LENS DISTORTION CORRECTION BY ANALYSING
THE SHAPE OF PATTERNS IN HOUGH TRANSFORM
SPACE

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Abstract

Many low cost, wide angle lenses suffer from lens distortion, resulting from a radial variation in the lens magnification. As a result, straight lines, particularly those in the periphery, appear curved. The Hough transform is a commonly used linear feature detection technique within an image. In Hough transform space, straight lines and curved lines have different shapes of peaks. This thesis proposes a lens distortion correction method named SLDC based on analysing the shape of patterns in the Hough transform space. It works by reconstructing the distorted line from significant points on the smile-shaped Hough pattern. It then optimises the distortion parameter by mapping the reconstructed curved line into a straight line and minimising the RMSE. From both simulation and correcting real world images, the SLDC provides encouraging results.

**Keywords:** Hough transform; barrel lens distortion correction; straight line; shape of peaks
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