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**Students' Perspective of a Mathematics Extension
Programme Designed with Special Interest in History**

**A thesis presented in partial fulfilment of the
requirements for the degree of
Master of Educational Studies (Mathematics)
at Massey University**

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2006

ABSTRACT

The current Mathematics in the New Zealand Curriculum (Ministry of Education, 1992) includes the development of mathematical talent as a major aim of mathematics education. In catering for the individual needs of all students, the document emphasizes that students with exceptional ability in mathematics must be extended and are not expected to repeat the work they have already mastered. Talented students should be exposed to broader, richer, and more challenging mathematical experiences, should be allowed to investigate whole new topics, and work at a higher conceptual level.

Despite a growing awareness among secondary school teachers of the needs of mathematically gifted and talented students in the New Zealand secondary school classrooms, there are few exemplars of how mathematics programmes can be adapted for class groups of talented students. This study involves an investigation based on student perceptions of a mathematics programme that build on specific interest of a whole class group of students.

The aim of this qualitative exploratory case study, undertaken in an urban secondary school for girls, was to seek students' views on a Year 10 mathematics extension programme. As part of their Year 10 general extension programme, they participated in mathematics extension and studied history as their chosen option. While all students in this class were academically talented and high achievers in their core subject areas, not all of them were equally talented, or equally interested in mathematics. The mathematics extension programme, designed by their mathematics teacher (the researcher), specifically integrated their interest in history.

Data was generated from student self-evaluation questionnaires at the beginning of the course, and student questionnaires and focus-group interviews at the end of the course. Students' written and verbal responses were analyzed and then conclusions drawn. The findings suggested that by approaching mathematics from a historical point of view and thereby building on their common interest, the programme of study facilitated the development of mathematical talent and supported students in developing interest and a positive disposition towards mathematics.

ACKNOWLEDGEMENTS

This thesis is completed with the support and encouragement of many people.

First, I would like to thank Associate Professor Glenda Anthony and Peter Rawlins, my MEdStuds(Mathematics) Thesis supervisors, for their encouragement and support during this research. Their generosity of time, their collective wisdom and expertise assisted me greatly in this work.

I would like to thank all my students of the Year 10 extension class who made this research possible. I would like to acknowledge their part in ensuring the research process was an enjoyable and exciting adventure.

My appreciation and thanks to my family and friends for their encouragement and support during the long hours of work, while as a full-time teacher, I completed this masterate degree.

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CHAPTER ONE: Introduction

1.1 Background to study

The curriculum document, Mathematics in the New Zealand Curriculum (1992), states that all students have the right to achieve to the maximum of their personal potential. Students should experience a range of mathematics which is appropriate to their level, interests and capabilities. In recommending the provision for the gifted and talented, the document states that students with exceptional mathematical abilities should be extended. In addition, the document highlights the fact that improved access is needed for girls in the field of mathematics, as the participation rate of female students in mathematics is still lower than that of male students. In regard to teaching mathematics to girls, suggested learning experiences include strategies that utilize girls' strengths and interests. This includes setting mathematics in relevant social contexts, involving students in co-operative learning tasks and in extended investigations.

The vision for education in New Zealand, as described in the New Zealand Curriculum (*draft*) (2006), is that our young people will be confident, connected, lifelong learners, and actively involved in a range of contexts. The aim is to build an education system where every student is stimulated to learn. As the proposed curriculum will allow greater flexibility for teachers to develop new and innovative teaching approaches, it is expected that all students will have the opportunity to engage in rich and authentic learning experiences. There are five key competencies identified in the curriculum draft. These are: managing self; relating to others; participating and contributing; thinking; using language, symbols, and texts. In terms of recommended effective pedagogy, the draft suggests: encourage reflective thought and action; make connections; provide multiple opportunities to learn; facilitate shared learning; enhance the relevance of new learning; create a supportive learning environment. The extension programme, offered to the Year 10 extension class of this research, was developed in the spirit of these curriculum guidelines.

1.2 The Gifted and Talented

Within the education literature the two terms, “gifted and talented”, tend to be used together to describe a certain group of students, who share some characteristics. Both concepts of ‘gifted’ and ‘talented’ refer to human abilities, both are normative and target individuals who differ from the norm or average, both refer to individuals who are ‘above’ the norm because of their outstanding behaviours.

However, some researchers in the literature offer clear distinctions for the terms gifted and talented. Gagne’s (1997) Differential Model of Giftedness and Talent (DMGT) can be used to define the distinction between the two concepts:

- **Giftedness** defines the possession and use of untrained and spontaneously expressed natural abilities (called aptitudes or gifts), in at least one ability domain, to a degree that places an individual at least among the top 10 percent of age peers.
- **Talent** defines the superior mastery of systematically developed abilities (or skills) and knowledge in at least one field of human activity to a degree that places an individual at least among the top 10 percent of age peers.

Accordingly, ‘giftedness’, as natural abilities, can be observed through the various tasks that confront children in the course of their development. High aptitudes or giftedness can be observed more easily in young children because environmental influences and systematic learning have only played a limited part in their development. High aptitudes or giftedness can manifest themselves in older children and in adults as well, through the facility and speed these individuals acquire new skills in any field of human activity. The easier and faster the learning process, the greater the natural ability.

Talents result from the transformation of high aptitudes into well trained or systematically developed skills particular to a special field of human activity. Measuring talent, in a school situation, is relatively simple. It corresponds to outstanding performance in the specific skills of any occupational field. During the developmental phase of any talent there are many opportunities for normative assessments: school exams, tests, competitions, scholarship. When normative assessments are not available, in an out-of-school situation, the assessment of talent

mostly relies on peers' or superiors' ratings, therefore their validity can be questionable. As talent is a developmental construct, the level of achievement can change as learning progresses.

In a school situation, students, who are identified as academically talented, usually obtain grades within the top 10 percent in their class in their first years at school. It is possible that for some reason their progress may slow down and they are removed from the talented group. The reverse is also possible. However, because of high correlation between yearly achievements, most talented students maintain their label during their formal schooling.

The concept of giftedness has increasingly become broader in scope from the beginning of the century to today, reflecting the diversity of abilities in society (McAlpine, 1996). In the identification of gifted students the transition from a single category concept (intelligence quotient) to a multi-category is a significant change. This multi-category approach toward the identification of giftedness is illustrated by works of educationalists, such as Renzulli and Gardner. Renzulli's Three Ring model (1986) identified above average ability, task commitment, and creativity as the three components that are necessary and sufficient to demonstrate giftedness.

Within Renzulli's model above general ability is characterised by high levels of abstract thinking, verbal and numerical reasoning, spatial relations, memory and word fluency. Specific ability requires the application of general abilities to specialized areas of knowledge and skills. Task commitment is one way of conceptualising motivation. It describes an ability for sustained motivation, and dedication that leads to excellence in the development of ideas and products. Creativity describes fluency, flexibility and originality of thought, and the ability to produce novel and effective solutions to problems and to create clever and unique products.

The multi-category approach is extended further in Gardner's (1983) Theory of Multiple Intelligences. Gardner identified nine intelligences, challenging the traditional view that emphasised scholastic intelligence alone in assessing giftedness.

These nine intelligences are:

1. Linguistic (mastery, sensitivity, desire to explore, and love of spoken words, spoken and written languages);
2. Logical-Mathematical (confront, logically analyse, assess and empirically investigate objects, abstractions and problems, discern relations and underlying principles, carry out mathematical operations, handle long chains of reasoning);
3. Musical (skill in producing, composing, performing, listening, discerning and sensitivity to the components of music and sound);
4. Spatial (accurately perceive, recognise, manipulate, modify and transform shape, form and pattern);
5. Bodily- Kinaesthetic (orchestrate and control body motions and handle objects skilfully, to perform tasks or fashion products);
6. Interpersonal (sensitive to, can accurately assess and understand others' actions, motivations, moods, feelings, and others' mental states and act productively on the basis of that knowledge);
7. Intrapersonal (ability to accurately assess, understand and recognise one's own motivations, feelings, and act productively on the basis of that knowledge);
8. Naturalistic (expertise in recognition and classification of natural objects, i.e., flora and fauna, or artefacts, i.e., cars, coins or stamps);
9. Existential (capturing and pondering the fundamental questions of existence, an interest and concern with 'ultimate' issues).

According to Gardner's Multiple Intelligences Theory, each type of intelligence is a relatively autonomous intellectual potential capable of functioning independently of others. However, it is not suggested that normally functioning individuals demonstrate intelligences that work completely independently of one another, in fact intelligences, in most cases, work in harmony with one another. What differs among individuals is their profile of intelligences at any given time. Individuals exhibit a more jagged profile of intelligences, indicating various strength and weaknesses. It cannot be assumed that an individual who displays exceptional linguistic and logical-mathematical skills will also demonstrate exceptional ability or even interest in, for example, kinaesthetic intelligence.

While acknowledging the importance of other essential factors in the identification of gifted and talented students, Fraser (1996) argued that the key characteristic is creativity. Gardner (1993) supported this notion, by stating that creativity can occur in any of the nine domains of intelligences, and that creativity represents the highest level of functioning in each domain.

Schiever and Maker (2003) offer a more academically oriented approach to giftedness and point toward some specific requirements of the mathematically gifted. They believed that the key concept in giftedness is the ability (as well as interest and willingness) to solve complex problems. Gifted people solve these complex problems in the most efficient, effective, ethical, elegant or economical way. Schiever and Maker identified a further key element: the enjoyment of challenges and complexity.

Identification of Mathematically Gifted

While the general characteristics of academically gifted students also apply to the mathematically gifted, Krutetskii (1976) and Greenes (1981) specify essential abilities that characterise mathematical thinking. These include the ability to generalize, to eliminate intermediate steps, to transfer ideas, to think flexibly, the desire to search for alternative solutions, the tendency to deal in the abstract, and to view the world mathematically, and the originality of interpretation.

However, the 'mathematically gifted' term by no means describes a homogenous group of students. Chang (1984) argued that essentially three different groups of students are identified here:

- 1) Students who learn the prescribed content well, perform accurately, but have difficulty when taught at a faster pace or at a deeper conceptual level.
- 2) Students who can learn more content and at a deeper level, who can reason well, and are able to solve more complex problems than their average peers.
- 3) Students who are highly talented, capable of performing well with little or no formal instruction at a higher level than their peers, and able to work expertly on complex, difficult mathematical problems.

Within school settings the identification of mathematically gifted students does not follow a well-defined path. From data analysis of a nation-wide survey, Winsley (2000) concluded that a consistent approach, toward the identification of the mathematically gifted students was lacking in New Zealand. Identification was largely left to individual teachers and to concerned parents. At the time of the study, there were no definitive tests available, however, several tests were recommended in official ministry guidelines.

In a recent nation-wide research study of identification of and provisions for gifted and talented students Bicknell and Riley (2005) found that the most often used methods for identification were teacher observation (94.1%) and achievement tests (89.7%). While teacher observation is a strongly supported method, Bicknell and Riley raised concerns regarding the effectiveness of this method, on the grounds of possible teacher bias, stereotyping, lack of sufficient knowledge about the mathematically gifted and the possibility of teachers focusing on a narrow set of skills only. Access is another dimension of the identification process for these gifted and talented students, as the selection process into programmes is strongly influenced by the provisions available at the particular school.

1.3 Research objectives

As Riley and Bicknell (2005) argued, there is no 'one-size-fits-all' solution to provisions for gifted and talented students. Gifted and talented students display differences in their intellectual, social, emotional or cultural profile. Therefore flexibility and variety is needed in the ways the schools meet the needs of the individual gifted and talented students. The authors warn, however, that flexibility could result in inconsistent and scattered approaches, so schools need to decide (based on their school culture) what strategies to provide and how to implement these in each classroom. Riley and Bicknell recommended further research, both, quantitative and qualitative to be undertaken in order to examine the cognitive and affective effects of strategies upon gifted and talented students; the ease and difficulty involved in the implementation of the strategies; the impact of using these strategies on teaching and on the perception of giftedness; student, teacher and parent perceptions on the usefulness, appropriateness and enjoyment of these strategies. As part of this research

agenda, this study seeks to examine one niche programme in Year 10 mathematics that involves deliberate integration of the history of mathematics.

The objective of this research was to explore students' perceptions of the mathematics extension programme they participated in during their Year 10 study. Specific research questions sought students' perceptions on the differences between the extension class and the mainstream class; on changes, due to their participation in the extension programme, in their view of mathematics and mathematics learning; on benefits gained from their participation in the programme; and on the bearing this extension programme had on their future plans.

The research setting

The location of this research was a large, urban secondary school for girls. The students, the subjects of this research, were members of the Year 10 extension class, the history option class. I, the researcher, was also the mathematics teacher of these students. I have been teaching the Year 10 mathematics extension class for a number of years. The composition of these classes varied from time to time, due to the variation of the selection method. During the year of this research, students were selected into this class based on their overall academic ability, and according to their interest, they all choose the history option. The academic achievement score was calculated as the average percentage score, at the end of Year 9, of the four core subjects: English, mathematics, science and social studies. Two extension classes were created: a history option and a geography option class. Students of these extension classes were mathematically able, but not all of them were mathematically gifted and talented.

The selection process influenced the type of extension programme I put together for these students. This combination, of high academic ability and interest in history, presented an opportunity for me to create an extension programme that integrated the history of mathematics into the Year 10 teaching programme. My goal was to cater for the needs of all my students and, while ensuring they achieved excellent grades, also to build on their common interest: history, allowing them to gain a greater and deeper understanding and appreciation of mathematics.

1.4 Outline of thesis

Chapter 2 reviews the literature from both an international and a New Zealand perspective and provides a theoretical background to the thesis. It describes the two main forms of provision for the gifted and talented: enrichment and acceleration. Well-known enrichment models, used in secondary schools, are discussed, and the issue of homogeneous grouping is addressed. As this research is built on the students' perspective, a review of literature is included on the value of student voice. The chapter continues with summarizing relevant research findings on the role of motivation and interest within teaching. Theoretical and practical aspects of curriculum integration follow, with specific reference to curriculum integration in mathematics. The enrichment based extension programme, used with Year 10 participants of this research, was designed to incorporate students' interest of history. A discussion on the key elements of integrating the history of mathematics in mathematics teaching and methods of integrating the history of mathematics in mathematics teaching conclude this chapter.

Chapter 3 outlines research methodology used for this research. Data gathering techniques are presented and justification is provided for the choice of particular techniques. A discussion is included on quality criteria such as reliability, validity, generalizability, triangulation, and ethical issues.

Chapter 4 describes the research process along with participants and settings of the research. The mathematics extension programme, used with the Year 10 extension class, is outlined, and data processing techniques are identified.

Chapter 5 provides details on research findings, based on students' responses to questionnaires and focus group interview discussions. Students' responses to questionnaires and focus group interview discussions were collated into broad categories and analyzed. Students' perceptions to the following aspects form the major outcome of this research:

:

- (a) Historical focus of the mathematics extension programme: on including the history of mathematics in the teaching of mathematics and on integrating humanities and sciences;
- (b) Learning in the mathematics extension class: on pace of learning, on academic focus, and on social and emotional aspects;
- (c) Long-term outcomes.

Chapter 6 discusses the results with emphasis on the major findings in the areas of: classroom situation; historical focus of the mathematics extension programme; learning in the mathematics extension class; long-term outcomes. Recommendations are made regarding further research opportunities.

Concluding thoughts, the final section, closes the study with my personal reflections as a teacher, regarding programme design, outcomes, and implications for the future.