

Copyright is owned by the Author of the thesis. Permission is given for a copy to be downloaded by an individual for the purpose of research and private study only. The thesis may not be reproduced elsewhere without the permission of the Author.

Forecasting Share Prices using Box-Jenkins methodology

A thesis presented in partial fulfilment  
of the requirements for the degree of  
Master of Science in Statistics  
at Massey University

Stuart Ian Young

1987

ABSTRACT

Share Market efficiency has been extensively tested in financial and economics literature since the 1960's. Most of this research supports the weak and semi-strong efficient market hypothesis. However Umstead (1975) and (1977) found that aggregate quarterly share prices are inefficient enough so that application of Box-Jenkins Time Series techniques to publicly available information could have permitted an investor to earn an excess or "above average" portfolio return.

This study undertakes a similar statistical investigation of aggregate quarterly share prices in a New Zealand setting (using the Reserve Bank of New Zealand Index). The period covered by the study is March 1961 to December 1986 i.e. a total of 104 Quarterly observations.

Initially, a univariate ARIMA model is built. Model parameters are estimated using only the first 76 observations (Mar 1961 - Dec 1979), and tested using the most recent 28 observations (Mar 1980 - Dec 1986). The model is evaluated (a) by computing a predicted  $R^2$ , over the test period, (b) by observing the "hit rate" for correct and incorrect trading decisions, and (c) by constructing suitable trading rule tests - i.e. simulating the results of three alternative portfolio strategies which could have been followed in the test period.

The model is found to be successful at forecasting share price changes one quarter ahead, when compared with a naive forecast.

The following conclusions are reached:

(i) The sequence of aggregate quarterly share prices is not best described as a Random Walk but rather as a seasonal moving average ARIMA process.

(ii) The sharemarket is efficient to the degree that significant trading benefits could not have been made over the test period utilising only publicly available information of previous prices (in the presence of 5 % round trip transaction costs and taxation).

These conclusions appear to support Fama's definition of a weak-form efficient market.

The latter section of the study is an attempt to build a Transfer Function Model relating changes in a leading indicator of business activity to subsequent changes in share prices i.e. use is made of other publicly available information besides historical share prices (testing the semi-strong form of the hypothesis).

A likely input series is the Money Supply as measured by M1 and/or M2. Cross (1985) provides a detailed review of empirical studies examining the relationship of share prices and Money Supply; his own results indicate that past changes in Money Supply appear to be significantly related to current share prices for the 1960 - 1982 period (S&P 500 Index on New York Stock Exchange)

It was not possible in the present study to construct an adequate Transfer Function Model which dynamically relates the two series and the conclusion reached in this study is:

The empirical results indicate that Money Supply (as measured by M1 or M2) do not significantly lead share price changes (as measured by the RBNZ Index).

ACKNOWLEDGEMENTS

I wish to thank Dr. R. Hugh Morton, Senior Lecturer in Statistics, and Mr. K. S. Birks, Senior Lecturer in Statistical Economics, (both of Massey University, Palmerston North) for their assistance and encouragement in the completion of this study.

Also I am grateful to Mr. Martin Young, of Young & Co. Sharebrokers, Palmerston North; Mr. K. Duggan, Market Analysis Section, Reserve Bank of New Zealand; Mr. C. Jones, Jarden's Investment Services, Wellington; and to Barclays NZ Ltd Investment Section for their help.

I am especially grateful to my wife, Jennifer, and to Sandra for their support.

TABLE OF CONTENTS

	Page
ACKNOWLEDGEMENTS . . . . .	iv
LIST OF FIGURES . . . . .	vii
LIST OF TABLES . . . . .	viii
LIST OF APPENDICES . . . . .	viii
Chapter	
One. INTRODUCTION . . . . .	1
1. Background . . . . .	1
2. Motivation . . . . .	3
3. Objective of the Study . . . . .	3
Two. HISTORICAL PERSPECTIVE . . . . .	5
1. Market Efficiency . . . . .	5
2. Random Walk Model . . . . .	7
3. Review of Empirical Studies . . . . .	8
4. Discussion . . . . .	9
Three. ARIMA MODELS . . . . .	11
1. Introduction . . . . .	11
2. Stochastic Models . . . . .	11
3. Simple Operators . . . . .	12
4. Linear Filter Model . . . . .	12
5. Autoregressive Models . . . . .	13
6. Moving Average Models . . . . .	14
7. Autoregressive-Moving Average Models . . . . .	15
8. Non-Stationary Models . . . . .	15
9. Seasonal Models . . . . .	16
Four. DATA CONSIDERATIONS . . . . .	17
1. General . . . . .	17
2. Share Price Index . . . . .	17
3. Deflation Index . . . . .	19
4. Transformation of Data . . . . .	19
5. Stationarity of Data . . . . .	21
Five. BASIC IDEAS IN THE MODEL BUILDING PROCESS . . . . .	23
1. Identification . . . . .	23
2. Estimation . . . . .	26
3. Diagnostic Checking . . . . .	26

Six. METHODOLOGY APPLIED TO RBNZ DATA . . . . .	29
1. Identification . . . . .	29
2. Estimation . . . . .	32
3. Diagnostic Checking . . . . .	33
4. Interpreting the Model . . . . .	35
5. Forecasting . . . . .	37
6. Testing the Model . . . . .	38
Seven. SUMMARY AND CONCLUSIONS . . . . .	45
1. Empirical Results . . . . .	45
2. Conclusions . . . . .	48
Eight. TRANSFER FUNCTION MODELS . . . . .	50
1. Introduction . . . . .	50
2. Preliminary Ideas . . . . .	52
3. Basic Forms . . . . .	53
4. Description of T F Methodology . . . . .	56
5. Leading Indicator (Money Supply ) . . . . .	61
6. Seasonal Adjustment . . . . .	65
7. Building the Shareprice Forecasting Model . . . . .	65
8. Discussion . . . . .	69
Nine. CONCLUDING REMARKS . . . . .	70
APPENDIX A . . . . .	71
APPENDIX B . . . . .	74
BIBLIOGRAPHY . . . . .	75

LIST OF FIGURES

<u>Figure</u>	<u>Between Pages</u>
1. Graph of Nominal RBNZ (March 1961-Dec 1986) - 104 Quarterly Observations . . . . .	18-19
2. Graph of Real RBNZ (deflated by CPI lagged One Quarter) - 104 Observations . . . . .	19-20
3. Plot of Group Means vs. Std Devs - 76 Obs (March 1961 - Dec 1979 ) . . . . .	30-31
4. Time Plot of First Differences of natural logarithms of Real RBNZ - 76 Obs . . . . .	30-31
5. SACF of above series . . . . .	30-31
6. SPACF of above series . . . . .	30-31
6a. Graph of forecasted RBNZ Mar1980 - Dec 1986 . .	37-38
7. Graph showing Quarterly multipliers of three alternative portfolios M, M -- I, I . . . . .	47-48
8. Time plot of natural logarithms of M2 - 104 Quarterly Observations . . . . .	65-66
9. First Regular Differences of above series . . .	65-66
10. SACF of First Regular Differences of initial 76 Obs from above series . . . . .	65-66
11. SCCF of input and output series . . . . .	66-67
12. SCCF of "prewhitened" input and output series . . .	68-69

LIST OF TABLES

Table	Between Pages
1. Period , Forecast RBNZ, Observed RBNZ . . . . .	37-38
2. Period, Observed RBNZ, Forecasted Dividend Rate, Forecasted Interest Rate . . . . .	41-42
3. Summary statistics for testing the Model . . . . .	45-45
4. Period, M, Indicator Variable, M $\leftrightarrow$ I, I, W1, W2, W3 . . . . .	45-46

LIST OF APPENDICES

	Page
<u>APPENDIX A</u> . . . . .	71

Contains:

- Nominal RBNZ Index - Quarterly Observations  
from March 1961 - December 1986
  
- Consumer Price Index - Quarterly Obs  
from March 1961 - December 1986
  
- Finance Company Average Interest Rates  
from March 1980 - December 1986 (3 month Term  
Deposits) (28 Obs)
  
- Percentage Quarterly Dividend Yield
  
- M1 and M2 Money Supply (Quarterly)  
- March 1960 to Dec 1986 (104 Obs)

<u>APPENDIX B</u> . . . . .	Page 74
-----------------------------	------------

- Datex Composite and Capital Index  
- March 1980 to Dec 1986 (28 Obs)

## CHAPTER ONE

### Introduction

#### 1. Background

The New Zealand sharemarket is an institution of considerable interest and importance to both Public and Private Sector alike. It generates, with regularity, a large mass of fairly reliable statistical material, which is readily available for analysis. Price data are usually of high quality and are easily available.

Granger and Morgenstern(1970) observe that until the 1960's most analysis of sharemarket prices was of a rather flimsy nature. Attempts to develop models on share prices using modern statistical methodology began to surface in the 1960's and 1970's, with economists investigating wide-ranging aspects of the market's performance.

The present study restricts attention to two economic assertions:

1. Future share prices (in their aggregate as represented by an Index) are predictable in a practically useful manner from their previous, mostly cyclical movements.
2. The price movements on the sharemarket in their aggregate are a predictor of, or are predicted by, fluctuations in the economy as a whole.

A review of research findings pertinent to the present study is presented in Chapter 2. Implicit in assertion 1 above are the following common beliefs:

(a) There are seasonal or cyclic variations in share prices e.g. there is a "year-end" price rise.

(b) Share prices can be predicted from "technical analysis" of price charts.

(c) It should be possible to predict future prices from past prices i.e. after suitable manipulation of data, regularities disclose themselves.

In this study, an empirical investigation of assertion 1 is carried out (Chapter 6). In particular, a Box-Jenkins approach to modelling is employed. A brief introduction to this topic is given in Chapters 3, 4, 5.

Implicit in assertion 2 i.e. sharemarket price cycles lead or lag general economic development, is the belief that one or more indicators of business activity can be used to forecast share prices more accurately. This is empirically investigated in Chapter 8 using the Box-Jenkins methodology to build a transfer function model relating changes in the money supply to subsequent share price changes.

## 2. Motivation

Professor D.A. Umstead(1975)(1977) developed and tested a model using quarterly data from the Standard and Poors (S&P) 500 Index from 1948-1974. He lists the following conclusions:

" The sequence of aggregate quarterly stock prices is not best described as a random walk, but rather as a sixteen quarter autoregressive model with drift. Stock prices are systematically related to leading elements of economic activity. Equity markets are inefficient to the degree that significant trading benefits could have been made over an extended period in the presence of 4% round trip transaction costs, utilizing only publicly available information." ....(Umstead 1975 p.4)

## 3. Objective of the Study

To examine empirically the issues raised by the following questions:

1. Is the sequence of share prices a Random Walk?
2. How efficient is the sharemarket in adjusting to new information?
3. Can share prices be forecasted successfully?
4. Are share price fluctuations related systematically to economic activity (as measured by Money Supply)?

The approach taken by Umstead (1975) & (1977) will be followed (with some modification), using historical data from the New Zealand Stock Exchange. Only a portion of the data is used in the model-building process. The remainder, retained as a "hold-out" sample, is used in attempting to simulate results of three alternative portfolio strategies:

Portfolio 1:

Buys-and-holds the sharemarket index for the test period.

Portfolio 2:

Switches between a fixed-interest rate investment and the sharemarket.

Portfolio 3:

Buys a fixed-interest rate security each quarter.

The switching decision is based on a comparison of the forecasted quarterly holding period return for equities with the forecasted (and actual) return for a fixed-interest investment.

## CHAPTER 2

### Historical perspective

#### 1. Market Efficiency

What theoretical basis is there for asserting that the sequence of share prices is unpredictable? Fama (1970), Dyckman, Downs and Magee (1975) and Granger and Morgenstern (1970) summarize and fully discuss this aspect. The predictability (or otherwise) of share prices is related to market efficiency i.e. How efficient is an equity market (like the sharemarket) at adjusting to new information?

Practitioners believe there exists sufficient inefficiency in price adjustment to offer an analyst a return in excess of the return commensurate with the portfolio risk (even after taking the cost of transactions into account) - in effect, above average returns.

Theoreticians argue that it is precisely because of astute investors that the market is efficient enough to exclude consistent excess returns.(Umstead 1975)

This disagreement on the degree of the equity valuation process has been the subject of extensive research by economists. Fama, an authority in the field, has defined an efficient

market to be one in which prices (or returns) "fully reflect" all available information (Fama 1970). He acknowledges that this statement is so general that it has no empirically testable implications e.g. "available information" is vaguely defined, so he attempted to make testing more possible by defining subsets of available information.

Early studies were concerned with "weak-form" tests (in which the information subset is just past price history). Most tests support the idea that the market is weak-form efficient (Cross 1984) i.e. no investor is able to earn excess returns by developing trading rules based upon historical prices (or returns) of the market itself.

Attention was then turned to "semi-strong-form" tests, where the information set is expanded to include any publicly available information e.g. share splits, annual reports, indicator variables, etc.

"Strong-form" efficiency requires that no investor be able to earn excess returns using any information whatsoever, publicly available or not publicly available. This is never tested, because "all" available information cannot be incorporated into a model.

In summary, the Efficient Market Hypothesis simply states that securities are correctly priced.

## 2. Random Walk Model

A description of this model is found in Malkiel (1985). A natural way of stating the random walk model is:

$$\text{Price at time } t = \text{Price at time } t-1 + \text{Residual at time } t$$

$$\text{or } P_t = P_{t-1} + E_t ; \text{ where } E(E_t) = 0 , \text{ cov}(E_t, E_{t-s})=0; s \neq 0$$

If this model is true, it immediately follows that the best predictor of tomorrow's price is today's price i.e. price changes cannot be predicted from previous prices.

This form of the Random Walk (used in most studies) merely requires the residuals to be uncorrelated; the series is said to be a martingale. If  $E_t, E_{t-s}$  are assumed independent, the  $P_t$  series is a strict random walk, while if  $E_t$  are also assumed identically normally distributed,  $P_t$  is called a Wiener process (Granger and Morgenstern 1970).

Essentially, the efficient market hypothesis maintains that share price changes are independent, identically distributed random variables i.e. the data presents consistent and strong support of a Random Walk model, and prohibits the existence of an excessive degree of dependence in successive price changes. Fama and Blume (1966) state that given the existence of an excessive degree of correlation in successive price changes, if the actual degree of dependence cannot be used to produce greater expected profits than a buy-and-hold policy, then a Random Walk model must be accepted by the investor.

### 3. Review Of Empirical Studies

The overwhelming majority of empirical research supports weak and semi-strong market efficiency. Bachelier (1900), Niederhoffer and Osborne (1966), Levy (1967), Fama (1970), Granger and Morgenstern (1970), Malkiel (1985) and many others have concluded in the main that equity markets are highly efficient at adjusting to new information. Levy found positive correlation in price movements and volatility over twenty-six week intervals where the correlation was great enough to be of use in share selection, but any positive gains were eliminated when transaction costs and the additional risk level were considered. Niederhoffer and Osborne cite two departures from randomness in the sequence of share prices:

(a) Price reversals are from two to three times more likely than price continuations.

(b) A continuation is slightly more frequent after a preceding continuation than after a reversal.

Umstead notes that unfortunately, no attempt is made to formulate a trading rule test to determine whether excess returns are possible. He also observes that in almost all research, the sequences of prices tested represented either daily, weekly or monthly prices i.e. mainly deal with short term effects.

The conventional economic time series model of trend, business cycle, seasonal and irregular components has not been successful when applied to U.K. share prices - Kendall (1953) in Praetz.

Spectral studies e.g. Granger and Morgenstern (1963), Godfrey Granger and Morgenstern (1964) have been largely unsuccessful in finding non-random patterns in price and volume series. The section of the power spectrum of most interest to economists, however, is the low frequency range within which are concentrated all the long-run and business cycle components (often difficult to deal with using Spectral Techniques).

Granger and Morgenstern attempted to study the low frequency range by using monthly data from the Standard & Poor's 500 Index from 1875-1952 and found evidence of a forty month cycle. After removal of trend, this accounted for less than ten percent of the variance of the series. Again, no attempt is made to check the profitability of this structure.

A review of research pertaining to money supply and share prices is presented in Chapter 8.

#### 4. Discussion

Umstead raises the possibility that the wrong tools have been applied to modelling share prices. Much of the empirical research has relied heavily on cross-sectional multiple regression techniques. These techniques are not equipped to explore the dynamic relationships among the variables over time. Various lag structures can certainly

be incorporated into regression models, but Umstead observes that the lag structure must be introduced by assumption. Periodogram analysis, together with the associated spectrogram ( see Granger and Newbold (1977) for details ) dont have this limitation, and are useful for identifying deterministic cyclical components in a series . However if the nature of the relationship is a correlation structure rather than a deterministic one, model identification and estimation built upon the estimated autocorrelation function rather than the estimated spectrogram should be more fruitful (Umstead p.17). Granger and Newbold (1977 p.114) believe that ... "virtually no series of any importance within the field of economics is deterministic for any information set, however widely defined."

In this study, we will view a sequence of share prices as a stochastic process. Each of the questions raised in the Objective of the Study concerns the behaviour of prices over time. This view suggests using a Box-Jenkins approach either in the Time Domain or the Frequency Domain.

Since the power spectrum is the Fourier transform of the autocovariance function (see e.g. Nerlove (1979) for details), the two functions are mathematically equivalent. Box and Jenkins observe the techniques should be regarded as complementary, not as rivals. Each approach has different strengths in explaining different aspects of the data. Granger (1977) and Nerlove (1979) both observe that in most cases, models estimated in the Time Domain and the Frequency Domain perform equally well. In this study, attention is restricted to the Time Domain only.

Robinson and Stamboulis (1975) have applied univariate Box-Jenkins time series techniques to Quarterly share prices and find similar results to Umstead.

## CHAPTER THREE

### A brief overview of ARIMA models

#### 1. Introduction

The idea of using a mathematical model to describe the behaviour of a physical phenomenon is well established. The model-building strategy adopted in this study was developed by Box and Jenkins (1976) who employ a powerful set of models called Autoregressive Integrated Moving Average models (ARIMA) for representing time series data.

This section aims to briefly outline a few important concepts, define key terms and establish symbolism and nomenclature. It is not intended in any way to be a complete summary of the method. What follows relies heavily on Box and Jenkins (1976), which remains the seminal work in the field. For a detailed treatment of the topic, the reader should consult this work. Other treatments of the Box-Jenkins procedure may be found in Nelson (1973), Cryer (1986), Makridakis (1985), Granger and Newbold (1977) or O'Donovan (1983).

#### 2. Stochastic models for time series.

A stochastic model (or probability model) is one that can be used to calculate the probability of a future value lying between two specified limits. ( Contrast this with a deterministic model where exact calculation is possible ). Thus a time series  $Y_1, Y_2, \dots, Y_N$  of  $N$  successive observations is regarded as a sample realization from an infinite population of such time series that could have been generated by the stochastic process. It is often useful to adjust the series to have zero mean i.e. let  $Z_1, Z_2, Z_3, \dots$  be deviations from the mean  $U_t$

$$\text{i.e.} \quad Z_t = Y_t - U_t$$

### 3. Some simple operators

Backward shift operator:  $B Y_t = Y_{t-1}$  ;  $B^n Y_t = Y_{t-n}$

Backward difference operator:  $\nabla Z_t = Z_t - Z_{t-1} = (1-B) Z_t$

Autoregressive operator:  $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p$

Moving average operator:  $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$

### 4. Linear Filter model

The stochastic models proposed by Box and Jenkins are based on the idea that a time series in which successive values are dependent may be regarded as being generated from a series of independent shocks,  $A_t$ , (often called "White Noise"), which are transformed to the process  $Z_t$  by a linear filter. These shocks should be viewed as random realizations of a fixed distribution. Mostly, this distribution is assumed  $N(0, \sigma^2 A)$ . The linear filtering operation simply takes a weighted sum of the previous observations so that :

$$Y = U_t + A_t + \psi_1 A_{t-1} + \psi_2 A_{t-2} + \dots \quad \{1\}$$

i.e.  $Y = U_t + \Psi(B)A_t$

where  $\Psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \dots$  is called the Transfer Function of the filter.

$Y_t$  is said to be stationary if the sequence of weights  $\psi_1, \psi_2, \dots$  is finite (or infinite and convergent).  $U_t$  is then the mean about which the process varies. Otherwise  $Y$  is non-stationary and the linear filter is called unstable.

### 5. Autoregressive models

In this model, the current value of the process is expressed as a finite, linear aggregate of previous values of the process and a shock  $A_t$  i.e.

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \dots + \phi_p Z_{t-p} + A_t \quad \{2\}$$

is called an Autoregressive (AR) process of order  $p$ , or more concisely

$$\phi(B) Z_t = A_t$$

The model contains  $p+2$  unknown parameters  $\phi_1, \phi_2, \dots, \phi_p, \sigma_A^2$ , which in practice have to be estimated from the data. (Here  $\sigma_A^2$  is the white noise variance). It is important to recognize that an AR model is a special case of the Linear Filter model {1}. Instead of  $Z_{t-1}$  in {2} we may write:

$$Z_{t-1} = \phi_1 Z_{t-2} + \phi_2 Z_{t-3} + \dots + \phi_p Z_{t-p-1} + A_{t-1}$$

This eliminates  $Z_{t-1}$  and introduces a second term in  $A_{t-1}$ . Repeating this for  $Z_{t-2}, Z_{t-3}$  etc. eventually yields an infinite series of  $A$ 's i.e.

$$\phi(B)Z_t = A_t \text{ becomes } Z_t = \Psi(B) A_t, \text{ where } \Psi(B) = \phi^{-1}(B)$$

As previously mentioned, for the process to be stationary,  $\phi$ 's must be chosen so that weights  $\Psi_1, \Psi_2, \dots$  form a convergent series. The mathematical implication is that zeros of  $\phi(B)$  must lie outside the unit circle. (For details see Box and Jenkins (1976)).

## 6. Moving Average models

Here the  $Z_t$  are linearly dependent on a finite number  $q$  of previous  $A$ 's:

$$Z_t = A_t - \theta_1 A_{t-1} - \theta_2 A_{t-2} - \dots - \theta_q A_{t-q} \quad \{3\}$$

is called a moving average (MA) process of order  $q$ , concisely written

$$Z_t = \theta(B)A_t$$

with  $q+2$  parameters  $U, \theta_1, \theta_2, \dots, \theta_q, \sigma_A^2$

It is possible to "invert" an MA model and express it as an AR model of infinite order. O'Donovan (1983) notes that in practical terms, an MA process is invertible if  $Z_t$  does not depend overwhelmingly on deviations (i.e the  $Z$ 's) in the remote past.

Certain conditions must be imposed on the parameters of MA models to ensure this invertibility. Mathematically, the roots of  $\theta(B) = 0$  must lie outside the unit circle. (Since  $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$  is finite, no restrictions are needed on the parameters of an MA process to ensure stationarity. A parallel argument states that an AR process is always invertible).

### 7. Mixed Autoregressive-Moving Average models.

Both AR and MA terms are included in the model i.e. ARMA (p,q)

$$Z_t = \phi_1 Z_{t-1} + \dots + \phi_p Z_{t-p} + A_t - \theta_1 A_{t-1} - \dots - \theta_q A_{t-q} \quad \{4\}$$

$$\text{or } \phi(B) Z_t = \theta(B) A_t$$

Mixed models are often found in economics and business series. The reasons for this are given without proof in Granger (1980):

If  $X_t \sim \text{ARMA}(p, q)$  and  $Y_t \sim \text{ARMA}(k, l)$ , then if  $X_t, Y_t$  are independent and  $Z_t = X_t + Y_t$ , then:

$$Z_t \sim \text{ARMA}(P, Q)$$

where  $P = p + k$

$Q = p + l$  or  $k + q$ , whichever is larger.

i.e. if  $X_t$  and  $Y_t$  are both AR(1) with different parameters, then  $Z_t$  will be ARMA(2,1). Since share price aggregates are being considered, if any of the individual series have an AR component their sum is a mixed process. The only case where a mixed model doesn't arise is if each of the components are MA's.

### 8. Non-stationary models.

Many time series encountered in practice may be termed "non-stationary" i.e. the process does not remain in equilibrium about a constant mean level. Even so, they exhibit homogeneity in the sense that, apart from local level, or perhaps local level and trend, one part of the series behaves much like any other part. Suppose some suitable (say d'th) differencing of the process is stationary. Denoting  $\nabla^d Z_t = (1-B)^d Z_t$  by  $\underline{W}_t$ , then a general model which can represent both stationary and homogenous nonstationary behaviour is:

$$\phi(B) (1-B)^d Z_t = \theta(B) A_t \quad \{5\}$$

or concisely  $\phi(B) W_t = \theta(B) A_t$

Box and Jenkins note that in practice  $d$  is usually 0, 1, or at most 2. The process defined by {5} is called an Autoregressive Integrated Moving Average process (ARIMA) of order  $(p,d,q)$ .

The concept of stationarity is treated more fully in the next chapter.

### 9. Seasonal models.

The main point about seasonal time series with period  $s$ , is that observations which are  $s$  periods apart are similar. Hence  $B^s Z_t = Z_{t-s}$  plays an important role in the analysis of seasonal series. Further, considering the series  $Z_t, Z_{t-s}, Z_{t-2s}, \dots$  may not be stationary, the simplifying operator

$$\nabla_s Z_t = (1-B^s) Z_t = Z_t - Z_{t-s} \quad \text{is useful.}$$

In periodic data, two time intervals are important. We expect relationships to occur between successive observations as well as between observations which are one period out of phase. A general (multiplicative) model of order  $(p,d,q) \times (P,D,Q)$  is written:

$$\phi(B) \Phi(B^s) \nabla^d \nabla_s^D Z_t = \theta(B) \Theta(B^s) A_t \quad \{6\}$$

where  $\Phi(B^s)$  and  $\Theta(B^s)$  are polynomials in  $B^s$  of degrees  $P$  and  $Q$  respectively, and {6} satisfies stationarity and invertibility conditions.

## CHAPTER 4

### Data Considerations

#### 1. General

Box and Jenkins(1976) state that at least 50, and preferably 100 observations or more, should be used when using their techniques for analysis of a time series. O'Donovan (1983) observes that at least six seasons' data is required for most seasonal series. Share market price data form a discrete series called a sampled time series, since although the price fluctuates more or less continuously during the trading day, reported prices are daily closing prices. Sharemarket indices form discrete series called aggregated time series (O'Donovan 1983) since values are accumulated over (nearly) equal intervals of time.

#### 2. Share Price Index

There are several sharemarket indices widely used in New Zealand. Each is calculated differently. Burrowes and Mulholland (1986) state that the overall aim of a share price index is to provide a summary measure of sharemarket behaviour. The measure is used for a wide variety of purposes, the two most important being:

(i) an indicator of business conditions (or anticipated business conditions)

(ii) as a benchmark for measuring portfolio performance or the performance of an individual stock.

For an index to be used for forecasting, it should be sensitive to daily changes taking place, which implies companies with infrequently traded shares should be excluded to avoid "damping" effects. However, too small a sample of companies may not detect important movements.

A further consideration is what type of index to use:

(a) Composite Index - measures the total increase in an investor's wealth i.e. takes account of capital gains, dividend income, bonus and cash issues etc.

(b) Capital Index measures only changes in shareprices i.e. takes account of capital gains only.

The popular indices, e.g. RBNZ, Barclays, NZSE, Datex, Jardens, Auckland Star and others all incorporate the above ideas in different proportions.

For the present study, the index selected is the RBNZ i.e. the Reserve Bank of New Zealand -see footnote[1]. According to the Reserve Bank Bulletin, May 1984 (p 193), the purpose of this index is to "provide an objective measure of movements in the prices of a representative range of shares which are caused by market conditions". A total of sixty listed companies are included in the index. No allowance is made for dividends, and it may broadly be described as a capital index which attempts to predict underlying market trends.

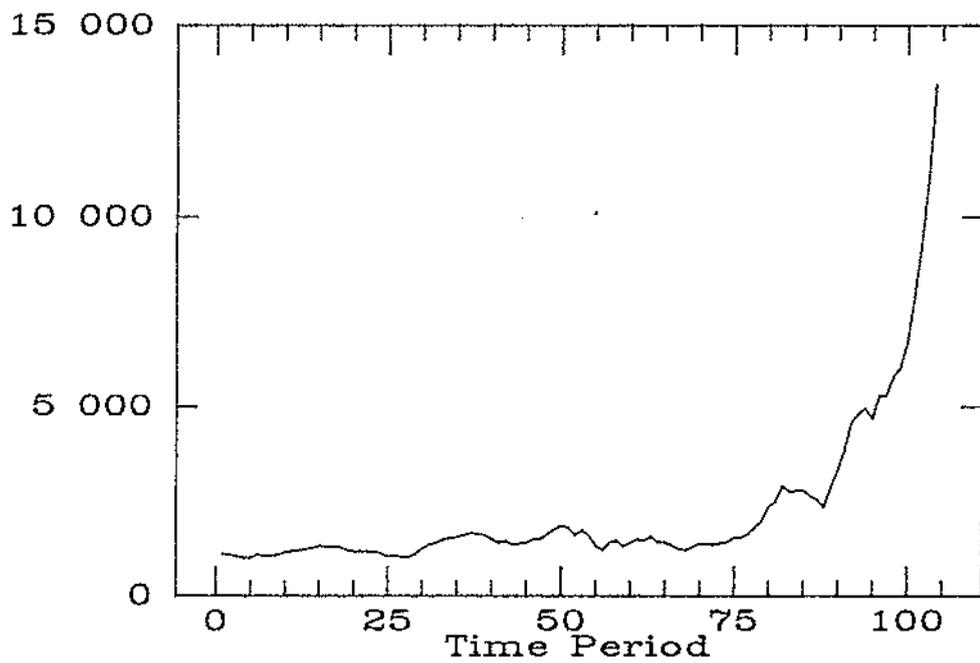
Thus aggregate quarterly observations of the RBNZ index were obtained from 1960 - 1986 (Dec) and the series was adjusted to the base period (Jan 1968 = 1000). These are the closing prices at the end of the last trading day of each calendar quarter. This gives a total of 104 quarterly observations, which are graphed in Fig. 1. Overleaf.

Footnote:

[1] The sample correlation between the RBNZ and BARCLAYS monthly series was calculated. The two series were found to be almost perfectly correlated.

# FIGURE 1: Nominal RBNZ

104 Quarterly Obs - Mar 1961 to Dec 1986



### 3. Deflation Index

The data is deflated using the Consumer Price Index (CPI) (Dec 1983=1000) lagged one quarter. The reason for lagging the CPI one quarter is because of publication delay (of up to two months). Essentially, the estimate of CPI for the present quarter is equal to the observed CPI for the previous quarter.

The deflated data (Dec 1983 prices) is graphed in Fig. 2. Overleaf.

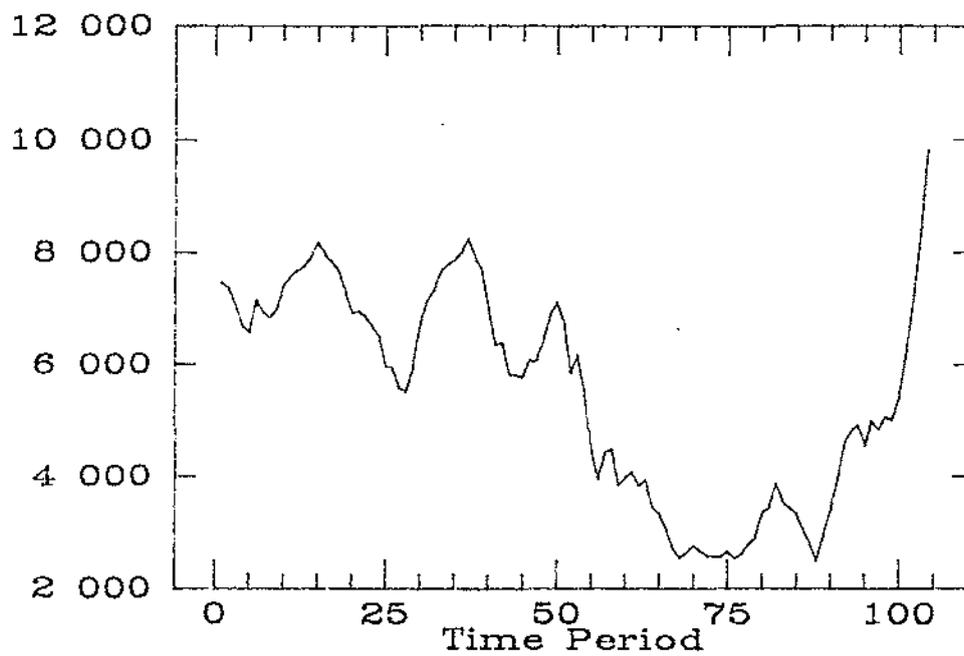
### 4. Transformation of Data

An important problem is whether to use the raw price index or some transformed version. A well known economic assumption is that the percentage price change, rather than absolute price change, reflects the effectiveness of the price change.

Granger and Morgenstern (1970) observe that it has become common practice to use the logarithm of prices when long series are considered, to compensate for the fact that such series will frequently contain trends both in mean and variance.

Moore (1962) has shown that the variability of simple price changes for a given share is an increasing function of the price level of the share; taking logarithms seems to neutralize most of this price level effect.

FIGURE 2: Real RBNZ  
Mar 1961 to Dec 1986



Letting  $P_t$  represent price at time  $t$ , consider the pair of equations:

$$P_t = P_{t-1} + E_t \quad \{7\}$$

and  $\log P_t = \log P_{t-1} + A_t \quad \{8\}$

where both  $E_t$  and  $A_t$  (the residual series) have zero means and are uncorrelated with earlier values.

Now, {7}: 
$$\frac{P_t}{P_{t-1}} = 1 + \frac{E_t}{P_{t-1}} \quad \text{or} \quad \log \frac{P_t}{P_{t-1}} = \log \left( 1 + \frac{E_t}{P_{t-1}} \right)$$

and {8}: 
$$\log \left( \frac{P_t}{P_{t-1}} \right) = A_t$$

so the two models would be identical if 
$$\log \left( \frac{1 + E_t}{P_{t-1}} \right) = A_t$$

If the percentage price change  $E_t/P_{t-1}$  is small (usually the case), then:  $E_t/P_{t-1} \sim A_t$ , or  $E_t \sim A_t P_{t-1}$

Assuming  $P_{t-1}$  is independent of  $A_t$ , the residual series  $E_t = A_t P_{t-1}$  will have zero mean and will be uncorrelated with earlier values. Thus the logarithmic transformation does not change the model fundamentally (except for large percentage price changes  $E_t/P_{t-1}$ ).

Other transformations are possible. A popular transformation is that suggested by Box and Cox (1964), who propose the power transformation:

$$Y = (X^\lambda + M) \quad \lambda \neq 0$$

-----

$$\lambda$$

or  $Y = \log X \quad \lambda = 0$ , with M a constant.

Many authors doubt the effectiveness of the transformation in the present context, see e.g. Granger and Nelson (1978), and simply try two or three common transformations e.g.(log, square root, reciprocal).

### 5. Stationarity of Data

A process is said to be strictly stationary if the joint distribution of  $Z_{t_1}, Z_{t_2}, Z_{t_3}, \dots, Z_{t_n}$  is the same as the joint distribution of  $Z_{t_1-k}, Z_{t_2-k}, \dots$  for all choices of  $t_1, t_2, \dots, t_n$  and all lags  $k$  (Cryer 1986). Effectively, this means:

(a)  $E(Z_t) = \mu_t = \mu$  (constant)

(b)  $\text{Var}(Z_t) = \gamma_{tt} = \gamma_0$  (constant) - where  $\gamma_{t,s} = \text{cov}(Z_t, Z_s)$

(c)  $\gamma_{s,t}$  depends on lag  $|t-s|$  not on actual times i.e.  $\gamma_{t,t-k} = \gamma_k$

(d)  $\rho_k = \text{corr}(Z_t, Z_{t-k}) = \gamma_k / \gamma_0$  ;  $\rho_0 = 1$

Note:  $\rho_k$ , the correlation coefficient between  $Z_t$  and  $Z_{t-k}$ , is called the autocorrelation at lag  $k$ .  $\rho_{-k} = \rho_k$ , so consider only positive lags. Thus if a process is strictly stationary and has finite variance, then the autocorrelation function (ACF) and the autocovariance function depend only on the time lag.

The process is said to be weakly stationary if:

$$(a) E(Z_t) = \mu_t = \mu \quad (\text{constant})$$

$$(b) \gamma_{t, t-k} = \gamma_{0, k} \quad \text{for all time } t \text{ and lag } k.$$

In the remainder of this study, the term stationary is used to represent weakly stationary data (a practice followed by most authors). Box and Jenkins show that in fact if the joint distribution of a process is assumed Normal, then the two definitions coincide.

There is a useful and simple technique for checking on stationarity. Divide the observed time series into subsets (4 - 10 observations) and calculate the mean and standard deviation for each subset. If the underlying stochastic process is stationary, all these means should have approximately the same value and so should all the s.d.'s. If s.d. is independent of the mean, a plot of mean vs. s.d. should be a random scatter about a horizontal line. An appropriate software package e.g. BMDP simplifies this task.

CHAPTER 5

Basic ideas in Model Building Process

Box and Jenkins stress that if two models appear to fit the data equally well, the one with fewest parameters will always be chosen. They propose a methodical, yet subjective, three-stage iterative procedure in the model-building process:

1. Identification

By this is meant using the data, as well as any information on how the data was generated, to suggest a subclass of tentative, parsimonious models. In effect, the task is:

(a) to identify a subclass of models from the ARIMA family {6}. Find  $d$  by differencing  $Z_t$  as many times as is needed to produce stationarity i.e. hopefully reduce to an ARMA process {4}.

(b) to identify  $p$  and  $q$  of the resultant stationary ARMA process.

The principal tools used in (a) and (b) are the sample, or estimated ACF, denoted by  $R(k)$ , and calculated as:

$$R(k) = \frac{\sum_{t=k+1}^N (Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=k+1}^N (Y_t - \bar{Y})^2} \quad \text{where } N = \text{no. of obs ,}$$

and the sample "partial" autocorrelation function (PACF), an estimate of the theoretical PACF. The theoretical PACF may be thought of as

the autocorrelation between any two variables  $Z_t$  and  $Z_{t-k}$ , with the effects of the intervening variables eliminated. The sample PACF, denoted  $R(kk)$  is an estimate of  $\rho_{kk}$  calculated from the observed time series values. The partial autocorrelations,  $R(kk)$ , are derived from the autocorrelations  $R(k)$  by means of the following equations:

$$R(1\ 1) = R(1)$$

$$R(2\ 2) = \frac{R(2) - R(1)^2}{1 - R(1)^2}$$

with more complicated equations for  $R(3\ 3)$ ,  $R(4\ 4)$ .. which will not be discussed here.

A plot of  $R(k)$  values against  $k$  is called the estimated Correlogram, while a plot of  $R(kk)$  against  $k$  is called an estimated Partial Correlogram.

The appearance of the SACF and the SPACF should provide clues about the choice of orders of  $p$  and  $q$  for the autoregressive and moving average operators, by comparing them with the (known) characteristic behaviour of the theoretical ACF and PACF of ARMA series i.e. the theoretical correlogram and partial correlogram take distinctive shapes for  $AR(p)$  and  $MA(q)$  models which are well documented e.g. Levenbach and Cleary (1984), Makridakis and Wheelwright (1983).

e.g. for a  $MA(q)$  series,  $\rho_k = 0$ ,  $k > q$

for an  $AR(p)$  series,  $\rho_{kk} = 0$ ,  $k > p$

Thus when confronted with a realization of an MA(q) model, the sample autocorrelations  $R(k)$  should all be near zero for  $k > q$ .

A Rule of Thumb for deciding whether  $R(k)$ , (or  $R(kk)$ ), is significantly different from zero, used by many authors including Cryer (1986) and O'Donovan (1983) is:

"If  $R(k)$  or  $R(kk)$  lies outside the limits  $\pm 1.96/\sqrt{N}$ , then either statistic is significantly different from zero and conclude  $\rho_k \neq 0$  or  $\rho_{kk} \neq 0$  respectively."

This follows from Bartlett's approximation (1946) for the variance of the estimated autocorrelation coefficient of a stationary Normal process at lags greater than  $q$ :

$$\text{Var}[ R(k) ] \sim \frac{1}{N} \left\{ 1 + 2 \sum_{k=1}^q R(k)^2 \right\} \quad k > q$$

and also Quenouille( 1949 ) who showed that

$$\text{Var} [ R(kk) ] \sim 1/N \quad ; \quad k \geq p+1$$

conditional on the process being AR(p).

The SACF and SPACF may be used not only to guess the form of the model but also to obtain approximate estimates of the model parameters. (These are often useful as starting values for the iterative procedures employed in estimation).

## 2. Estimation

At this stage, the tentative model is fitted to the data and its parameters estimated. Most software packages minimise the unconditional sum of squares function through backforecasting of presample observations. The optimization procedure has many variations, but is usually a version of Marquardt's nonlinear procedure which amounts to a compromise between the methods of Gauss-Newton and Steepest Descent (Makridakis 1984). A whole book could easily be devoted to this topic i.e. making efficient use of the data in order to make inferences about parameters conditional on the adequacy of the tentative model. A detailed description of the estimation process is beyond the scope of this study. Interested readers should consult Box and Jenkins (1976).

## 3. Diagnostic Checking

Granger (1980) outlines a strategy to check if the model(s) selected and estimated really do fit the data adequately, or if further models should be considered. He observes that the model should be tested outside the sample period i.e. one set of data is used to identify and estimate a model and a later set is used to evaluate it. Frequently, there is insufficient data for this approach and all analysis is done on the same data set.

The usual test is to fit a model, estimate the residuals from it, form the correlogram of these residuals, and then see if this correlogram suggests that the residuals are a white noise as they should be. However, if the sample is used for all purposes, then this test is rather weak and lacking in power.

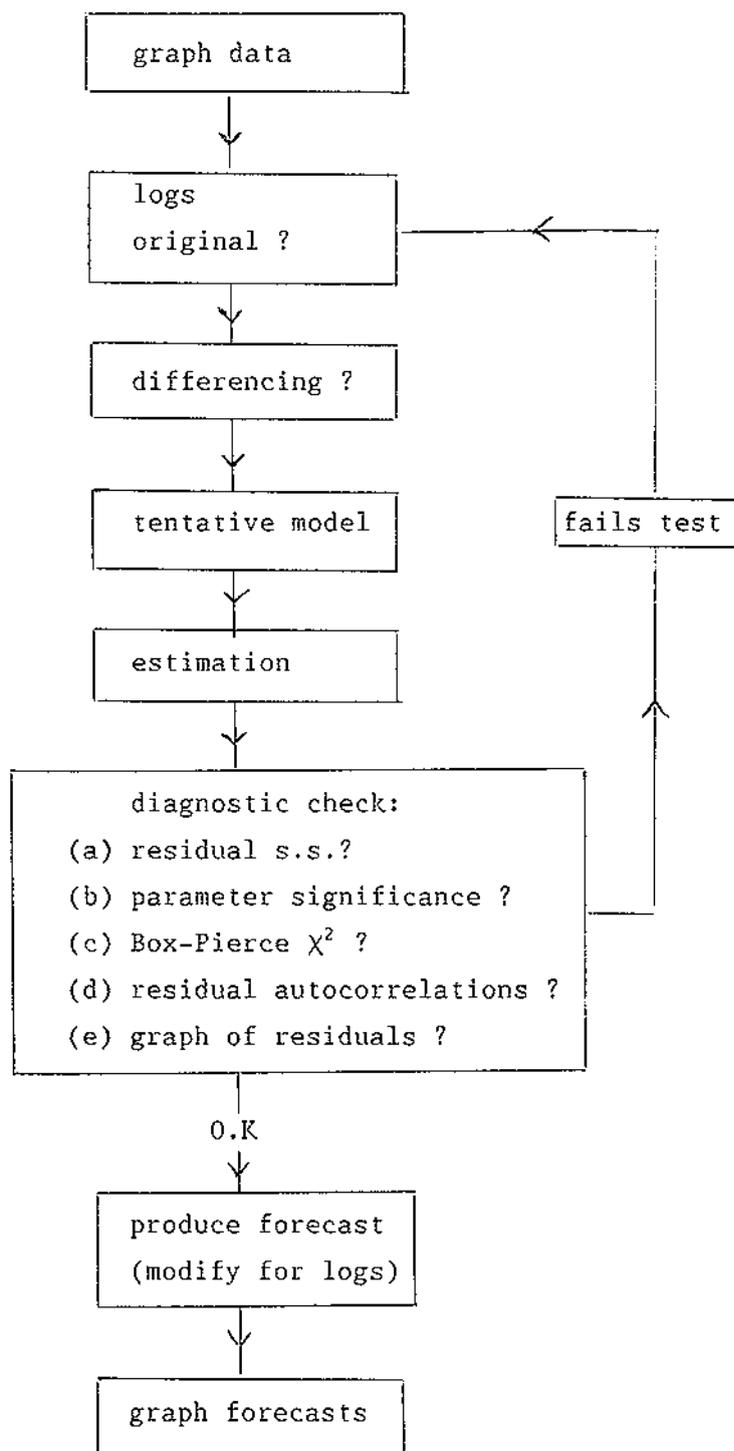
Cryer (1986) gives details of a Portmanteau Test which takes into consideration the magnitudes of the residual autocorrelations as a group. This test uses the Modified Box-Pierce statistic:

$$Q = N(N+2) \sum_{k=1}^L \frac{R(k)^2}{(N-k)} ; \text{ where } L = \text{max lag}$$

If the correct ARMA(p,q) model is estimated, then for large N, Q has a  $\chi^2$  (L-p-q) distribution. Fitting an erroneous model will tend to inflate Q. Thus if Q exceeded an appropriate critical value the current model is rejected.

An additional approach is to fit a slightly higher-order model and then to see if the extra parameters are significantly different from zero. For instance, if the model fitted is ARMA(1,1) then the test involves fitting ARMA(2,1) and ARMA(1,2) models. The significance of the coefficients of the extra terms as well as the stability of all coefficients must be carefully examined. Details of this approach are in Box and Jenkins (1976).

Andersen and Weiss (1984) p.183 summarise the steps often followed in practice by means of the diagram overleaf:



## CHAPTER 6

### Methodology

#### 1. Identification

Recall that the information set consists of 104 quarterly observations of the RBNZ Index (Dec 1983 prices) from 1961(Mar) to 1986(Dec). The CPI observation used to deflate the data is lagged one Quarter so that it may be assumed known at the time of the price observation i.e.

$$\text{at time } t, \text{ RBNZ}(t) = \text{Nominal RBNZ}(t) / \text{CPI}(t-1)$$

This data is divided into two sets:

(a) The first 76 observations, 1961(Mar) - 1979(Dec), are used in the model-building process.

(b) The remaining 28 observations, 1980(Mar) - 1986(Dec), are used only for testing the model outside the sample, and play no part in identification or parameter estimation.

From Fig.2 between pages 19 and 20, it appears the data is non-stationary in both mean and variance. This is confirmed by plotting the SACF which dies away slowly. Also, by dividing the time series into 11 subsets of 6-7 observations each, and plotting group means vs. std. devs. it is apparent that the variance is increasing with the mean. Various data transformations were applied (see footnote [1]) in attempting to achieve stationarity.

The final choice is to take first regular differences of the natural logarithms of the data. Fig 3 overleaf (group means vs. std devs) approximates a random scatter about a horizontal line.

A time plot of the transformed data is given in Fig. 4 ; the SACF and SPACF are shown in Figs. 5 and 6 respectively. The time plot appears to be stationary in the mean, and both SACF and SPACF die down quickly to zero.

If  $Y_t$  represents Real RBNZ at time  $t$ , the transformed stationary series to be modelled may be expressed as:  $(1-B) \ln Y_t$ .

Turning attention to the SACF and SPACF:

1. Each has significant lags at  $k = 1, 6, 12$ .
2. The most significant autocorrelation is  $R(6) = - 0.306$
3. Disregarding spikes at lags 6 and 12, both appear either:
  - (i) to cut off after lag 1, or
  - (ii) to tail off toward zero.
4. Seasonal differencing does not appear to be required, since the significant correlations at lags 6 and 12 appear to cut off after 12 or are dying away quickly.
5. The sample mean is not large compared with the sample std. dev. Thus no constant term will be included in the model i.e assume  $U_t = 0$ , all  $t$ . See O'Donovan for details. [2]

Figure 3: Group Means vs. Std Devs  
 76 Obs Mar 1961 to Dec 1979  
 Group Size = 7

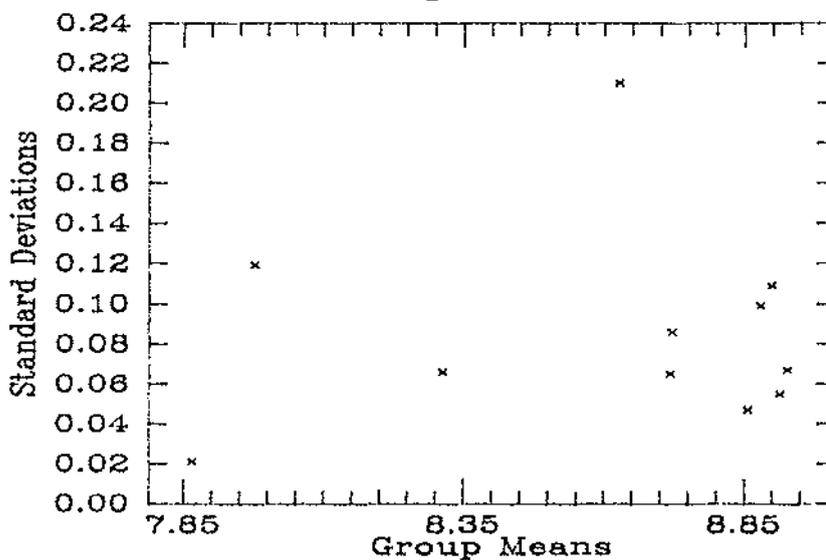


FIGURE 4: First Differences of Natural Logarithms  
 of Real RBNZ 76 Obs

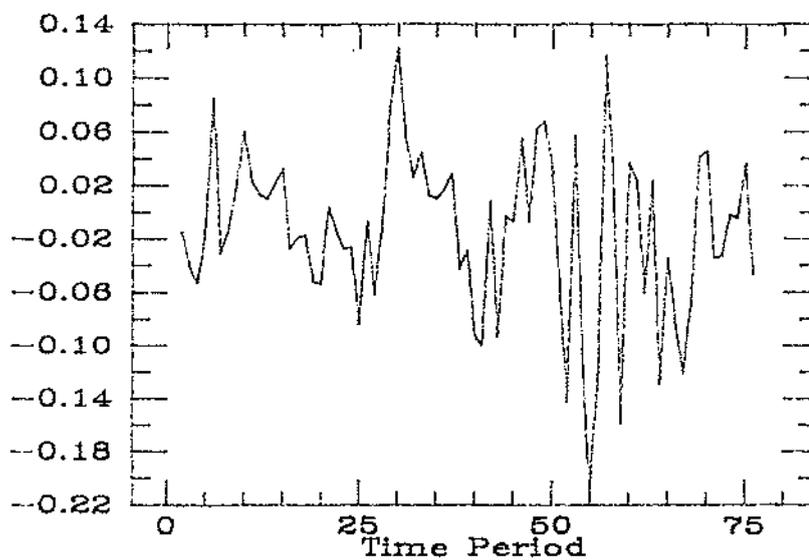
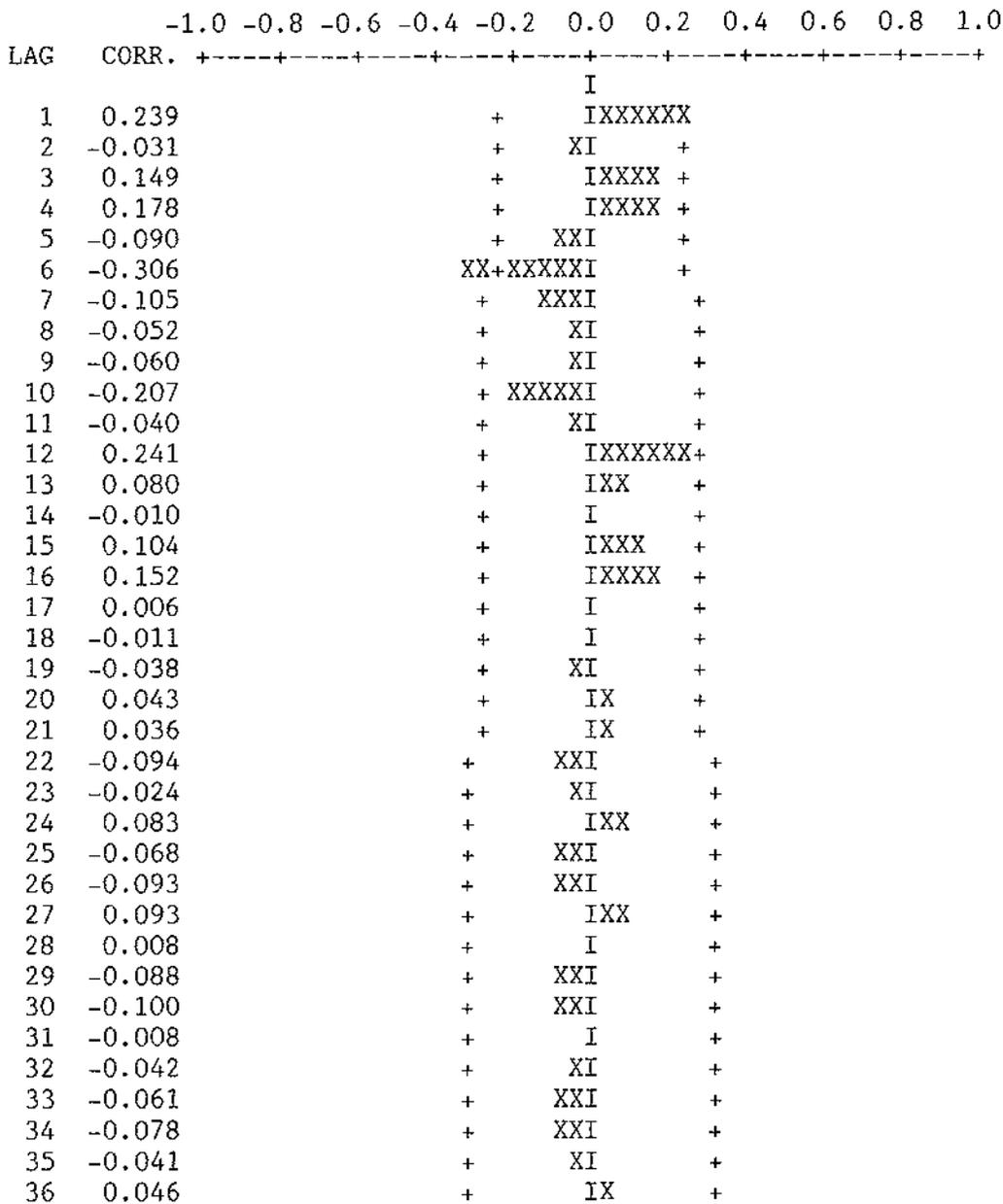


FIGURE 5

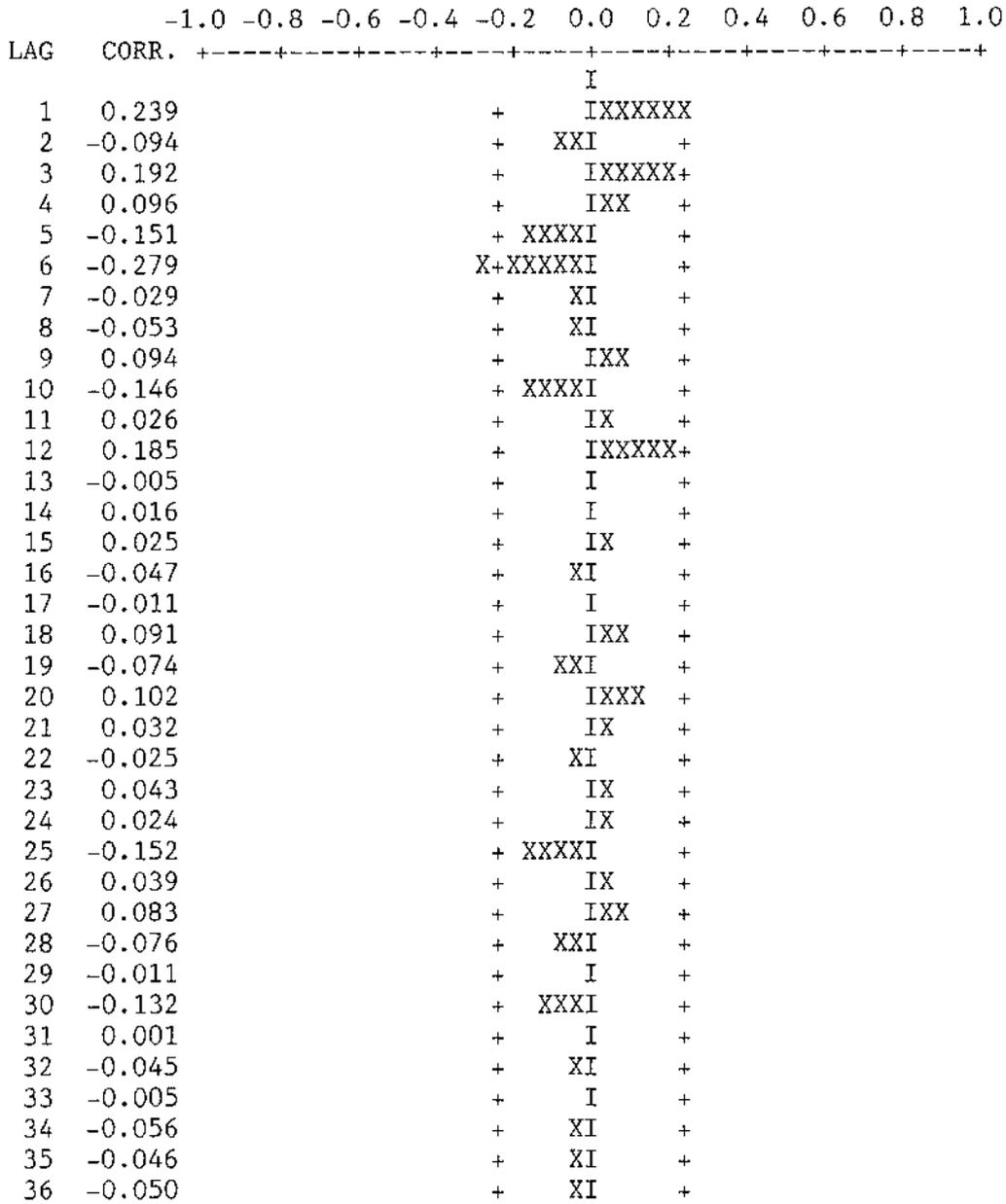
PLOT OF AUTOCORRELATIONS - RBNZ (Real)



MEAN OF THE (DIFFERENCED) SERIES = -0.0143  
 STANDARD ERROR OF THE MEAN = 0.0073  
 T-VALUE OF MEAN (AGAINST ZERO) = -1.9699

FIGURE 6

PLOT OF PARTIAL AUTOCORRELATIONS - RBNZ (Real)



MEAN OF THE (DIFFERENCED) SERIES = -0.0143  
 STANDARD ERROR OF THE MEAN = 0.0073  
 T-VALUE OF MEAN (AGAINST ZERO) = -1.9699

The tentative model chosen [3] is:

$$W_t = (1 - \theta_1 B)(1 - \theta_6 B^6)(1 - \theta_{12} B^{12})A_t$$

where  $W_t = (1-B) \ln Y_t$ ;  $A_t$  is white noise

$$\text{i.e. } (0 \ 1 \ 1) (0 \ 0 \ 2) 6$$

#### Footnotes

[1] A second plot of group means vs. std. dev. was drawn on a logarithmic scale. Log group std. devs. was regressed on log group means, the observed beta coefficient is approximately 1.5 which implies  $(1-\beta) = -0.5$ . This in turn indicates a reciprocal square root transformation ( see BMDP manual 1985 p. 112 for details ).

A Box Cox transformation approach yielded a lambda approximately equal to one, which indicates no transformation of the data is necessary.

Arbitrarily, a square root transformation and a reciprocal transformation were also investigated.

[2] Since the t-ratio for the sample mean is 1.96, a constant term was also included in the model and found to be insignificant.

[3] The tentative conclusion is that the SACF cuts off after one regular lag and two "seasonal" ( six-quarterly ) lags i.e. lag 12, and the SPACF dies away quickly.

## 2. Estimation

The parameters of the model were estimated using a standard MINITAB software package. Starting values were not required. This resulted in the model:

$$Wt = ( 1 + .301 B ) ( 1 - .2622 B^6 ) ( 1 + .3467 B^{12} ) At$$

[2.66]
[2.29]
[2.96]

All the t-ratios (in square brackets) lie outside the limits  $\pm 1.96$ ; each parameter estimate is significantly different from zero (O'Donovan) i.e. there is no indication that  $\theta_1$ ,  $\theta_6$ , or  $\theta_{12}$  should be excluded from the model.

Modified Box-Pierce chisquare statistic results:

Critical Values (0.05)	Q
-----	----
$\chi^2 ( 9 ) = 16.9$	6.9
$\chi^2 (21) = 32.7$	12.8
$\chi^2 (33) = 47$	20.8
$\chi^2 (45) = 61$	23.7

Each of the observed values is not significant at the 5 % level and so the model cannot be rejected at this level. (Note Q = 20.8 for the first 36 autocorrelations. If the residual series were white noise, an observed Q statistic this large would be expected in more than 95 percent of cases.)

The overall  $R^2$  for the model when fitted values are regressed against observed values is 97.0 %.

### 3. Diagnostic checking

#### Residual Analysis

The usual diagnostic check of individual residuals reveals no evidence of inadequacy of fit. The SACF and SPACF of the residuals shows no significant correlations at the 5 % level. When the residuals are plotted with time, and also against fitted values, no unusually suspicious values emerge.

A histogram of the standardized residuals approximates a normal curve. A plot of std. res. vs. normal scores appears to be linear. Cryer (1986 p.42) lists critical values for the Normal-Scores Correlation Test for Normality. The appropriate critical values are:

<u>sample size</u>	<u>significance level</u>		
	<u>0.01</u>	<u>0.05</u>	<u>0.1</u>
75	.976	.984	.987

Since the observed correlation of 0.995 does not fall below any of the critical values, normality would not be rejected at any of these levels of significance. A Runs test does not suggest a lack of independence. To summarise, the residuals appear to behave like independent (normal) random variables.

### Overfitting

All overfits are rejected either because the parameter estimate of the new parameter is not significantly different from zero or because the residual variance is increased.

### Sub-periods

A method of roughly checking as to whether the model structure or the model parameters change with time is to split the time period into smaller intervals.

The period Mar 1961 - Dec 1979 was divided in half and the model fitted to the resulting sub-periods i.e. Mar 1961 - June 1970, and Sept 1970 - Dec 1979. The model appeared to perform adequately in each sub-period.

### Conclusion:

There is little ( or no ) evidence to suggest the model chosen is not adequate. The (0 1 1 ) (0 0 2) 6 ARIMA model is thus retained for forecasting purposes.

#### 4. Interpreting the model

This result appears to support the idea that barely discernible "business cycles" exist in the NZ economy, and that each economic expansion will sooner or later be followed by a contraction. It seems reasonable to suppose that share prices are linked to this business cycle, since the value placed on shares represents a discounting of expected cash flows.

The parameter  $\theta_1$  may arise since as Chatfield (1982) observes share prices are affected by a variety of 'random' events such as strikes, government decisions, etc., and such events will not only have an immediate effect but may also effect share prices to a lesser extent in a subsequent period.

There is a well known effect called the 'Fisher' effect, discussed in Praetz (1982), who notes that certain shares are infrequently traded due to their market being rather thin. Their movements tend to lag behind those of the market as a whole.

It could be hypothesized that the observed six quarter seasonal structure in share price changes arises because of the tri-ennial general election cycle i.e. 12 quarters. Fig 5 shows that values every six quarters are significantly negatively correlated, while those lagged by twelve quarters are significantly positively correlated.

One interpretation is that Government policies and decisions are designed so that that the period immediately prior to a general election is likely to be one of expansive business activity. Contractions and "belt-tightening" would be most evident mid-way between elections.

Investors who are impressed by any pre-election stimuli may overestimate share values and cause prices to increase. Umstead observes that an investor who recognises these periods of overvaluation and anticipates the subsequent correction in the market can realize a higher return with less risk by using this knowledge in his decision process.

A possible method of testing this election cycle hypothesis is simply to compare average returns for the five quarters before each election with the five periods after each election for the last eight elections.

The Arithmetic Mean quarterly returns (price change only)for each period is:

<u>Sample</u>	<u>Mean Return</u>
Pre-Election	-0.02
Post-Election	-2.70

The t-statistic for the difference between these two sample means is 1.675 which is significant at the .05 level (but not at the .025 level).

5. Forecasting - see Table #1 overleaf and Fig. 6a

One-step-ahead forecasts of the holdout sample (28 observations from 1980 (march) - 1986 (dec) ) are made e.g. a forecast for the 1980 March quarter is made. Then the observed 1980 March value is incorporated into the information set, new parameter estimates are calculated, (using same model structure), and a forecast for 1980 June is made. The observed 1980 June value is now incorporated into the data set, the parameters re-estimated etc.

The model appears to be successful at forecasting in so far that the parameter estimates are relatively stable throughout the 28 forecasts, and the model appears adequate each iteration (as tested by the Q statistic which remains non-significant ). The Mean Absolute Percentage Error (MAPE) of the simulated forecasts is 6.97 .

Discussion

The simplest structure of the transformed stationary share price series is the Markov model:

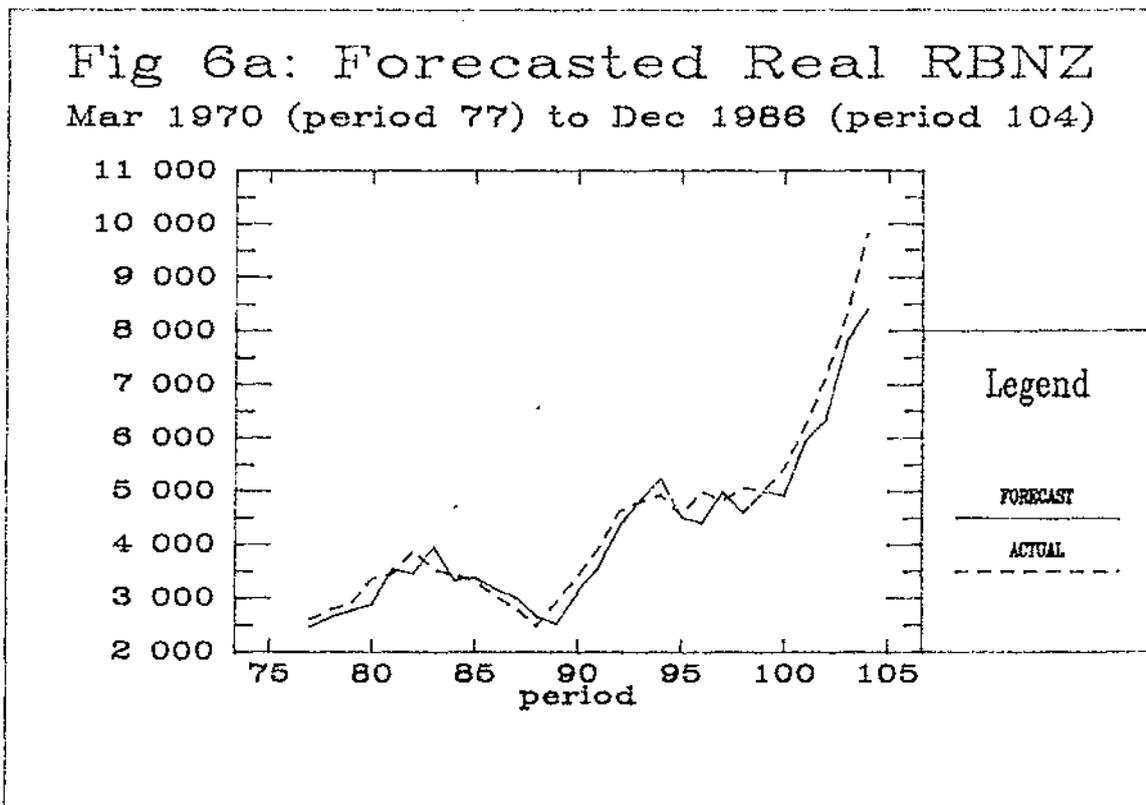
$$W_t = A_t \quad ( \text{recall } W_t = (1-B) \text{ Ln } Y_t )$$

This model assumes that change from  $Y_{t-1}$  to  $Y_t$  is due only to  $A_t$ ; there is nothing in the historical structure of the series that is useful in predicting  $Y_t$  from  $Y_{t-1}$  and the best forecast of the next price is the current price. Using such a model to produce one-step-ahead forecasts for the 28 observations in the holdout sample gives a (MAPE) of 8.59.

Thus the selected model reduces the MAPE for the 28 one-step-ahead forecasts by 1.62 - it remains to be seen whether the reduction in error variance can be made profitable.

TABLE #1

PERIOD	FORECAST RBNZ	ACTUAL RBNZ
76	*	2546.67
77	2469.61	2616.37
78	2648.50	2788.25
79	2775.11	2909.50
80	2888.72	3363.90
81	3540.53	3446.13
82	3452.18	3882.04
83	3954.67	3536.08
84	3331.96	3430.52
85	3386.03	3328.55
86	3175.64	3039.35
87	3011.92	2802.65
88	2658.15	2479.79
89	2524.10	2954.40
90	3125.67	3403.91
91	3564.87	3922.69
92	4337.90	4622.60
93	4826.31	4816.00
94	5242.93	4929.49
95	4520.72	4544.22
96	4398.49	5000.00
97	4989.82	4830.90
98	4592.08	5070.05
99	4994.98	5017.50
100	4911.62	5431.47
101	5938.85	6233.94
102	6361.98	7175.19
103	7808.00	8304.90
104	8403.82	9813.73



## 6. Testing the model

### Discussion

It is commonly agreed that the best test of a time series model is to compare model forecasts with genuine future observations. Unfortunately, with the present series it will take too long to make valid ex post tests of the mode, and a holdout sample is used instead. The procedure adopted in this study is that the holdout sample plays no part whatsoever in the model-building process i.e. it tries to avoid the possibility of a researcher inadvertently capitalizing on chance while developing a model.

Whitcomb (1977) is critical of the fact that Umstead used the full data set to identify the model and only then removed the "holdout" sample when estimating parameters. Umstead thus makes use of information which would not be available at the time of forecasting. As will be realized, this approach has not been followed here.

Another criticism some might make of Umstead's procedure is that, because no definite picture of model structure emerges from the SACF and SPACF under scrutiny in his investigation, he adopts an exhaustive estimation search procedure i.e. systematically expanding from simple to more complex models. This, he states, is to ensure proper model identification is achieved. Twenty one models are sifted through before one is chosen. Arzac (1977) is uneasy about this process since spurious relationships may be incorporated into the model. In this study, the procedure is:

(a) Select an adequate model (from two or three) which is most indicated by the historical data

(b) Use this model for forecasting. No return to choose a second adequate model is allowed.

The model is tested by (i) simulation (ii) computing the predicted  $R^2$  over the test period (iii) evaluating the "hit" rate for forecasting "up" and "down" markets.

(i) Simulation

The results of three alternative portfolio strategies are simulated during the 28 quarterly period 1980(Mar) - 1986(Dec). Assume each portfolio starts with an initial wealth of \$10 000 on March 31, 1980.

Portfolio One (M): Buys-and-holds the market (M) i.e. the RBNZ index, throughout the entire period. Also any dividends are re-invested in M (after tax)[1].

Portfolio Two (M ↔ I): Switches between the market (M) and fixed rate interest (I) i.e. Finance company 3 month term deposits. All income is re-invested.

Portfolio Three (I): Buys fixed-interest (I) each quarter i.e 3 month deposits with finance companies. Any interest accrued is re-invested (after tax).

The switching decision in Portfolio Two is based on a comparison of the forecasted quarterly return for equities (using the ARIMA model developed earlier) with the forecasted (and actual) return from the 3 month term deposits. Naturally, transaction costs (TRANS) must be taken into account. These are assumed as being 2.5 % to move the portfolio from M to I or vice versa.

(a) Thus:

$$\hat{R}M_t = (1 - \text{TRANS}) \left[ \begin{array}{c} \hat{M}_t + \hat{D}_t \\ \text{---} \\ M_{t-1} \end{array} \right] \quad \text{--- (9)}$$

$\hat{R}M_t$  = forecasted return for the portfolio if it is invested in the sharemarket for time period t.

TRANS = 0, if portfolio is in M at time t-1  
 = .025, if portfolio is in I at time t-1.

$\hat{D}_t$  = forecasted quarterly tax-paid dividend rate at time t. [2]

$M_{t-1}$  = observed market (i.e. as represented by the Real RBNZ index) at the end of time period t-1. [3]

$\hat{M}_t$  = forecasted RBNZ (using ARIMA model) at end of time period t.

(b) Next:

$$\hat{R}It = ( 1 - \text{TRANS} ) \left[ 1 + \frac{91}{365} \hat{I}t \right] \quad \text{--- (10)}$$

$\hat{R}It$  = forecasted ( and actual ) return for the portfolio if it is invested in fixed-interest rate deposits for time period t.

TRANS = 0 ,      if portfolio is in I at time t-1  
          = .025,      if portfolio is in M at time t-1.

$\hat{I}t=It-1$  = forecast (and actual) average tax-paid interest rate offered by finance companies for 3 month term deposits at the end of period t-1. [4]

Table #2 is given overleaf, showing calculated values.

(c) Each quarter  $\hat{R}Mt$  is compared with  $\hat{R}It$ . If  $\hat{R}Mt > \hat{R}It$  , then portfolio Two either stays stays in the sharemarket or moves entirely from fixed-interest investment to the market.

Similarly, if  $\hat{R}It > \hat{R}Mt$ , then portfolio Two remains in I or switches from M to I.

PERIOD	ACTUAL RBNZ	TABLE #2	
		TAX-PAID DIVIDEND	TAX PAID INTEREST
76	2546.67	0.0139888	0.05700
77	2616.37	0.0072332	0.06000
78	2788.25	0.0053406	0.05750
79	2909.50	0.0138535	0.06000
80	3363.90	0.0151250	0.05675
81	3446.13	0.0067344	0.05625
82	3882.04	0.0037323	0.06000
83	3536.08	0.0080662	0.06065
84	3430.52	0.0107184	0.06375
85	3328.55	0.0087851	0.07000
86	3039.35	0.0053030	0.07125
87	2802.65	0.0117121	0.07125
88	2479.79	0.0104979	0.07125
89	2954.40	0.0153897	0.06375
90	3403.91	0.0133006	0.07000
91	3922.69	0.0096146	0.06625
92	4622.60	0.0089700	0.04875
93	4816.00	0.0059693	0.05000
94	4929.49	0.0029479	0.05125
95	4544.22	0.0078361	0.05875
96	5000.00	0.0121356	0.06125
97	4830.90	0.0028203	0.07500
98	5070.05	0.0047376	0.07750
99	5017.50	0.0068094	0.07750
100	5431.47	0.0101158	0.07750
101	6233.94	0.0041208	0.08000
102	7175.19	0.0025888	0.07000
103	8304.90	0.0049521	0.06875
104	9813.73	0.0057926	0.06875

(d) Let  $W_t$  denote wealth at end of time period  $t$ . Each quarter, the accumulated wealth of each portfolio is calculated as follows:

Portfolio One ( M ): [5]

$$W_t = W_{t-1} \left[ \begin{array}{c} M_t + .975 D_{t-1} \\ \text{---} \\ M_{t-1} \end{array} \right] \text{---}\{11\}$$

Portfolio Three ( I ):

$$W_t = W_{t-1} \left[ \begin{array}{c} 1 + .91 I_{t-1} \\ \text{---} \\ 365 \end{array} \right] \text{---}\{12\}$$

Portfolio Two ( M -- I ):

Depending on the switching decision, either {11} or {12} is used to calculate  $W_t$ .

(ii) Predicted R<sup>2</sup>

The Predicted R<sup>2</sup> is the square of the correlation between the series of 28 one-quarter-ahead forecasts and the actual price changes that occurred.

(iii) "Hit Rate"

Umstead's intuitive approach is followed in evaluating the Hit Rate for the Model. A simple test is proposed: define an up market and a down market as those which have a return above and below the Median return respectively, and then compute the success of the Model in predicting these events.

Footnotes

[1] The Tax Rate is taken as 50c/dollar for all calculations. This is obviously a subjective value: it pre-supposes that the investor has a regular income exceeding \$30 000, and that income from interest or dividends attracts a high taxation rate. Prior to 1985 the rate was approximately 55c/dollar; since then income over \$30 000 is taxed at 48c/dollar.

[2] The forecasted dividends to be paid out each quarter are estimated as the dividends payable in the previous quarter i.e.  $\hat{D}_t = D_{t-1}$ . Umstead (1977) makes the point that the dividend component of market returns is relatively stable and the variation in total returns from quarter to quarter is due almost entirely to price fluctuations. Thus a forecast of next quarter price is essentially a forecast of total return if the current dividend yield is added to the forecasted price change.

This is confirmed in Horsfield and O'Dea (1983) p.35, who list the yearly dividend yields in New Zealand from 1971 - 1981. These yields show little significant fluctuation. For the present study, the historical quarterly dividend yields for 1980 - 1986 were calculated from the Datex Composite and Datex Capital Indices for that period. A listing of the Index is given in Appendix B.

[3] As published on the last trading day of each quarter. Note that when testing the model, the observed Real RBNZ is calculated by deflating with the current CPI, i.e. CPI is no longer lagged one quarter as it was during the model-building process.

[4] The quarterly interest rate is calculated by averaging the Maximum and Minimum Rates quoted in the appropriate Reserve Bank Bulletins. A listing of the rates is given in Appendix A.

[5] A transaction cost of 2.5 % is deducted from Portfolio One in the first and last quarters of the test period i.e. periods 77 and 104.

CHAPTER 7Summary and ConclusionsResults(i) Simulation

Table 3 below gives summary statistics for the quarterly percentage returns :

Table #3

<u>Portfolio</u>		<u>Arithmetic</u>	<u>Geometric</u>	<u>Std.</u>	<u>Wealth</u>
<u>Number</u>		<u>Mean Return</u>	<u>Mean Return</u>	<u>Dev.</u>	
M	1	1.055	1.060	.096	\$45348
M ↔ I	2	1.048	1.050	.073	\$36000
I	3	1.016	1.016	.022	\$15653

A complete listing is given in Table #4 , overleaf.

The entries under the M, M ↔ I, and I columns are the appropriate quarterly percentage returns for each period.

It is evident that Portfolio 1 attained the highest growth during the test period, and Portfolio 2 outperformed Portfolio 3.

Umstead p.439 notes that since we are hypothesizing that samples are not drawn from the same population, a formal test of significance of mean returns may not be valid. (i.e. pooling of variance estimates requires that samples are drawn from same underlying population)

TABLE # 4

Time Period	Indicator Variable						
	M		M---I	I	W1	W2	W3
			\$10 000				
Mar]							
1980]							
77	1.00942	0	1.01421	1.01421	\$10094.2	\$10142.1	\$10142.1
78	1.07090	0	1.01496	1.01496	10809.9	10293.8	10293.8
79	1.05699	0	1.01434	1.01434	11425.9	10441.4	10441.4
80	1.17092	0	1.01496	1.01496	13378.8	10597.6	10597.6
81	1.03101	1	1.00523	1.01415	13793.7	10653.1	10747.6
82	1.13013	1	1.13013	1.01402	15588.7	12039.3	10898.3
83	0.91875	1	0.89578	1.01496	14322.1	10784.6	11061.3
84	0.98060	0	0.98974	1.01512	14044.3	10673.9	11228.5
85	0.97884	0	1.01589	1.01589	13747.1	10843.6	11407.0
86	0.91829	0	1.01745	1.01745	12623.8	11032.8	11606.0
87	0.93354	0	1.01776	1.01776	11784.8	11228.7	11812.1
88	0.89504	0	1.01776	1.01776	10547.9	11428.1	12021.9
89	1.20640	0	1.01776	1.01776	12725.0	11631.1	12235.4
90	1.16512	1	1.13599	1.01589	14826.1	13212.8	12429.8
91	1.16178	1	1.16178	1.01745	17224.7	15350.4	12646.7
92	1.18717	1	1.18717	1.01652	20448.6	18223.5	12855.7
93	1.04766	1	1.04766	1.01215	21423.2	19092.0	13011.9
94	1.02644	1	1.00078	1.01247	21989.6	19106.9	13174.1
95	0.92948	0	0.98746	1.01278	20438.9	18867.3	13342.5
96	1.11213	0	1.01465	1.01465	22730.7	19143.7	13537.9
97	0.96893	0	1.01527	1.01527	22024.5	19436.0	13744.7
98	1.05412	0	1.01870	1.01870	23216.4	19799.5	14001.7
99	0.99627	0	1.01932	1.01932	23129.8	20182.0	14272.2
100	1.09237	0	1.01932	1.01932	25266.3	20571.9	14547.9
101	1.15176	1	1.12297	1.01932	29100.7	23101.6	14829.0
102	1.15351	1	1.15351	1.01995	33568.0	26647.9	15124.8
103	1.16227	1	1.16227	1.01745	39015.1	30972.1	15388.7
104	1.16233	1	1.18733	1.01714	\$45348.4	\$35999.8	\$15652.5
[Dec					!-----!	!-----!	!-----!
1986]					Portfolio	Portfolio	Portfolio
		0 = Interest			1	2	3
		1 = Market					

(ii) Predicted R<sup>2</sup>

The predicted R<sup>2</sup> for the model is 96.4%.

(iii) Hit Rate

The median quarterly return for the Real RBNZ Index (price change only) over the 104 quarters of the study period was -0.16 %. Over the test period (28 quarters), the model forecasted 14 "up" markets and 14 "down" markets, with twenty one out of these twenty eight forecasts correct, i.e. a "Hit Rate" of 0.75 . Umstead makes an assumption that the probability of success in each trial is 0.5 (see footnote [1]). The probability of observing at least 21 successes out of 28 trials is 0.007. Thus if the above assumption is valid, the probability that this result occurred by chance is .007

Footnotes

[1] Both E.R.Arzac and Whitcomb, in a discussion of Umstead's Paper, believe the binomial test applied in the previous section to be inappropriate i.e. the assumption that the probability of a correct Market vs. Fixed Interest decision is 0.5, is not valid.

Professor Whitcomb states .." it is well known that rising portions of business cycles and share market cycles are longer than falling portions." The assumption that decisions are independent from Quarter to quarter are also doubtful.

### Discussion

Interesting points to note are:

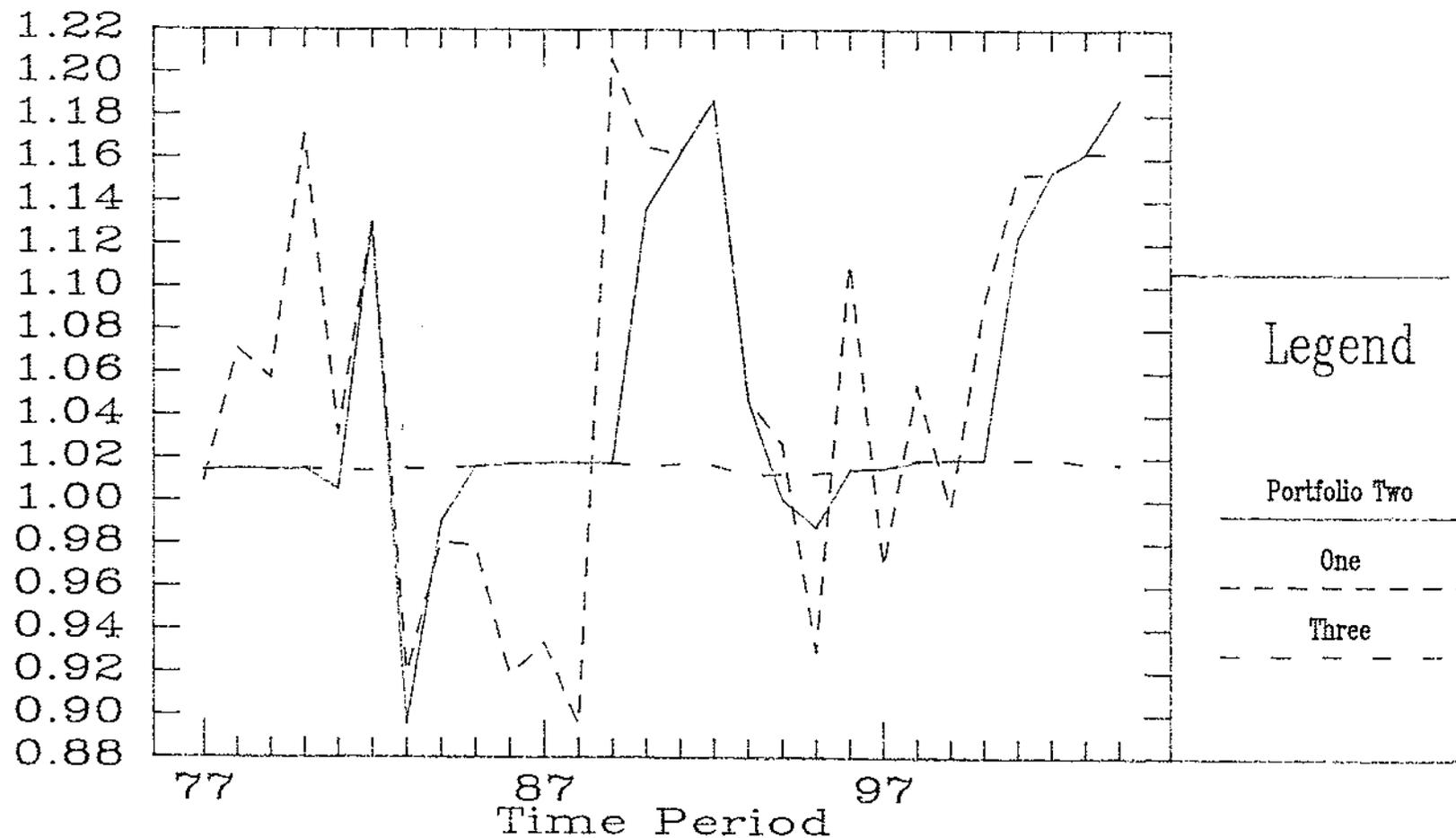
(a) If a major market drop is arbitrarily defined as a quarterly percentage return of less than 0.95, the model manages to avoid 4 of the 5 major declines of the test period.

Similarly, if a major market improvement is taken as a return of greater than 1.10, then 8 out of 11 such "boom" periods are anticipated by the model.

(b) Twenty eight decisions were generated by the model, and of these, twenty were correct i.e. Portfolio Two was correctly placed in the investment area which actually showed the greatest return for the quarter.

(c) Figure 7 overleaf is a graph showing the quarterly returns of each Portfolio. Portfolio Three (fixed interest) returns are comparatively stable. Portfolio One shows very erratic returns. The "damping" effect of transaction costs on Portfolio Two is noticeable, showing that a switch only occurs in fairly extreme conditions.

FIGURE 7: Comparison of Portfolios  
Quarterly Returns



## 2. Conclusions

Several authors believe that Box Jenkins models are not very promising for short term prediction, as prices, after first differencing, usually have little correlation left to model. Even a very large serial correlation coefficient of 0.4 at lag 1 reduces the standard deviation of one-period-ahead prediction by only 8% (Praetz).

The empirical results appear to support Professor Umstead's conclusion that business cycle related information does not become available in isolated, discrete parcels which the market instantly absorbs with perfect efficiency. The information rather unfolds gradually in a cyclical pattern over time.

In a small economy like New Zealand's, where Government expenditure and policy has a major impact, share prices appear to respond in a predictable manner (to a small degree) to this cyclical flow of information. By using a sophisticated statistical technique, widely available historical data and fundamental concepts, share price changes may be slightly more accurately predicted than by using a naive forecasting method.

This would conflict with the Purist's definition of the weak form of an efficient market. However, the results also indicate that the degree of inefficiency in the market is not sufficient for an investor to make consistent excess returns by examining historical prices only, when transaction costs are considered.

Many researchers would claim that this minor degree of inefficiency does not disprove the efficient markets hypothesis in its weak form. The hypothesis states that all methods of deciding when to buy, and sell, over a particular time period, and using past prices to evaluate the decision, are inferior to the strategy of buying at the beginning of the period and selling at its conclusion (Fama 1970).

Only a minor degree of dependence has been found in the sequence of share price changes. So the Random Walk Hypothesis still holds as a first approximation to the underlying reality. It says that future price changes and therefore future prices cannot be predicted from past price changes.

## CHAPTER EIGHT

### 1. Introduction

When a time series is subjected to exogenous influences that make an ARIMA model inadequate, the effect of the exogenous disturbances may be combined with the ARIMA model. If the exogenous variable is a nonrandom process, such as a nonrandom binary variable (consisting of 0's and 1's), we refer to it as the intervention component in the model. This approach is widely used to analyze the effect of external managerial decisions. Interested readers may consult Shahabuddin (1980) who presents an analysis of shareprices using Time Series with Intervention Analysis.

If the exogenous variable is itself a random time series, the model is called a Transfer Function (TF) Model, or occasionally a Multivariate ARIMA (MARIMA). Makridakis et al.(1983) observe that TF models combine some of the characteristics of univariate ARIMA models and some of the characteristics of multiple regression analysis i.e. blend a time series approach with the causal approach [1]. Because of the increased complexity of the methodology, attention is restricted in this study to the bivariate case i.e. two time series.

Frequently, forecasts of a time series  $Y_t, Y_{t-1} \dots$  may be improved by using information from some associated series  $X_t, X_{t-1}, \dots$ . This is especially so if changes in  $Y_t$  tend to be anticipated by changes in  $X_t$ . Economists call  $X_t$  a "leading indicator" for  $Y_t$ . Thus leading indicators are series where changes in pattern precede those of the specific series forecast.

An output time series  $Y_t$ , ( in the present case the RBNZ quarterly index ), is presumed to be influenced by firstly, an (observed) input time series  $X_t$ , and many other (unobserved) inputs which are treated collectively and called "noise"  $N_t$  [2]. This superimposed noise  $N_t$  is assumed independent of  $X_t$ , and corrupts the true relationship between input and output.

The system is dynamic i.e. the Transfer Function distributes the impact of  $X_t$  over several future time periods. The objective is to identify the role of a leading indicator  $X_t$  in determining the variable of interest  $Y_t$ . Box and Jenkins(1976) is the primary source and devote two chapters to Transfer Function models. A simpler and less rigorous treatment of the topic is given in Makridakis (chapter 10).

#### Footnotes

[1] Granger(1969) has suggested a definition of causality in which  $X$  is said to cause  $Y$  if present and past values of  $X$  can be used to obtain more accurate forecasts of future values of  $Y$  than forecasts obtained by using past values of  $Y$ .

[2] By ignoring  $N_t$ , and generating  $X_t = A_t$ ,  $A_t$  white noise, it is clear that an ARIMA model is a special case of a TF model.

## 2. Preliminary Ideas

The Transfer Function itself [1] may be written as:

$$Y_t = v_0 X_t + v_1 X_{t-1} + v_2 X_{t-2} + \dots + v_k X_{t-k}$$

$$Y_t = (v_0 + v_1 B + v_2 B^2 + \dots + v_k B^k) X_t$$

$$Y_t = v(B)X_t \quad \text{---- (1)}$$

Note the last shorthand notation, where  $v(B)$  is the Transfer Function. The series of values (weights)  $v_0, v_1, v_2, \dots, v_k$  is called the Impulse Response Function of the system. Thus the TF equates a change in  $Y_t$  to a weighted average of current and previous changes in  $X_t$ .

The linear filter linking  $X_t$  and  $Y_t$  is said to be stable [2] if  $v(B) = (v_0 + v_1 B + v_2 B^2 + \dots)$  converges for  $|B|$  less than or equal to 1.

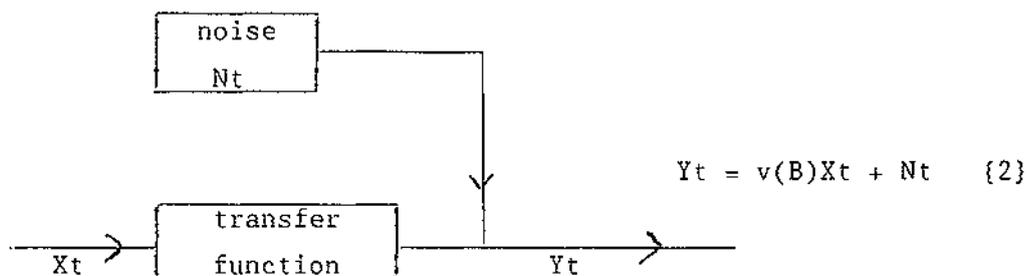
### Footnotes

[1] In this study, attention is restricted to linear, discrete transfer function models. These models often adequately represent commonly occurring continuous dynamic situations and are parameter parsimonious. (By discrete is meant that observations of input and output are made at equispaced intervals of time).

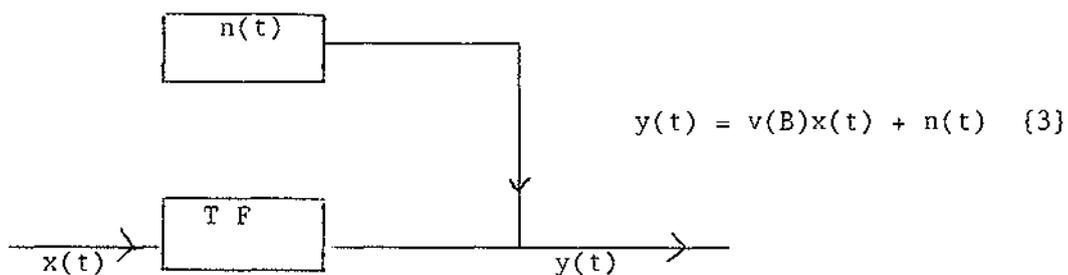
[2] The requirement of stability for the discrete TF model parallels that of stationarity for the ARMA stochastic models. In general, for stability the requirement is that the roots of the characteristic equation  $d(B) = 0$  (defined in footnote [1] of next section) with  $B$  regarded as a variable, lie outside the unit circle.

### 3. Basic Forms of the Transfer Function Model

If it is assumed noise  $N_t$  is independent of the level of  $X_t$  and is additive with respect to the influence of  $X_t$ , then:



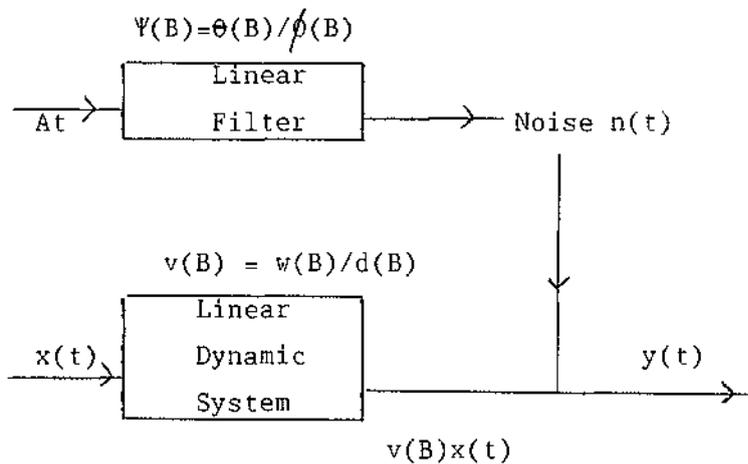
The input and output series are often transformed and/or differenced to achieve stationarity of the overall system. Let  $x(t)$ ,  $y(t)$  and  $n(t)$  denote transformed values of  $X_t$ ,  $Y_t$  and  $N_t$  respectively. Box and Jenkins show that  $y(t)$  and  $x(t)$  satisfy the same TF model as  $Y_t$  and  $X_t$  i.e.



The transfer function model is also written in the following more parsimonious form: (see footnote [1] for explanation of symbols)

$$y(t) = \frac{w(B)}{d(B)} x(t-b) + \frac{\theta(B)}{\phi(B)} A_t \quad \{4\}$$

$A_t = \text{"white noise"}$



In practice it is necessary to estimate the transfer function  $\Psi(B) = \theta(B)/\phi(B)$  of the linear filter describing the noise, in addition to the transfer function  $v(B)$  which describes the dynamic relationship between the input and output.

Footnotes

[1] Suppose that  $n(t)$  can be described by  $\phi(B) n(t) = \theta(B)At$ , so that rearranging gives:

$$n(t) = \frac{\theta(B)At}{\phi(B)}$$

Also,  $v(B)$ , an infinite series, can frequently be represented with brevity and sufficient accuracy by the ratio of two polynomials of low degree in  $B$ :

$$v(B) = \frac{w(B)}{d(B)} \quad \text{where}$$

$$w(B) = w_0 - w_1B - w_2B^2 - \dots - w_sB^s \quad \text{and}$$

$$d(B) = 1 - d_1B - d_2B^2 - \dots - d_rB^r$$

To summarise: (  $r, s, b$  ) are parameters of the TF model relating  $y(t)$  and  $x(t)$  where  $r$  is the degree of the  $d(B)$  function,  $s$  is the degree of the  $w(B)$  function and  $b$  (constant) refers to the delay registered in the subscript of the  $x(t-b)$  term in equation {4} i.e. the delay of  $b$  periods before  $X$  begins to influence  $Y$ .

(  $P_n, Q_n$  ) refer to the parameterization of the ARMA noise model, and must not be confused with similar symbols (  $P_x, Q_x$  ) for the parameterization of the input series  $X_t$  to be discussed shortly.

#### 4. General description of the TF model building methodology

The main steps in the process are (i) identification (ii) estimation (iii) diagnostic checking and (iv) usage.

##### (i) Identification

The objective is to identify a combined transfer function-noise model

$$Y_t = \frac{w(B)}{d(B)} X_{t-b} + N_t$$

for a linear system where  $N_t$  is assumed (statistically) independent of  $X_t$  and is generated by an ARIMA process. Initially, the aim is to tentatively specify the orders  $r$  and  $s$  of the operators  $d(B)$  and  $w(B)$  in the transfer function component of the model and to make initial guesses for the parameters  $d_1, d_2, \dots, d_r$  and  $w_1, w_2, \dots, w_s$ . Also, initial guesses of the parameters ( $P_n, D_n, Q_n$ ) of the ARIMA process describing the noise at the output are made. Initial estimates of  $\theta_1, \theta_2, \dots, \theta_{Q_n}$  and  $\phi_1, \phi_2, \dots, \phi_{P_n}$  are required for that model. The procedure is to:

- (a) derive estimates  $\hat{v}_j$  of the impulse response weights  $v_j$  in {3}
- (b) use these  $\hat{v}_j$  to guess  $r, s$  and the delay parameter  $b$ .
- (c) estimate  $d_1 \dots d_r$  and  $w_1 \dots w_s$  using results of (a) and (b).

Box and Jenkins give details of this procedure (p.378,379) and observe that it is lengthy and cumbersome in general, and does not provide efficient estimates.

Identification of the impulse response weights  $v_j$  is considerably simplified when the input series is "white" noise. Usually,  $X_t$  follows some other stochastic process, but it can be made more manageable by "prewhitening" it - effectively removing all known pattern - and then applying the same transformation to the output. This prewhitening is accomplished by building a univariate model of the input series using the Box and Jenkins technique.

Suppose the suitably transformed and differenced input process  $x(t)$  is stationary and is, say, ARMA ( $P_x, Q_x$ ) i.e.

$$\phi_x(B) x(t) = \theta_x(B) A_t, \text{ so that: } \frac{\phi_x(B)x(t)}{\theta_x(B)} = A_t$$

which approximately transforms the correlated input series  $x(t)$  to the random white noise series  $A_t$ .

Now, to preserve the integrity of the functional relationship we apply the same transformation to the output series  $y(t)$ . The idea is summarized in a figure from Makridakis (p.491):

$$\begin{array}{l} \text{input } x(t) \text{ ----} \rightarrow \text{ transfer function } \text{----} \rightarrow \text{ output } y(t) \\ \\ \text{input } \left[ \begin{array}{l} \phi_x(B)x(t) \\ \theta_x(B) \end{array} \right] \text{ --} \rightarrow \text{ transfer function } \text{--} \rightarrow \text{ output } \left[ \begin{array}{l} \phi_x(B)y(t) \\ \theta_x(B) \end{array} \right] \end{array}$$

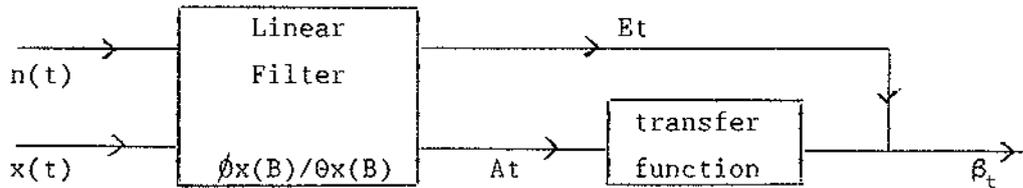
This gives the "prewhitened" output series  $\beta_t$ :

$$\beta_t = \frac{\phi_x(B) y(t)}{\theta_x(B)}$$

Thus {3} may be written:

$$\beta_t = v(B) A_t + E_t \quad \text{-----}{5}$$

where  $E_t$  is the transformed noise series:  $E_t = \frac{\phi_x(B)n(t)}{\theta_x(B)}$



Box and Jenkins show that after prewhitening, the Cross Correlation Function CCF [1] between the prewhitened input,  $A_t$ , and the correspondingly transformed output,  $\beta_t$ , is directly proportional to the impulse response function (details in footnote [2]). The factor of proportionality is the ratio of the standard deviations of the two series. In practice, the theoretical CCF  $\rho_{xy}(k)$  is not known and so the SCCF is used to give:

$$\hat{v}_k = \frac{R_{xy}(k) S_y}{S_x} \quad k = 0, + 1, + 2, \dots$$

Once the  $\hat{v}_k$  are calculated, then the sample crosscorrelations are examined, and  $(r, s, b)$  are tentatively estimated. This stage is subjective and requires both experience and care. Details of TF models for all combinations of  $r=0,1,2$  and  $s=0,1,2$  are given in Box and Jenkins p.349-352 both in Table and Figure form. For a complete description of the estimation technique the reader should consult this work; what follows overleaf is a brief outline of main ideas.

r s b

0 0 b The impulse response function consists of a single value  $v_b = w_0$ . The output is proportional to the input but is displaced by  $b$  time intervals.

$$Y_t = w_0 B^b X_t$$

0 s b The instantaneous input will be delayed  $b$  intervals and will be spread over  $s+1$  intervals.

$$Y_t = (w_0 - w_1 B - \dots - w_s B^s) B^b X_t$$

1 0 b The impulse response tails off exponentially from the initial starting value  $v_b = w_0$ .

$$(1 - d_1 B) Y_t = w_0 B^b X_t$$

1 s b There are  $s$  values  $v_b, v_{b+1}, \dots$  to  $v_{b+s-1}$  for the impulse response function which do not follow a pattern, followed by exponential fall off beginning with  $v_{b+s}$ .

$$(1 - d_1 B) Y_t = (w_0 - w_1 B - \dots - w_s B^s) B^b X_t$$

2 1 b  $(1 - d_1 B - d_2 B^2) Y_t = (w_0 - w_1 B) B^b X_t$

The next step is:  $(P_n, Q_n)$  for the ARIMA( $P_n, 0, Q_n$ ) model of the noise series are estimated. From {3} :

$$n(t) = y(t) - v(B)x(t)$$

so that the  $n(t)$  values are obtained and analysed using conventional ARIMA techniques. Finally, in the identification stage, preliminary estimates of the parameters  $w_0, \dots, w_s$ ;  $d_1, \dots, d_r$ ;  $\theta_1, \dots, \theta_{Q_n}$ ;  $\phi_1, \dots, \phi_{P_n}$ ; are made. These are used as starting values for the estimation stage.

## (ii) Estimation

The Marquardt algorithm is used to proceed iteratively to better estimates. This consists of simultaneously estimating all parameters by least squares fitting. This involves considerable computation and is therefore done using a computer.

(iii) Diagnostic Checking

The adequacy of the model is then determined by examining the estimated ACF of the residuals and the estimated CCF of the prewhitened input and the residuals. Evidence of inadequacy implies re-identification of  $r$ ,  $s$  and  $b$ .

(iv) Usage

The TF model is used to forecast future values if checking reveals no inadequacies.

Footnotes

[1] The cross correlation  $\rho_{xy}(k)$  between  $X_t$  and  $Y_t$  defines the degree of association between values of  $X_t$  and  $Y_{t+k}$  where  $k=0, \pm 1, \pm 2 \dots$

In general,  $\rho_{xy}(k) \neq \rho_{xy}(-k)$  in contrast to the ACF i.e. is not symmetric about  $k=0$ . However,  $\rho_{xy}(k) = \rho_{yx}(-k)$ .

Now,  $SCCF = R_{xy}(k) = \hat{\rho}_{xy}(k) = \frac{C_{xy}(k)}{S_x S_y}$ , where

$$C_{xy}(k) = \frac{1}{n} \sum_{t=1}^{n-k} (X_t - \bar{X})(Y_{t+k} - \bar{Y})$$

The standard error of  $R_{xy}(k)$  is approximately equal to  $1/\sqrt{n-k}$  if  $X_t$  and  $Y_t$  are independent and one series is white noise.

[2] Multiply both sides of {5} by  $A_{t-k}$ , and take expectations.

This gives  $E(A_{t-k}, \beta_t) = v_k \sigma_A^2$ ; so that using above:

$$v_k = \frac{\rho_{A\beta}(k) \sigma_\beta}{\sigma_A}$$

## 5. Leading Indicator

Cross(1984) found a significant relationship between changes in the money supply and changes in the monthly S&P 500 Share Index for the period 1960-1982. He observes that past changes in the money supply, as measured by M1 and M2 [1] appear to be significantly related to current share prices for the period (1960-1982).

Homa and Jaffe (1971), using econometric techniques, estimated a constant lag between the money supply and an index of share prices. The relationship was used as a signal to switch from shares to treasury bills and back. They found that a switching policy yielded a higher rate of return (8.6% as opposed to 7.6%) than a simple buy and hold strategy even when the commission costs of switching were considered.

In this study, the money supply is selected as a possible leading indicator of aggregated quarterly share prices and will form the input series  $X_t$  discussed previously. Data for the Quarterly money supply as measured by M1 and M2 from 1961 - 1986 was collected from appropriate issues of the Reserve Bank Bulletin [2].

The series are lagged one quarter to allow for publication delay, so that each observation of the money supply series would have been known at the time of the quarterly RBNZ Index observation. The appropriate observations of M1 and M2 are shown in Appendix A.

## Discussion

The relationship between money and share prices has been the focus of much interest and controversy. The issue of whether current share prices are related to current, past or future changes in money supply appears to be not yet settled.

Early empirical studies such as those by Sprinkel (1971), Keran (1971), Hamburger and Kochin (1972) and others concluded generally that changes in money supply lead changes in share prices.

Later studies conducted by Pesando (1974), Cooper (1974), Kraft and Kraft (1979), Davidson and Froyen (1982) and others, began to question the methodology and conclusions of the earlier studies. Sprinkel's tests are criticized as being merely visual and impressionistic. He relies on simplistic discussions: for example growth in the money supply slowed at time  $t$  and share prices turned down shortly thereafter; or he uses graphs that seem to indicate a lead-lag relationship between money supply and an index of share prices. Many researchers warn against trying to build models from graphs as being a dangerous practice - see for example, Downing (1982).

Cross comments that these later writers feel that a relationship in which share price changes lagged money supply changes is inconsistent with a growing body of literature supporting the efficient market hypothesis. Briefly, their reasoning was that in a world where markets quickly incorporate all relevant current and past information, and where investors base their actions on predictions of the future, share prices could not lag money supply.

Many of these later empirical results were interpreted as showing that shareprice changes were related to future money supply changes and not past money supply changes. Some went so far as to say that this meant that share price changes caused money supply changes. Other studies concluded that shareprice changes were related only to current money supply changes, and not to either future or past money supply changes.

Cross provides a detailed discussion and summary of the studies and issues outlined above. Although his own empirical results support the early writers conclusions, he notes (p.145) "...future money supply changes, at least as measured by monthly M2, also appear to be significantly related to current stock prices for some periods studied."

Recall the efficient market hypothesis states that no one should be able to earn "excess" profits from studying past data. Cross points out that simply because a model (using past money supply data as an input) may be built to produce a "good fit" to actual past data, this is not in conflict with the hypothesis. The model must be able to predict the future course of share prices significantly better than another investor who does not use such a model i.e. "excess" profits are earned by using the model to form "trading rules". The significance of the improvement is the critical factor.

Footnotes

[1] M1 comprises notes and coins held by the public plus deposits on Trading and Savings bank cheque accounts ( net of Government and other financial institutions' deposits with Trading banks ).

M2 is M1 plus demand and call deposits of POSB and Trustee Savings banks, Private Savings banks, official money market dealers, Finance Companies and current accounts of Stock and Station agencies ( net of call and demand deposits of all these financial institutions with each other).

M3 is, roughly speaking, M2 plus all term deposits at the institutions listed in the above paragraph.

M1 and M2 are loosely termed "narrow" money, while M3 is "broad" money. In an article in the Reserve Bank Bulletin of April 1982 (p.118), the conclusion is reached that a narrower definition of money supply (M1 and perhaps M2) is more suitable when examining the relationship between the level of economic activity and the money supply.

[2] The money supply as measured by M1 and M2 is only available from 1967, when these measures were introduced. Estimated values for M1 from 1960-1967 were obtained from August 1984 Reserve Bank Bulletin article " A Quarterly Money Supply series for New Zealand before 1967" and M2 early values from the Reserve Bank publication: "Long Term Statistical Series" (January 1978)

## 6. Seasonal Adjustment

The M1 and M2 series are both seasonally adjusted using the well-known Ratio-to-Moving Average Classical Decomposition approach. Details of the procedure may be found in Makridakis, Wheelwright and McGee (1983).

Firstly a centred moving average of length four is computed for the series. The ratios of observed values to moving averages are calculated; these ratios vary around 1 indicating the effects of seasonality on the average deseasonalised values.

The Medial average of the ratios for each season is found by excluding the largest and smallest values and then finding the Arithmetic Mean. This gives four seasonal indexes which are adjusted to give a sum of four. Original observations are divided by the appropriate index to yield a "seasonally adjusted" value.

## 7. Building the Shareprice Forecasting Model using M2 series

Although Umstead asserts that it is common in Time Series Analysis to utilize the full series ( including the test sample ) in the identification stages of an analysis, the same procedure adopted earlier is used i.e. only the first 76 observations are used to build the model, the last 28 are retained to test it.

A plot of the logarithms of the seasonally-adjusted M2 series (Fig 8 overleaf) and its SACF reveals that this series requires further differencing to achieve stationarity ( the SACF does not dampen out quickly ). Both the graph in Fig.9, as well as the SACF of the first differences of the series, shown in Fig.10, suggest a series which appears to be stationary.

FIGURE 8: Natural Logarithms of M2  
76 Seasonally Adjusted Obs

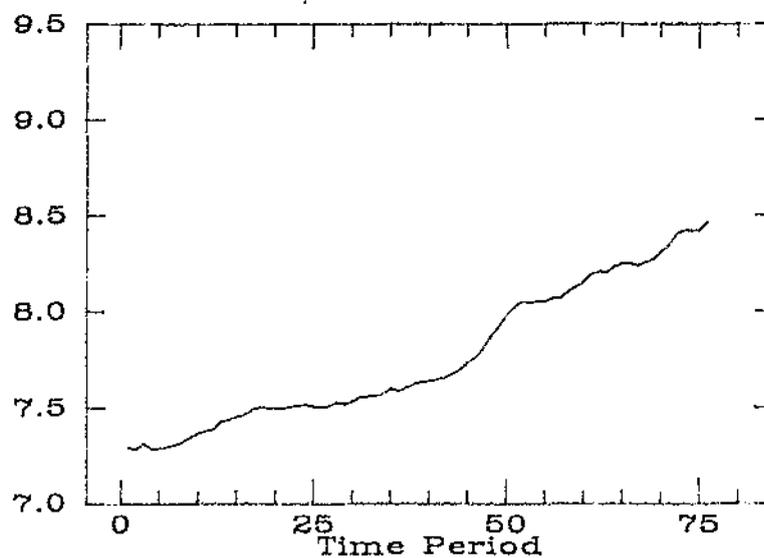


FIGURE 9: First Differences of Log e M2

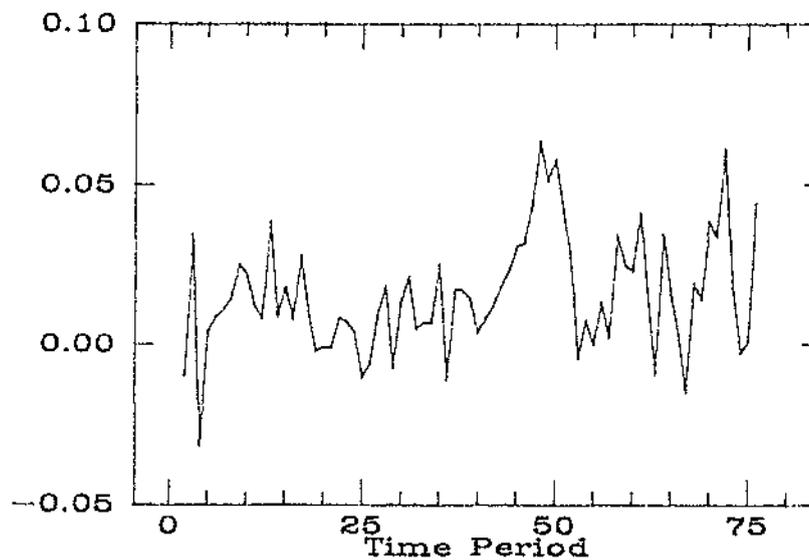
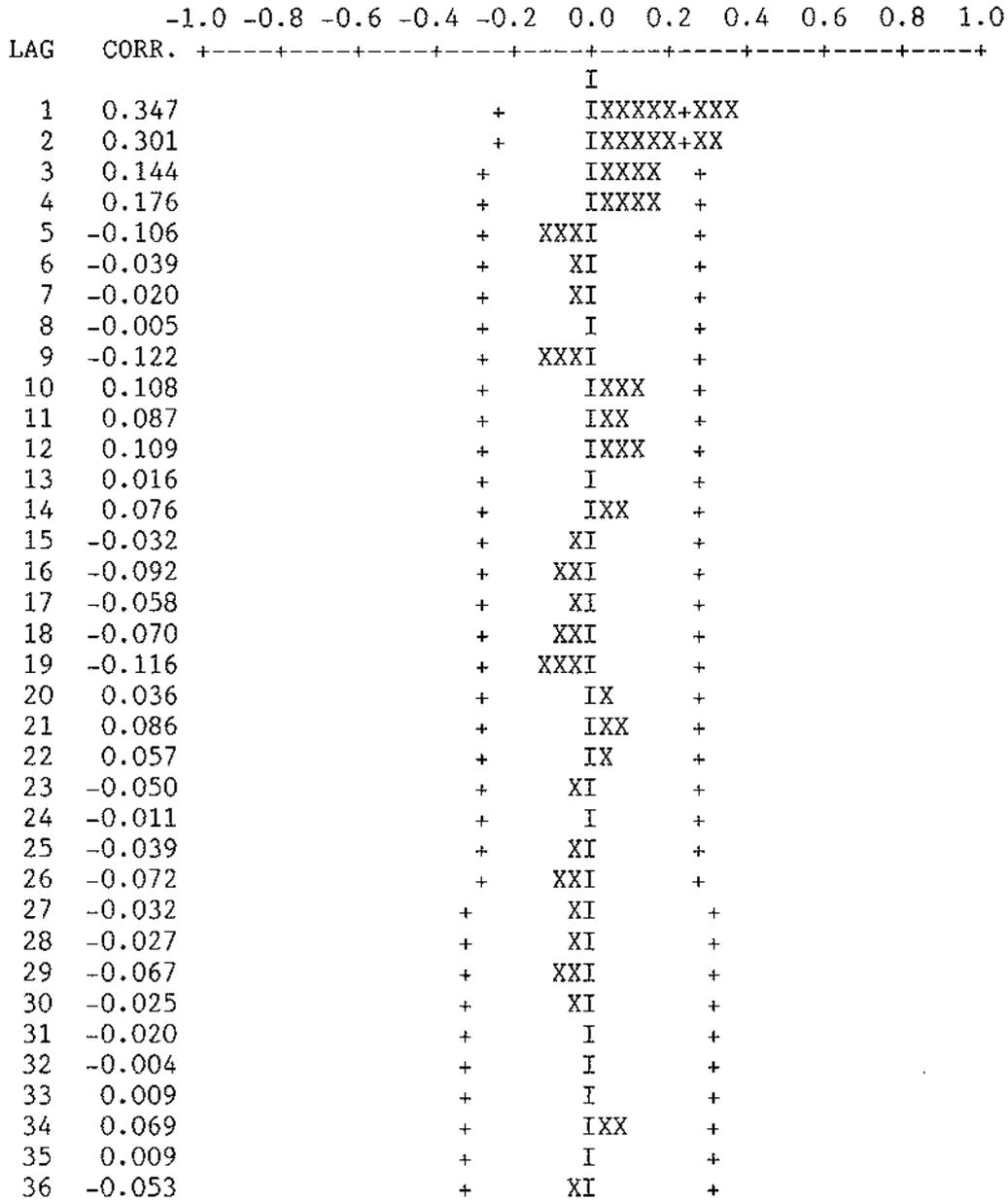


FIGURE 10

PLOT OF AUTOCORRELATIONS - M2 (Seasonally Adjusted)



MEAN OF THE (DIFFERENCED) SERIES = 0.0156  
 STANDARD ERROR OF THE MEAN = 0.0021  
 T-VALUE OF MEAN (AGAINST ZERO) = 7.4005

To summarize - it is assumed the input series ,  $x(t)$ , where

$$x(t) = (1-B) \ln X_t$$

i.e.(first differences of natural logarithms of seasonally adjusted M2) - is stationary. Also the output series ,  $y(t)$ , where

$$y(t) = (1-B) \ln Y_t$$

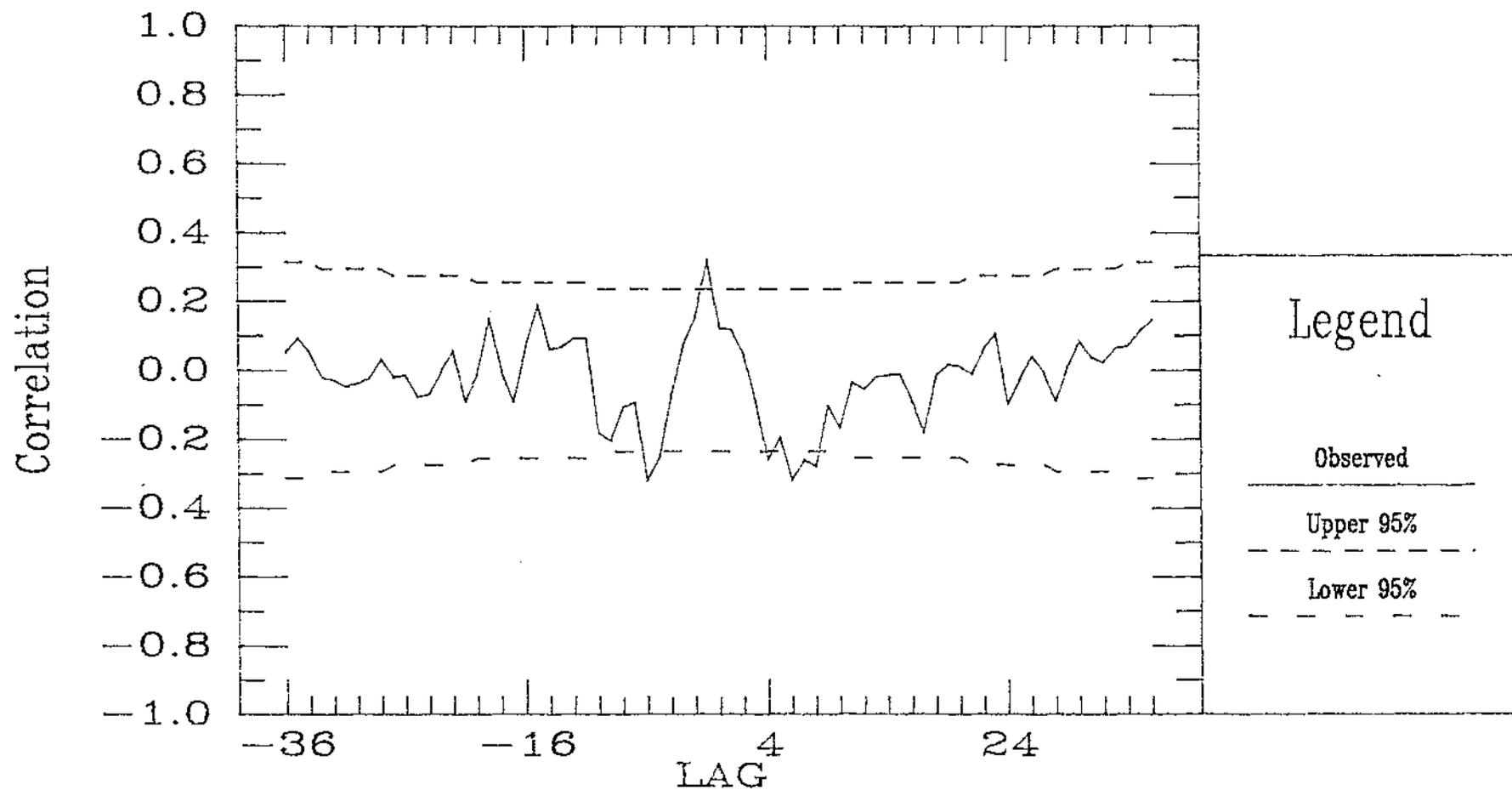
(first differences of natural logarithms of Real RBNZ Index) - is stationary.

The estimated Cross Correlation Function between  $x(t)$  and  $y(t)$  is shown in Fig.11, overleaf, and indicates that stationarity has been obtained for the entire system. First regular differences of logs of the input and output series achieves stationarity.

It is noted that significant cross correlations between  $x(t)$  and  $y(t)$  occur at lags of -6, -1, and +6. It appears then that the two series may be significantly negatively cross correlated when either is lagged by six quarters i.e. an increase in either series is followed by a decrease in the other 6 quarters later. However, recall that in order to obtain more efficient estimates, the input series must be 'prewhitened', and not too much notice should be taken of these significant lags i.e. the interpretation of the SCCF can be very misleading unless the prefiltering procedure described earlier is used. See footnote [1].

An attempt is now made to build an adequate ARIMA model for the input series ( M2 ) and obtain the prewhitened input series (i.e. residuals). From Fig.9, the SACF, and the SPACF, an ARIMA ( 0 1 2 ) model is suggested. Also since the T-value of the mean (against zero) is 7.4 a constant term is included.

FIGURE 11: SCCF of input and output series  
input = M2  
output = RBNZ



Proposed Model Structure:

$$x(t) = (1-\theta_1 B) (1- \theta_2 B ) A t + \theta_0$$

The process appears to be adequately fitted by the model:

	<u>Parameter Estimate</u>	<u>T-Ratio</u>
$\theta_1$	-0.28	-2.33
$\theta_2$	-0.22	-1.84
$\theta_0$	0.015	5.11

Modified Box-Pierce chisquare statistic results:

<u>Critical Values ( .05)</u>	<u>Q</u>
$\chi^2$ (10) = 18.3	13.3
$\chi^2$ (22) = 33.9	21.5
$\chi^2$ (34) = 48	28.3
$\chi^2$ (46) = 62	40.4

Each of the observed values is not significant at the 5 percent level.

Overfits of (0 1 3) and (1 1 2) are rejected and a diagnostic check of the residuals of the model reveals no strong evidence of inadequacy of fit. The autocorrelation of the residuals is essentially zero.

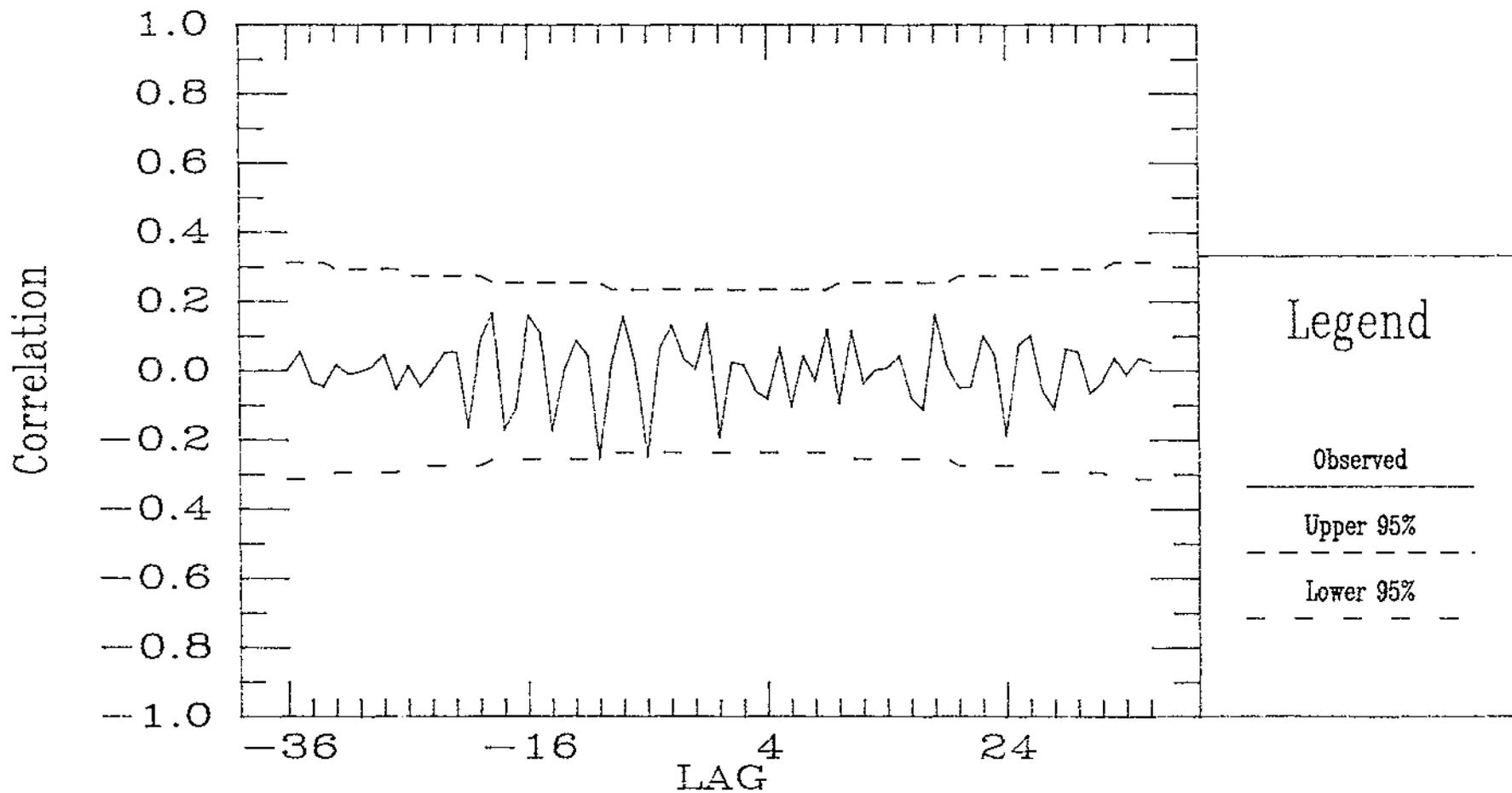
The above estimated model is therefore used to prewhiten the input series and reduce it to white noise. The output series (RBNZ) is now filtered by the above estimated ARIMA (0 1 2) model. The SCCF for the prewhitened input and "prewhitened" output is shown in Fig. 12 overleaf. A BMDP software package is used for this procedure.

There appear to be significant correlations at lags negative 6 and negative 10; these relationships are not useful for forecasting share price changes since they do not represent a leading relationship. The crosscorrelations in Fig. 12 do not indicate that M2 leads RBNZ i.e. the Money Supply (as measured by M2) does not appear to have a significant impact on Sharemarket Index. Similar results are obtained for the M1 series.

#### Footnotes

[1]Chatfield(1980) states that although the SCCF is asymptotically an unbiased and consistent estimator of the CCF, it can be shown that successive estimates are themselves autocorrelated. In addition, the variances of the estimators depend on the ACF's of the two components. Thus it is possible for two series, which are actually uncorrelated, to give rise to apparently 'large' cross-correlation coefficients which are actually spurious. So if a test is required for non-zero correlation between two time series, both series should first be filtered to convert them to white noise before computing the SCCF.

FIGURE 12: SCCF of "PREWHITENED" input output  
Input = white noise  
Output = Filtered RBNZ



## 8. Discussion

Downing and Pack(1982) attempted to build an ARIMA TF model describing indoor temperature in an experimental house as an endogenous variable impacted by the exogenous outdoor temperature. Despite scientific, graphical and statistical evidence for the existence of a relationship none can be identified using the prewhitening identification technique of Box and Jenkins.

They found that a very important factor in obtaining a clear picture of reality through prewhitening is that the additive noise contribution to output variability be considerably less than the input series contribution. The ability to pick up the input signal diminishes very quickly with a slight increase in the additive noise process. They maintain that this phenomenon is often found in economic data and is the weakness in determining causal relationships.

Thus failure to identify a relationship might be viewed as being related to the relative variability of the output and input series.

For the present study, the variance of the output series is about thirty times that of the input series. The effect of altering the relative variability of the two series was investigated, but no conclusive results were obtained.

## CHAPTER NINE

### Concluding Remarks

(i) The Transfer Function Methodology may be extended to involve two or more exogenous variables - however this greatly complicates the procedure and there does not as yet appear to be suitable computer software available.

(ii) Rather than limiting the statistical theory to linear TF, identification and estimation of non-linear impulse response functions may be of value. Moore and Shiskin (1967) found that Money Supply leads share prices in a non-linear fashion and this could be explored using TF methodology.

(iii) Naturally there are other leading indicators which may be investigated, such as Interest Rate, Unemployment, Inflation etc. For example, Malkiel and Quandt extended the study of Homa and Jaffe (which had not gone beyond 1969) and tested their hypothesis on 1954-1970 data. They also used revised money supply data. In testing the hypothesis, Malkiel and Quandt found that monetary policy did not predict share market behaviour well enough to outperform a buy and hold strategy. They went on to test an alternative model, using the interest rate and other economic variables related to corporate profits and sales. It was found that this alternative formulation outperformed predictions based on money supply.

Branch (1976) in turn criticizes the Malkiel study for using quarterly data. He asserts that monthly or even weekly data is needed, since the lag in response may be shorter than a quarter. Many filter rules are made with a good deal of hindsight, and Branch concludes that leading indicators are of rather limited value in share market selection.

The results obtained in the present study appear to point up the need for an integrated approach to investment decisions and timing. Relying purely on actual monetary policy or forecasts for timing is inappropriate.

(iv) There are those who are cautious about the effectiveness of the TF methodology. Chatfield observes that most examples used to demonstrate the procedure usually have exceptionally high correlation between input and output which means that virtually any identification procedure will give good results.

Cross notes that in attempting to filter or prewhiten the data, and removing either trends or irregular variations, we tend to remove information which would logically be utilized by investors and which is part of relationships between variables. This seeking of statistically "antiseptic" results changes the variables under investigation to the point where they may no longer coincide with the variables purportedly examined, and may produce more ill effects than good. The methodology should be used cautiously, to avoid the possibility of "a sophisticated attempt at producing a naive result".

## APPENDIX A

TIME PERIOD	DATE	NOMINAL RBNZ	CPI -1	REAL RBNZ	INTEREST P.A.	Percent DIVIDEND Quarterly	M1	M2
1	1961.1	1089	146	7458.90	*	*	689	1458
2	1961.2	1080	147	7346.94	*	*	685	1458
3	1961.3	1036	147	7047.62	*	*	642	1471
4	1961.4	996	149	6684.56	*	*	712	1490
5	1962.1	991	151	6562.91	*	*	649	1454
6	1962.2	1079	151	7145.70	*	*	677	1481
7	1962.3	1053	152	6927.63	*	*	647	1459
8	1962.4	1046	153	6836.60	*	*	713	1548
9	1963.1	1074	154	6974.03	*	*	677	1543
10	1963.2	1140	154	7402.60	*	*	705	1593
11	1963.3	1174	155	7574.19	*	*	678	1572
12	1963.4	1197	156	7673.08	*	*	737	1657
13	1964.1	1217	157	7751.59	*	*	729	1674
14	1964.2	1252	158	7924.05	*	*	744	1705
15	1964.3	1302	159	8188.68	*	*	722	1692
16	1964.4	1290	162	7962.96	*	*	778	1783
17	1965.1	1282	164	7817.07	*	*	745	1782
18	1965.2	1268	165	7684.85	*	*	758	1818
19	1965.3	1204	165	7296.97	*	*	703	1768
20	1965.4	1155	167	6916.17	*	*	766	1847
21	1966.1	1167	168	6946.43	*	*	703	1793
22	1966.2	1165	170	6852.94	*	*	732	1826
23	1966.3	1141	171	6672.51	*	*	704	1792
24	1966.4	1118	172	6500.00	*	*	776	1881
25	1967.1	1028	172	5976.74	*	*	706	1809
26	1967.2	1051	177	5937.85	*	*	695	1816
27	1967.3	1010	181	5580.11	*	*	668	1787
28	1967.4	1009	183	5513.66	*	*	762	1903
29	1968.1	1095	184	5951.09	*	*	685	1836
30	1968.2	1245	185	6729.73	*	*	702	1879
31	1968.3	1343	188	7143.62	*	*	690	1871
32	1968.4	1393	190	7331.58	*	*	756	1966
33	1969.1	1481	193	7673.57	*	*	713	1924
34	1969.2	1524	196	7775.51	*	*	731	1956
35	1969.3	1556	198	7858.58	*	*	723	1955
36	1969.4	1599	200	7995.00	*	*	777	2021
37	1970.1	1656	201	8238.80	*	*	764	1998
38	1970.2	1619	205	7897.56	*	*	788	2052
39	1970.3	1597	208	7677.88	*	*	765	2028
40	1970.4	1484	212	7000.00	*	*	838	2128

## Appendix A continued

TIME PERIOD	DATE	NOMINAL RBNZ	CPI -1	REAL RBNZ	INTEREST P.A.	Percent DIVIDEND Quarterly	M1	M2
41	1971.1	1400	221	6334.84	*	*	802	2085
42	1971.2	1444	226	6389.38	*	*	833	2131
43	1971.3	1344	231	5818.18	*	*	821	2115
44	1971.4	1370	236	5805.08	*	*	920	2263
45	1972.1	1389	241	5763.49	*	*	908	2268
46	1972.2	1492	245	6089.80	*	*	959	2364
47	1972.3	1500	248	6048.39	*	*	978	2406
48	1972.4	1616	251	6438.25	*	*	1165	2681
49	1973.1	1750	254	6889.76	*	*	1135	2742
50	1973.2	1848	260	7107.69	*	*	1244	2934
51	1973.3	1795	267	6722.85	*	*	1270	2984
52	1973.4	1591	273	5827.84	*	*	1422	3209
53	1974.1	1728	280	6171.43	*	*	1298	3105
54	1974.2	1581	287	5508.71	*	*	1351	3160
55	1974.3	1314	294	4469.39	*	*	1285	3079
56	1974.4	1203	305	3944.26	*	*	1436	3263
57	1975.1	1397	315	4434.92	*	*	1332	3178
58	1975.2	1456	324	4493.83	*	*	1424	3321
59	1975.3	1292	337	3833.83	*	*	1407	3318
60	1975.4	1393	350	3980.00	*	*	1580	3550
61	1976.1	1483	364	4074.18	*	*	1596	3596
62	1976.2	1457	380	3834.21	*	*	1655	3686
63	1976.3	1559	397	3926.95	*	*	1548	3558
64	1976.4	1415	410	3451.22	*	*	1792	3852
65	1977.1	1404	421	3334.92	*	*	1689	3803
66	1977.2	1327	432	3071.76	*	*	1689	3849
67	1977.3	1232	453	2719.65	*	*	1586	3694
68	1977.4	1190	469	2537.31	*	*	1820	3938
69	1978.1	1285	486	2644.03	*	*	1720	3881
70	1978.2	1370	495	2767.68	*	*	1843	4074
71	1978.3	1358	508	2673.23	*	*	1834	4107
72	1978.4	1347	521	2585.41	*	*	2189	4566
73	1979.1	1381	535	2581.31	*	*	2035	4518
74	1979.2	1406	547	2570.38	*	*	2072	4549
75	1979.3	1526	572	2667.83	*	*	2005	4437
76	1979.4	1528	600	2546.67	11.40	2.80	2371	4850

## Appendix A continued

TIME PERIOD	DATE	NOMINAL RBNZ	CPI -1	REAL RBNZ	INTEREST P.A.	Percent DIVIDEND Quarterly	M1	M2
( "holdout sample" )								
77	1980.1	1630	623	2616.37	12.00	1.45	2147	4660
78	1980.2	1804	647	2788.25	11.50	1.07	2294	4860
79	1980.3	1961	674	2909.50	12.00	2.77	2184	4730
80	1980.4	2348	698	3363.90	11.35	3.02	2482	5100
81	1981.1	2495	724	3446.13	11.25	1.35	2452	5120
82	1981.2	2896	746	3882.04	12.00	0.75	2650	5440
83	1981.3	2744	776	3536.08	12.13	1.61	2566	5350
84	1981.4	2765	806	3430.52	12.75	2.14	2861	5704
85	1982.1	2786	837	3328.55	14.00	1.76	2878	5752
86	1982.2	2626	864	3039.35	14.25	1.06	2906	5368
87	1982.3	2542	907	2802.65	14.25	2.34	2643	5781
88	1982.4	2331	940	2479.79	14.25	2.10	3030	5733
89	1983.1	2851	965	2954.40	12.75	3.08	3002	5628
90	1983.2	3312	973	3403.91	14.00	2.66	2940	5592
91	1983.3	3856	983	3922.69	13.25	1.92	2955	5662
92	1983.4	4581	991	4622.60	9.75	1.79	3480	6181
93	1984.1	4816	1000	4816.00	10.00	1.19	3251	5955
94	1984.2	4964	1007	4929.49	10.25	0.59	3267	5988
95	1984.3	4676	1029	4544.22	11.75	1.57	3140	5567
96	1984.4	5300	1060	5000.00	12.25	2.43	3866	6302
97	1985.1	5285	1094	4830.90	15.00	0.56	3338	5658
98	1985.2	5790	1142	5070.05	15.50	0.95	3557	5848
99	1985.3	6021	1200	5017.50	15.50	1.36	3466	5653
100	1985.4	6697	1233	5431.47	15.50	2.02	4342	6687
101	1986.1	7861	1261	6233.94	16.00	0.82	4059	6427
102	1986.2	9256	1290	7175.19	14.00	0.52	4186	6503
103	1986.3	11004	1325	8304.90	13.75	0.99	4514	6856
104	1986.4	13435	1369	9813.73	13.75	1.16	5032	7518

APPENDIX BDatex Composite and Capital Index - Sept 1979 to Dec 1986

CAPITAL	COMPOSITE
1137	1199
1127	1222
1202	1321
1312	1456
1400	1594
1651	1928
1830	2163
2071	2464
1988	2405
2095	2586
1995	2508
1911	2429
1904	2477
1791	2382
2184	2978
2629	3664
3113	4409
3634	5226
3713	5402
3887	5687
3904	5801
4351	6606
4364	6663
4732	7288
4900	7646
5559	8829
6457	10328
7819	12560
8986	14559
10865	17772

## BIBLIOGRAPHY

- ANDERSEN A. and Weiss A.  
Box Jenkins Forecasting Technique in Makridakis (1984) - Chapter 5
- ARZAC E.R.  
Forecasting Stock Market Prices: Discussion.  
Journal of Finance 22 no. 2 1977 p. 445
- BACHELIER L. (1900)  
Theory of Speculation. Paris: Gouthier-Villars in Umstead (1975)
- BARTLETT M.S.  
On the theoretical specification of Sampling Properties of  
Autocorrelated Time Series. Journal of Royal Statistical Society  
108: 93, 1945 in Box and Jenkins.
- BOX G.E.P. and Jenkins G.M. (1976)  
Time Series Analysis: Forecasting and Control, Revised ed.  
San-Fransisco: Holden-Day.
- BMDP Statistical Software.  
Edited by W.J.Dixon. London: University of California Press 1985
- BRANCH B. (1976)  
Fundamentals of Investing. New York: J.Wiley & Sons
- BURROWES A.W. and Mulholland R.D. (1986)  
Investing on the New Zealand Sharemarket. Auckland: Macmillan.
- BOWERMAN B.L. and O'Connell R.T (1979)  
Time Series and Forecasting: an Applied Approach.  
Massachusetts, Duxbury Press.
- BOX G.E.P. and COX D.R.  
An Analysis of Transformations.  
Jour. Royal Stat. Soc. B26:211,1964.
- CHATFIELD C. (1982)  
The Analysis of Time Series. 2nd ed. London: Chapman & Hall.
- COOPER R.V.L.  
Efficient Capital Markets and the Quantity Theory of Money.  
Journal of Finance 19 887-908, 1974. in Cross(1985)
- CROSS S.J.  
Money and Stock Prices. Dissertation, Doctor of Arts, Tennessee  
State University, 1985.
- CRYER J.D. (1986)  
Time Series Analysis. Duxbury Press, Boston.
- DAVIDSON L.S. and Froyen R.T.  
Monetary Policy and Stock Returns: Are Stock Markets Efficient?  
Federal Reserve Bank of St. Louis Review 64:3-12,1982 in Cross 1985
- DOWNING D.J. and PACK D.J.  
The Vanishing Transfer Function.  
in Time Series Analysis: Theory and Practice 1. Ed. O.D. Anderson.  
Proc of the Int. Conference held in Valencia, Spain in June 1981.  
Amsterdam: North-Holland.
- DYCKMAN T.R. et al. (1975)  
Efficient Capital Markets and Accounting: a Critical Analysis  
Eaglewood Cliffs NJ: Prentice-Hall in Umstead (1975)
- FIRTH M. (1977)  
The Valuation of Shares and the Efficient Markets Theory  
London: Macmillan.
- FAMA E.F.  
The Behaviour of Stock Market Prices. Journal of Business 38 1965

## Bibliography (cont.)

- FAMA E.G. and Blume  
Filter Rules and Stock Market Trading. Journal of Business 39  
226-241, 1966.
- FAMA E.F.  
Efficient Capital Markets: A Review of Theory and Empirical Work.  
Journal of Finance 25 No. 2: 383-420, 1970.
- GRANGER C.W.J. and Morgenstern O.  
Spectral Analysis of New York Stock Market Prices. Kyklos 16  
1-27, 1963. in Umstead (1975)
- GRANGER C.W.J.  
Investigating the Future. Statistical Forecasting Problems. 1968
- GRANGER C.W.J. and Morgenstern O. (1970)  
Predictability of Stock Market Prices. Lexington, Massachusetts.  
D.C. Heath
- GRANGER C.W.J. and Newbold P. (1977)  
Forecasting Economic Time Series. New York. Academic Press.
- GRANGER C.W.J. (1980)  
Forecasting in Business and Economics. New York. Academic Press.
- GODFREY M.D. et al.  
The Random Walk Hypothesis of Stock Market Behaviour. Kyklos 17  
1-29, 1964. in Umstead (1975)
- HAMBURGER M.J. and Kochin L.A.  
Money and Stock Prices: Channels of Influence. Journal of Finance  
27: 231-249, 1972 in Cross(1985).
- HEBDEN J. (1981)  
Statistics for Economists Oxford: Phillip Allen.
- HOMA K.E. and Jaffee D.M.  
The Supply of Money and Common Stock Prices. Journal of Finance 26  
1045-1066, 1971.
- HORSFIELD A.K and O'Dea D.J  
New Zealand Institute of Economic Research. Unit Paper No. 18  
Equity Investment in New Zealand. 1983
- KENDALL M. (1976)  
Time Series. 2nd ed. London: Griffin.
- KRAFT J. and Kraft A.  
Determinants of Common Stock Prices: A Time Series Analysis.  
Journal of Finance 22 No. 2 1977.
- KERAN M.W.  
Expectations, Money and the Stock Market. St. Louis Review.  
Jan 1971. in Cross (1985).
- LEVENBACH H. and Cleary J.P. (1984)  
The Modern Forecaster: the forecasting process through Data  
Analysis. Belmont California: Lifetime Learning Publications.
- LEVY R.A.  
Relative Strength as a Criterion for Investment Selection.  
Journal of Finance 22: 595-610, 1967.
- MAKRIDAKIS S. et al. (1983)  
Forecasting: Methods and Applications. 2nd ed. New York:  
Wiley & Sons
- MAKRIDAKIS S. et al. (1984)  
The Forecasting Accuracy of Major Time Series Methods.  
Chichester, New York: Wiley.

## Bibliography (cont.)

- MALKIEL B.G. (1985)  
A Random Walk down Wall Street. 4th ed. New York: Norton.
- MOORE G.H. and Shiskin J.  
Indicators of Business Expansion and Contraction New York.  
 National Bureau of Economic Research: 1967. in Umstead.
- NERLOVE M. et al. (1979)  
Analysis of Economic Time Series: A Synthesis. New York: Academic  
 Press.
- NELSON C.R. (1973)  
Applied Time Series Analysis for Managerial Forecasting.  
 San Francisco: Holden-Day.
- NIEDERHOFFER V. and Osborne M.  
 Market Making and Reversal of the Stock Exchange. Journal of  
the American Statistical Association 61: 897-916, 1966.
- O'DONOVAN T.M (1983)  
Short Term Forecasting: An Introduction to the Box-Jenkins Approach  
 New York: J. Wiley & Sons
- PESANDO J.E.  
 The Supply of Money and Common Stock Prices: Further observations  
 & Econometric Evidence. Journal of Finance 29: 900-21, 1974.  
 in Cross(1985)
- PRAETZ P.  
 Time Series Models for Stock Market Prices.  
 in Time Series Analysis: Theory and Practice 1. Ed. O.D. Anderson.  
 Proc of the Int. Conference held in Valencia, Spain in June 1981.  
 Amsterdam: North-Holland.
- QUENOILLE M.H.  
 Approximate tests of correlation in Time Series. Journal of  
the Royal Statistical Society B 11: 68-84, 1949. in  
 Granger and Newbold (1977)
- RESERVE BANK OF NEW ZEALAND - No. 10  
Essays on Economic Practice. 1973
- ROBINSON P.B. and Stamboulis  
 On the Stochastic Nature of Stock Prices. Unpublished  
 working paper New York University. in Umstead (1975)
- SHAHABUDDIN S.  
 Analysis of Stock Prices using Time Series with Intervention  
 Analysis. American Statistical Association - Procedures  
of Business and Economic Statistics Section. 1980
- SPRINKEL B.W.  
Money and Markets: A Monetarist View. Homewood, Ill.  
 Richard D. Irwin, 1971 in Cross(1985)
- UMSTEAD D.A.  
Forecasting Stock Market Prices. Dissertation, Ph.D.,  
 University of North Carolina, 1975.
- UMSTEAD D.A.  
 Forecasting Stock Market Prices. Journal of Finance 22 no.2  
 1977.
- WHITCOMB D.K.  
 Forecasting Stock Market Prices: Discussion.  
Journal of Finance 22 no. 2 1977 p. 442