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Static and Dynamic Imaging using Magnetic Field Gradients

静态与动态核磁成像

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by

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Abstract

The theory and techniques of NMR imaging are described together with a detailed description of the Filtered Back Projection (FBP) technique used in an existing NMR imaging system.

The existing 'static' NMR imaging system has been modified to be capable of performing 'dynamic' NMR imaging experiments, as well as better 'static' NMR imaging experiments.

The potential of NMR microscopy in the imaging of both the static spin distribution $P(\mathbf{r}_0)$ and the dynamic spin correlation function $P(\mathbf{r}_0|\mathbf{r},t)$ has been investigated. Both homogeneous and inhomogeneous systems have been studied. Detailed theoretical analysis and experimental considerations of dynamic imaging experiments have been given.

A transverse resolution of 15 μm for a 1 mm slice thickness is obtained from a static imaging experiment of a phantom using the modified system. The rabbit trachea imaging experiment has revealed the asymmetrical collapse of tracheas under negative pressures, a collapse which had previously been considered as symmetrical process.

The Poiseuille flow experiment has involved the first simultaneous measurement of flow and diffusion at the microscopic level. Maps of two dimensional distribution functions of flow and diffusion are given by this experiment, highlighting this totally non-invasive dynamic imaging technique.

As an example of dynamic imaging, the wheat grain experiment has displayed the flow and diffusion maps within a single wheat grain *in vivo*.

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a	RF coil radius	29
a_{mj}	Complex admixture amplitudes of a spin system	3
A	An operator representing an observable quantity	5
A	Signal amplitude	44
A	Cross sectional area	108
B_{eff}	Effective field in the rotating frame	7
$\mathbf{B}$	Magnetic field	6
B_0	Amplitude of the main magnetic field	3
$\mathbf{B_0}$	Main magnetic field, directed along the z axis	3
B_1	Amplitude of the transverse rf field $\mathbf{B_1(t)}$	7
$\mathbf{B_1(t)}$	RF field (in the transverse plane)	7
D	Self-diffusion coefficient	32
$\mathbf{D}$	Self-diffusion tensor	33
D_e	Extra broadening due to velocity spread	47
$E(m_j)$	Energy eigenvalues of a spin system	3
f	Spectrometer frequency	29
F	Noise figure of the spectrometer	16
$F\{\ \}$	Fourier transform of the function in { }	23
$F_s\{\ \}$	sin transform of the function in { }	23
$F_c\{\ \}$	cos transform of the function in { }	23
g	Amplitude of PGSE gradient	32
$\mathbf{g}$	PGSE gradient in dynamic imaging	32
g_m	Maximum gradient employed in dynamic imaging	40
G	Amplitude of field gradient	32
$\mathbf{G}$	Field gradient	19
$h(t)$	Fourier transform of frequency domain function H(f)	38
H_I	Imaginary part of the discrete function H	42
H_R	Real part of the discrete function H	42
$H(f)$	Fourier transform of time domain function h(t)	38
$\mathcal{H}$	Hamiltonian operator	3
$\mathcal{H}_1(t)$	Perturbation term in Hamiltonian operator	7
i	$(-1)^{1/2}$	4
I	Function selected in dynamic imaging analysis program	91
$Im[\ ]$	Imaginary part of a complex function in [ ]	36
j	Spin quantum number	3
$\mathbf{J}$	Spin angular momentum operator	3
k	Frequency domain (digital) variable	38
$\mathbf{k}$	Static reciprocal space vector	24
k_B	Boltzmann constant	5
K	Numerical factor in the calculation of S/N	16
l	Length of the pipe in Poiseuille sample system	119
L	Length of the conductor	16
m_j	Azimuthal quantum numbers	3
$\mathbf{M}$	Macroscopic magnetization vector	5
M_0	Magnitude of **M** in the equilibrium state	6
$M_\perp$	Transverse component of **M**	9
n	Time domain (digital) variable	38
n_D	Maximum number of data images	36
n_1	A constant in the 'tube law'	109

N	Number of spins per unit volume	5
N	Total number of digits in time domain	38
N_h	Number of hydrogen nuclei per unit volume	16
N_p	Number of projections	29
N_{acc}	Number of accumulations per projection	25
p	Perimeter of the conductor	16
P	Transmural pressure difference	109
P_1	Constant asymptotic pressure	109
P_s	Self-correlation function of the nuclear spin	33
P^*	Filtered profile	25
Q	Quality factor of the coil	16
Q	Volume amount of fluid	120
$\mathbf{q}$	Dynamic reciprocal space vector	34
$\mathbf{r}$	Position vector	18
R	Attenuation factor	33
R	Radius of the pipe in dynamic imaging experiment	120
$\mathrm{Re}[\ ]$	Real part of a complex function in [ ]	24
S	Fluid displacement vector	33
$S(t)$	FID signal	24
S^*	Complex conjugate of S	25
t_p	Duration of the pulse	9
t_{rep}	Repetition time of in experiments	25
T	Absolute temperature of a spin system	5
T	Time domain sampling interval	38
T	Sampling time in imaging experiments	96
T_c	Probe temperature	16
T_s	Sample temperature	16
T_1	Spin-lattice relaxation time	10
T_2	Spin-spin relaxation time	10
T_2^*	Transverse relaxation time	10
$\mathrm{Tr}(\ )$	Trace of the operator in ( )	4
$U_E(t)$	Evolution operator	4
$U_{Rz}(\theta)$	Rotation operator	4
$\mathbf{v}$	Velocity of fluid flow	33
V_c	Volume of the coil	16
V_s	Sample volume	16
w	Weight of the fluid	120
α	A variable in discrete Fourier transform	38
γ	Gyromagnetic ratio	3
δ	Duration of the PGSE pulse	32
η	Fraction of the coil volume occupied by the sample	16
η	Dynamic viscosity of the fluid	120
θ	Rotation angle of the magnetization vector	9
λ	Wave length	61
μ_0	Permeability of free space	16
$\boldsymbol{\mu}$	Magnetic moment vector	3
ν	Kinematic viscosity of the fluid	119
ξ	Complex FID signal	35

ρ	Density operator	4
ρ	Density of the fluid	119
ρ_I	Imaginary part of nuclear spin density	35
ρ_R	Real part of nuclear spin density	35
ρ_T	Resistivity of the conductor	16
$\rho(\mathbf{r})$	Nuclear spin density	24
σ	RF coil proximity factor	16
$\sigma(n)$	Noise function	127
τ	Short time interval	10
$\boldsymbol{\omega}$	Larmor precession frequency	7
$\boldsymbol{\omega}_0$	Larmor precession frequency due to $\mathbf{B}_0$	4
ω_1	Larmor precession frequency due to B_1	8
ω_{eff}	Precession frequency in the rotating frame	7
Δ	Separation of the PGSE pulses	32
ϕ	Projection angle in imaging experiment	24
$\lvert j\ m_j>$	Basis eigenket set of a spin system	3
$\lvert\psi>$	General quantum state of a spin j system	3
$\overline{<A>}$	Ensemble average of the observable quantity A	4
$\overline{\lvert a_{m_j}\rvert^2}$	Normalized population in the eigenstate $\lvert j\ m_j>$	5
Δf	Bandwidth of the receiver	16
Δh	Height difference	120
Δv	Velocity spread between the adjacent pixels	47
Δx	Transverse resolution	29
ΔZ	Slice thickness	29
ΔE	Energy difference between the two adjacent states	7
ΔP	Pressure difference along the length of the pipe	119
$\Delta\phi$	Step angle in imaging experiment	25
∇B_0	Magnetic field gradient	19
∇P	Pressure gradient	119
$\hbar$	Planck's constant divided by 2π	3

Chapter 1 Introduction

1.1 Introduction

Nuclear Magnetic Resonance (NMR) Imaging is a non-invasive technique which gives the spatial distribution of the NMR signal intensity or other NMR parameters in a heterogeneous sample. The first experimental demonstration of the feasibility of macroscopic NMR imaging was given by Lauterbur in 1972 (1,2).

In conventional NMR it is usual to place the sample, which is homogeneous and small, in a very uniform magnetic field, so that the resonant frequency depends upon the external field modified slightly by the local environment. NMR spectra obtained in this way yield details of the local molecular environment.

By contrast, NMR Imaging concerns a sample which is heterogeneous, and usually not small. Furthermore, the sample is placed in a deliberately non-uniform magnetic field, which enables the hetero-structure of the sample to be derived and displayed.

Many different techniques have been described for NMR Imaging(3,4,5). Among these the Projection Reconstruction technique, originally from X-ray Tomography, is the most sensitive one(6).

The proton (^1H) is the most commonly used nucleus when doing imaging experiments, Hydrogen being the most abundant element in the living systems. ^1H is isotopically almost 100% abundant, and has the highest magnetic moment among stable nuclei, thus yielding optimum sensitivity. ^{19}F and ^{31}P nuclei are next in sensitivity and have some practical interest. Other nuclei are, in practice, difficult to image.

Traditionally NMR imaging reveals some stationary distribution functions of a nuclear spin system, for example, the spin density distribution. Such imaging is termed **'static' NMR imaging** in this thesis.

By incorporating the Pulse-Gradient-Spin-Echo (PGSE) technique, the NMR imaging can describe time-dependent functions. This technique is termed **'dynamic' NMR imaging**. Simultaneous imaging of flow and diffusion at the microscopic level can be performed using this new technique, which has been demonstrated by some imaging experiments in this work.

1.2 Organisation of the Thesis

This thesis is divided into 6 chapters.

Chapter 2 provides a description of NMR and NMR imaging. One of the most commonly used imaging techniques, **Filtered Back Projection (FBP)**, is described in detail. The theory of dynamic imaging is discussed extensively in this chapter.

In Chapter 3 a brief description of an existing static NMR microscopic imaging system is given first, followed by some developments and modifications to this system which form part of the present work. These have improved this system and enabled the performance of the flow and diffusion imaging experiments.

The static imaging experimental results are presented in Chapter 4, while the dynamic results are in Chapter 5.

A brief summary and some comments about possible future work are given in Chapter 6.

Appendix A gives the complete software listings for the flow and diffusion imaging experiments. Appendix B gives the software listings for the simulating the uniformity of G_y field gradient.