked correctly minus the number marked incorrectly. Therefore, it will not be to your advantage to guess unless you have some idea whether or not the piece is correct.

You will have 8 minutes for each of the two parts of this test. Each part has 2 pages. When you have finished Part 1 (pages 2 and 3), STOP. Please do not go on to Part 2 until you are asked to do so.

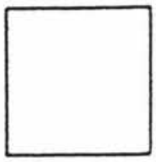
DO NOT TURN THIS PAGE UNTIL ASKED TO DO SO.

Part 1 (8 minutes)



GO ON TO THE NEXT PAGE.

Part 1 (continued)



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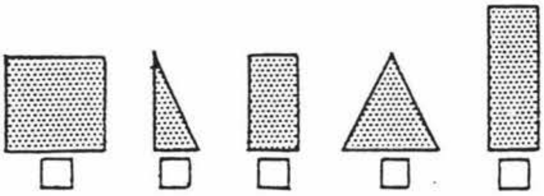

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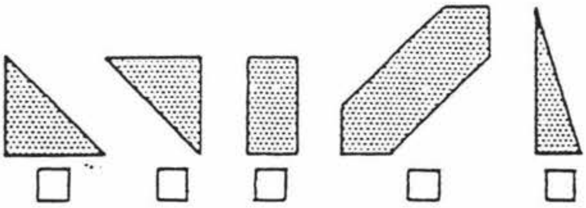

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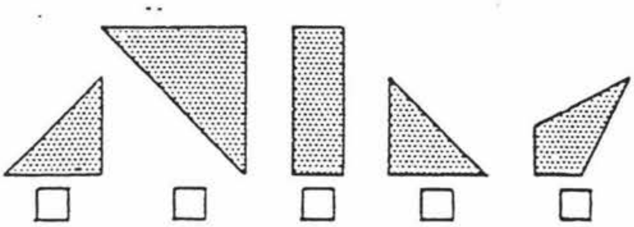

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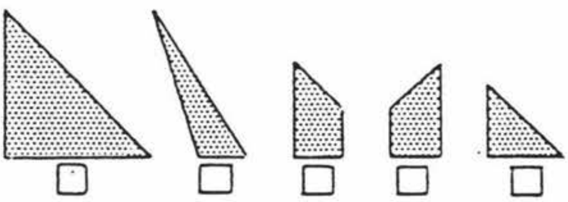

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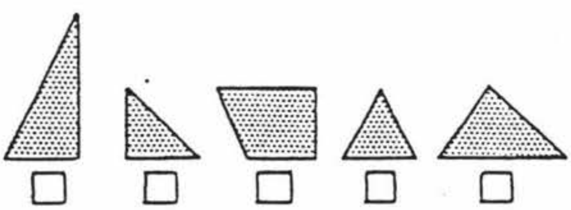

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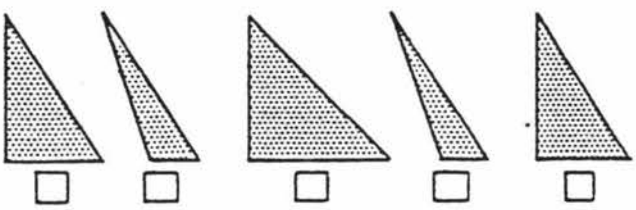

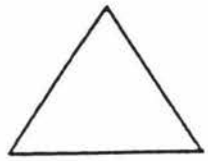

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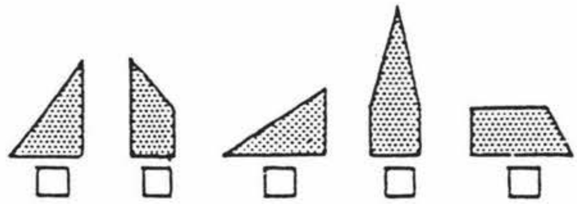
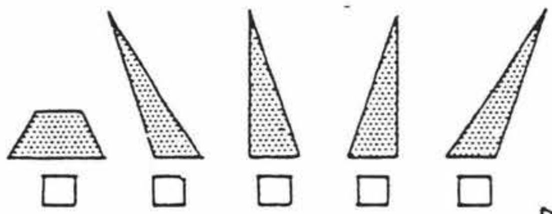
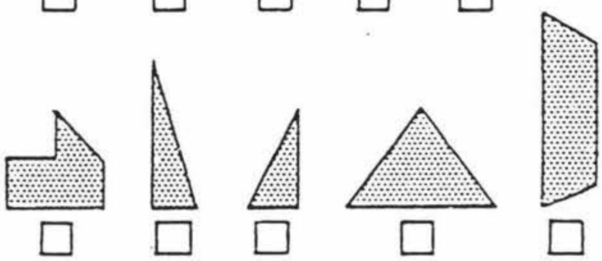


20.


21.


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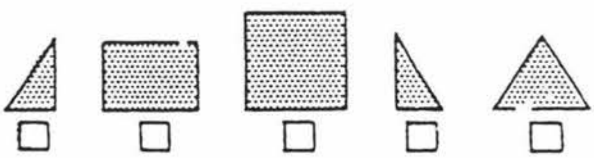

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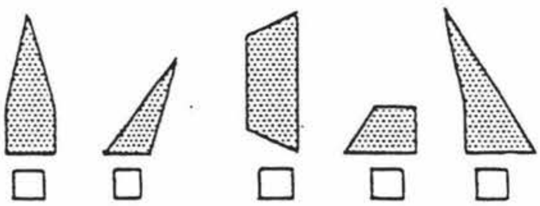
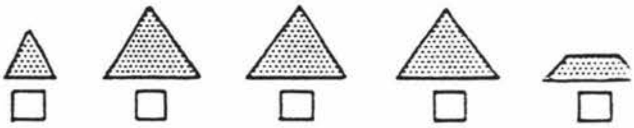
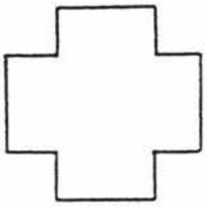


24.


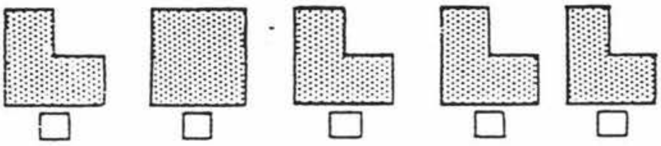
DO NOT TURN THIS PAGE UNTIL ASKED TO DO SO.

STOP.

Part 2 (8 minutes)



25.



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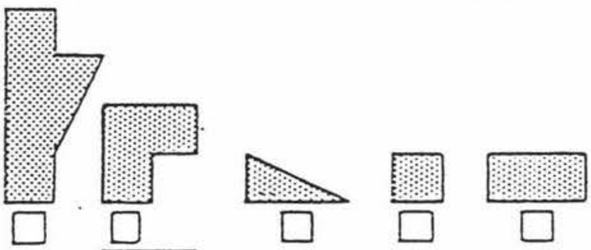
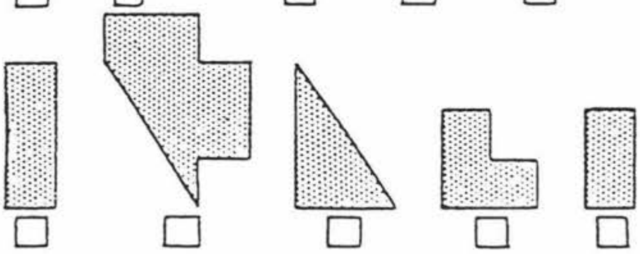
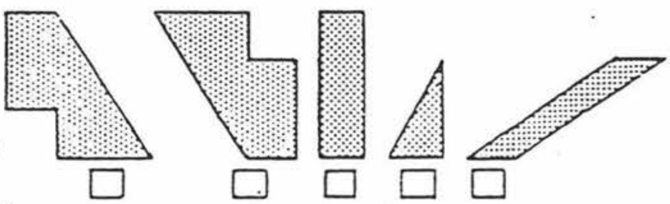


27.



28.



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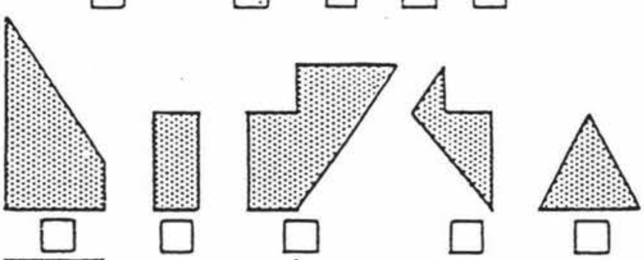

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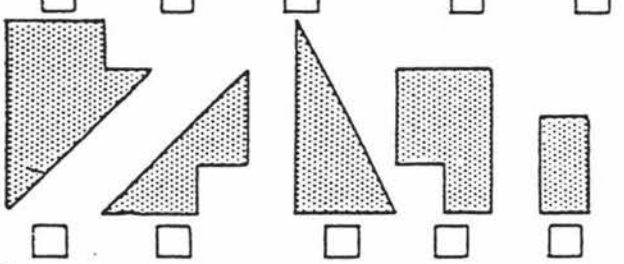

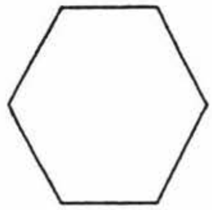

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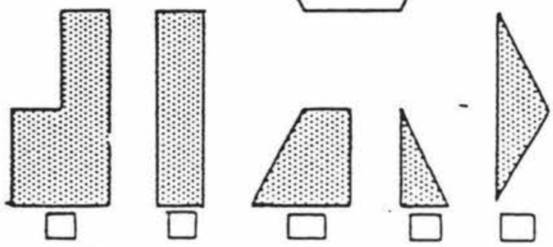

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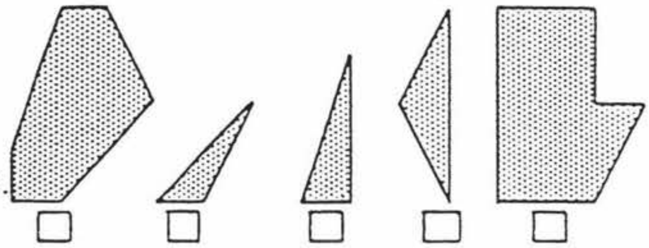

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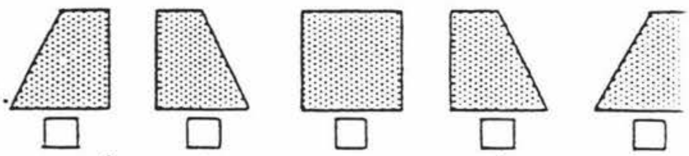

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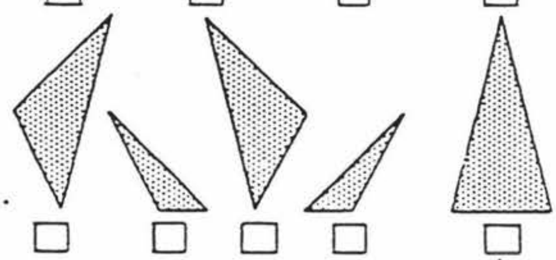

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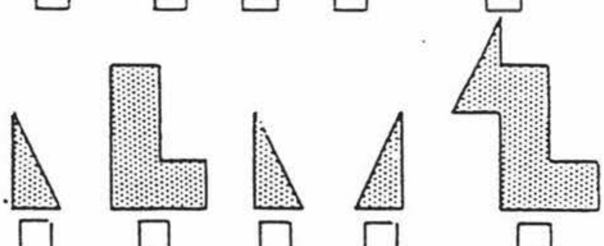

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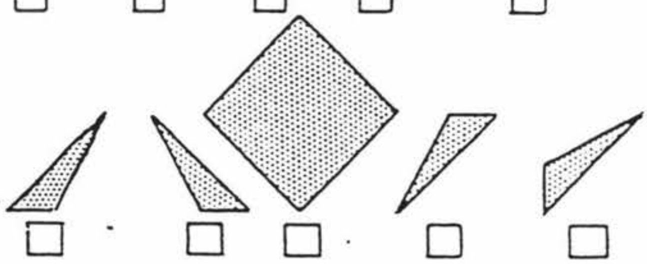
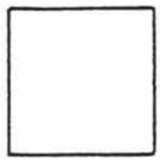
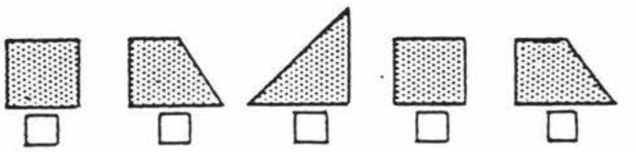
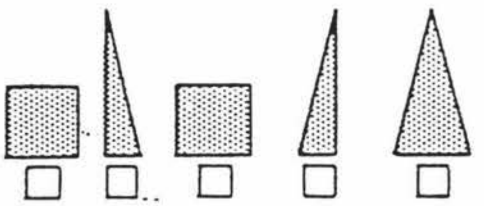


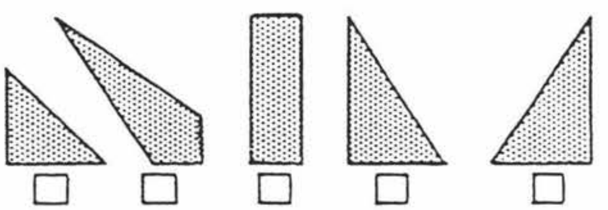
GO ON TO THE NEXT PAGE.

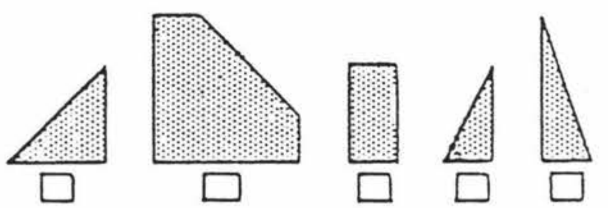
Part 2 - (continued)

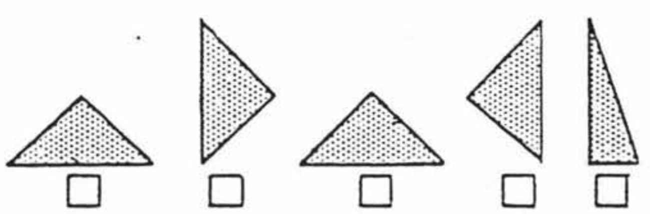


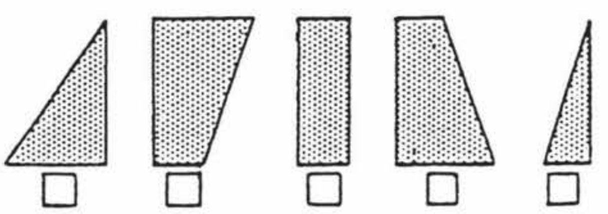
37. 

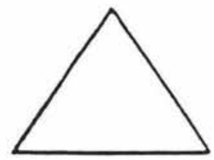
38. 

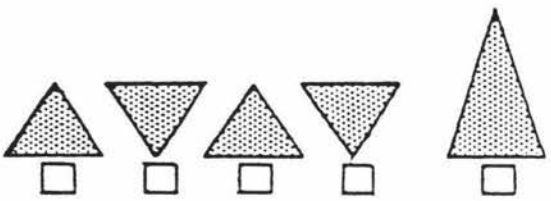
39. 

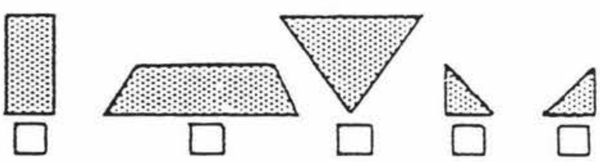
40. 

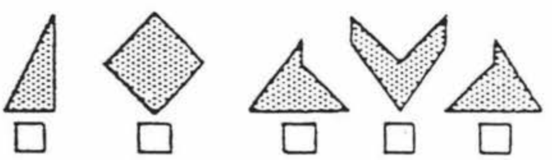
41. 

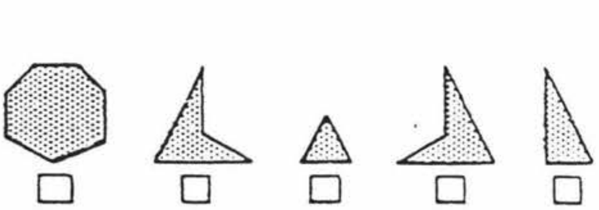
42. 

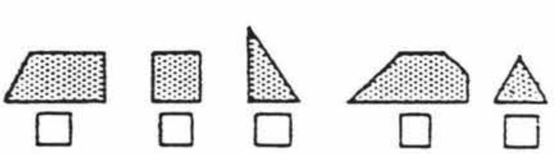


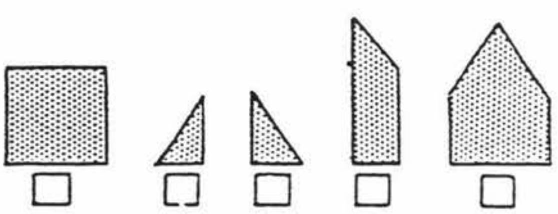
43. 

44. 

45. 

46. 

47. 

48. 

DO NOT GO BACK TO PART 1, AND

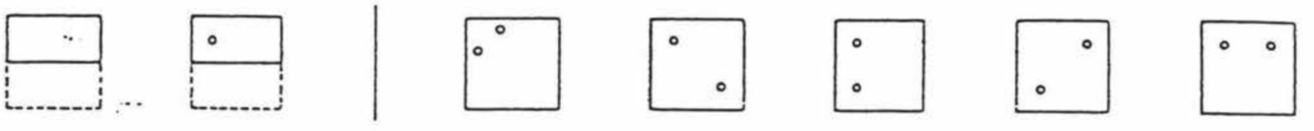
DO NOT GO ON TO ANY OTHER TEST UNTIL ASKED TO DO SO. STOP.

PAPER FOLDING TEST — VZ-2

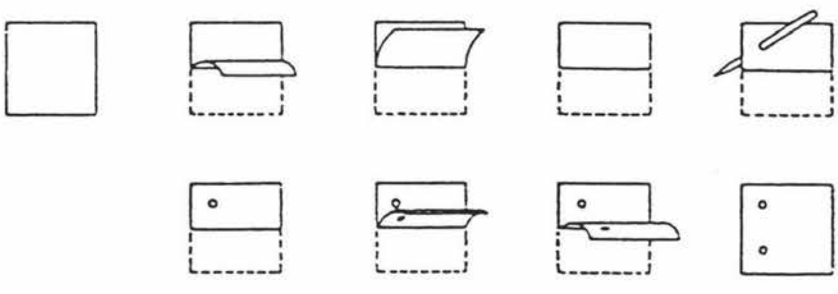
In this test you are to imagine the folding and unfolding of pieces of paper. In each problem in the test there are some figures drawn at the left of a vertical line and there are others drawn at the right of the line. The figures at the left represent a square piece of paper being folded, and the last of these figures has one or two small circles drawn on it to show where the paper has been punched. Each hole is punched through all the thicknesses of paper at that point. One of the five figures at the right of the vertical line shows where the holes will be when the paper is completely unfolded. You are to decide which one of these figures is correct and draw an X through that figure.

Now try the sample problem below. (In this problem only one hole was punched in the folded paper.)

A B C D E



The correct answer to the sample problem above is C and so it should have been marked with an X. The figures below show how the paper was folded and why C is the correct answer.



In these problems all of the folds that are made are shown in the figures at the left of the line, and the paper is not turned or moved in any way except to make the folds shown in the figures. Remember, the answer is the figure that shows the positions of the holes when the paper is completely unfolded.

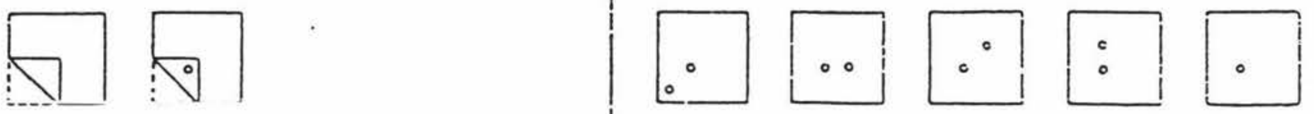
Your score on this test will be the number marked correctly minus a fraction of the number marked incorrectly. Therefore, it will <u>not</u> be to your advantage to guess unless you are able to eliminate one or more of the answer choices as wrong.

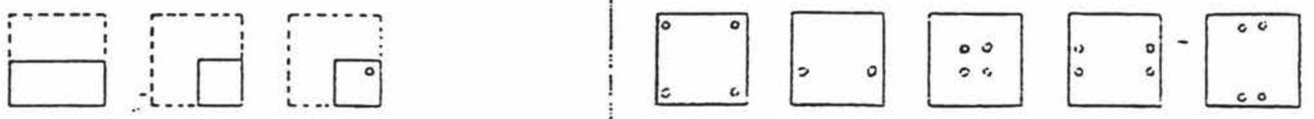
You will have 3 minutes for each of the two parts of this test. Each part has 1 page. When you have finished Part 1, STOP. Please do not go on to Part 2 until you are asked to do so.

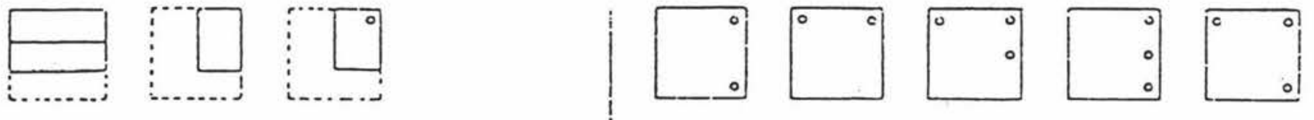
DO NOT TURN THIS PAGE UNTIL ASKED TO DO SO.

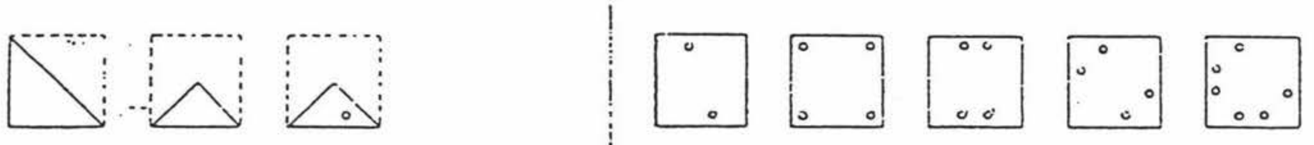
Part 1 (3 minutes)

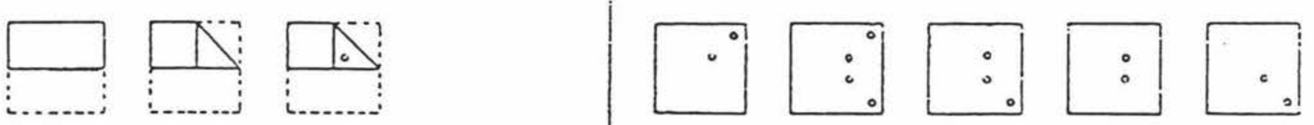
A B C D E

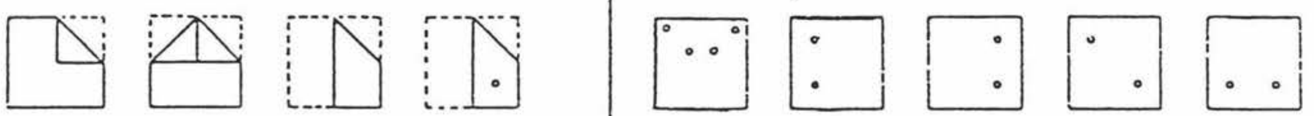
1 

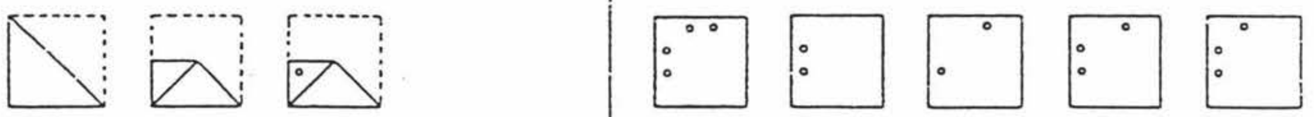
2 

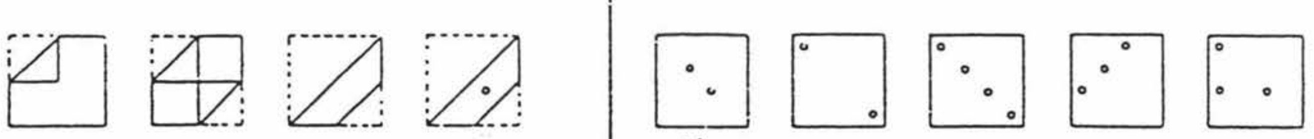
3 

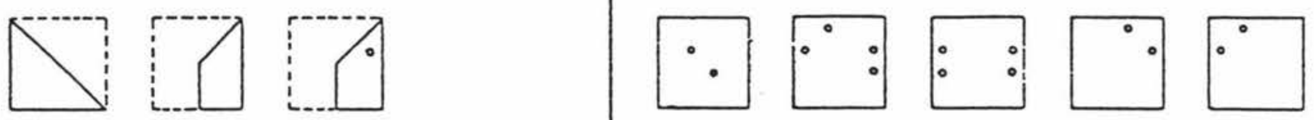
4 

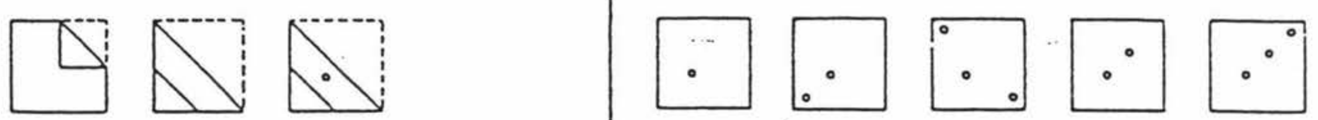
5 

6 

7 

8 

9 

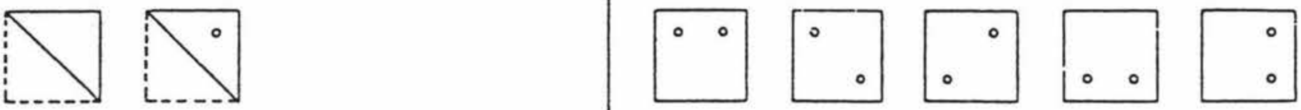
10 

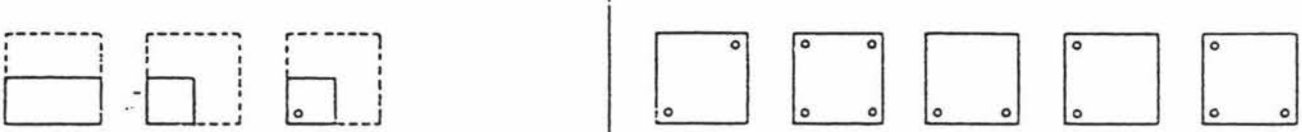
DO NOT GO ON TO THE NEXT PAGE UNTIL ASKED TO DO SO.

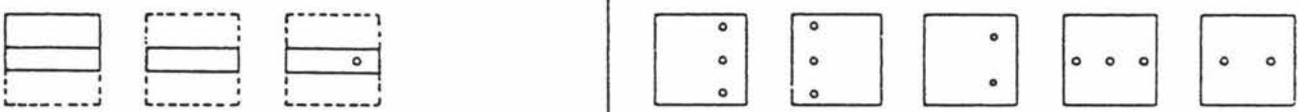
STOP.

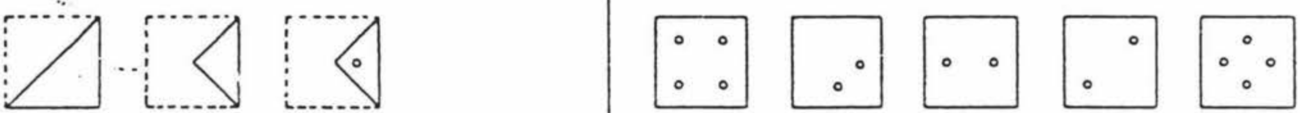
Part 2 (3 minutes)

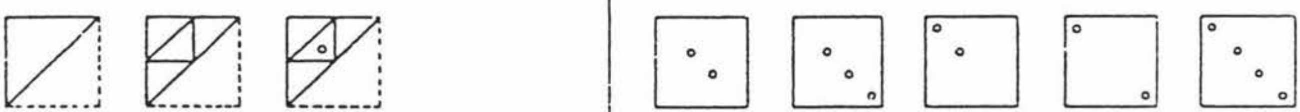
A B C D E

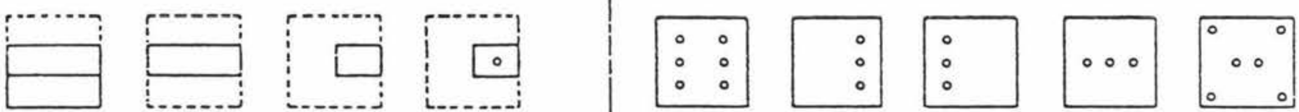
11 

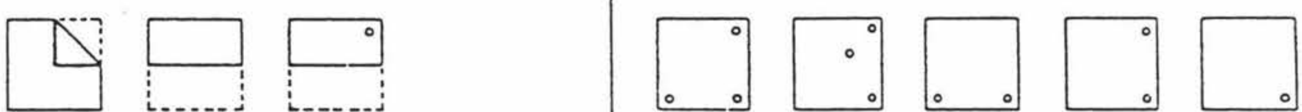
12 

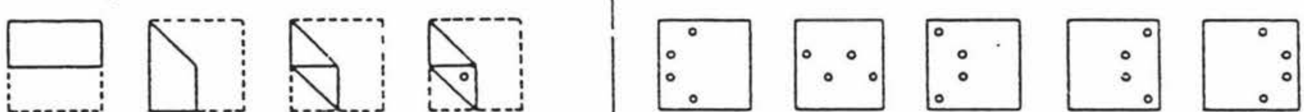
13 

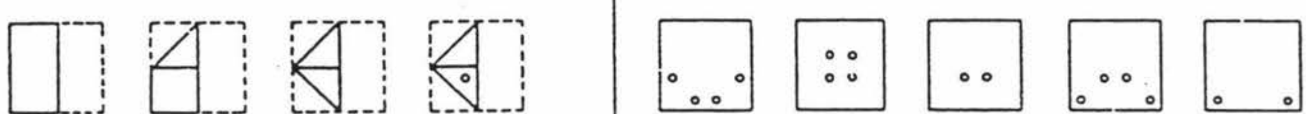
14 

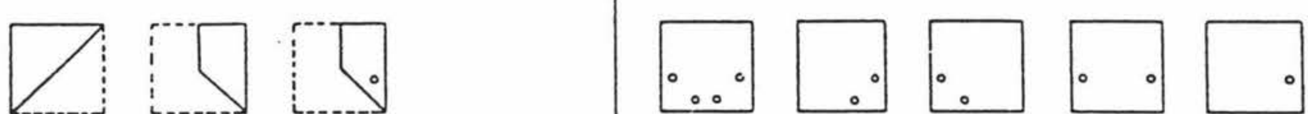
15 

16 

17 

18 

19 

20 

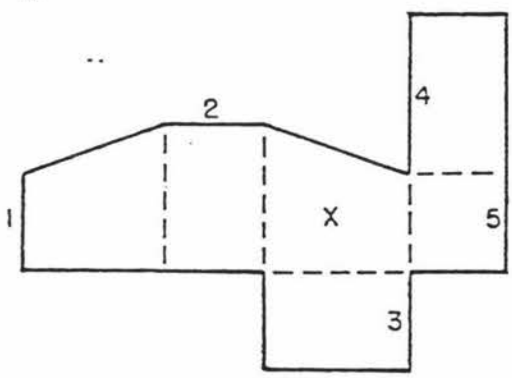
DO NOT GO BACK TO PART 1, AND

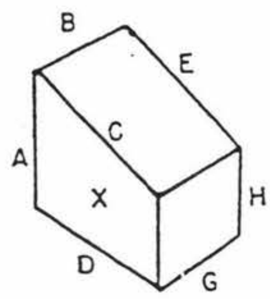
DO NOT GO ON TO ANY OTHER TEST UNTIL ASKED TO DO SO. **STOP.**

SURFACE DEVELOPMENT TEST — VZ-3

In this test you are to try to imagine or visualize how a piece of paper can be folded to form some kind of object. Look at the two drawings below. The drawing on the left is of a piece of paper which can be folded on the dotted lines to form the object drawn at the right. You are to imagine the folding and are to figure out which of the lettered edges on the object are the same as the numbered edges on the piece of paper at the left. Write the letters of the answers in the numbered spaces at the far right.

Now try the practice problem below. Numbers 1 and 4 are already correctly marked for you.





1:	H
2:	
3:	
4:	C
5:	

NOTE: The side of the flat piece marked with the X will always be the same as the side of the object marked with the X. Therefore, the paper must always be folded so that the X will be on the outside of the object.

In the above problem, if the side with edge 1 is folded around to form the back of the object, then edge 1 will be the same as edge H. If the side with edge 5 is folded back, then the side with edge 4 may be folded down so that edge 4 is the same as edge C. The other answers are as follows: 2 is B; 3 is G; and 5 is H. Notice that two of the answers can be the same.

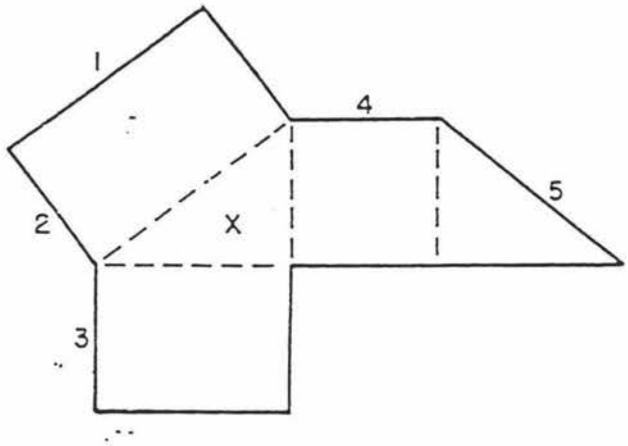
Your score on this test will be the number of correct letters minus a fraction of the number of incorrect letters. Therefore, it will not be to your advantage to guess unless you are able to eliminate one or more of the answer choices as wrong.

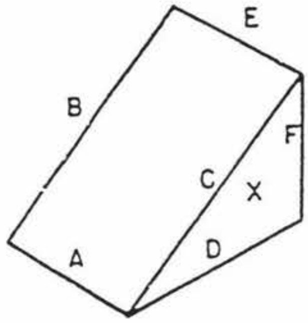
You will have 6 minutes for each of the two parts of this test. Each part has 2 pages. When you have finished Part 1 (pages 2 and 3), STOP. Please do not go on to Part 2 until you are asked to do so.

DO NOT TURN THIS PAGE UNTIL ASKED TO DO SO.

Part 1 (6 minutes)

1





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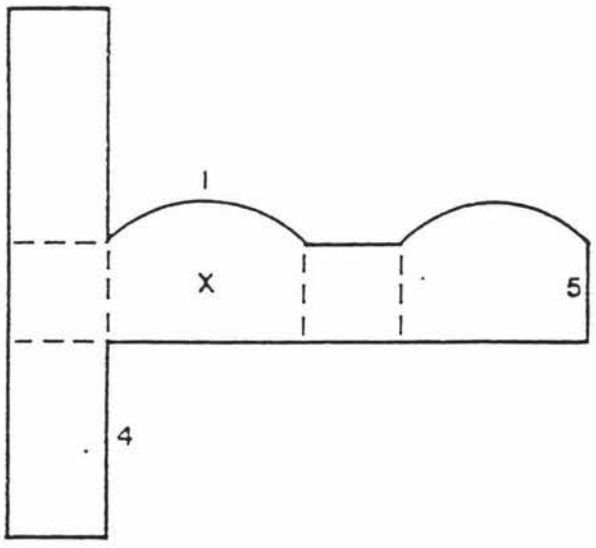
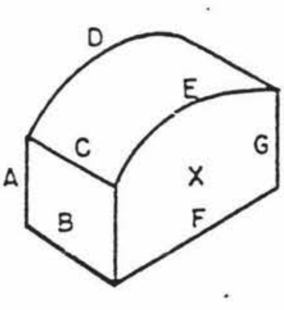
2





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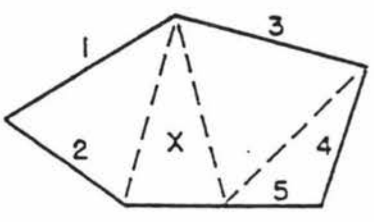
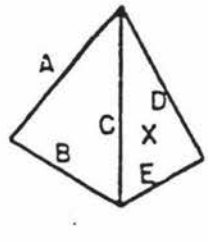
5:	

3





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2:	
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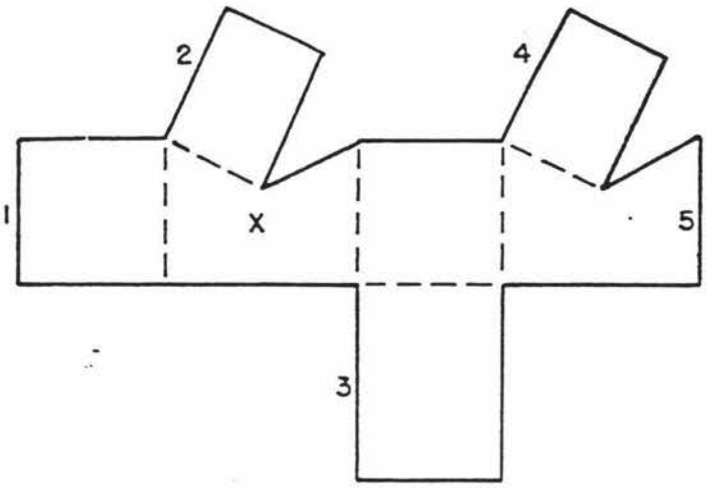
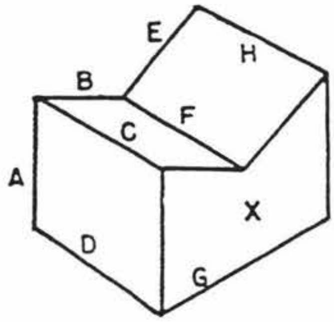
GO ON TO THE NEXT PAGE

Part 1 (continued)

4





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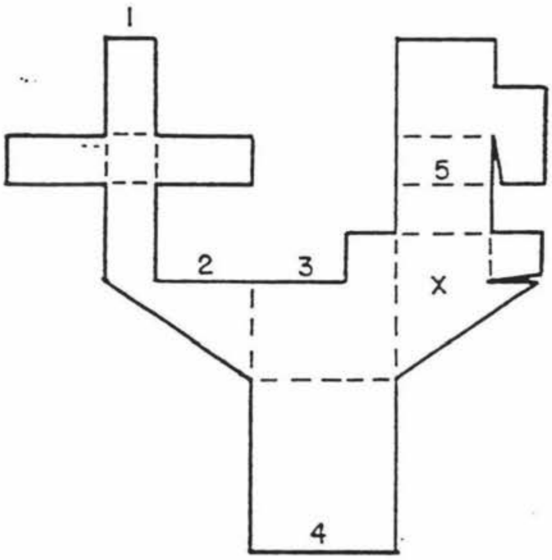

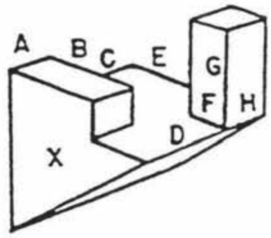

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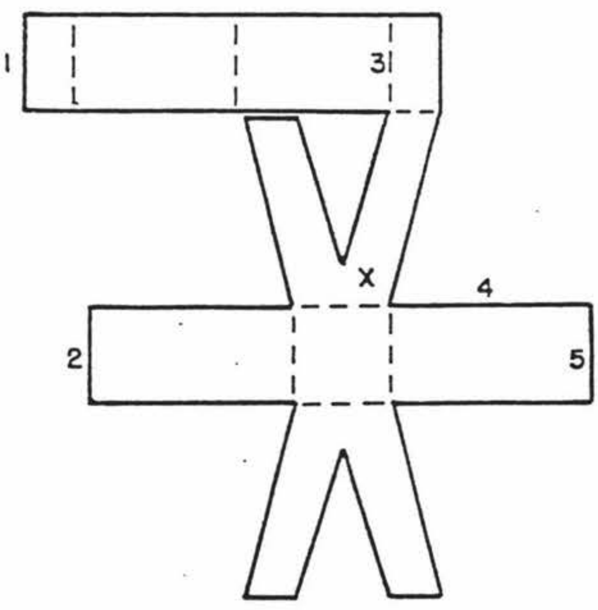
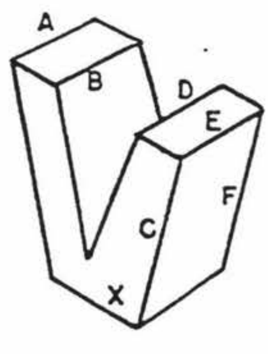
6





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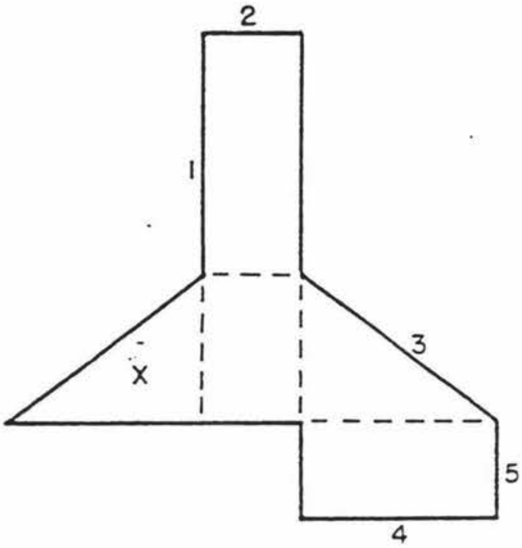
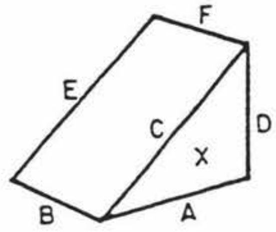
DO NOT GO ON TO THE NEXT PAGE UNTIL ASKED TO DO SO. STOP.

Part 2 (6 minutes)





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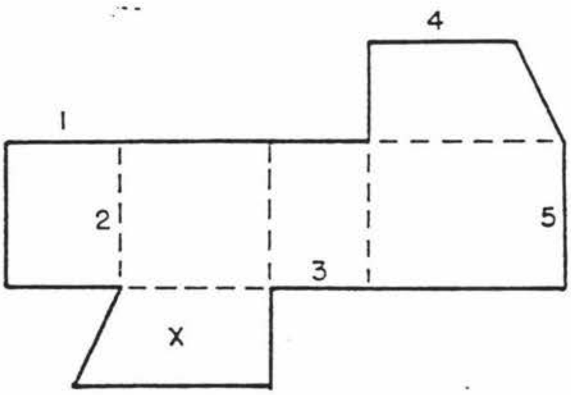

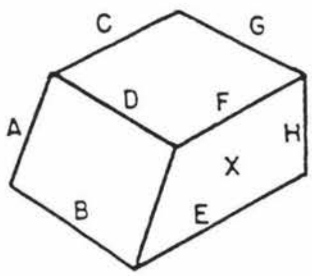

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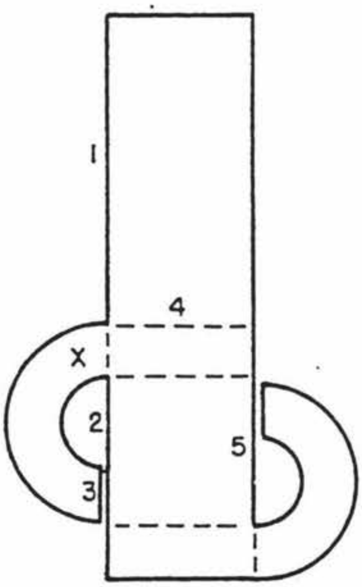

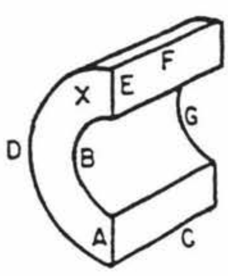

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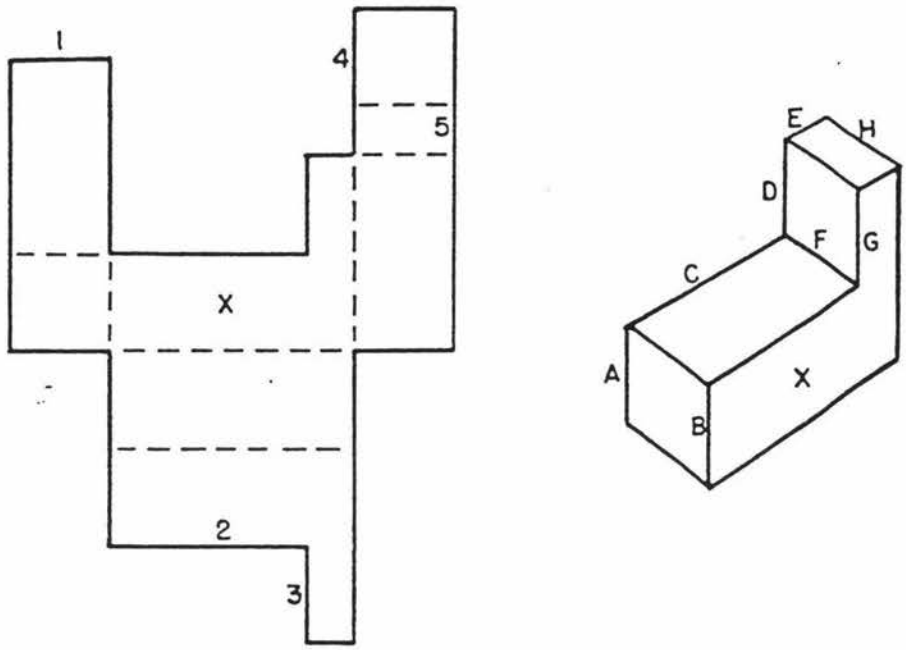
Part 2 (continued)

10



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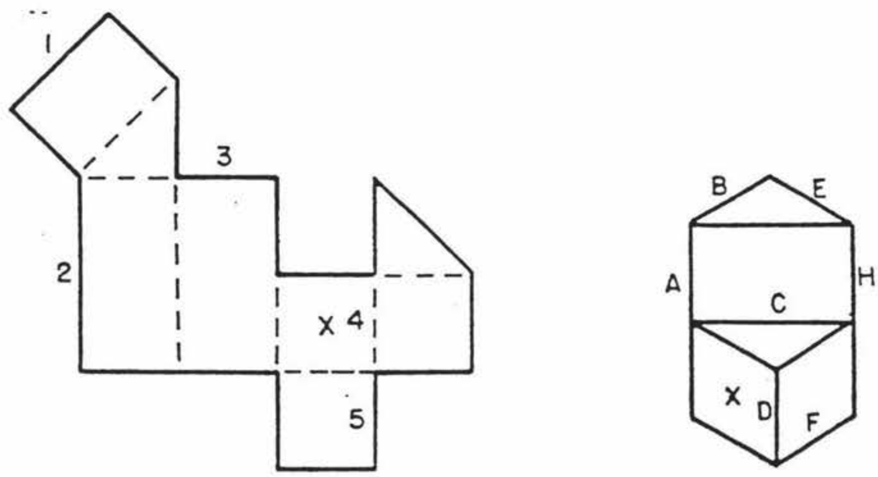

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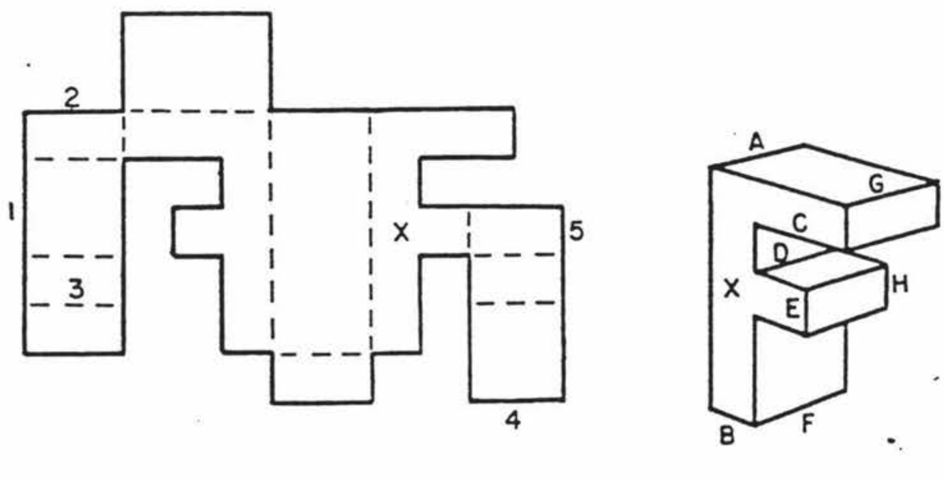

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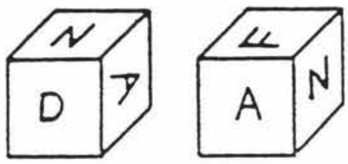
DO NOT GO BACK TO PART 1, AND

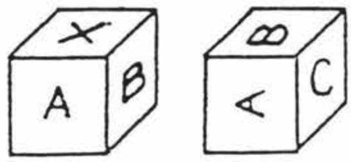
DO NOT GO ON TO ANY OTHER TEST UNTIL ASKED TO DO SO. STOP.

CUBE COMPARISONS TEST -- S-2 (Rev.)

Wooden blocks such as children play with are often cubical with a different letter, number, or symbol on each of the six faces (top, bottom, four sides). Each problem in this test consists of drawings of pairs of cubes or blocks of this kind. Remember, there is a different design, number, or letter on each face of a given cube or block. Compare the two cubes in each pair below.



S D



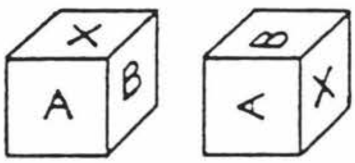
S D

The first pair is marked D because they must be drawings of <u>different</u> cubes. If the left cube is turned so that the A is upright and facing you, the N would be to the left of the A and hidden, not to the right of the A as is shown on the right hand member of the pair. Thus, the drawings must be of different cubes.

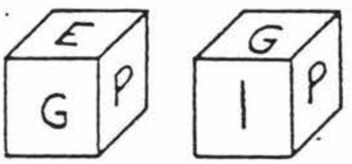
The second pair is marked S because they could be drawings of the same cube. That is, if the A is turned on its side the X becomes hidden, the B is now on top, and the C (which was hidden) now appears. Thus the two drawings could be of the same cube.

<u>Note</u>: No letters, numbers, or symbols appear on more than one face of a given cube. Except for that, <u>any</u> letter, number or symbol can be on the hidden faces of a cube.

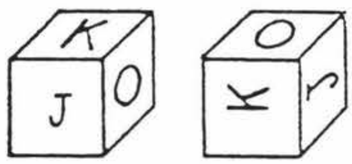
Work the three examples below.



S D



S D



S D

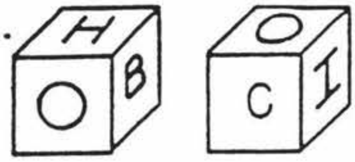
The first pair immediately above should be marked D because the X cannot be at the peak of the A on the left hand drawing and at the base of the A on the right hand drawing. The second pair is "different" because P has its side next to G on the left hand cube but its top next to G on the right hand cube. The blocks in the third pair are the same, the J and K are just turned on their side, moving the O to the top.

Your score on this test will be the number marked correctly minus the number marked incorrectly. Therefore, it will <u>not</u> be to your advantage to guess unless you have some idea which choice is correct. Work as quickly as you can without sacrificing accuracy.

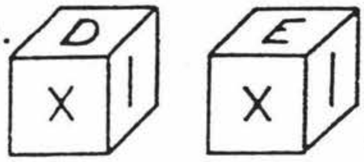
You will have <u>3 minutes</u> for each of the two parts of this test. Each part has one page. When you have finished Part 1, STOP.

DO NOT TURN THE PAGE UNTIL YOU ARE ASKED TO DO SO.

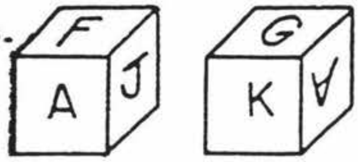
Part 1 (3 minutes)

1. 

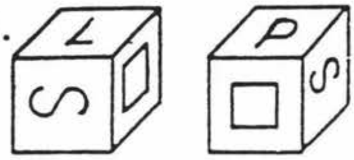
S ▭ D ▭

2. 

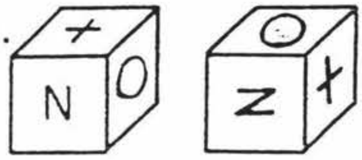
S ▭ D ▭

3. 

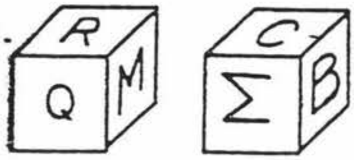
S ▭ D ▭

4. 

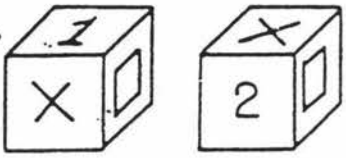
S ▭ D ▭

5. 

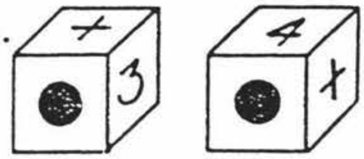
S ▭ D ▭

6. 

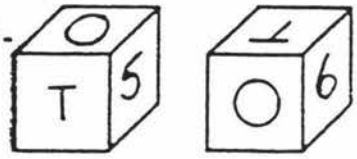
S ▭ D ▭

7. 

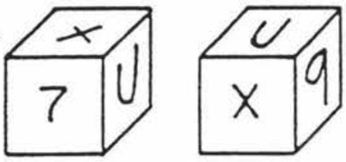
S ▭ D ▭

8. 

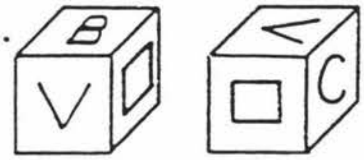
S ▭ D ▭

9. 

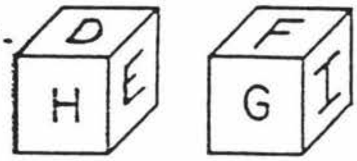
S ▭ D ▭

10. 

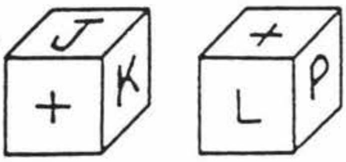
S ▭ D ▭

11. 

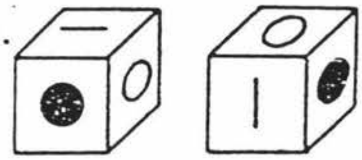
S ▭ D ▭

12. 

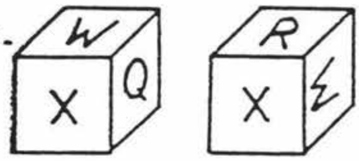
S ▭ D ▭

13. 

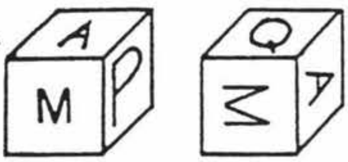
S ▭ D ▭

14. 

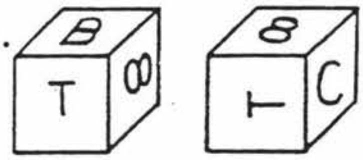
S ▭ D ▭

15. 

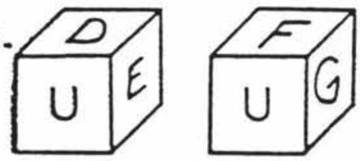
S ▭ D ▭

16. 

S ▭ D ▭

17. 

S ▭ D ▭

18. 

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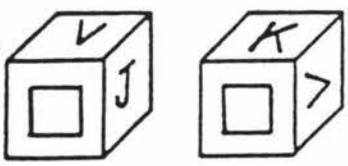
19. 

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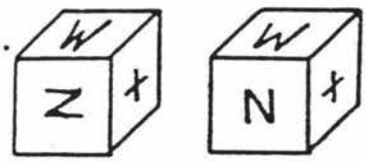
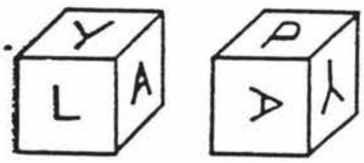
20. 

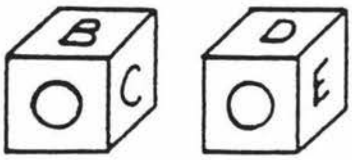
S ▭ D ▭

21. 

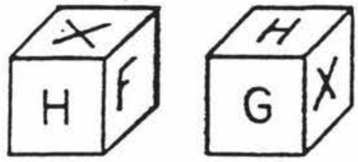
S ▭ D ▭

DO NOT GO ON TO THE NEXT PAGE UNTIL ASKED TO DO SO. STOP.

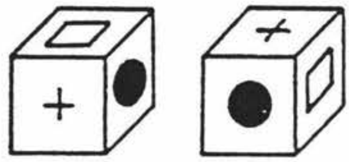
Part 2 (3 minutes)

22. 

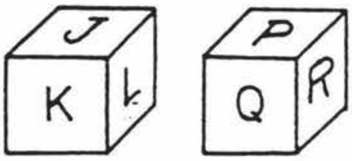
S ☐ D ☐

23. 

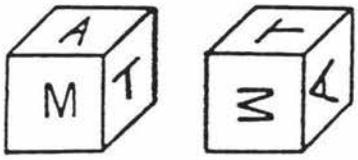
S ☐ D ☐

24. 

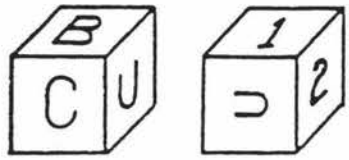
S ☐ D ☐

25. 

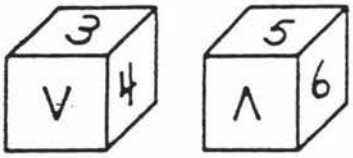
S ☐ D ☐

26. 

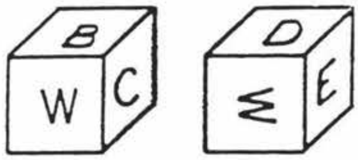
S ☐ D ☐

27. 

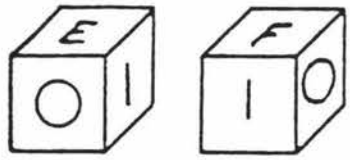
S ☐ D ☐

28. 

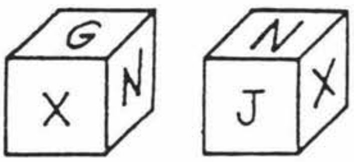
S ☐ D ☐

29. 

S ☐ D ☐

30. 

S ☐ D ☐

31. 

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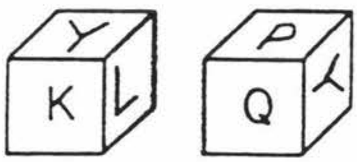
32. 

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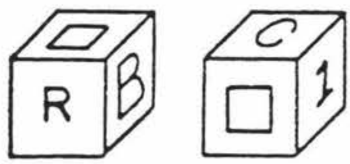
33. 

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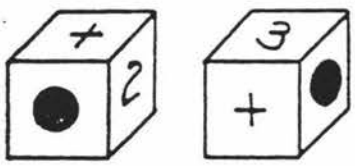
34. 

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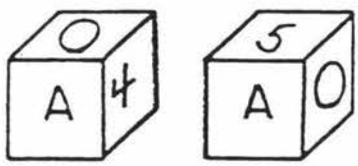
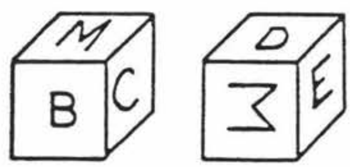
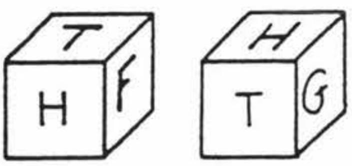
35. 

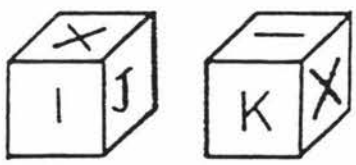
S ☐ D ☐

36. 

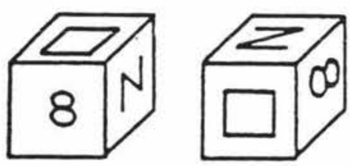
S ☐ D ☐

37. 

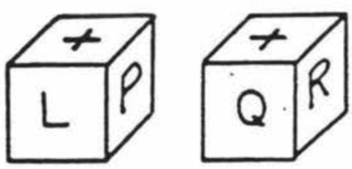
S ☐ D ☐

38. 

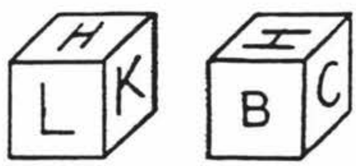
S ☐ D ☐

39. 

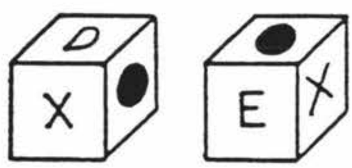
S ☐ D ☐

40. 

S ☐ D ☐

41. 

S ☐ D ☐

42. 

S ☐ D ☐

DO NOT GO BACK TO PART 1 AND

DO NOT GO ON TO ANY OTHER TEST UNTIL ASKED TO DO SO. STOP.

CARD ROTATIONS TEST — S-1 (Rev.)

This is a test of your ability to see differences in figures. Look at the 5 triangle-shaped cards drawn below.

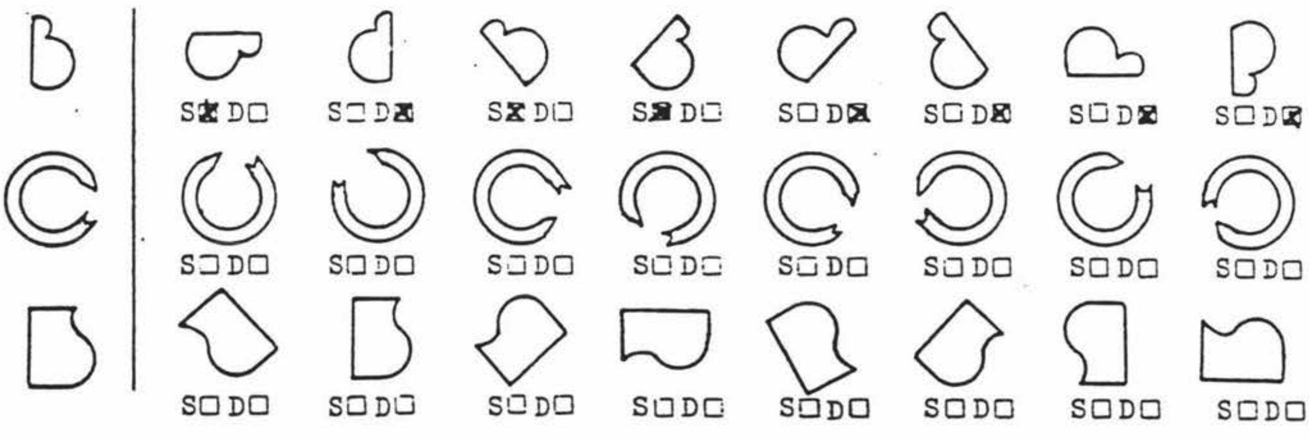
All of these drawings are of the same card, which has been slid around into different positions on the page.

Now look at the 2 cards below:

These two cards are not alike. The first cannot be made to look like the second by sliding it around on the page. It would have to be flipped over or made differently.

Each problem in this test consists of one card on the left of a vertical line and eight cards on the right. You are to decide whether each of the eight cards on the right is the same as or different from the card at the left. Mark the box beside the S if it is the same as the one at the beginning of the row. Mark the box beside the D if it is different from the one at the beginning of the row.

Practice on the following rows. The first row has been correctly marked for you.

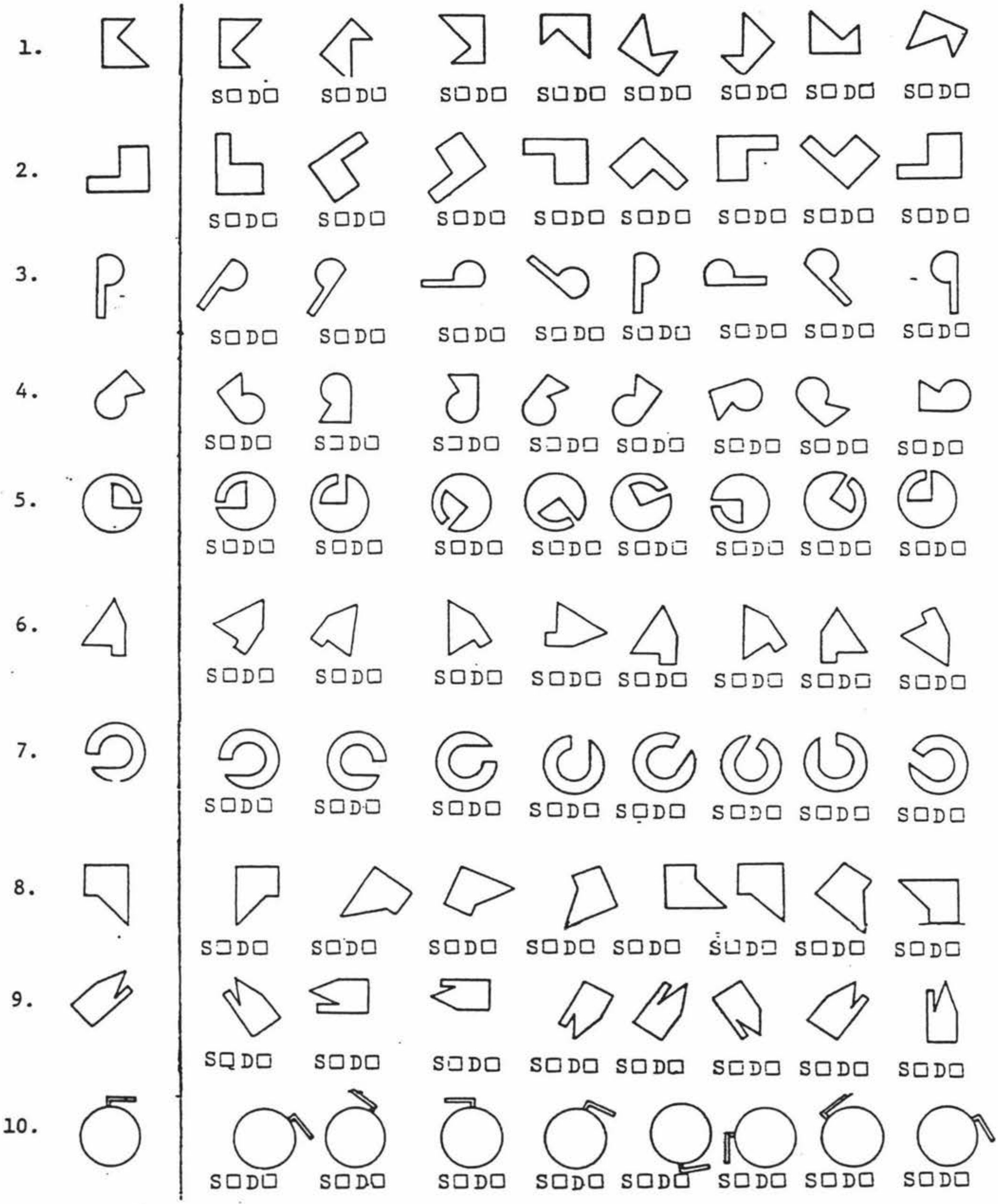


Your score on this test will be the number of items answered correctly minus the number answered incorrectly. Therefore, it will not be to your advantage to guess, unless you have some idea whether the card is the same or different. Work as quickly as you can without sacrificing accuracy.

You will have 3 minutes for each of the two parts of this test. Each part has 1 page. When you have finished Part 1, STOP. Please do not go on to Part 2 until you are asked to do so.

DO NOT TURN THIS PAGE UNTIL ASKED TO DO SO.

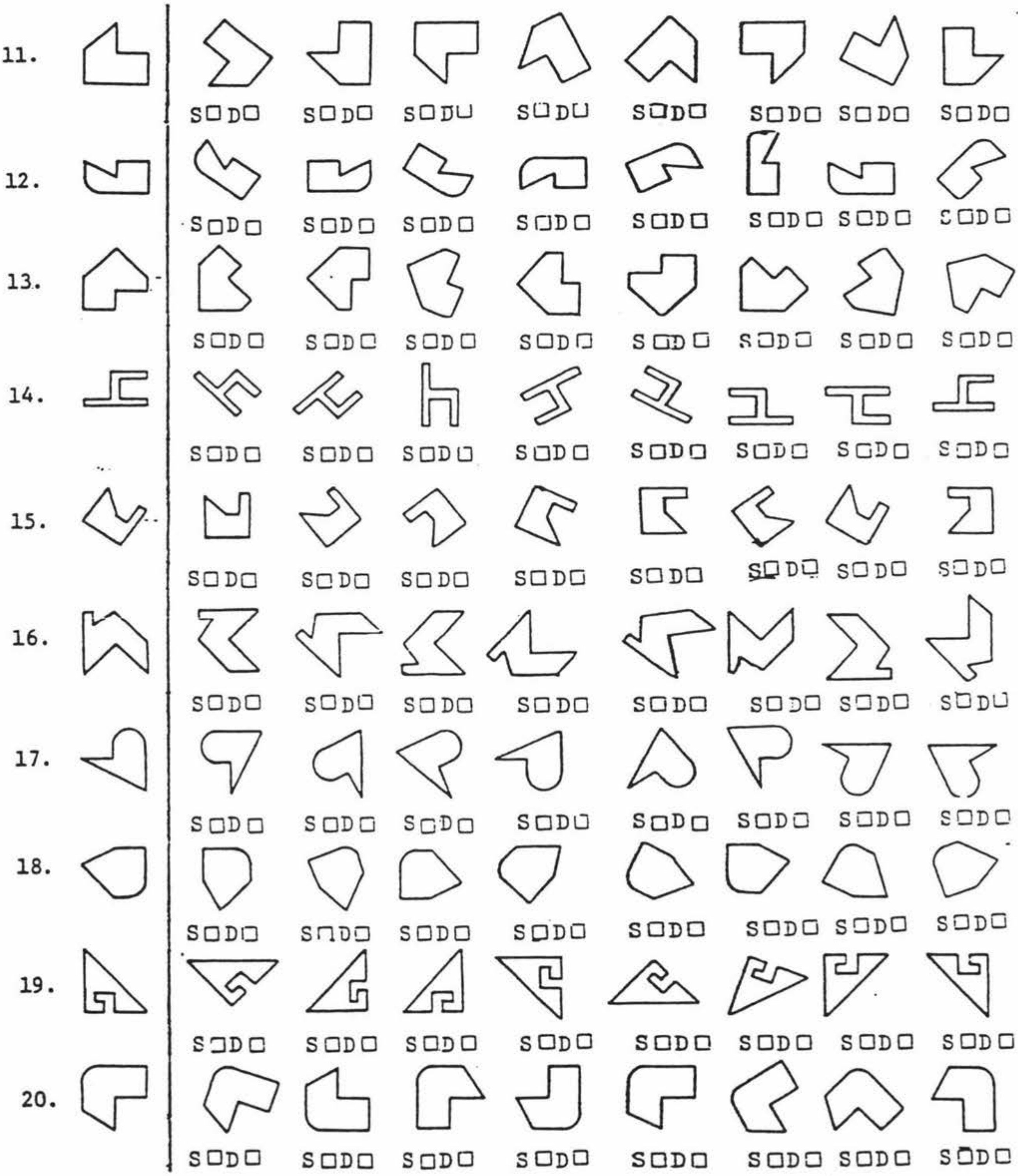
Part 1 (3 minutes)



DO NOT TURN THIS PAGE UNTIL ASKED TO DO SO.

STOP.

Part 2 (3 minutes)



DO NOT GO BACK TO PART 1 AND

DO NOT GO ON TO ANY OTHER TEST UNTIL ASKED TO DO SO. STOP.

Appendix C

Table C1 contains the output of the discriminant function analysis classification table. This table descibes the probabilities of being placed into a group based on the discriminant function.

Table C1 SPSS PC output of the discriminant function analysis classification table

Case Number	Mis Val	Sel	Actual Group	Highest Probability Group	P(D/G)	P(G/D)	2nd Highest Group	P(G/D)	Discriminant Scores
1			2	2	.9081	.9190	1	.0810	-1.1613
2			2	2	.9190	.8782	1	.1218	-.9442
3			2	2	.5065	.9728	1	.0272	-1.7102
4			2 **	1	.4345	.6349	2	.3651	.2644
5			2	2	.4992	.6845	1	.3155	-.3702
6			2	2	.5484	.7175	1	.2825	-.4457
7			2 **	1	.4130	.6167	2	.3833	.2273
8			2 **	1	.3287	.5361	2	.4639	.0692
9			1	1	.8903	.9225	2	.0775	1.1838
10			2	2	.9329	.9140	1	.0860	-1.1300
11			2	2	.7447	.9463	1	.0537	-1.3716
12			2	2	.5319	.9706	1	.0294	-1.6710
13			2	2	.7715	.9424	1	.0576	-1.3363
14			1	1	.0114	.9994	2	.0006	3.5755
15			1	1	.8447	.8555	2	.1445	.8500
16			1	1	.7070	.9514	2	.0486	1.4218
17			1	1	.0447	.9983	2	.0017	3.0530
18			2 **	1	.5660	.7285	2	.2715	.4720
19			1	1	.3984	.9812	2	.0188	1.8903
20			1	1	.0126	.9994	2	.0006	3.5421
21			1	1	.9924	.8973	2	.1027	1.0364
22			2	2	.2632	.9893	1	.0107	-2.1648
23			1	1	.0594	.9978	2	.0022	2.9313
24			2	2	.4249	.9793	1	.0207	-1.8438
25			1 **	2	.3195	.5263	1	.4737	-.0503
26			1	1	.4380	.9783	2	.0217	1.8214
27			1	1	.9396	.9126	2	.0874	1.1217
28			1	1	.1090	.9961	2	.0039	2.6488
29			1	1	.6476	.9587	2	.0413	1.5029
30			2	2	.6340	.9602	1	.0398	-1.5220
31			2	2	.8995	.9207	1	.0793	-1.1721
32			1	1	.4848	.6740	2	.3260	.3473
33			1	1	.1458	.9947	2	.0053	2.5003
34			2 **	1	.5693	.7305	2	.2695	.4768
35			1	1	.2184	.9915	2	.0085	2.2767
36			1	1	.6810	.7905	2	.2095	.6349
37			2	2	.6564	.9576	1	.0424	-1.4907
38			2	2	.6009	.9638	1	.0362	-1.5689
39			1	1	.3449	.5529	2	.4471	.1014
40			2	2	.3842	.9822	1	.0178	-1.9161
41			2	2	.6075	.9631	1	.0369	-1.5595
42			2	2	.5767	.7350	1	.2650	-.4877
43			2	2	.7975	.9385	1	.0615	-1.3025
44			1	1	.4470	.6450	2	.3550	.2855

Table C1 continued

Case Number	Mis Val Sel	Actual Group	Highest Probability Group P(D/G)	P(G/D)	2nd Highest Group	P(G/D)	Discriminant Scores
45	2	2	.7229	.8094	1	.1906	-.6913
46	2 **	1	.5680	.7297	2	.2703	.4748
47	1 **	2	.5482	.7174	1	.2826	-.4454
48	1	1	.0179	.9992	2	.0008	3.4138
49	1	1	.9261	.8801	2	.1199	.9531
50	2	2	.5902	.9649	1	.0351	-1.5844
51	1	1	.4345	.6348	2	.3652	.2643
52	2	2	.6238	.9613	1	.0387	-1.5363
53	2	2	.8399	.8538	1	.1462	-.8438
54	2	2	.8258	.9339	1	.0661	-1.2659
55	2	2	.9722	.9056	1	.0944	-1.0807
56	1	1	.4861	.6750	2	.3250	.3494
57	1 **	2	.4863	.6751	1	.3249	-.3497
58	1	1	.9073	.8748	2	.1252	.9294
59	1	1	.8770	.8658	2	.1342	.8911
60	2	2	.5164	.6964	1	.3036	-.3969
61	2	2	.5078	.9727	1	.0273	-1.7082
62	1	1	.7815	.8331	2	.1669	.7685
63	2	2	.8281	.9335	1	.0665	-1.2630
64	2	2	.9287	.9149	1	.0851	-1.1353
65	1 **	2	.7665	.8273	1	.1727	-.7489
66	2	2	.7548	.9449	1	.0551	-1.3582
67	1 **	2	.7999	.8399	1	.1601	-.7924
68	1	1	.7024	.8004	2	.1996	.6638
69	2	2	.9484	.9108	1	.0892	-1.1106
70	1	1	.9574	.9088	2	.0912	1.0992
71	1	1	.9156	.8772	2	.1228	.9399
72	2	2	.7993	.9382	1	.0618	-1.3002
73	2	2	.6367	.9599	1	.0401	-1.5183
74	2	2	.3570	.5649	1	.4351	-.1248
75	1	1	.7731	.9422	2	.0578	1.3342
76	2	2	.8969	.9212	1	.0788	-1.1755
77	1 **	2	.3103	.5163	1	.4837	-.0313
78	1	1	.1508	.9945	2	.0055	2.4827
79	1	1	.7955	.8383	2	.1617	.7867
80	1 **	2	.3585	.5663	1	.4337	-.1276
81	1	1	.1858	.9930	2	.0070	2.3689
82	1	1	.8737	.8647	2	.1353	.8869
83	1	1	.6481	.9586	2	.0414	1.5023
84	1 **	2	.8122	.9361	1	.0639	-1.2835
85	1 **	2	.6908	.7951	1	.2049	-.6481
86	2	2	.9814	.9035	1	.0965	-1.0692
87	1 **	2	.3090	.5150	1	.4850	-.0286

Table C1 continued

Case Number	Mis Val	Sel	Actual Group	Highest Probability Group	P(D/G)	P(G/D)	2nd Highest Group	P(G/D)	Discriminant Scores
88			2	1 **	.5512	.7193	1	.2807	-.4499
89			2	2	.3841	.9822	1	.0178	-1.9163
90			2	2	.6598	.9572	1	.0428	-1.4861
91			1	1	.2181	.9915	2	.0085	2.2776
92			2	2	.8007	.8402	1	.1598	-.7934
93			2	2	.9522	.9100	1	.0900	-1.1058
94			2	2	.2349	.9907	1	.0093	-2.2338
95			2	2	.6558	.9577	1	.0423	-1.4916
96			1	1	.3595	.9838	2	.0162	1.9622
97			2	2	.5274	.9710	1	.0290	-1.6778
98			1	2 **	.7731	.8298	2	.1702	.7575
99			1	1	.7708	.8290	2	.1710	.7545
100			2	1 **	.3462	.5541	1	.4459	-.1038

Classification Results -

Actual Group	Number of Cases	Predicted Group Membership 1	2
Group 1	50	38	12
(above 75th percentile)		76.0%	24.0%
Group 2	50	7	43
(below 25th percentile)		14.0%	86.0%

Percent of grouped cases correctly classified: 81.00%