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Shaping Student Views on Mathematics:

Influences on Year 5 and 6 Students' Mathematical Dispositions and Mindsets towards Learning

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Abstract

There exists a contrast in students' views surrounding what it means to do mathematics, what their role is as learners of mathematics, and its place within their lives. While some students see mathematics as a creative, deep phenomenon to be explored, discussed, and relevant in understanding life, many come to see it as a dull subject, full of memorised facts to be recited, and eventually useful in getting a job. Likewise, while some students hold a growth mindset, seeing hard work, struggle, and perseverance as essential for growth, others come to view ability as being fixed, and something that cannot be changed.

Through a social constructivism lens, and with selection of a qualitative case study approach, this study explored the different factors that influenced a group of Year 5 and 6 students' mathematical dispositions and mindsets towards learning with particular interest in how they viewed and reacted to mistakes. In total, 41 year 5 and 6 students participated in this study with data being collected through the use of student questionnaires, semi-structured interviews, and student self-reflections.

In examining literature surrounding the formation of mathematical dispositions and mindsets towards learning, several factors such as the tasks students engage in, teacher interactions, grouping, assessment practices, and family were found to have an influential role. As students described their views and experiences of mathematics, the importance of the teacher and family were revealed. Considerations into the type of tasks, the formation of groups, use of assessment, and the positioning of students at school and at home were identified and analysed.

In understanding where these contrasting mathematical dispositions and mindsets stem from, teachers and family are more equipped to foster positive mathematical dispositions in students and mindsets and create a culture that best supports the learning of mathematics.

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Chapter One: Introduction

1.1 Introduction

This chapter provides a background to the current study, firstly looking at the New Zealand context in which the research takes place. In Section 1.2, the importance of students' mathematical dispositions and mindsets are discussed in terms of its impact on their achievement and engagement in future mathematics education. Section 1.2 also illustrates the need for understanding the influence different factors such as the teacher, task, grouping, assessment, and family have on students' mathematical dispositions and the mindsets they develop. The aim and research questions of this study will be outlined in Section 1.3, and important terms used in this thesis will be defined and clarified in section 1.4. Finally, in Section 1.5, an overview of the chapters in this thesis will be presented.

1.2 Background to study

The word mathematics has the power to elicit very different emotions with different students. While some hold views of a dull subject, filled with anxiety and little relevance to real life, others see it as a much more creative, deep phenomenon that can be used to understand the world and the way things work (Boaler, 2010; Grootenboer & Marshman, 2016). In knowing where these contrasting perceptions of mathematics stem from, teachers and family are more equipped to foster positive dispositions in students and create a culture that best supports the learning of mathematics.

Like many other western countries, New Zealand students share a downward trend in mathematics achievement and enjoyment as they go through school. This often culminates as adults who frequently claim a dislike or incompetence in mathematics, and often choose not to pursue it in future study (Mutodi & Hlanganipai, 2014). Recent findings from the National Monitoring Study of Student Achievement (NIMSSA) have shown that while 81% of New Zealand Year 4 students were performing at their expected level, this drops off to only 41% by Year 8 (Education Review Office, 2018). These findings also signalled that by Year 8, students had become less positive about mathematics and its purpose within their lives. This coincides with a meta-analysis conducted by Sullivan et al. (2013) who found that the middle years of schooling (Years 5 to 9) are an especially critical point for the formation of students' dispositions and it is during this time that students often develop an anxiety towards mathematics.

The importance of having a positive mathematical disposition is well researched (Beyers, 2011; Grootenboer & Marshman, 2016; Kilpatrick, Swafford, & Findell, 2001) with students' beliefs about the nature of mathematics and their own role as a learner being found to have a strong influence on whether students take advantage of opportunities to learn mathematics. Use of these opportunities such as spending time on task, listening to explanations, exploring solutions, or conjecturing and justifying are some of the most important predictors of student achievement (Beyers, 2011; Grootenboer & Marshman, 2016; Kilpatrick et al., 2001). Those students, who for whatever reason, have an absence of certain dispositional cognitive functions, often have limited opportunities to extend their mathematical knowledge and understanding (Beyers, 2011). Similarly, Boaler (2010) identifies students who hold a growth mindset and view mathematics as a learnable and flexible skill are more likely to persevere through challenges and enjoy exploring mathematics. Given then, the contrasting views students have towards mathematics while at school, and the impact this can have on their capacity to learn, it is important that we investigate where these dispositions and mindsets originate from (Grootenboer & Marshman, 2016).

Studies such as (Beyers, 2011; Boaler, 2002; Franke & Carey, 1997; Grootenboer & Marshman, 2016) suggest several factors such as teacher beliefs, classroom practices, and parent input have a significant effect on the formation students' mathematical dispositions. As students go through school, these collective experiences and interactions influence how they position themselves in the classroom and come to interpret mathematics (Mutodi & Hlanganipai, 2014).

There seems however to be limited examples of New Zealand research around how each of these specific factors shape students' mathematical dispositions and how students interpret these different experiences that they have with mathematics.

Gaining students' point of view is important according to Taylor, Hawera, and Young-Loveridge (2005), as they "often have an awareness of the social and organisational matters that can affect their learning" (p. 728). The current study aims to add insight into what New Zealand Year 5 and 6 school students think about mathematics and how these dispositions are formed.

As part of understanding students' mindsets towards learning, I am interested to examine students views around mistakes and the part they play in learning mathematics. Boaler (2010) demonstrates how important mistakes are for brain development and the learning of mathematical concepts, yet many students are very quick to hide the idea that they have made a mistake. Students often regard mistakes as an indicator of low ability and in turn, miss out on opportunities to deepen their thinking and connect ideas (DeBrincat, 2015). It is for this reason that this study will also delve into the idea of learning from mistakes and what students' reactions and views on them can tell us about their understanding.

1.3 Research objectives

The primary purpose of this study is to explore the different factors that influence Year 5 and 6 students' mathematical dispositions and mindsets towards learning. This will help develop existing ideas of how teachers and family can build positive mathematical disposition and mindsets in students, and in turn, support achievement. The secondary aim of this study is to understand students' views towards mistakes, and what occurs when mistakes are made in mathematics so they can be used productively to support the learning of mathematical concepts.

To meet the purpose of this study, the following research questions have been addressed:

1. What are students' current mathematical dispositions and mindsets towards learning?

2. How do factors such as the teacher, task design, grouping, assessment, and family influence students' mathematical dispositions and mindsets towards learning?
3. What are students' views and reactions to mistakes within mathematics?

1.4 Definition of terms

To help develop a shared understanding of ideas being described in this study between the author and the reader, important terms are outlined and defined:

- **Mathematical disposition,** as used in this study, is the collection of attitudes and beliefs about mathematics that everyone holds. It refers to what students believe their role as learners of mathematics is, what it means to do mathematics, and what place mathematics has in the world (Dossey, 1992; Gresalfi, 2009). Students with a positive mathematical disposition are characterised with the ability to persevere with challenges, have belief in their capability, reason ideas, be flexible with numbers, and have a positive outlook on mathematics.
- **Mindset** is often described as either fixed or growth. Those who hold a fixed mindset believe that ability is fixed, and you are either smart or you are not. Examples of students with a fixed mindset are those who give up if they make a mistake and often avoid challenges in favour of easier work that they know they can succeed with. Growth mindset, on the other hand, is the idea that ability can be grown with hard work and struggle. Students with a growth mindset are more willing to take risks, make mistakes, and persevere through challenges (Boaler, 2013).
- **Mistakes** in mathematics can refer to several different types of errors. Computational or calculational errors occur when numbers have been incorrectly added, subtracted, multiplied, or divided and can often arise when students rush. Misconceptions, on the other hand, refer to students misleading ideas or misapplication of concepts. A common example of this is when students apply whole number thinking to fractions and decimals. These

misconceptions can reveal a lot about a student's understanding and will be the main form of mistake referred to in this study (Rushton, 2014).

1.5 Overview of chapters

Chapter two outlines and gives a brief history of the theoretical framework, social constructivism, which underpins this study. The contrasting views of mathematics are then examined. Following on from this, the importance of having a growth mindset and the role of mistakes in mathematics are analysed with respect to student learning. Finally, the different factors such as task design, grouping, assessment, and family are explored in depth, using New Zealand and international literature to help understand how these may influence students' mathematical dispositions.

Chapter three sets out the research design and methodology for this study. A justification for utilising a qualitative case study approach and the role of the researcher is given. The setting, sample and schedule are then established while the data collection tools, and analysis process are explained. Finally, the reliability, validity, and ethics of this study are discussed with reference to considerations made throughout the research process.

Through analysing collected questionnaire and student self-reflection data, Chapter four and five discuss the students' mathematical dispositions and mindsets towards learning, how different factors have influenced their mathematical dispositions and mindsets, and how the students view and react to mistakes in mathematics. Chapter six examines 3 students' experiences with mathematics, unpacking their mathematical dispositions and mindsets and how they have come to view mathematics in the way that they do.

Chapter seven discusses the implications of the results; how teachers and parents can help foster positive mathematical dispositions and mindsets in students, and how mistakes can be utilised and acknowledged as a part of mathematics learning. The limitations of the study and opportunities for further research will then be outlined before concluding thoughts are given.

Chapter Two: Literature review

2.1 Introduction

As previously highlighted, contrasting perceptions of what mathematics is, and what it means to do mathematics exist. While the importance of students developing positive mathematical dispositions is well documented, there continues to be a disturbing trend of students disliking mathematics and choosing not to pursue it further through life. The following literature review seeks to unpack the different factors that influence students' mathematical dispositions.

In section 2.2 the theoretical framework of this study, social constructivism, is outlined, giving a brief history and implications for the development of mathematical dispositions. The differing views of mathematics are identified in section 2.3. Using literature, these differing views are analysed further in sections 2.3.1 and 2.3.2 with respect to teaching practices, emerging themes, and resulting dispositions. In section 2.4, growth mindset, and its implications for student learning are discussed. Following this, in section 2.5, is the role that mistakes play in student learning of mathematics and their relationship with mindset. Section 2.6 and resulting subsections analyse the influence that specific factors such as the teacher's role, task design, grouping, and assessment have on mathematical dispositions. Finally, section 2.7 discusses the importance of home and family in developing positive mathematical dispositions.

2.2 Social constructivism theory

To understand the development of mathematical dispositions, a learning theory that accounts for the complexity of mathematics education is required. Through the lens of the social constructivism learning theory, students do not begin life with inherent positive or negative dispositions towards mathematics. It is instead through the experiences and interactions they have with mathematics that shift their attitudes and self-concepts (Palincsar, 2005). Of note in this study, is the idea that the mathematical dispositions of students are malleable, not permanent, and able to be affected by both internal and external factors throughout their lives (Gresalfi, 2009; Hall, 2016).

Traditionally, Piaget's constructivism theory had been the significant paradigm for the theories of learning mathematics. Although variations of constructivism exist, these learning theories view student learning and knowledge of the world as being constructed internally, filtering new experiences through previous understandings (Cobb, 1994). The extremes of this learning theory, according to Cobb and Steffe (1983), can be likened to a solo piano player or lone scientist, being devoid of social interaction. While Piaget's constructivism implies that knowledge is constructed internally, it does implicate interaction with the outside world in some form.

The social constructivism learning theory was developed out of a need to acknowledge both the individual sense making aspects, and the social processes, that are essential to the learning of mathematics (Ernest, 1994). This multidisciplinary account of mathematics learning has drawn inspiration from several theories such as aforementioned constructivism theories and Vygotskian social theories. This social constructivism theory uses conversation as a central metaphor; that being conversation of the mind, and conversation between learners (Ernest, 2006).

Learning through the lens of social constructivism is perceived as an active process, relying on a combination of cognitive, and affective functions rather than simply being acquired (Gresalfi, 2009; Palincsar, 2005). Ernest (2006) highlights the complexities of catering for these different domains with consideration of relationships, roles, materials, discourse, content, and modes of communication in the classroom being significant.

A key feature of social constructivism, and of importance to this study, is the view towards errors and misconceptions. Based on the idea that students need to internalise new information and filter it through existing understanding, Cobb (1994) believe that errors cannot be avoided. It is therefore considered 'normal' that students make errors and have misconceptions as they rationalise their own thinking and make sense of new concepts. Through the lens of social constructivism, these errors are opportunities for discussion and shared understanding between learners.

2.3 The differing views of mathematics

Much like the philosophy of mathematics, the nature of mathematics itself has been the focus of much debate over the past few decades with differing views coming from policy makers, teachers, and mathematicians (Boaler, 2002). How society perceives the nature of mathematics has a strong influence on how school curriculum and instruction is developed. Dossey (1992) believes that only in understanding these different conceptions, can we develop and successfully implement effective mathematics programmes in schools. Young-Loveridge et al. (2006) believe that this, for the past few decades, has not been the case, with different perceptions having created a society where the teaching of mathematics and the true nature of mathematics do not always align.

While mathematics education has largely been focused on preparing students for employment or future mathematics study, this has often been taught in ways that subtly affect students' mathematical dispositions (Sullivan et al., 2013). Boaler (2010) believes that students often come to see the nature of mathematics as just "numbers" or "lots of rules" and misalign success with memory and speed. This is in contrast to asking mathematicians about the nature of mathematics where they will normally respond with ideas of "the study of patterns" or "a set of connected ideas" (Boaler, 2010). Presmeg (2002) notes the importance of getting this connection between school and the true nature of mathematics right, acknowledging that students commonly develop a dislike for mathematics as they progress through school and in turn, often miss out on what the opportunities and experiences mathematics can provide. So what experiences of mathematics are students getting, and how does this influence their mathematical disposition and mindsets?

2.3.1 Performance mathematics

An experience that many students internationally, and in New Zealand, have of mathematics, is one tailored towards performance. Typically comprised of ability grouping, standardised testing, and a strong emphasis on basic facts and procedural work, these more traditional classrooms have dominated mathematics education for the past few decades (Boaler, 2002). While this form of mathematics education is often held in contrast to more inquiry or reform based classrooms where students work in mixed ability groups, explore deep and complex problems, and use multiple methods, Boaler (2010) believes that both forms can include effective and ineffective teaching. Some teachers, who may be described as traditional due to lecturing students, who in turn work individually or with students at their 'ability level', may also ask great questions and engage students in interesting mathematical problems. Boaler does however go on to describe traditional teaching practices which may contribute to students' narrow perception of mathematics. This being mathematics classrooms which revolve around the demonstration of methods for a large majority of class time followed by students working through sets of identical questions without opportunity for discussion or exploration of ideas (Gresalfi, 2009). Students in such classrooms quickly learn that in order to be successful, they must listen to, and copy the teacher carefully (Boaler, 2010; Taylor et al., 2005). While this passive form of receiving knowledge may suit some students, many students miss out on chances to engage in sense making, reasoning, and questioning; all of which are important aspects of mathematics (Sullivan et al., 2013).

In a study conducted by Young-Loveridge et al. (2006), 459 New Zealand students from Years 2 to 8 (6 to 13-year-olds) were asked the question, "what is maths all about?" This study was comprised of students from 12different schools, in which half were taught through the Numeracy Development Project, and half were not. While the Numeracy Development project was introduced with the aim of developing students' understanding of numbers and their ability to use numbers to solve problems, Young-Loveridge et al. (2006) found that inclusion in the Numeracy Development project showed few differences when comparing student responses. Table 2.1, showing students' responses, illustrates what many students in New Zealand have come to perceive mathematics as.

Table 2.1 Percentages of Students Who Mentioned Each Category as a Function of Year Group and Project Status (NDP vs Non-NDP) (Young-Loveridge et al., 2006, p. 58).

Aspect mentioned	Years 2–4 NDP	Years 5–6 NDP	Years 5–6 non-NDP	Years 7–8 NDP	Years 7–8 non-NDP
Mathematical content	48%	48%	44%	52%	49%
Learning	59%	35%	55%	16%	18%
Thinking	9%	3%	5%	3%	4%
Problem solving	0%	8%	5%	10%	16%
Utility – here and now	3%	12%	6%	4%	3%
Utility – in the future	9%	27%	44%	23%	16%
Enjoyment	6%	9%	3%	1%	3%
Non-responders	12%	19%	5%	17%	12%
Number of students	37	121	62	123	116

Most notable of the data was students' responses regarding mathematical content. Young-Loveridge et al. (2006) writes how many student responses reflected the view that mathematics is all about computation, with many students giving examples of times-tables being a fundamental aspect of being good at mathematics. Grootenboer and Marshman (2016) found similar trends in their meta-analysis of four different studies, indicating that almost 70% of the students involved thought that times-tables and basic facts were the most important thing they had leaned in mathematics, while almost all other responses listed them as being important. While students in this study believed those who could quickly and publicly recite their times tables were regarded as "brainy", times-tables were also a key factor in students' responses about not liking mathematics (Grootenboer & Marshman, 2016). It was noted that "generally times-tables are taught and learned in a rote fashion with the emphasis on accurate and speedy recall. In other words, it seemed primarily about efficient memorisation, and this is not a particularly mathematical process" (Grootenboer & Marshman, 2016, p. 49). While basic facts in themselves may be important for students to understand and use, the idea that these are the peak of mathematics learning needs to be challenged, especially given students negative views towards them (Boaler, 2010; Gresalfi, 2009; Grootenboer & Marshman, 2016).

While the New Zealand Curriculum outlines that “mathematics is the exploration and use of patterns and relationships in quantities, space, and time” (Ministry of Education, 2014), very few students analysed by either Young-Loveridge et al. (2006) or Grootenboer and Marshman (2016) talked about the nature of mathematics in this way. There were however, a number of students who made comments about its usefulness in the future with reference to getting a job. These answers, unsurprisingly, became more sophisticated from the older students as they talked about handling money and being independent (Young-Loveridge et al., 2006).

Interestingly, a large number of respondents from Young-Loveridge et al. (2006) study appeared to give no view about the nature of mathematics. While this, was not analysed in terms of gender, ethnicity, or ability, Young-Loveridge et al. (2006) suggests that these children may do mathematics without much opportunity to discuss what they are actually learning. Franke and Carey (1997) shares the idea that students who perceive mathematics as a given, and the answer as either right or wrong, are more unlikely to feel compelled to make judgements about mathematical ideas and engage in mathematical discussions.

As with previously mentioned studies, Boaler (2002), who analysed two different approaches to mathematics education, found that students with a more algorithmic or traditional form of mathematics education seemed to have “created an important distinction in their minds between what they perceived as the algorithmic demands of school mathematics and the completely separate demands of the real world” (Boaler, 2002, p. 123). These students, according to Boaler (2002), did not hold the view that algorithms were useful tools for solving different mathematical problems but were rather abstract entities, used to answer textbook questions. When interviewed, these students reported that they often invented their own methods in real life situations to try and work problems out and seldom used taught strategies at school (Boaler, 2002).

Looking at a classrooms transition from a more traditional mathematics programme towards more inquiry based mathematics, Hunter and Anthony (2011) describe how students’ mathematical dispositions and perceptions of mathematics changed over the course of the year. Initially, the teacher taught content in a procedural manner followed by questioning of teacher selected students about the strategy’s steps. These responses were in turn evaluated by the teacher with no discussion around other ways of reasoning or other mathematical content (Hunter & Anthony, 2011). Interestingly, although some students stated that they liked mathematics, they could not describe the reasons for this, or expressed more ambivalent views about why they liked it. Some examples of not liking mathematics seemed to hinge on students perceived inability to make sense of what the teacher had explained; an underlying theme that these studies share. Learning mathematics, here in the initial setting of this classroom, was all about being a good listener of the teacher. The teacher's role according to these students, was to tell them what to do when they got stuck, and showing them a range of different strategies that they could then be questioned on (Hunter & Anthony, 2011).

These studies, taking a snapshot of student dispositions and views towards the nature of mathematics in more traditional based programmes, gives an understanding of just how influential classroom practice can be. Sullivan et al. (2013) believes that if we are to foster positive mathematical dispositions in our students then teaching needs to reflect the real nature of mathematics; the goal of many reform and social constructivism classrooms.

2.3.2 Mathematical freedom

With a rapidly changing world and job market, it is impossible to know which mathematical methods will be most helpful in the future. While previously, the world seemed to respect people who could calculate quickly, it has become a job which computers and machines were built for. It is therefore no longer enough for students to just perform algorithms and recall procedures (Grootenboer & Marshman, 2016). It is instead, those who think deeply, creatively, and can use their knowledge in unfamiliar situations, that go on to do amazing things with mathematics (Boaler, 2016; Sullivan et al., 2013). More than ever, we need students who can reason their ideas, be flexible with numbers, and have dispositions that will allow them to persevere through challenges (Boaler, 2010).

Through the lens of social constructivism, classrooms can support the development of these positive mathematical dispositions through problem solving, reflection, acknowledgement of different strategies, and collaboration towards a shared understanding (Cobb, 1994). Positive mathematical dispositions do not often form through the teaching of isolated facts and routines, which Boaler (2016) compares to pieces of a dismantled bike. They instead grow from the exploration of interconnecting ideas, much like that of an assembled bike where all the pieces work together (Department for Education and Child Development, 2017; Sullivan et al., 2013).

In a study by Franke and Carey (1997), 36 first grade students' perceptions of mathematics were examined after being taught in reform classrooms. Within these classrooms, students had opportunities to solve a variety of contextualised problems with the understanding that they could be solved in a number of ways. An important aspect of the classroom culture here was that the students were encouraged to share their own thinking and reasoning, rather than just the answers (Franke & Carey, 1997).

What was evident from the students' responses in this study was the lack of attention to the traditional views of mathematics such as getting the correct answer, being quick, and doing bookwork (Franke & Carey, 1997). This was shown with only nine students (25%) believing correct answers or speed and accuracy were determining factors in one's success. It was further emphasised with over half of the students reporting about successful students being able to solve problems and share their strategies to their peers and teacher.

While not the only determining factor, the dispositions of these students were largely influenced by their teacher's shared belief that communication was a central part of solving the problems. According to Franke and Carey (1997), by the end of the study it was not only the teacher that shared these positive views of mathematics, with note that "the children were resourceful in their problem-solving approaches, recognized and accepted a variety of solutions, and assumed a shared responsibility with the teacher for their mathematics learning" (p. 8).

Interestingly, when questioned, the students could also go beyond describing the strategies they use in the problem and used vocabulary that showed understanding of what responsibilities they had as learners and how they perceived the tasks (Franke & Carey, 1997). The study holds a unique position; examining students emerging mathematical dispositions at the beginning of their schooling and gives us an understanding of what students' views can be when given these conditions early on.

Focussing on a similar classroom environment, a case study by Hunter and Anthony (2011) goes into depth on how students' dispositions and perceptions of mathematics can be developed through intervention and the transforming of classroom culture in a New Zealand school. Through the use of small groups, students worked collaboratively to solve contextualised problems, constructing different ways to represent their thinking and with an expectation that they present to a larger group after shared understanding was reached (Hunter & Anthony, 2011). Students in this study came to see mathematics as being more enjoyable and noted a change in their relationships with mathematics as the classroom moved more towards becoming more inquiry based. No longer did the students contribute learning solely to the teacher but rather acknowledged the contributions of their classmates and themselves. This, according to Hunter and Anthony (2011) was attributed to being "a part of a community where learning mathematics was an active process that involved them engaging with their own reasoning and the reasoning of others" (p. 109). This idea of community is echoed by Boaler (2016) who states that mathematics is a human activity, a social phenomenon, and one that is part of our culture. In requiring students to be creative, reason, connect ideas and use multiple methods, like the students studied in this classroom, mathematics is also being taught closely to how it is found in the real world (Boaler, 2010).

Similar to the themes found within Hunter and Anthony (2011), Boaler (2002) describes the mathematical dispositions of students at Phoenix Park. These students, Years 8 to 10, worked on open-ended mathematics projects in every lesson. During this time, students worked collaboratively and in mixed ability groups. Findings from this study indicated how confident students were in using mathematics in new, and unfamiliar situations, using what they had and applying it in new ways. This ability to adapt their thinking and be flexible with numbers indicates that they had also learned mathematics in a way that bridges the gap that generally exists between the classroom and real world situations (Boaler, 2002).

Similarities exist between the students within these studies, both in the way they were taught, and the dispositions that they formed. They all had opportunities to solve real world problem, make conjectures, explore and refine ideas, work collaboratively, and struggle. These classrooms and teaching practices allow opportunities for students to learn in the most productive way, with freedom. More importantly, these students have the best chance to enjoy mathematics for what it truly is (Gresalfi, 2009; Sullivan et al., 2013).

2.4 Growth Mindset

Despite the best intentions of many parents, educators, and researchers, there still exists a widespread belief that you either have a 'math brain' or you do not. This fixed mindset, which permeates throughout society, can be strongly linked to underachievement in mathematics, limiting students' potential to learn before they even begin (Boaler, Chen, Williams, & Cordero, 2016). According to Ernest (2006), the idea that people who can do mathematics are simply clever and learning mathematics is a question more of ability than effort is a myth, and one that causes mathematics anxiety in adults and students alike. While many adults accept their lack of accomplishment in mathematics as being down to their inherent mathematical ability which they have little control over, new research has demonstrated this does not have to be the case (Anderson, Boaler, & Dieckmann, 2018).

A range of studies in neuroscience (Dweck, 2012; Moser, Schroder, Heeter, Moran, & Lee, 2011) have recently shown that like all other subjects, mathematics is learned through hard work and practice. These studies have also shown the damage a fixed mindset has on students' potential to create new brain pathways and how this limits new mathematics learning (Boaler, 2013). Dweck (2012) discusses how these fixed mindsets often develop in students who have had their work praised from an early age, being described openly as smart or clever. While this praise often comes from well-meaning teachers and parents, it fails to attribute success to anything tangible such as hard work or struggle. The problem with such praise, according to Boaler (2013) is that “as soon as students fail at a task they infer that they are not smart after all. The damage of fixed ability thinking harms all students; it is communicated through the practice of ability grouping, even when the idea that is communicated is that members of the group are smart” (p. 147). It is these students that feel they need to maintain a level of smartness, tending to avoid challenge, and instead choosing easier tasks which they know they will succeed at.

Students however, who hold a growth mindset, see mathematics ability as malleable, able to be improved through hard work and practice. Dweck (2012) explains how these students often seek to understand mathematical content and are far more willing to take risks, make mistakes, and struggle compared to those who were focused on performance. The importance of this is emphasised by Boaler (2019) who states that when the brain is put under load, that being struggling with a task or trying to understand a misconception, one of three things happen. Either connections will be formed between the brain’s synapses, brand new pathways will be made, or existing pathways will be strengthened. These connections form a deeper understanding of mathematical concepts and better prepare students to make sense of incoming information (Dweck, 2012; Granberg, 2016).

Anderson et al. (2018) writes how students who were given growth mindset intervention earned higher grade averages and reported greater engagement and enjoyment of mathematics. It was also noted the importance of the teacher during these interventions as their actions and own beliefs about growth mindset weighed heavily on students’ attitudes. Many teachers during this study talked about the importance of having a growth mindset with students but yet did not change their teaching practice to reflect these views. This, according to Anderson et al. (2018), creates frustration in students as they receive conflicting messages. Boaler (2013) outlines the many ways teachers can convey subtle suggestions about what it means to do mathematics and whether students’ ability is fixed or changeable. These suggestions are made through the tasks that students engage in, the grouping of students, the questions they ask, and the norms they establish in the classroom. It is important then that teachers set high expectations of students, sharing the idea that with a growth mindset, their potential is limited only by what they think they can do (Boaler, 2019).

2.5 Mistakes and their place in mathematics

Already alluded to is the idea of mistakes being beneficial for brain development and growth. For those with a fixed mindset who believe that intelligence is stable, mistakes indicate lack of ability (Moser et al., 2011). However, studies such as Moser et al. (2011) have found some comparisons in the neural connections between people who hold a growth mindset and those with a fixed mindset. By examining event-related potentials, which probe neural mechanisms, Moser et al. (2011) were able to study peoples' reactions to mistakes that they made. During their study they found a greater awareness of mistakes for people who held a growth mindset compared to a fixed mindset. Interestingly, those with a growth mindset also possessed the ability to recover from these mistakes, correcting them and learning from them. Dweck (2012) believes that this research has massive implications for teaching and learning. It tells us that the ideas we have about learning, our own ability, and mistakes, especially when we approach challenges, can change the workings of our brain (Boaler, 2013).

If we are in fact wanting to take advantage of the development to learning that mistakes provide, then students should be engaging in challenging work that results in mistakes, rather than producing pages and pages of correct answers (Antlová, Chudý, & Peng, 2016; Boaler, 2013; Kapur, 2010). According to Granberg (2016), it is when students are making, discovering and correcting mistakes, that they are engaged in 'productive struggle'. This productive struggle, in turn, helps restructure the connections in the brain in more useful, and powerful ways, helping assimilate new information, ideas and facts (Granberg, 2016; Kapur, 2010).

So why do authentic mistakes occur in the first place? Granberg (2016) describes how when students' prior knowledge is insufficient to understand new information or assimilate it, students attempt to construct their own interpretation that connects with their current understanding. These misconceptions or mistakes, according to Schleppenbach, Flevares, Sims, and Perry (2007), are more incomplete as opposed to incorrect, while DeBrincat (2015) describes them as an internal struggle between what we believe and the external reality. This also poses the argument for some, that being wrong is a challenge to who we are because we link our knowledge to our self-awareness (DeBrincat, 2015). For those who hold a fixed mindset, mistakes are indeed personal, a reflection of their own preparation and intelligence. The importance of building growth mindsets in students and developing a classroom culture that values mistakes is therefore unquestionable, especially if we want students who are confident and able to persevere through challenges.

Despite the value of mistakes, they are often a source of frustration for teachers and students in mathematics, and are countered by teachers providing students with instructions on how to fix them (Granberg, 2016; Kapur, 2010; Lischka, Gerstenschlager, Stephens, Barlow, & Strayer, 2018). In doing this, teachers take away the ability for students to become productive strugglers (Granberg, 2016). In using mistakes as springboards for inquiry, teachers can instead engage students in discussions around their own, and others thinking. This requires teachers to provide a safe environment where they can explore their mistakes without judgement (Schleppenbach et al., 2007).

The issue of time is one that is often cited by teachers, with the time needed to unpack all misconceptions being impractical in a mathematics lesson. Lischka et al. (2018) poses that although all mistakes should be inspected by the student making the mistake, not all mistakes are worthy of discussion by the whole class. Both Lischka et al. (2018) and Willingham, Strayer, Barlow, and Lischk (2018) outline several considerations teachers need to make when deciding what mistakes are inspection worthy. This includes whether the mistake will help with the class's understanding of the mathematical concept, whether the mistake is representative of a large group in the class, or whether it is simply a fundamental misconception of the mathematical concept being taught. No matter what the reason for addressing the mistake, it is important that these rich mathematical conversations occur, as one student's mistake could lead to another student's clarity (DeBrincat, 2015; Schleppenbach et al., 2007).

2.6 School factors

For most, school is where mathematics is first encountered in a formal setting. The nature of this environment therefore has a strong influence on how students come to view mathematics, what it means to do mathematics, and its place within the world (Franke & Carey, 1997; Taylor et al., 2005). Students do not simply just learn new concepts and facts in classrooms; they learn to how to be a mathematics learner and define their identities with mathematics. What this looks like will largely depend on a number of factors. The following sections expands on these specific teaching and classroom factors that influence students' mathematical dispositions and mindsets.

2.6.1 The role of the Teacher

The first factor is the role that the teacher plays in developing students' mathematical dispositions. According to Boaler (2003), the common perception that to teach well, teachers simply need to know a lot, is incorrect. "Teaching is not a knowledge base, it is an action, and teacher knowledge is only useful to the extent that it interacts productively with all of the different variables in teaching" (Boaler, 2003, p. 12). Teacher's instructional practice however, is also largely connected to their own beliefs, views, and attitudes around mathematics and education (Ernest, 2006; Gagatsis & Kyriakides, 2010). For teachers who hold compatible attitudes to that of the curriculum, implementation will be a lot more effective.

As already discussed, the teacher's views and handling of mistakes in the classroom can send clear messages about their value in learning mathematics. Boaler (2013) shares how teachers can reposition mistakes in the classroom by simply not crossing 'wrong answers' but rather with a comment or gold star. DeBrincat (2015) identifies how teachers in many classrooms can shift the focus away from the makers of mistakes, to the mistakes themselves and their solutions. In doing this, time is not wasted tiptoeing around mistakes and the embarrassment of them, but rather places value in them and student's risk taking (DeBrincat, 2015). This risk taking however, requires that classroom norms are established.

Classroom norms are the shared expectations of a classroom and are concerned with how students interact with each other and engage with mathematical tasks. These norms can be separate from the practices institutionalised by wider society according to Cobb (1994) and are negotiated between the teacher and students. Hunter and Anthony (2011) demonstrate how "classroom norms which hold all students accountable for sense making and the sense making of others during mathematical activity are pivotal factors in the strong sense of competence established by community members" (p. 113). This accountability can be achieved through teachers holding high expectations of their students, encouraging reflection, and establishing participative norms (Mariva & Rinante, 2019; Taylor et al., 2005).

Teachers' can deflect responsibility for understanding concepts back on to students, developing tools for students to overcome problems and giving them time to productively struggle (Kapur, 2015). An example of this is found in Boaler (2003) where students asked "is this correct?" Where the teacher could have replied with yes or no, she posed the questions "have you tried it with other numbers?" or "can you draw it in a diagram?" Not only does this go beyond simply asking what the student thinks, it gives students the opportunity to reason their ideas and check for themselves (Boaler, 2003; Dossey, 1992). Education Review Office (2018) also identified teachers encouraging growth mindsets by avoiding immediately offering solutions to problems and instead reminding them to listen carefully to other group member's contributions. This was also accompanied by students helping others understand their solutions and ideas. The teachers here are able highlight the norm of being a community of learners, all being responsible for the learning of one another (Education Review Office, 2018).

Ensuring that all students have opportunity to engage in mathematical discourse and participate in group discussions can, at times, require careful and subtle intervention from the teacher (Sullivan et al., 2013). When disparities in status between students exist, or if articulate students dominate discussions with their ideas and strategies, teachers may need to elevate the status of students who are seen to be low achievers by their peers (Hunter & Anthony, 2011). Calling attention to these low status students' ideas and suggestions can assign competence to them as a mathematical thinker and group participant.

The teachers' overall goal in mathematics can be likened to a conductor, orchestrating productive classroom discourse (Kooloos, Oolbekkink-Marchand, Kaenders, & Heckman, 2019). It requires knowledge of the students, consideration of the task, and use of talk moves such as re-voicing to steer discussion towards big mathematical ideas. Although these practices and norms may take time to establish within a classroom, they allow teachers to instil positive views and dispositions of mathematics in students (Diachuk, 2019; Sullivan et al., 2013).

2.6.2 Task design

"Knowing mathematics is equated with doing mathematics" (Dossey, 1992, p. 44). With this, the activity or task that students participate in can have a profound effect on what 'doing' mathematics means to them. It is important then that we provide tasks which promote positive mathematical dispositions and allow students to enjoy mathematics for what it is (Boaler, 2003). Copying methods into books and answering near identical questions quickly purveys the idea that to be successful in mathematics, you need to simply watch the teacher and copy what they do. This also raises the concern of 'how students cope when they are away from the source of authority, in this case, the teacher and text books (Boaler, 2010). This reliance on teacher knowledge leaves little room for development of mathematical dispositions as students become passive receivers of knowledge rather than users of knowledge.

Closed tasks, which usually require repetition of standard procedures, share little in common with what actual mathematicians do in their job, and are unique to classroom settings according to Sullivan et al. (2013). Even if students seem to understand how to use the procedure in repeated questions, Boaler (2010) argues that the neural pathways these tasks create in the students' minds are like pathways in sand, easy to wash away. Instead, allowing students to encounter mathematics in multiple ways such as through games, building, discussion, words, pictures, and graphs, helps build stronger neural pathways that are easily connected to other experiences and are far more accessible (Boaler et al., 2016).

A mathematical task, according to Kooloos et al. (2019), can be "regarded as a problem if students do not have easy access to a solution method" (p. 4). These problems need to ideally be accessible to all students, with a high ceiling, where students can be extended, and a low floor in which all students can engage. This not only increases the success students can experience but send messages that mathematics is an open and growth subject (Anderson et al., 2018; Sullivan et al., 2013). While there may be some methods that are more efficient for some problems, allowing students to generate their own ideas first and then connecting to them with methods is a far more beneficial exercise then prescribing a strategy to them. Franke and Carey (1997) highlights that open-ended problems also allow more insights into students' perceptions about mathematics as they are required to talk about their thinking with teachers. This provides a much clearer picture of student understanding than any textbook answer could provide.

Ensuring learning has meaning to students can often be tricky for teachers. Therefore, the role of context in learning experiences, according to Boaler (2003), is a major one. "If the students' cultural and social values are valued in the mathematics classroom, through the use of appropriate contexts, then their learning will have more meaning for them" (Maxwell, 2001, p. 5). Contextualising problems also has the benefit of giving students the opportunities to be the experts (Cobb, Boufi, McClain, & Whitenack, 1997). According to Lotan (2003), "by assigning such tasks, teachers delegate intellectual authority to their students and make their life experiences, opinions, and points of view legitimate components of the content to be learned" (p. 72). In knowing the students' prior knowledge, experiences, and cultures, teachers are able to get a better sense of what contexts would work for students which emphasises the need for building relationships (Taylor & Cowie, 2006).

Finally, in the construction of tasks, is the opportunity to include visual components. Boaler et al. (2016) discuss how when content is taught visually, it reduces the common issue of problems being too 'hard' or too 'easy'. Boaler et al. (2016) also believes that the status differences that often exist between students seem to disappear when content is taught in this way. Unfortunately however, "students who display a preference for visual thinking are often labelled as having special educational needs in schools, and many young children hide their counting on fingers, as they have been led to believe that finger counting is babyish or just wrong" (Boaler et al., 2016, p. 1). In celebrating students' visual approaches when formulating ideas and sharing their thinking, teachers can share the idea that mathematics is not all about memorisation (Dossey, 1992).

2.6.3 Grouping

Recent findings from the Programme for International Student Assessment (PISA), which compares student achievement and mathematical understanding between students from around the world, showed that New Zealand had one of the highest rates of ability grouping in mathematics (Ministry of Education, 2017). This ability grouping, which is widely supported by teachers and has been a long held practice in New Zealand, refers to students being grouped with other students of a similar 'level' (Hunter, Hunter, & Anthony, 2020). However, this form of grouping is cautioned against by Education Review Office (2018), who believe there is little evidence that it is effective in enhancing student learning.

Many teachers, according to Blatchford, Baines, Kutnick, and Martin (2001), group by notions of ability because they think they can set more targeted work for students. This work is often cited as being matched to students' ability and as way of catering to the wide range of abilities that exist between students in a classroom (Hunter et al., 2020). While this comes with pressure of set standards and a need to raise the achievement of all children, evidence suggests that many students perceived as low ability and in low groups, find the work they are given as often being too easy (Blatchford et al., 2001).

Issues around the use of ability grouping stem from the lack of opportunity that are provided to lower ability groups (Education Review Office, 2018; Hunter et al., 2020) and the mindset that being in a lower or higher group develops in students (Boaler, 2013). Even students who are placed in higher ability groups are disadvantaged by the expectations placed on them to succeed (Hunter et al., 2020). Ability grouping students at a young age makes it difficult for them to move to a higher level as they are separated from other's that could stimulate their thinking, are given less challenging work, do not experience the full curriculum, and are given the perception that they are not as able as the students in higher groups (Boaler, 2010; Education Review Office, 2018). Even with the use of names for the different groups such as the triangles and squares, students are quickly able to identify the group hierarchy and perceived ability (Boaler, 2013).

In no other subject is streaming so prevalent than with mathematics. In abandoning this practice and using mixed ability grouping, teachers open up a range of opportunities for students and authentic contexts for tasks (Sullivan et al., 2013). With the need to hear and explain ideas being important for mathematical understanding, situating students in mixed ability groups allows struggling learners to hear higher order thinking. This also extends 'higher level' students with the expectation that they need to explain their ideas so all members in their group understand. This causes students to think about their strategies in different ways and deepen their understanding of strategies (Boaler, 2013; Diachuk, 2019; Hunter & Anthony, 2014). Education Review Office (2018) discusses how many students who had previously been considered as lower ability students and in the bottom groups could now share their confidence and enjoyment of mathematics after working within flexible, mixed ability grouping. This notion is continued by Hunter et al. (2020) who shares how teachers perceptions of students and their capabilities in mathematics changed when students worked in mixed ability grouping. The development of social skills that emerged through mixed ability grouping and collaboration tasks also meant that students constructed stronger and more positive mathematical dispositions.

The use of small mixed ability groups is one that helps students take risks, share their ideas, ask questions, and participate in mathematical discussion, all without being in the public view (Hunter & Anthony, 2014). Students in a study by Hunter and Anthony (2011) indicated that small problem solving groups provided a safe setting for them, allowing them to be more comfortable when constructing and trialling their explanations. This was accompanied by an increase in mathematical conversation that occurred in these groups and students finding value of being a part of a community of learners. Students' expressed ideas around being able to help others when they are confused and being able to ask questions if they did not understand what someone was explaining. All these practices allow for clarification of students' understandings and builds a mutual responsibility for learning (Hunter & Anthony, 2014).

2.6.4 Assessment and speed

More than any other subject, mathematics has a culture of regular testing (Boaler, n.d.). While teachers have come to view mathematics as a performance subject, many see testing as just a part of doing mathematics and do not consider the impact these tests have on students' views, especially for those who are slow, deep thinkers (Boaler, 2014). While gathering information about students' understanding and development is essential for teaching and learning, many testing practices that exist do little more than incite mathematics anxiety in students (Amador & Lamberg, 2013; Diachuk, 2019).

Assessment practices centred around speed and rapid recall, according to Boaler (n.d.) have had a lasting, negative impact on student dispositions for the past decade with evidence suggesting that it affects students right across the achievement range. When students become stressed, such as during answering mathematics questions under time pressure, the working memory part of the brain becomes blocked, limiting students' access to memorised facts (Beilock & O'Callaghan, 2011). Interestingly, for those students who are not reliant on memorisation and have a deeper understanding of numbers, pressure did not seem to have as much impact on their ability to solve problems (Beilock & O'Callaghan, 2011; Grootenboer & Marshman, 2016). Despite this, it is important that we consider why we are testing students and ways in which we can gain the greatest understanding of their learning without undoing the positive dispositions we are trying to instil in our students (Boaler, n.d.).

Formative assessment, which informs teachers of students' progress and can help determine students' next steps is useful for teachers and can be done through multiple means, including observation. Summative assessment on the other hand is used to summarise where students learning has 'ended up' and gives an overall level at the end of learning (Boaler, n.d.). An issue is raised however, with many teachers' using summative assessment formatively. This practice sees students given a grade or score whilst learning is still taking place and material is still being taught. As already discussed, this often leads to students being grouped by ability and in turn, gives a comparison between students abilities that are visible to the students themselves (Boaler, n.d.).

To judge students understanding of mathematics through narrow, procedure based, multichoice questions, disregards a lot of the mathematical practices that we want to be looking for in students. In assessing for learning, students receive diagnostic feedback on specific ideas for them to try, which is probably the greatest gift teachers can give students according to Boaler, Dance, and Woodbury (2018). This feedback may take longer for teachers than ticks and crosses but does not need to be done as frequently and lasts longer. In a study Pulfrey, Buchs, and Butera (2011), summative grades that were normally given to students were replaced with diagnostic feedback. Results showed significant achievement increases, especially with the top twenty five percent and bottom twenty five percent of students. Elawar and Corno (1985) saw 18 primary school teachers in Venezuela given professional development to provide diagnostic feedback on Year 6 students mathematics homework. Results also showed significant improvements in both achievement and attitude towards mathematics compared to those who received regular grades. The advantages of providing such feedback is emphasised by Boaler (n.d.) who discusses Finland's high scoring in international mathematics tests and their teachers' rich feedback given to students which is gained through formative practices. This, according to Boaler (n.d.) was because students are "taught to believe in their own capabilities; they had been given helpful, diagnostic information on their learning; and they had learned that they could solve any question, as they were mathematical problem solvers" (p. 2).

While for some countries or schools, testing is compulsory, there are still several things teachers can do to help promote a growth mindset and positive mathematical dispositions. Education Review Office (2018) encourage use of formative assessment where possible so students understand what they know, what they don't know, and the path between the two. Boaler (n.d.) promotes sharing grades with school administrators if needed but not with the students. Providing feedback on mathematical practices, not just conceptual learning, is also a good way to promote the mathematical practices that are so vital to learning with positive dispositions.

2.7 Importance of home

As learning and association with mathematics does not stop the moment students leave school, the importance of home and the interactions students have with family cannot be ignored. Of importance to this study is Rokeach (1968b) proposition of how beliefs are formed. While primary beliefs are developed through personal experience such as being engaged in a mathematical task or discussion, derived beliefs develop indirectly from the beliefs of people of importance in the students' life such as parents and other family members (Rokeach, 1968a). Research into the influence of parent attitudes in relation to student motivation shows increased mathematics anxiety and limited resilience in students of parents who hold negative attitudes towards mathematics (Department for Education and Child Development, 2017; McLeod & Adams, 1989). These attitudes and views that parents hold influence students through the ways they teach their children and can often cause conflicting messages between school and home (Maxwell, 2001). Department for Education and Child Development (2017) notes however, that if parent fears around mathematics can be reduced, and if positive experiences of mathematics are shared between parents and students, then home can support student learning at school.

The Education Review Office (2018) gives examples of parents including their children in their day to day decisions, making links between their work, home and mathematics. Sharing experiences such shopping, games and puzzles, measuring while baking, and working out how long it will take to get to different destinations, shares the idea that mathematics is everywhere and in all we do (Boaler, 2019). Parents can also share the same positive messages about mathematics that teachers can by encouraging students to persevere with challenges, supporting them but not giving them the answer, and not associating mathematics with speed.

2.8 Summary

Fostering positive mathematical dispositions in students is not only important for their learning of mathematics but in the way they interact with it throughout their lives. Through the lens of social constructivism, the learning and development of mathematical dispositions comes through both the internal workings of the mind, and the social interactions that they have with their teacher, family, and peers. In sharing messages that mathematics is a growth subject which you can improve through hard work and perseverance, students come to see and use mistakes as being a valued part of learning.

Teachers play a large part in the development of students' mathematical dispositions; conveying their own beliefs through their actions, assigning competence to students, positioning them to participate in mathematical discourse, and designing group worthy tasks which have multiple entry points and allow students to productively struggle. While traditional grouping methods can often give ideas of ability and status differences between students, mixed ability grouping can provide opportunities for students to work collaboratively, ask questions of others, extend their own thinking, and support one another. Similarly, assessment can be conducted in a variety of ways that share different ideas of what it means to do mathematics. In assessment methods that focus on speed and accuracy, students often conclude that mathematics is all about memorisation and recall. In using formative assessment to provide diagnostic feedback, students understand what they know and what they need to improve upon to achieve their goals.

Finally, in examining the different factors, is that of home, which plays a vital role in the formation of mathematical dispositions. Including students in day to day activities that include mathematics such as baking, shopping, games, and puzzles, they come to see mathematics as a relevant, valued part of their lives. The complexity of mathematics education cannot be overlooked with such a variety of factors effecting mathematical disposition. The aim of this study is therefore to gain students' perspectives about mathematics to understand how their mathematical dispositions and mindsets are influenced by the different factors found at school and home.

Chapter Three: Research Design and Methodology

3.1 Introduction

Previously highlighted is the importance of developing positive mathematical dispositions in students and the factors that play a role in forming these dispositions. Within the studies reviewed, many describe types of classroom environments that are conducive to the formation of positive mathematical dispositions but are limited in describing how each factor such as the teacher, task, grouping, and assessment influence these dispositions. This following chapter sets out to develop a research design and methodology to collect and analyse data on students' mathematical dispositions and mindsets, how the different factors influence these dispositions, and finally, students' views and reactions to mistakes within mathematics.

Section 3.2 begins with the justification of utilising a qualitative case study methodology, giving an overview of qualitative research, its role within mathematics education, and the features of case studies which apply to the current research. Section 3.3 outlines the role of the researcher in qualitative research as well as the advantages and considerations to having an insider role within the school where the study took place. The setting, sample and schedule are established in section 3.4. In section 3.5, the data collection tools utilised are described, with an overview of how the data will be analysed in Section 3.6. Section 3.7 is concerned with how this research will maintain reliability and validity while Section 3.8 looks at the ethical considerations that need to be made throughout the study.

3.2 Justification of methodology

Given the complexity of mathematics education and the many factors that are present in the formation of mathematical dispositions, a research design and methodology that accounts for these complexities is needed. The following section outlines the justification for using a qualitative case study approach for this study.

Often, qualitative research is defined by comparing it to quantitative research. Leavy (2014) however believes that qualitative research needs to be understood for its own merits and traditions rather than through describing what it is not. According to Punch and Oancea (2014) “qualitative research is, by and large, naturalistic, preferring to study people, things, and events in their natural settings.” (p. 146). Being able to study people in normal situations is not only reflective of everyday life, but allows researchers to gain a deeper understanding of how and why people behave and respond the way they do given their situations (Miles, Huberman, & Saldaña, 2014).

Bryman (1988); Creswell (2013) believe that often, qualitative research hinges on the social constructivism view that in interacting with the world and through their previous experiences, students construct their own reality and interpret the world in different ways. This orientation, in which this study is positioned, assumes that there is no single reality available to observe but rather multiple realities that need to be interpreted (Merriam & Tisdell, 2016).

With its goal of understanding how people make sense of their experiences and lives, qualitative research does so by gathering rich, descriptive data from the point of view of the participants and from within their natural environment (Merriam, 1998; Merriam & Tisdell, 2016). With mathematics education being increasingly recognised for its socially situated context, qualitative research has become the predominant research paradigm in mathematics education in the past decade (Ernest, 1997; Hunter, 2002). Given then, that this study is examining how different factors effect students’ mathematical dispositions and how they respond to mistakes within mathematics, the use of a qualitative approach is well suited.

Although all qualitative research designs share the goal of searching for meaning and understanding, different forms of qualitative research have additional dimensions (Merriam & Tisdell, 2016). Several different forms of qualitative study were considered for this research such as phenomenology, which seeks to understand the underlying essence of a phenomenon, and narrative analysis, which uses peoples stories to understand how they make meaning of their experiences; ultimately a case study approach was selected.

As the name implies, case studies seek a deep understanding of a particular case or cases. Merriam and Tisdell (2016) believe case studies to be more a choice of 'what' is to be studied rather than a methodological choice. They go further to define the 'what' as being a bounded system. This could be an individual, role, event, institution, or community if there is a limit or boundedness to the number of people involved or finite time for observations (Barth & Thomas, 2012). Simply put, if there is no end to the number of people who could be interviewed or observations that could occur then the phenomenon cannot be bounded enough to be considered a case (Merriam & Tisdell, 2016). In this study, the case consists of three Year 5 and 6 classrooms at a single school.

Another reason for the selection of a case study design is its ability to study a phenomenon holistically in authentic, natural settings and with the use of multiple data collection tools (Merriam, 1998). It is in these natural settings that the researcher becomes the primary instrument of data collection and analysis, being flexible and responsive in their approach. This is ideal for an education setting where arising data can be expanded upon and explored further (Punch & Oancea, 2014). During this study, interesting misconceptions and consequential learning were able to be analysed further which may have otherwise gone untapped in other forms of research. Yin (2009) does however caution that although researchers can be flexible, in order to gain rich, descriptive data, they need to be aware of their position and have a clear criterion for interpreting their findings.

3.3 Role of the researcher

The role of the researcher in qualitative research is a unique one. While being responsible for the design of the study, the researcher is also the primary instrument for gathering and interpreting data (Merriam, 1998). The researcher's role is not fixed however, needing to adapt and shift throughout the lifecycle of the research and utilise a variety of data collection methods.

Throughout the study, the researcher's role is to ensure that validity and reliability is maintained. Yin (2009) identifies how being clear on the reasoning for the research being conducted, developing research questions that guide the study, and engaging with available literature throughout the research process is essential. Punch and Oancea (2014) illustrate the importance of researchers being unbiased and being able to conduct the research with as little influence on the environment as possible. This is so the phenomenon can be studied and observed in a way that is as close to natural as possible.

While being adaptable and responsive to data, the researcher can come into the study with biases and a theoretical framework which informs the way they conduct the study. Merriam and Tisdell (2016) believe that it is important to identify these biases that the researcher may have and be transparent about how these may shape their collection and interpretation of data rather than trying to eliminate them altogether. These biases or the positioning that the researcher may have can relate to their beliefs, sense of identity, their personal experiences or even race, gender, and socioeconomic status.

My positioning in this study is influenced by my role as a teacher within the school where the study took place. I had taught at the school for the five years prior to the research taking place, of which the majority have been with Year 5 and 6 students. It was during this time that my interest in students' views towards mathematics and mistakes developed. During the conduction of this research however, I did not have my own classroom due to being on study leave and was instead teaching part time as the school's CRT (Classroom release time) teacher. This had me teach STEM (Science, technology, engineering, and mathematics) throughout the school from New Entrant level right through to Year 8. While this position as an "insider" had many advantages, I had to make careful considerations throughout the study to ensure validity and reliability (Merriam, 1998).

This insider role allowed for prior understanding of Year 5 and 6 student learning trajectories and helped in co-constructing mathematics tasks that elicited misconceptions and mistakes. This co-construction of mathematics tasks helped minimise any assumptions I held about students' prior knowledge and ensured alignment with the classroom's current programme.

Another advantage afforded to being an “insider” is already having a report with the teachers and students (Merriam, 1998). While this allowed for open and natural conversations, I needed to establish a new role as a researcher and kept reflections throughout the data gathering process. This helped minimise any bias and maximise data gathering opportunities (Yin, 2009).

3.4 Setting, Sample, and Schedule

When selecting the setting, sample, and schedule in qualitative research there are important considerations to be made. Qualitative research often consists of non-random, purposeful sampling opposed to the more random sampling strategies of quantitative research (Merriam & Tisdell, 2016). When researching mathematical dispositions, where there exists a wide range of classroom cultures, practices, and students available for analysis, the selection of an appropriate sample is crucial.

The research was conducted at a full primary school (Years 1 to 8) in Northland, New Zealand. This school is situated 10 minutes outside of a main city but is classified as a rural school, with a mixture of farming families as well as students who live in town. The school has a population of 300 students with approximately one-third Maori. Three classrooms were selected to be part of this study with information and consent forms sent home after discussion with the Principal and Teachers of the classes. In total, 41 Year 5 and 6 students (Aged 9 to 11) participated in this study.

The three classrooms involved in this study, although differing slightly in programmes, planned collaboratively and had many similarities. Lessons began with a quick warm up in the form of 5x5 grid (Basic facts), maths game, or revision activity which could involve a class discussion. Ability grouping was used for the teaching of mathematics strategies, with all students working with the teacher at least once a week. These groups were formed from assessment data collected at the beginning of each unit (Number and algebra, measurement, geometry, and statistics). Students who were not working with the teacher rotated through a range of different activities including group maths games, working independently or with a buddy answering closed textbook questions, answering revision questions from the board, or online maths programmes such as Maths Whizz. Some lessons however were conducted using whole class teaching. These lessons normally revolved around strand mathematics such as measurement, geometry, and statistics and would involve practical, hands on activities such as measuring, making shapes and nets, and creating graphs. Basic facts sheets were also sent home each week and testing occurred once a week as part of their home learning programme.

The selection or sampling of this school and classrooms was done for several reasons, with one of them being convenience. Creswell (2013) outlines convenience sampling as being exactly what the name implies, being based on location, availability of sites and participants of convenience. With myself as the researcher teaching at the school part time, it was easily accessible and allowed me to plan data gathering around the classroom's schedules so minimal disruption to classroom programme was made. The benefits of this convenience sampling were also apparent when the schedule had to be adapted with the closure of schools due to covid-19. The selection of the school however was not only made because of its convenience, with Merriam and Tisdell (2016) stating "although some dimension of convenience always figures into sample selection, selection made on this basis alone is not very credible and is likely to produce information-poor rather than information-rich cases" (p. 97).

The key reason for selection of this school was its characteristics that made it a typical sample. Typical samples are a purposeful sampling strategy that is employed when you want to highlight what is normal, or average (Merriam & Tisdell, 2016; Patton, 2005). Students in this school have a mixture of backgrounds, ethnicities, and have had exposure to a range of different teaching practices which are common within New Zealand. In selecting a typical sample, Patton (2005) identifies how samples can also be selected because they are not in any major way atypical or extreme. Following these guidelines, the school chosen has characteristics that will allow for generalisations to be made to some degree for Year 5 and 6 students in New Zealand.

With the setting selected there was still the consideration of who to include in interviews, questionnaires, and self-reflections. The "second-tier" sampling method according to Merriam and Tisdell (2016) needs to include enough participants to ensure saturation of information. In other words, sampling until there is no new information or insights emerging. To recognise data saturation however, data analysis needed to be conducted alongside the data collection. Being inductive and dynamic during data collection also meant that the original sample of six students interviewed was increased to twelve (Four from each classroom). These students were selected with discussion with the teachers to ensure a variety of perceived abilities and backgrounds.

To gain a broader understanding of students' mathematical dispositions, questionnaires were given to all students who returned consent forms. All 3 classrooms participated in the problem solving tasks but only those who returned consent forms and who wished to complete a self-reflection on their mathematical mistakes did so. Below is a summary of the participants.

Table 3.2 Summary of participants

DATA COLLECTION ITEM	YEAR LEVEL	GENDER	ETHNICITIES
Semi-structured Interviews	Year Five: 5 Year Six: 7	Female: 7 Male: 5	NZ European: 9 Maori: 3
		Total: 12	
Questionnaires	Year Five: 19 Year Six: 22	Female: 21 Male: 20	NZ European: 28 Maori: 12 Korean: 1
		Total: 41	
Student self-reflections (Voluntary)		Total: 30	

The data was gathered over a four-week block at the end of term two, 2020. This block, which was originally scheduled for the end of term one, had to be moved due to covid-19 and the consequential closing of schools within New Zealand. In considering a new time to gather data, the choice was made to move it to the end of term two when classroom programmes were well established, there was no assessment, and students were back into routine. Below is a summary of the data collection schedule.

Table 3.2 Summary of data collection schedule

DATE			DATA COLLECTION ITEM/ACTIVITY		
			Wednesday	**Thursday**	**Friday**
TERM TWO	Week 9	10/06/2020	Problem Solving Task One/Student self-reflection (Class One) Questionnaires (Class One)	Problem Solving Task One/Student self-reflection (Class Two) Questionnaires (Class Two)	Problem Solving Task One/Student self-reflection (Class Three) Questionnaires (Class Three)
	Week 10	17/06/2020	Semi-structured interviews (Class One)	Semi-structured interviews (Class Two)	Semi-structured interviews (Class Three)
	Week 11	24/06/2020	Problem Solving Task Two/Student self-reflection (Class One)	Problem Solving Task Two/Student self-reflection (Class Two)	Problem Solving Task Two/Student self-reflection (Class Three)
	Week 12	01/07/2020	Catch up questionnaires	Catch up interviews	

3.5 Data collection

Data, according to Merriam and Tisdell (2016), is nothing more than ordinary bits of information found within the environment studied. This information however, can be collected through multiple data collection methods, which help to ensure validity and gain a deeper understanding of the case (Merriam, 1998). This case study employed questionnaires, semi-structured interviews, and student self-reflections, which were triangulated to build a detailed picture of students' mathematical dispositions and mindsets, the influence of different factors on these, and their views towards mistakes.

3.5.1 Questionnaires

The first data collection method, the questionnaire (Appendix A), was conducted with all 41 students who returned consent forms. These questionnaires aimed to gain data about students' mathematical dispositions and mindsets, the different factors that influence these, and their views towards mistakes.

Questionnaires have the advantage of being standardised, allow for open responses to a range of topics, while also being reliable, cheap, and comparatively straightforward to analyse. This is however, provided that time is taken to develop, pilot, and further refine the questionnaire; thinking about how much strain is put on the respondent and the types of questions used (Cohen, Manion, & Morrison, 2018).

Like interviews, questionnaires capture a glimpse into the thinking of participants. Questionnaires however, allow for a larger amount of participants than can be afforded by interviews within the same timeframe (Cohen et al., 2018). Questionnaires also hold the advantage of being completed at the participants own pace and with little influence from the researcher.

Due to the age of the students that participated in this questionnaire, I was available to help students access the survey through google forms and clarify any questions that they were unsure about. To help minimise the extent to which I needed to intervene, the questionnaire was piloted prior to participants being given access. This was done by giving the questionnaire to two family friends of a similar age to the participants but who were not part of the group used in this research (Hamilton & Corbett-Whittier, 2013). The answers given were examined to see if the data would be suitable for analysis, while feedback was also collected on the format and wording of the questions to ensure clarity and minimise bias.

Both open-ended and scale questions were utilised in this questionnaire. Opening with more general questions allowed participants to share their ideas on the nature of mathematics before more specific questions were posed about their thoughts on different tasks, grouping, assessment, mathematics outside school, and finally mistakes. Using scale questions allowed students to pick a position from strongly disagree through to strongly agree, with neutral being in the middle. These scale questions intend to understand attitudes and opinions towards statements such as '*there should only be one away to solve a maths problem'* and were recorded for analysis as a ordinal score between 1 to 5 (Miles et al., 2014).

Throughout the administering and collecting of the questionnaires, no names were recorded or received so results were kept anonymous, helping invoke more honest answers and guaranteeing confidentiality (Merriam & Tisdell, 2016). Once all questionnaires were complete, the data was transferred into an excel sheet where the questions and answers could be analysed and coded.

While these questionnaires allowed for a large amount of data to be collected, there was little opportunity to expand on any relevant information or interesting answers that arose. It is for this reason that the questionnaires were used in conjunction with other data gathering tools, including semi-structured interviews. This allowed for more rich, descriptive data to triangulate and build upon (Merriam, 1998).

3.5.2 Semi-structured interviews

When attempting to gain an understanding about students' perceptions of mathematics, Taylor et al. (2005) suggest that the students' own voices may be one of the most important sources. Cohen et al. (2018) details how students being the best source of information about themselves does rely on the researcher's ability to enter the students' world and see the situation through their eyes.

Many researchers (Cohen et al., 2018; Leavy, 2014; Merriam & Tisdell, 2016; Punch & Oancea, 2014) note that one of the most common and central instruments for understanding the lives of people is the interview. Conversation between people has, for as long as we know, been the way we learn about others; the way the think, feel, act, and construct reality. This in turn has been refined throughout the past few decades to become what we know as qualitative interviews (Leavy, 2014).

Interviews, according to (Patton, 2005), help researchers find out information that cannot be directly observed. Through having conversations, be it with a purpose, researchers can understand participants thoughts, feelings, and perceptions and enter their world view (Merriam & Tisdell, 2016). Depending on the situation and information required, interviews can take on several forms, each with varying degrees of flexibility.

The use of semi-structured interviews with more open ended questions, according to Merriam and Tisdell (2016), usually provide more accurate data compared to the use of closed questions with no possibility for exploration of ideas. These semi-structured interviews have flexibility in the wording of the questions, allowing for the participant to expand on ideas of interest to the study (Boaler, 2016).

The interview conducted in the current study utilised three different types of questions described by Merriam and Tisdell (2016). These include experience questions (*When you are stuck in mathematics, what do you normally do*), feeding questions (*How do you feel about mathematics*), and finally, hypothetical questions (*Can you describe the perfect maths class to me*). The full range of questions allowed data to be gathered on all the research questions of this study and allow for descriptive analysis of cases.

Like any other data collection method, considerations need to be made so the data collected is valid, reliable, and ethical. The interviews were conducted in a breakout room next to the classrooms of the students. This was chosen as it was a familiar location for students while having little risk of interruption and minimal travel time (Cohen et al., 2018). Questions were piloted prior to participants being interviewed which not only allowed me to practice asking the questions but also helped understand which questions were confusing or led to little usable data. Interviews were kept light-hearted and open with my aim to help create an atmosphere of trust (Merriam, 1998). Interviews were recorded to avoid relying on memory and allow the interviews to flow naturally. All recordings were stored securely until the completion of the data analysis (Bakker, 2018).

3.5.3 Student self-reflections

The final data collection method was the student self-reflections (Appendix E). These were completed after the problem solving tasks (Appendix C and D) and on a voluntary basis (Provided participants had also returned consent forms). The primary aim of these self-reflections was to gather data on students' views and reactions towards mistakes during mathematics.

The tasks themselves were utilised because they elicited common mistakes Year 5 and 6 students have about graphs and decimals. Students were given individual thinking time before discussing and reasoning their ideas with their group, and later sharing their collective understandings with the class. The teacher's role here was to facilitate group discussions and orchestrate groups sharing with the class, unpacking misconceptions any groups or students had.

Student self-reflections were completed directly after the problem solving tasks to minimise the time lapse between making a mistake and reflecting on it. Students were given the opportunity to either voice record their responses to questions or record them on paper. Student responses, both written and voice recorded, were transcribed onto an excel sheet to be coded and analysed. Students were also given the choice in the questions to respond to. This allowed students to answer questions they felt comfortable with, as well as only answer questions that were relevant to their experience with the problem solving task.

3.6 Data analysis

Data analysis is the process that helps make sense of, and gives meaning to, the collected data (Merriam & Tisdell, 2016). Bogdan and Biklen (2006) describe data analysis as organising data, reducing it, coding it, and searching it for patterns and themes. This process, in qualitative research, relies on the researchers own sense making, understandings, experience, and judgement, as there is no statistical test to help identify what is pieces of data are significant (Patton, 2005).

As data was collected in this study, it was organised and prepared for analysis. This included downloading student questionnaire data, transcribing interviews from voice recordings, transcribing student self-reflections from voice recordings and written responses, and finally formatting it all onto excel spreadsheets. Having a shared format for all data sets allowed for straightforward collation and triangulation (Leavy, 2014).

Analysing qualitative data can be a time consuming and often overwhelming task for researchers with the huge amount of data that can quickly amass (Cohen et al., 2018). Analysing data frequently and before all data had been collected, the problem of data overload was minimised. This process of analysing data alongside data collection also allowed for progressive focusing; the selection of key ideas for future investigation (Miles et al., 2014).

At the beginning of data analysis, student responses to the questionnaires and student self-reflections were coded openly, looking at bits of data and deriving tentative codes (Merriam & Tisdell, 2016). This was done by highlighting words or phrases and assigning them a code which was normally a short summary of the idea or even the word itself. Once initial codes were assigned I begun data reduction, in which codes were collapsed or merged into themes and were assigned a colour for easy reference. Cohen et al. (2018) describes this stage of data analysis as distilling data from the complexity of the findings into key points of the phenomenon in question. This did not mean however, that data was disregarded, but rather reclassified and reduced without violating the original idea. Having two columns, one with initial codes and another with themes, gave me a clear picture of how the original data was reduced and categorised into a theme, enabling me to reassign or replace codes if I felt that meaning was lost from its original context.

Coding the semi-structured interviews also started as a very open process, where all data that might be useful was considered, but eventually becoming more deductive towards the end of data analysis and the point of data saturation. It was here that I was looking for more evidence that supported the final set of themes (Merriam & Tisdell, 2016). To help the coding process for all three sources of data, I created analytic notes on the criteria for assigning themes to the data (Punch & Oancea, 2014). Creating these analytic notes from literature helped me clarify my own thinking and ensured data was exhaustive and mutually exclusive; being able to be placed into one theme without needing to be refine the themes (Merriam & Tisdell, 2016).

As Leavy (2014) notes, research findings themselves don’t have any meaning until the researcher reflects on, and makes sense of, them. Reflecting on the data using literature helped formulate a discussion of the themes and answer the research questions. Student questionnaire and student self-reflection data was used to analyse student dispositions and mindsets, the factors that influence these, and student reactions and views on mistakes in mathematics. Semi-structured interview data was used to focus in on 3 student cases and delve into their mathematical experiences; exploring how they came to view mathematics in the way that they did.

3.7 Validity and reliability

As qualitative research is based on assumptions about reality, the rigor of the research relies on the researcher themselves, their interaction with the participants, and their interpretation of the data. Although Creswell (2013) believe that qualitative researchers can never truly capture an objective reality, there are a number of strategies that can be employed to help increase the credibility of the findings.

Internal validity concerns itself with the question of how well the research findings match reality. Ensuring the research has internal validity allows the reader to be confident that the results are true for the participants and understand how the researcher came to such conclusions. As previously discussed, and illustrated by Merriam and Tisdell (2016), is the importance of firstly being explicit about the researcher's role and their relationship with the those being studied. Flyvbjerg (2006) holds the view that if the researcher is clear about their intentions and background, then case studies can actually lean towards falsifying the researchers' preconceived notions and contain less bias than other research types.

Another common strategy that researchers can utilise to ensure internal validity is member checks. This process involves providing participants with preliminary analysis of interviews they were part of or responses from questionnaires and solicit feedback. While this helps minimise misinterpretation of what participants said or wrote, and aims to better capture participants perspectives, it can be problematic (Cohen et al., 2018; Hamilton & Corbett-Whittier, 2013; Merriam & Tisdell, 2016).

Allowing participants to correct or amend data opens up the possibility for them to withdraw comments due to subsequent events, feel embarrassed or nervous about what they said, as well as being reliant on their memory of the interview or questionnaire. Because of the age of the participants and the process of creating a thesis, member checks were not used and the analysis was instead peer reviewed (Hamilton & Corbett-Whittier, 2013; Merriam & Tisdell, 2016).

Triangulation of data was utilised to help ensure validity and reliability. This process involves comparing different data sets gathered by multiple means and can help overcome the biases or boundedness of some data collection methods (Merriam, 1998). In this study, questionnaires, semi-structured interviews, and student self-reflection data were all considered individually before being analysed and discussed together. This helped build a more reliable picture of student mathematical dispositions and mindsets and the influences on them (Patton, 2005).

Whereas internal validity is concerned with how well findings match reality, external validity refers to how well results can be generalised to other populations or settings. While not all qualitative researchers are concerned with generalisability and solely seek to understand a case in its entirity, Merriam and Tisdell (2016); Punch and Oancea (2014) believe that every case can, theoretically, be an example of something else. The goal of qualitative research is to explore the richness of the case and provide, what Ernest (1997) refers to as, thick desciption. Through providing thick description, which could be in the form of destailed descriptions and evidence such as quotes, the reader can understand the case and assess the similarities between the study and other situations.

Based on the premise that there is only one reality, which can be replicated if studied under the same conditions, many argue that qualitative research lacks the ability to achieve reliabilty (Merriam, 1998; Yin, 2009). While reliability may be problematic for qualitative research due to human behaviour being dynamic and unable to be fully isolated, Lincoln and Guba (1985) suggest that realiability for qualitative research be thought of more as 'dependability' or 'consistancy'. Instead of expecting the same results to be gained from other researchers, it is more a question of whether, given the data collected, the results make sense, are consistant, and dependable (Merriam & Tisdell, 2016). Again, enhancing reliability requires researchers to be explicit in their positioning and assumptions, haing well documented proceedures, and triangulating results (Merriam, 1998).

3.8 Ethical considerations

The need to follow ethical practices and minimise potential harm at all stages of research, especially when conducting research involving children, is essential (Hamilton & Corbett-Whittier, 2013). This study utilised The Massey University Code of Ethical Conduct for Research, Teaching and Evaluations Involving Human Participants as the guideline to ensure ethical issues were identified, considered, and minimised. The primary ethical concerns considered in this study were maintaining autonomy, avoidance of harm, consent, privacy, and confidentiality.

Ethics approval was sought through Massey University prior to any data collection. Through this process, ethical concerns were identified and analysed which deemed the study to be of low ethical risk. As a current practicing teacher and New Zealand Educational Institute (NZEI) member, I also needed to uphold the NZEI Code of Ethics. The values found within this code guided how I conducted myself throughout the research; showing collective responsibility for quality education, acting responsibly, being honest and integral, and seeking equal opportunities for students (NZEI Te Riu Roa, 2008).

Approval from the Principal of the school and teachers of the participants was sought prior to any data collection. Discussing the research aims and process with these key stakeholders allowed them to support the research by organising suitable times for data collection, distributing and collecting consent forms, descriptions of their classroom programmes, and the facilitation of the problem solving tasks.

As all participants were under the age of sixteen, and therefore considered children, the explicit consent of the parents and caregivers was sought. All participants were given an information and consent form to take home for their parents and caregivers (Appendix F). These information sheets contained information about myself as the researcher, the reasons for the study taking place and their child's invitation to participate, an explanation about what was involved, and how much time was required in participating.

In the request to participate in the study, students and their parents had the right to decline participation, the right to withdraw from the study at any stage with no consequence, provide information on the understanding that the students' name would not be used or identified, and that they would be given access to a summary of the research findings on conclusion of the study (Leavy, 2014). While these rights were outlined on the information sheet and care was taken to ensure this was provided in an easy to understand manner, my email address and phone number was provided if parents and caregivers had any further questions about the study. Massey University Ethics contact information was also given if parents or caregivers had questions or concerns that they wanted to raise with someone other than the researcher.

As it was important to establish a distinct role as a researcher, and due to the nature of this research, no academic data was used during this study. All data collected was through the three identified collection tools: Student questionnaires, semi-structured interviews, and student self-reflections. This ensured the expectation that outside of the data collection times, such as during normal class time, any information disclosed or said in passing would not be used as part of the study (Hamilton & Corbett-Whittier, 2013).

To minimise disruption to class programmes, the problem solving tasks were done in class time. All students (With and without consent) completed the problem solving tasks, but only those who had their consent forms signed and permission given, completed a student self-reflection. These student self-reflections were also only voluntary and up to the student to decide what questions fitted with their experiences during the task.

An anticipated ethical dilemma was the transition between being a teacher and a researcher. As such, no evaluations of teaching and learning programmes were made outside the scope of this research and no discussion about student comments were made with any teachers or staff members (Cohen et al., 2018). All data collected was held securely throughout the entirety of the research.

3.9 Summary

While several research approaches were considered, a qualitative case study approach was utilised for this study as it allows for a deep understanding of how and people behave and respond the way they do given their situations. As the predominant research paradigm of mathematics education, a qualitative case study approach also allowed for the study of student mathematical dispositions and mindsets in an authentic, natural setting.

The researcher's role during qualitative research is not fixed, needing to be the data gatherer, ensure validity and reliability, and transcribe and analyse the data. My position in this study was also influenced by my role as a teacher (on part-time study leave) within the school where the study took place. This had several advantages but also required considerations to be made to ensure the study remained ethical, valid, and reliable.

The selection of the setting and sample was done because its characteristics were common throughout New Zealand schools and through providing rich, descriptive data, generalisations could be made to some extent. Forty-one students in total participated in the study with explicit consent being obtained from parents and caregivers prior to any data collection. The data of students' mathematics dispositions and mindsets, and their views towards mistakes was collected through questionnaires, semi-structured interviews, and self-reflections after problem solving tasks. This data was then transcribed, analysed, and reported on to answer the research questions.

At all stages of the research, validity, reliability, and ethical dilemmas needed to be considered and addressed. Being explicit about the researcher's role, having clear reasons for the study, collecting data from multiple methods, and providing thick description were all essential for ensuring validity and reliability. This study followed ethical practices outlined by Massey University and NZEI to minimise potential harm to participants. As such, the participants had the right to refuse participation, withdraw at any stage, and provide information on the understanding that the students' name would not be used or identified.

Chapter Four: Influences on Students' Mathematical Dispositions

4.1 Introduction

Through analysing and reporting on their responses to questionnaires about different aspects of mathematics, this chapter aims to understand students' current mathematical dispositions and how factors such as the teacher, task, assessment, and home have influenced these views.

Firstly, section 4.2 and resulting subsections will explore students' views on the nature of mathematics. That being, what does it mean to do mathematics and what place does it have within their lives. This gives us an understanding of whether the mathematics tasks that these students do at school reflects their nature of mathematics, or whether other factors such as assessment and home also play an influential role.

Students' enjoyment of mathematics will then be analysed in section 4.3, unpacking whether they hold positive views or show a dislike towards mathematics. From analysing students' favourite and least favourite aspects of mathematics, as well as their overall enjoyment of mathematics, we can understand what types of activities build positive associations with mathematics and reflect the positive dispositions that we are trying to build.

Finally, in section 4.4, and importantly for the learning of mathematics and the development of lifelong learners, is how students' see their role as a learner and the role of their teacher. Knowing what roles exist in the classroom through the eyes of Year 5 and 6 students allows us to understand how likely these students are to persevere with challenge and utilise mathematics when there is no source of authority such as a teacher or textbook.

4.2 Student views on the nature of mathematics

Already discussed is the importance of how students view the nature of mathematics, being seen as either as a connected, deep phenomenon, with uses throughout students' lives (Mathematical freedom), or as a narrow subject full of rules to be learned and performed at school and eventually being helpful in getting a job (Performance mathematics). This section looks to identify how their schooling and home have influenced a group of Year 5 and 6 students' views towards the nature of mathematics and understand whether these views are aligned with performance mathematics or mathematical freedom.

Table 4.1 shows the percentages of students whose responses indicated views of performance mathematics, freedom mathematics, or more ambivalent views that could not be classified as either. To be noted is that some students gave more than one response which could be coded into more than one category. Hence the totals add up to more than 100%.

Table 4.1 Student responses to questions about different aspects of mathematics (Performance mathematics responses vs Freedom mathematics responses)

Question	Performance mathematics	Freedom mathematics	Ambivalent or no views
What words can you think of that describe mathematics?	27% (n=11)	17% (n=7)	85% (n=35)
What are the most important things you have learned in mathematics?	71% (n=29)	15% (n=6)	22% (n=9)
Why do we learn mathematics?	71% (n=29)	17% (n=7)	25% (n=10)
How do we know if someone is good at mathematics?	76% (n=31)	49% (n=20)	2% (n=1)
To be good at mathematics you need to solve problems quickly	27% (n=11)	46% (n=19)	27% (n=11)
There should be only one way to solve a mathematics problem	12% (n=5)	71% (n=29)	17% (n=7)

4.2.1 Is knowing mathematics equated with doing mathematics?

Responding to the question *'What words can you think of that describe mathematics'*, 11 students (n=41) indicated mathematics was directly associated with mathematical content. These responses were consistent across this group of students, with mention of addition, subtraction, multiplication, division, and equations. These responses were also largely related to number and operations rather than any strand of mathematics.

While the New Zealand Curriculum outlines that "mathematics is the exploration and use of patterns and relationships in quantities, space, and time" (Ministry of Education, 2014), very few students in this study talked about the nature of mathematics in this way. These responses were also similar to both Young-Loveridge et al. (2006) and Grootenboer and Marshman (2016) who found a common perception that mathematics is predominantly about times tables and numbers, while very few thought of any other strand of mathematics such as geometry, measurement or statistics.

While the prominence of times table and number responses was comparatively smaller in this study to those of Young-Loveridge et al. (2006) and Grootenboer and Marshman (2016), student responses to the question 'what are the *most important things you have learned in mathematics'* is more telling about the value that basic facts really holds. Nineteen (n=41) of the student responses indicated that basic facts were one of the most important things they had learned in mathematics, while 3 students made comment on being able to quickly recall these facts. Boaler (2010) identifies how students often come to see mathematics in this way, misaligning success with accuracy and speed, from the tasks they do in class.

When examining the tasks these students engaged in, basic facts practice and testing could be found as part of warm-ups and their home learning programme. Completed on a regular basis at the start of mathematics lessons, students filled out 5x5 grids as quickly as they could, aiming to beat their previous time. Although the time allocated to these tasks was relatively small, many students it seems, had come to associate getting their basic facts correct with the pinnacle of learning mathematics.

This idea of accuracy was echoed in students' comments on '*how do we know if someone is good at mathematics*'. Twenty-one students (n=41) shared ideas of good mathematicians being able to get mathematics answers correct:

SQ.4: *They get all of the maths right.*

SQ.8: *They will be able to get the answers to questions easily.*

SQ.30: *They always have their hand up and get things right.*

For these students, who exemplified typical responses, being good at mathematics was being able to answer questions with ease and consistency. There were however, only eight students (n=41) who emphasised speed being a determining factor in what makes a successful mathematician. This idea of needing to be quick to be a successful mathematician is further challenged through responses to the statement *'to be good at mathematics you need to solve problems quickly'*. Nineteen students (n=41) disagreed or strongly disagreed with this statement while 11 students agreed or strongly agreed that being quick was a needed skill to be good at mathematics. Eleven students had neutral views about speed and mathematics. These responses signify that although all these students engage in basic facts practice and testing and many have developed a belief that mathematics is all about accuracy, relatively few carry the associated belief of needing to calculate and solve mathematics quickly (Franke & Carey, 1997).

With ideas of accuracy being more prevalent than speed in these students' views about the nature of mathematics, other activities that these students engage in must be examined. Boaler (2002), who found similar beliefs in students, identified that closed tasks, similar to the one these students engage in, had an effect on their views of mathematics over time. Boaler (2002) concluded that many students do mathematics without opportunities to value other aspects such as risk taking, asking questions, reasoning, and instead simply seeking to get answers correct. According to Sullivan et al. (2013), these closed tasks are also unique to the classroom and share little in common with what actual mathematicians do.

While there was a large amount of time that the students involved in this study completed these closed activities, such as working through text books or answering questions using the procedure they had just learned with the teacher, there were also some opportunities to engage in other, more open activities. Discussions were had at the beginning of some maths lessons, where the teacher felt that the ideas shared would benefit the whole class. These would often be during strand mathematics such as geometry, measurement, or statistics, and would be accompanied with whole class activities such as making shapes or nets, measuring objects around the classroom, or creating graphs. Maths games were also part of maths rotations used when students were not working with the teacher. These opportunities for students to encounter mathematics in multiple ways helps build stronger neural pathways, according to Boaler et al. (2016), making learning more accessible for students. The inclusion of these activities in classroom programmes also help share the idea that mathematics is not all about memorisation (Dossey, 1992).

For students to think creatively and use mathematics in unfamiliar situations, they need to see mathematics as open and not set procedures to be recited (Boaler, 2002). In responding to the statement *'there should be only one way to solve a mathematics problem',* 29 (n=41) students had views on there being multiple ways to solve mathematical problems. This is in contrast to only 5 students who had the view that mathematics problems should only be solved with one solution and 7 students who showed no views either way. This view of mathematics problems having multiple solutions indicates that for most of these students, using a single learned procedure was not necessarily the way problems should be solved.

The mathematics that these students engaged overall in was similar to those described by Education Review Office (2018), being made up of whole class teaching strategies and students working individually or with peers while the teacher occupied themselves with teaching strategies to ability groups. Education Review Office (2018) also go on to describe the changes in student views that can occur when they are able to work with others outside of their group.

Considering then, that these classrooms employed both whole class teaching and ability grouping strategies, what ideas do students have about where they learn best? When responding to the question *'Where do you feel you learn best',* 21 of the students (n=41) believed they learned best either in groups, with a buddy, or both. Reasons for these views were mainly around being able to receive or give support:

SQ.16: *Because I'm not so good at maths and I ask a lot of questions.*

SQ.25: *So I can talk to my group and tell them my ideas.*

SQ.28: *So I can get more ideas and learn new things.*

These responses, which are in line with a social constructivism view, indicate that many students valued the social aspects of mathematics, welcoming the opportunity for discussion and collaboration. Being able to collaborate, ask questions and hear other's thinking allows students to rationalise their own ideas and is essential to the learning of mathematics (Ernest, 2006).

In contrast, 18 students (n=41) believed that they learned best independently, citing being able to focus and not be distracted from others. These students, while also talking about a social aspect of mathematics, saw it as disadvantageous and something to be avoided. Cobb (1994) highlights how through developing classroom norms, students can successfully work collaboratively, ask questions, and reason ideas; maximising their opportunities to understand mathematics at a deeper level.

Although all these students engaged in similar tasks, and several influences could be found, the dichotomy of views tells us there is more at play in how these students have come to view the nature of mathematics.

4.2.2 Messages given through assessment?

With the prevalence of mathematics assessment in New Zealand and around the world, what messages are they conveying about the nature of mathematics to students? Assessment procedures based on memorisation and speed, often incite anxiety in students, limiting their ability to access the working memory part of their brain and rationalise their thinking (Beilock & O'Callaghan, 2011). Furthermore, many students across the achievement range, who undertake such assessments often come to view mathematics as being all about memorisation and speed (Boaler, n.d.). Already established is how few of the students in this study viewed speed as a determining factor in being good at mathematics. Are there however, other messages being conveyed through assessment, and what are students' thoughts about the purpose of assessment?

Students in this study engaged in basic facts testing each week as part of their home learning programme. Basic facts lists taken home were practiced during the week and then tested on a Friday. These lists normally contained 10 basic facts questions such as 9 times tables or addition to 20 which were derived from the numeracy project and separated into different stages. While there was no time limit for testing, rapid recall was encouraged both at school and at home.

E-asTTle tests were the main form of assessment used throughout the year, requiring students to complete an online mathematics test with a mix of word problems, multiple-choice questions, ordering tasks, and closed questions. Within the year, students undertook eight e-asTTle tests with pre and post tests for number and algebra, statistics and probability, measurement, and geometry.

Pre tests were conducted prior to the teaching of any concepts and was used to examine students prior understanding and form ability groups. Grouping using assessment data is a common practice in New Zealand, according to Ministry of Education (2017), and is cited by some teachers as a way of catering for a wide range of abilities. Post tests were used at the end of the unit such as statistics and probability or measurement to record students' progress and identify any gaps that remain. This data was used to inform the teachers' overall judgement on students' progress and was shared with parents and students during interviews.

Noticeable, was the amount of emotive responses to the question '*what words can you think of that describe tests*.' These responses covered a wide range of views, with 8 students (n=41) describing tests as being stressful, 12 viewing tests as boring, and 5 describing tests as enjoyable. This contrast in views continues with students' responses that included aspects of testing difficulty. Out of the 8 students that described aspects of difficulty (n=41), 4 talked about tests being hard or challenging while 4 citied tests as being easy. While there seems to be a contrast in enjoyment and the perceived difficulty of assessments, what are students views on the reasoning for assessment and do they align with the ideals of providing feedback and supporting learning or are they viewed simply just a part of what it means to do mathematics?

Responding to the question *'why do you think we do tests',* 9 students (n=41) shared ideas of finding out what group they should be in:

SQ.5: *To see if we should go higher up/down to a different group.*
SQ.13: *To get put in the right group.*

This view of mathematics being a performance subject, has several implications for students and their mathematical dispositions. Firstly, many students who do not achieve highly in these tests are often placed in lower groups, giving them and their peers, the perception that they are not as capable (Boaler, 2013). These students, who may be deep, slow thinkers, may in turn miss out on opportunities to hear higher order thinking, be given less challenging work, and not experience the full curriculum (Education Review Office, 2018). Those students who are able to perform under test conditions tend to end up in higher groups. These students, according to Blatchford et al. (2001) often don't get opportunities to reason their ideas and think about mathematics in different ways. These test results it seems, may indirectly play a role in the opportunities and experiences that some students have with mathematics.

Similarly, 6 different students (n=41) believed that the purpose of testing was to find out your 'level' or 'stage.' Although these responses did not detail what levels or stages meant, students were aware of stages of the Numeracy Project and subsequent Number framework. This framework was established as part of an initiative by the Ministry of Education to develop the mathematics teaching capability of New Zealand primary school teachers and help parents and students understand the requirements for the Number strand of the New Zealand Curriculum (Young-Loveridge et al., 2006). Separated into eight stages, The Number Framework introduces strategies and required knowledge that students are taught as they move through the different stages. While progress through the stages would indicate an expansion of learned strategies and knowledge for students, Boaler (2016) cautions the teaching of strategies and knowledge in isolation. The problem has arisen that these strategies are often taught without context and used exclusively to solve closed problems which are similar in nature, causing students to see mathematics as narrow and fragmented (Young-Loveridge et al., 2006). According to Boaler et al. (2018), judging students through narrow procedure based work also disregards many of the mathematical practices that we want to reinforce.

Fourteen students (n=41) talked about assessments being used to see improvement that they had made. These responses included comments such as:

> SQ.20: *So teachers can see how much we have learned.*
> SQ.22: *To see how much we have improved.*
> SQ.31: *So the teachers know where you are at in your learning.*

Many of these comments refer to the teacher being the benefiter of this information. While no elaboration was given about what the students believe their teachers do with this information, 16 students (n=41) commented on tests helping their learning in some way. These comments were vague in nature however, not detailing how they helped their learning except for 2 students who talked about identifying their next steps. Education Review Office (2018) encourages the use of formative assessment where possible, allowing students to understand what they know, what they do not know, and the path between the two. It seems that although formative feedback was occurring to an extent, students were not fully aware of this link between assessment and their next steps.

From the responses to these questions, students' views on assessment seemed to be aligned with a more traditional or performance mathematics approach, being useful for sorting students into ability groups and seemed to reinforce a narrow perception of mathematics. It seems that for many of these students, testing was a just part of learning mathematics, helping improve their learning in unknown ways and sharing the idea that getting a good score and moving up levels is an important part of doing mathematics.

4.2.3 Connections between school mathematics and the outside world

The link between school mathematics and the mathematics outside of the classroom is an important, but not always realised, connection for students. Students often come to create a distinction in their minds between the demands of school mathematics and what occurs in the real world, making it difficult to capitalise on experiences they have outside of the school setting (Boaler, 2002; Presmeg, 2002). In examining what students believe the reasons for learning mathematics are, we can build a clearer picture of what role it plays in their lives and what the nature of mathematics is to them.

When responding to the question *'why do we learn mathematics'*, a common theme emerged of mathematics being useful for the future. Twenty-nine students (n=41) commented on needing it when they were older. For example, three different future focused reasons were provided:

> SQ.17: *In some jobs you need to know lots of different types of maths.*
> SQ.30: *So we are ready for high school.*
> SQ.26: *So we know how much money we have when we go to the supermarket.*

Clearly, these illustrated a utilitarian approach to the use of mathematics including handling money, employment, and future mathematics education. Grootenboer and Marshman (2016) reports on similar findings in their meta-analysis, highlighting that while it was clear that students had some general ideas about the application and uses of mathematics, they tended to be unsophisticated. These comments indicate that while there is a belief that mathematics is useful, it is more for the opportunities that it might create in getting a job or with future study rather than for the actual knowledge and understanding of mathematics now.

While mathematics education has traditionally been tasked with preparing students for employment and future mathematics study, very few students talked about it being relevant for their lives now (Sullivan et al., 2013). Only 2 students in this study made comments about how learning mathematics was useful for their lives here and now, with both relating to helping on the farm:

SQ.4: *So I can help Dad count the cows in the milking shed.*

SQ.31: *I need it to count the posts when we do fencing.*

These students, in recognising the inherent value of mathematics for their lives here and now, have greater scope for connecting the learning at school with real experiences, and in turn deepening their understanding of mathematics (Boaler, 2002). Maxwell (2001) believes that this connection between school and home can also be strengthened though the use of contextualised, relevant problems. While use of these problems was not apparent within this study, Lotan (2003) demonstrates how small changes in task design give intellectual authority to students, making their life experiences and interests a valid part of learning.

When posed the question *'outside of school, where do you see mathematics'*, several different themes emerged. Twenty students (n=41) gave responses related to doing activities around the house, 8 of which referred to 'baking with Mum' and 3 about 'playing games'. Three students shared comments about seeing mathematics when working on the farm and another 4 students talked about seeing mathematics in their parent's jobs such as building or working in the orchard. Rokeach (1968a) highlights the importance of the interactions between students' families and mathematics with the understanding that beliefs can be indirectly developed through seeing how others interact with mathematics in their lives. In seeing their parents interact positively with mathematics and use it throughout their daily lives, students can come to see the value that it should have and build similar dispositions (McLeod & Adams, 1989). This idea is developed further by Education Review Office (2018) with how including their children in everyday discissions involving mathematics such as baking, shopping, games and puzzles, parents share the idea that mathematics is everywhere and in all that we do.

Again, the utilitarian view of mathematics was apparent in responses to the question *'outside of school, where do you see mathematics',* with 14 students commenting on seeing mathematics when using money or for travel such as speed signs and distances.

While not all these instances of mathematics are relevant for students now, most students again seemed to be aware of their importance for when they were older.

Finally, in responding to *'outside of school, where do you see mathematics'*, 12 students commented simply with 'everywhere'. From the viewpoint of students being lifelong learners and having positive dispositions towards mathematics, this is encouraging (Boaler, 2002). While indeed there is scope for more specific connections to be made between school and home and more contextualised, relevant problems to be used, these students' views overall seemed to support the idea that mathematics is not just about textbook answers and numbers and has wider applications in life.

4.3 Enjoyment of mathematics

While not as stable as beliefs, students' enjoyment of mathematics is an important and integral part of learning mathematics. Grootenboer and Marshman (2016), who found an age-related decline in students' enjoyment of mathematics as they go through school, believe that if students are enjoying the mathematics they are doing, these emotions can, over time, form into more permanent dispositions. In unpacking the following responses, we assume that these are offered as students' overall enjoyment of mathematics.

Unlike responses to '*what words can you think of that describe tests*', students made no mention of stress when describing mathematics in general. Similarly, while 12 students (n=41) also responded with views of boredom in testing, only 3 did the same when describing mathematics in general. These responses indicate a comparative dislike of the testing aspect of mathematics. These views of testing, however, did not seem to effect students' overall enjoyment of mathematics with 21 students (n=41) giving positive responses to the question '*what words can you think of that describe mathematics.'* Of these positive responses, 18 students used the word 'fun', and 3 students used the word 'cool'. Looking at these responses, the mathematics that these students engage in seem to be viewed in a positive light.

Table 4.2 shows students' responses to the questions *'what are your favourite mathematics activities', and 'what are your least favourite mathematics activities'.*

Table 4.2 Student responses to the questions about favourite and least favourite mathematics activities

Activity	Favourite Activity	Least Favourite Activity
Multiplication	29% (n=12)	29% (n=12)
Division	7% (n=3)	29% (n=12)
Addition	7% (n=3)	10% (n=4)
Subtraction	10% (n=4)	10% (n=4)
Decimals, fractions, and percentages	2% (n=1)	2% (n=1)
Geometry	7% (n=3)	-
Computer programmes e.g. Maths Whizz	10% (n=4)	7% (n=3)
Open problems	10% (n=4)	-
Closed problems	7% (n=3)	10% (n=4)
Hard problems	10% (n=4)	7% (n=3)
Easy problems	2% (n=1)	10% (n=4)
Visual and hands on activities	10% (n=4)	-
Science	10% (n=4)	-
Games	24% (n=10)	-
Tests	-	15% (n=6)

Initially evident was not the differences between students' favourite and least favourite mathematics activities but rather the similarities between the two. Activities involving multiplication, addition, and subtraction all had very similar amounts of responses for being some students' favourite activities in mathematics, and others' least favourite activities. This scenario, where some view certain activities as their favourite activity while others dislike them raises further questions about whether students' enjoyment was related previous experiences, their achievement, confidence, or whether they are not getting the same opportunities.

Looking at students' enjoyment of number related activities, division clearly had a more negative connotation with 12 students listing it as their least favourite activity while only 3 listed it as their favourite. This dislike of division may come stem from students learning how to calculate division facts but not understanding what division actually is or its connection with multiplication (Boaler, 2015). Out of the 52 least favourite activities described by students, 32 of these were related to multiplication, division, addition, and subtraction. This tells us although students previously identified basic facts (including multiplication, division, addition, and subtraction) as being one of the most important things they had learned in mathematics, it was also one of their least favourite activities to engage in.

What was also clear in examining these results, is that some responses were unique to either students' favourite activities or least favourite activities. For example, geometry (n=3), open problems (n=4), visual and hands activities (n=4), science (n=4), and games (n=10) were only listed as an activity students like, showing a positive association with them. These activities are also associated with a more inquiry or reform approach to teaching, being open, integrated, and allowing students to explore mathematical concepts through more creative means (Boaler, 2010). Interesting to note is Grootenboer and Marshman (2016) findings that many students, when engaging in games or more hands on activities, seemed to think that they were not really engaged in meaningful mathematics learning. Many studies such as Beyers (2011); Boaler (2002); Hunter and Anthony (2011), however, have demonstrated the need for such activities in developing students' conceptual knowledge.

In analysing responses about students' favourite and least favourite activities, it seems that there was a divide in what students find enjoyable. While some students found more traditional tasks in mathematics enjoyable such as closed tasks and basic facts, some viewed these as their least enjoyable and preferred exploring challenging, open problems. The following section aims to understand if this distinction in enjoyment is reflected in what students believe their role is as a learner of mathematics.

4.4 Roles within the classroom context

An important aspect of students' mathematical dispositions, and one that has implications for their lifelong engagement with mathematics, is the role that they believe their teacher, and themselves have in learning. Taylor et al. (2005) describe how as children experience different classroom settings, they become more aware of the social and organisational structures that exist. While the studies reviewed indicate the importance of discussion, reasoning, and teaching practices that help develop higher order thinking, some students view learning mathematics as a more passive endeavour. Through students' own views on these different roles that exist, we are able to identify whether their dispositions allow for lifelong learning or whether they are more reliant on a source of authority such as a textbook or teacher to instruct them.

4.4.1 Passive receivers of knowledge

When analysing students' responses to questions concerning what roles exist in the classroom, two distinct views were found. The first view found, and more in line with traditional views, was of the teacher being the source of knowledge and the students themselves being the passive receivers of this knowledge. When questioned *'what is your job as a learner during mathematics'*, 10 students (n=41), who showed this more passive view of learning, shared responses about listening and paying attention to the teacher. Along similar lines, 2 students responded with needing to 'follow instructions' and 'do what I'm told.'

These students seemed to believe that their teacher was not only responsible for asking them questions, the teacher was also the source of information, strategies, and ideas. As a result of listening carefully, these students believed information would be imparted to them and they would be successful in mathematics. Taylor et al. (2005) identified that students who thought in this way, often placed blame on the teacher when they struggled in mathematics, not considering the idea that they themselves might need to be an active part of learning (Taylor et al., 2005, p. 730). Consequently, the question is raised to whether these roles are preventing these students from exploring mathematical ideas and are giving students a reliance on the teachers' ability to communicate ideas.

4.4.2 Active negotiators of mathematics

There were 24 students who expressed views of themselves being an active part of learning mathematics. Eleven of these students, when responding to the question *'What is your job as a learner during mathematics',* talked about needing to 'try their hardest' or 'do their best'. It can be assumed that regardless of the mathematics these students were engaged in, they saw their own role as completing it to the best of their ability. While their hardest' during closed tasks would require students to concentrate and focus on accuracy, 'trying their hardest' in open, inquiry tasks may require students to ask good questions and focus on reasoning their ideas.

Four students responded to the question *'What is your job as a learner during mathematics'* with needing to understand the mathematics they were doing:

SQ.8: *Learn everything in my own time.*
SQ.12: *To understand what I'm trying to learn.*
SQ. 13: *To find out different strategies and how they work.*

As evident from current literature (Boaler, 2019); Sullivan et al. (2013), these views of needing to think deeply, and understand the underlying concepts are positive dispositions that allows students to connect ideas and use mathematics in unfamiliar situations. Four students described their job as a learner of mathematics as asking good questions. Through asking good questions, students can not only clarify their own thinking, they provide other students the opportunity to reason their ideas. In line with a social constructivism view, 5 students also commented on their job as a learner of mathematics being to help others learn. These responses included:

SQ.4: *Talk about the answers so others understand.*
SQ.9: *Teach others.*
SQ.27: *Help others learn.*

While there was little elaboration in how students were to help others learn, the view that their job as a learner of mathematics did not just include their own learning is encouraging. This idea of being a community of learners is inherently part of mathematics according to Boaler (2016) who notes the social nature of mathematics and how in requiring students to reason ideas, support others, and be creative, they learn mathematics in a similar way to how it is found in the real world.

In supporting these students views of trying hard, asking good questions, and supporting others is the responses to the question *'How do we know if someone is good at mathematics'.* Eight students responded to this question with 'they work hard', 6 responses of being able to use different strategies, 2 responses about helping others, and 2 responses about being able to ask good questions. This reinforces the idea held by Boaler (2010); Sullivan et al. (2013) who believe students who think deeply, creatively, collaborate, and persevere through challenges are the future mathematicians we need in an ever changing world.

4.5 Summary

Throughout the data, several themes emerged concerning the mathematical dispositions that these students held and the factors that have influenced these. There seemed to be a common perception that mathematics was about aspects of number and operations rather than any strand of mathematics or exploration of patterns as detailed in The New Zealand Curriculum. Furthermore, when responding to questions about the most important things students had learned and what makes a good mathematician, students indicated recalling basic facts correctly was a highly important skill. While these responses of accuracy were similar to findings from Grootenboer and Marshman (2016); Young-Loveridge et al. (2006), students in this current study did not hold the same level of belief the speed was important in being a successful mathematician.

Overall, these students seemed to have had positive views towards mathematics but had noticeably divided views on what aspects of mathematics were enjoyable. For some students, aspects of number and more traditional practices were enjoyable while others appreciated the opportunities to engage in more open, hands on activities and cited closed activities as being their least favourite. This contrast in views seemed to reflect the mixture of tasks that students engaged in, largely consisting of closed tasks but with some opportunities to engage in class discussions and more open activities.

In examining what roles existed in these classrooms, two different viewpoints emerged. One view, which seemed to align with the closed, more teacher lead tasks that these students engaged in, was that of being a passive receiver of knowledge. In order to be successful, these students saw their role as needing to listen carefully, pay attention to the teacher, and follow their instructions. Many of these students also commented on preferring to work independently, believing they could focus on their work without distraction. In contrast to this was another large group of students who saw their role as needing to think deeply and understand concepts in their entirety through trying hard, asking good questions, and helping others to learn.

Although assessment practices did not seem to impact students' overall enjoyment of mathematics, responses concerning the purpose of it imply its influence on students' mathematical dispositions. Central reasons for testing, according to these students, was to find out their levels, how much progress they had made, and determine their groups. Through being used in the forming the groups, assessment also may have indirectly played a role in the opportunities and experiences that some of these students had with mathematics. It seems from these responses that assessment seemed to reinforce a more narrow view of mathematics, sharing the idea that getting a good score and moving up levels is an important part of mathematics.

A large number of students in this study shared utilitarian views of mathematics being useful in the future such as with employment, handling money, and future education rather than any major application here and now. These views seemed to stem from where they saw mathematics outside of school with comments of shopping, parents' jobs, and home learning. Many students detailed how they used mathematics for enjoyment such as baking and games. Encouragingly, some students also noted that mathematics could be found everywhere. Through all these responses, students have provided us with some important insights into their experiences and have raised some questions that warrant further investigation.

Chapter Five: Students' mindsets and their views on mistakes in mathematics

5.1 Introduction

As previously identified, the way students view their own ability to learn and how they value mistakes can have significant implications for their learning of mathematics. For those that view mathematical ability as fixed, mistakes are seen as indicators of their inability and are to be avoided. These students, according to Boaler (2013), tend to favour easier tasks which they know they will be successful in and often give up when faced with challenges. Those who hold a growth mindset, however, see challenges and mistakes as tools to help their brain grow. Through hard work, perseverance, and risk taking, these students seek to understand and connect mathematical concepts rather than simply recite them. This in turn affords these students greater opportunities to engage in meaningful mathematics and learn.

Section 5.2 examines students' responses to questions concerning their own perceived ability, the potential for anyone to learn mathematics, and the indicators of a successful mathematician. In knowing these views, we can determine whether these students hold a fixed mindset or a growth mindset and what implications this may have for their learning. Subsection 5.2.1 analyses the messages that are being shared through grouping and students' understandings of why they are in their group for mathematics. Through analysing students' questionnaire responses and their self-reflections collected after two problem solving tasks, section 5.3 and subsequent sections identify students' views and reactions towards mistakes. Finally, subsection 5.3.4 details the learning that emerged after mistakes were made in two problem solving tasks, helping illustrate what role mistakes can have in the learning of mathematical concepts.

5.2 Students' mindsets towards learning

Already discussed, but important in determining students' mindsets, is what attributes they believe successful mathematicians have. In responding to the question *'how do we know if someone is good at mathematics',* 21 students (n=41) talked about getting answers correct. For those whose focus is solely on accuracy, making a mistake can be seen as an indicator of their inability which can not only be devastating for students' perceptions of themselves but their motivation as well (Boaler, 2013). While these responses alone are not a clear indication of a fixed mindset, it does raise questions about how these students react when mistakes are made.

More in line with a growth mindset were 8 students' responses of 'working hard' being a sign of a good mathematician. This view is consistent with Boaler (2019), who illustrates how when students work hard and when their brain is put under load, they form new connections or strengthen previous connections in their brain. In contrast to a more fixed mindset, students who persevere and work hard through challenges are also more likely to take risks and learn from their mistakes, deepening their understanding of mathematical concepts (Dweck, 2012).

Table 5.1 outlines students' responses to the statements *'I am good at mathematics'* and *'Anyone can be good at mathematics'.* This helps to illustrate students' perceived ability and gives an indication of their mindset towards learning mathematics.

Table 5.1 Student views about their ability in mathematics and the ability for anyone to be good at mathematics

Response	I am good at mathematics	Anyone can be good at mathematics
Strongly agree	24% (n=10)	68% (n=28)
Agree	34% (n=14)	15% (n=6)
Neutral	31% (n=13)	10% (n=4)
Disagree	7% (n=3)	2% (n=1)

Strongly disagree	2% (n=1)	5% (n=2)

In responding to the statement *'I am good at mathematics'* 24 students (n=41) saw themselves as either good or very good at mathematics. A further 13 students' responses indicated they believed they were 'okay' at mathematics while only 4 students perceived their ability as low. From these responses, it is clear that most of these students view themselves as being above average mathematicians. Dweck (2012) explains how students can come to view themselves in this light either through their experiences in mathematics, or the praise they receive. While not ideal, but common in the developing of students' mindsets, is praise from well-meaning teachers or parents who openly describe students as clever or smart without attributing success to anything tangible such as perseverance. While students can often identify as being good at mathematics from such praise, it is only when they make mistakes and struggle that their true capability is shown (Boaler, 2013).

Looking at students' responses to the statement *'anyone can be good at mathematics',* we can see 34 students (n=41) agreed or strongly agreed, 4 students had neutral views, and only 3 students disagreed or strongly disagreed. This demonstrates that a large number of students, larger than just the group identifying as good mathematicians, thought that anyone could learn and be good at mathematics. Anderson et al. (2018) writes how students who hold this growth mindset, seeing ability as changeable, have higher reported engagement levels and enjoyment of mathematics. Those students who may not see themselves as good mathematicians yet, still have the ability improve through hard work and practice according to these students' responses. While the students in this study have an overall view of mathematics being accessible and achievable for anyone, there remains the question of whether this would continue to be the case if students' perceived themselves as struggling learners, rather than being good at mathematics. In identifying students' actual reactions and responses to challenges and mistakes in the coming sections, we can examine whether these responses match reality.

5.2.1 Messages about student ability given by grouping

With ability grouping being widely used in New Zealand, including the classrooms in this case study, what messages are being conveyed to students about their ability and their capacity to learn. When questioned *'why do you think you are in your group for maths',* 15 students (n=41) responded with ideas of levels or ability. For example, four different reason were provided:

SQ.1: *So you can work with people as smart as you.*

SQ.4: *Because the class has different levels for different people.*

SQ.24: *To work with the people that are in the same stage.*

SQ.25: *To learn with other people you can compete with.*

These responses seem to signify that these students' viewed learning mathematics in a more traditional sense, needing to be taught specific concepts and strategies depending on their stage or level. While these responses, in no way, imply a belief that you cannot grow or improve, it does indicate a belief that you need to have a certain ability to learn certain things. This raises issues around the lack of opportunity for those who are placed into lower groups to hear higher order thinking, engage in rich tasks, and experience the full curriculum (Education Review Office, 2018). Even those placed in higher ability groups are disadvantaged according to Hunter et al. (2020), who writes about the expectations placed on these students to maintain their progress and achievement.

In believing their group or level is an indication of their ability, students quickly determine whether they are good or not so good at mathematics and make comparisons between themselves and those around them. Even with the use of different group names, students are quickly able to identify the group hierarchy and perceived ability (Boaler, 2013). If students, who are placed in higher levels, begin to struggle, or make mistakes, they can begin to believe they are not so smart after all and hide their mistakes or favour easier work that they know they will be successful in.

While many of the responses described above do mention learning being done with others of the same ability, it is far from suggesting learning should occur in a collaborative and supportive way. There were however 7 students (n=41) who responded with ideas of helping others when questioned *'why do you think you are in your group for maths'.* For example, four different reason were provided:

SQ.10: *To help me and them.*
SQ.22: *Because I can help people that need help.*
SQ.26: *To work as a team.*
SQ.28: *To help each other learn.*

It seems that for these 7 students, their group placement did not indicate their ability but rather was an opportunity to work with a group of students and help each other learn. Hunter and Anthony (2014) writes how often small groups are safe settings for students to construct ideas, trial them, and have conversations about them without judgement. In creating mixed ability groups, students studied by Hunter and Anthony (2014) found value in being a part of a community of learners and expressed ideas of being able to help each other and being mutually responsible for learning. It seems that through encouraging students to work with others outside of their 'ability', more students can come to view groups as a tool for learning rather than just an indication of their ability.

When responding to the question *'why do you think you are in your group for maths',* 9 students (n=41) gave vague responses of 'to learn'. Similar to responses about '*why do you think we do tests',* students seemed to have the idea that groups were there to help their learning but were unable to communicate how and why this was so. Finally, were 10 responses of not being sure why they were in their group for mathematics. It seems that for many of these students, groups were simply a part of what it means to do mathematics and did not question its purpose.

5.3 Mistakes in mathematics

Mistakes, while being beneficial for brain development and useful in the learning of mathematical concepts, are not always viewed in a positive light. For those with a fixed mindset, who view intelligence as being fixed, mistakes are an indication of their inability and are to be avoided (Moser et al., 2011). While previous sections identified a large number of students that believed they were good at mathematics and that everyone had the ability to be good at mathematics, what are their views towards mistakes, and does this align with their previous responses?

5.3.1 Views towards mistakes

Responding to the statement '*mistakes are just a part of learning mathematics',* 32 students strongly greed, 6 students agreed, 2 student had neutral views, and 1 student strongly disagreed. These responses align closely with a growth mindset and indicate an overall acceptance of mistakes within these classrooms. There were, however, mixed views on how making mistakes made students feel. When completing their self-reflections and questioned '*how do mistakes make you feel about mathematics*', 10 students (n=30) responded in positive ways. Three examples of positive responses were provided:

> SR.10: *They remind me that I am still learning.*
> SR.27: *You can't learn without mistakes.*
> SR.30: *It makes sense if you keep trying.*

These responses seem to enforce the idea that mistakes are to be encouraged and utilised. This is also a sign that these students held a growth mindset, seeing mistakes as part of grasping with mathematical concepts and through persevering, they would eventually gain understanding.

Responding with more negative emotions to the question '*how do mistakes make you feel about mathematics*', were 8 students (n=30) who talked about being embarrassed, feeling sad, and being frustrated. These responses suggest that these students do not feel comfortable with the idea of being 'wrong'. DeBrincat (2015), who describes mistakes as an internal struggle between what we believe and the external reality, poses the argument that for those that hold a fixed mindset, these mistakes are a challenge to their own intelligence and often invoke feelings of sadness or frustration. Boaler (2013) describes how these feelings surrounding mistakes may also be caused by the reactions of others such as laughing or teasing. It is important then, no matter what the source of these negative feelings, that teachers' help develop a classroom culture that values mistakes and where mistakes can be explored without judgement (Schleppenbach et al., 2007).

5.3.2 Noticing mistakes

As mistakes are only valuable if noticed and explored, the following section analyses students' self-reflections concerning how they noticed their mistakes in the two problem solving tasks. In total, 30 students reported on making at least one mistake over the two problem solving tasks.

When asked *'what mistake did you make',* 4 students (n=30) described how they misread the question while 26 students described mistakes that could be classed as misconceptions. This latter form of mistake refers to students' misapplication of concepts such as applying whole number thinking to fractions or decimals and can reveal a lot about a student's understanding (Rushton, 2014). These mistakes are ideal for unpacking and exploring according to DeBrincat (2015) and can be the source of rich mathematical discussion. Some examples of misconceptions were provided:

SR.1: *Not knowing what 2/1 was and thinking it was a small fraction.*

SR.4: *We thought 0.125 was the biggest because it has more numbers than 0.13.*

SR.26: *We picked the graph with the three different lines because it went low when she was running slowly and it was higher when she sped up.*

These examples illustrate common misconceptions that many students have when encountering fractions or graphs. These misconceptions, however, can often go overlooked within a classroom setting and not be drawn upon for discussion or exploration, remaining correct in students' minds. In responding to the question *'how did you know you had made a mistake'* during their self-reflection, thirteen students (n=30) reported that the teacher had told them they had the wrong answer. Before analysing how these mistakes were handled in the following section, it seems that many students were unaware of their mistake until the teacher noticed them.

Five students (n=30) reported that they noticed their mistakes after others shared their solutions while another 4 students identified that their mistakes were brought to their attention by members of their group. This further highlights the importance of sharing and exploring mistakes, as DeBrincat (2015) writes, one student's mistake could lead to another student's clarity.

Finally, in examining how mistakes were noticed in these problem solving tasks were 3 students who described how they self-corrected their own mistakes. While only a small number, Moser et al. (2011) describes how people who held a growth mindset also had a greater awareness of their mistakes and possessed the ability to recover and learn from these mistakes. It seems that for this group of students, they were actively involved in the mathematics task and were rationalising their thoughts.

5.3.3 Reactions to mistakes

Perhaps the greatest indicator of students' mindsets and views towards mistakes is how they react to making them. Those who made mistakes in the two problem solving tasks had the opportunity to reflect on what they did when they made the mistake. Twenty-six students (n=30) shared how they tried to fix their mistake. Of these 26 responses, 3 students described how they listened to other's ideas to understand how they solved the problem, 4 students asked for help from their group on where they went wrong, while the remaining 19 students talked simply about trying again. In responding to this same question, 4 students had a very different reaction to their mistakes, describing how they gave up. Examples of giving up were provided:

> SR.5: *I just put my head on the table and waited for the teacher to help me.*
> SR.7: *Nothing, I just waited until time was up.*
> SR.14: *I looked at the teacher, but they weren't looking so I did nothing.*

When checking these 4 students' responses to the question '*how do mistakes make you feel about mathematics*', all 4 indicated a negative view towards mistakes with mention of embarrassment and frustration. It seems that these students had developed a reliance on the teacher to provide the answers or had come to view mistakes as an indication that they were not good enough to solve the problem. This fixed mindset and reliance on the teacher raises concerns around what happens when there is no source of authority such as a teacher (Boaler, 2010).

As students navigated these problem solving tasks with individual thinking time and then group discussion, understanding the whole groups' reactions to students' mistakes tells us a lot about the classroom culture towards mistakes. When asked *'what did your group do when you made a mistake'* 13 students (n=30) responded with how their group helped them understand their mistake, with one response describing how they all were responsible for making sure everyone understood the solution and why it didn't work before. In developing a classroom culture where this shared responsibility occurs, Franke and Carey (1997) note the need for the teacher to explicitly share the importance of communication and of being a community of learners.

Although still trying to help, 7 students responses described how other group members took over solving the problem after they had made a mistake, writing down the 'correct' answer without discussion or explanation. With discussion and the reasoning of ideas being an important part in learning mathematical concepts, this dominance of some students took away the opportunity for other students to learn from their mistakes (Sullivan et al., 2013).

Five students' responses described negative reactions to their mistakes from their group members. These reactions included laughing and moaning, which, according to Boaler (2013) may contribute to more negative views of mistakes over time. These responses, however, were relatively few compared to the positive reactions to mistakes from group members.

Finally, and importantly, due to some students' reliance on the teacher, is how the teacher reacted to mistakes. Of the 13 students (n=30) that noticed their mistake only after the teacher pointed it out, 7 students described how the teacher told them the correct way to solve it. Two examples of the teacher telling students the answer were provided:

SR.1: *Told me where the fraction went and why it was bigger than the other ones.*

SR.8: *Showed me why it wasn't the right graph.*

In these examples, explanations are given as to why the solution was not correct. While students were given the opportunity to generate their own solutions first before the right answer was explained to them, it does not take full advantage of the mistake, nor allow for students to productively struggle (Granberg, 2016). There were however 6 students who commented on how the teacher helped them work through their mistakes without revealing the solution. Two examples of the teacher helping were provided:

SR.6: *Helped us understand what we did wrong by making us draw the fraction out.*

SR.26: *Didn't give us the answer and kept asking questions about what the graph would look like if she sprinted or stopped.*

This questioning and deflection of responsibility back to the students, as suggest by DeBrincat (2015), helps students develop tools to overcome problems and gives them more opportunities to productively struggle. In the case of SR.26, students were required to make conjectures and in turn, make generalisations about graphs. It is through such cognitive demand, according to Kapur (2015), that students are learning deeply and developing growth mindsets.

5.3.4 Learning that emerged

Of the 30 students who reported on making a mistake during the two problem solving tasks, 22 students described how they learned from their mistake when completing their self-reflections. Some examples of learning from mistakes were provided:

SR.4: *0.13 is greater than 0.125 because the 3 is worth more than the 2 even though 0.125 has more numbers.*

SR.9: *That there is a thing called improper fractions.*

SR.25: *Percentages can be worth the same as fractions and decimals.*

SR.28: *That when something moves quicker, the line on the graph gets steeper because it is going further in less time.*

It was clear that although these students engaged in the same problems, there were very different learning outcomes for different students. This not only demonstrates how mistakes can be used to support the learning of a wide range of mathematical concepts, but it also shows how with careful selection of tasks, all students can engage and learn together on the same task.

5.4 Summary

While examining these students' mindsets towards learning and consequently, their views on mistakes, several important themes were identified. Most of these students saw themselves as good mathematicians, with an even larger number of students believing that anyone could be good at mathematics through hard work and practice. While this is indicative a growth mindset, there was a common perception that the aim of grouping was to allow students to learn with others of a similar ability and learn concepts and strategies at their 'level' or 'stage'. While ability grouping, which is common practice in New Zealand, is often done to cater for a mixture of abilities, it shares the message to students that their capability in mathematics is equivalent with the group they are in. This also raises questions around the lack of opportunities for those who are placed into lower groups as well as what happens to higher grouped students when they begin to struggle and inevitably make mistakes.

While a large group of students were unable to communicate the purpose of grouping or gave vague views of it being helpful, there was a small number of students who held a view that grouping provided opportunities to learn collaboratively. Overall, the students' views on grouping seemed to match more traditional practices and while not implying a belief that ability is fixed, it did little to encourage a growth mindset.

Although many students held the belief that mistakes are just a part of learning, there were divided feelings towards making them. While some students held positive feelings towards making mistakes, others shared feelings of embarrassment and frustration which may have resulted either from the students themselves holding a fixed mindset, or from the reactions of others. This illustrates the importance of the teacher in constructing a classroom culture where mistakes are valued and can be explored without judgement.

Many of the mistakes that were described by students in this study were noticed by the teacher before being conveyed to the students. According to the students' self-reflections, the teachers' reactions to these mistakes fell into one of two categories. Some students described how the teacher told them the correct solution, explaining why their first solution was not correct while some students described how the teacher helped them work through their mistakes without revealing the solution. In not revealing the solution but rather posing questions and extending students thinking, the teacher took full advantage of the mistakes and in turn helped students develop problem solving tools and growth mindsets.

Although some students seemed rely on the teacher to help them when they made mistakes, most tried to fix the mistake themselves. Strategies for doing so included listening to other's ideas and asking for help from their group members. While a small number of group members reacted negatively, most tried to help either by explaining why the solution did not work, or by taking over. In examining the learning that occurred from these mistakes, we can see very different learning outcomes which helps demonstrate the power of mistakes in learning mathematical concepts.

Chapter Six: Semi-structured interviews with students

6.1 Introduction

The following chapter examines 3 students' responses to semi-structured interview questions, building on the previous findings around students' mathematical dispositions, mindsets towards learning, and their views and reactions to mistakes. These 3 cases have been chosen because they exemplify the importance of different factors in developing mathematical dispositions and mindsets and give us a deeper understanding of the current findings.

6.2 Traditional views of mathematics

The following case analyses responses from a semi-structured interview with Anthony, a year 6 student who was identified by their teacher as being in the top group. Firstly, when describing the nature of mathematics, Anthony talked about strategies and techniques:

Interviewer: *So what do you think mathematics is all about?*

Anthony: *It's about learning new strategies and techniques that help you work things out.*

Interviewer: *What sort of strategies do you learn?*

Anthony: *Like how to add big numbers together in parts because it's easier to work out.*

Similar to the theme found throughout this study, Anthony seemed to view mathematics as being about operations and aspects of number. Anthony also shared a positive outlook towards mathematics in general, except for noting his dislike of times tables and preference for solving large addition and subtraction problems. This was reflected in a conversation about what made him feel successful in mathematics:

Interviewer: *What kind of things make you feel successful in mathematics?*

Anthony: *Being the first one to answer a really hard question! It makes me feel confident!*

Interviewer: *It is important to be quick at answering questions?*

Anthony: *Yeah, normally in basic facts and that. I like getting big questions right more than just basic facts though.*

Interviewer: *Why are big questions better to get right?*

Anthony: *It shows people that you are smart because most people don't get the right answer right away.*

From these responses, it seemed that Anthony also viewed basic facts as important and did view speed as part of being good at mathematics. When discussing the purpose of tests, Anthony shared how it was important for seeing how much you have improved and to see how many groups you can move up or if you need to step back. This view of testing, that was shared by a large number of students in this study, influences students' mindsets according to Boaler (2013) who describes how students who are in lower groups come to perceive that they are not so good at mathematics. In the case of Anthony, this idea of grouping seemed to cause pressure on him to remain at the same level or move up and he was quick to explain how struggling and making mistakes was something that made him feel unsuccessful.

In discussing Anthony's family and mathematics, conflicting messages were uncovered:

Interviewer: *Can you tell me a bit about your family and mathematics? Do they like it?*

Anthony: *Yeah, my Mum does! She uses it heaps at work.*

Interviewer: *Ah, do you get to do lots of maths with her at home?*

Anthony: *A little bit but she keeps telling me to do it different ways.*

Interviewer: *What do you mean by different ways?*

Anthony: *I have a strategy that I stick to, but she gets me to change it because it looks messy. So she gets me to you put the numbers above the other numbers…[Goes on to describe algorithms].*

Interviewer: *Oh, algorithms. Do you get to do them at school to?*

Anthony: *Sometimes but I normally just add big numbers in parts like the tens and then the hundreds and do it Mum's way at home.*

While the teaching of these algorithms was done in trying to support Anthony's learning, it has created separate demands between school and home in his mind. In explaining the connections between the two mathematical strategies or processes, mathematics could be seen as universal rather than unique to the setting (Maxwell, 2001).

In sharing his experiences with mathematics in class and then explaining what a perfect mathematics class would be for him, Anthony described more traditional practices. This included smaller groups for those who struggle and being able to work in pairs or by themselves for those who are good at mathematics. When asked about if he likes working in groups, Anthony talked about how he prefers to work by himself, especially on hard questions where he struggles to explain his ideas and get through to others. Cobb (1994) highlights the of importance of reasoning ideas and how students can learn this skill through modelling by the teacher and classroom norms that hold all students accountable for the sense making of others.

Anthony did, however, value the communication of others when he becomes stuck during problems, commenting on how he asks his friends or the teacher for help. This was followed up with mention that he tries his best to work it out himself first, so his friends do not think he is just copying. From this semi-structured interview, it seems that Anthony's experiences both at school and at home had fostered more traditional views of mathematics which revolved around operations, speed, and being a more individual process for the most part.

6.3 Conflicting views of mathematics

The following analysis of an interview with Madison, a year 6 student who was identified as a struggling learner, demonstrates how mathematical dispositions and mindsets can change, given exposure to different classroom practices and tasks.

Beginning the interview, a discussion was had around Madison's views and feelings towards the nature of mathematics:

Interviewer: *What do you think mathematics is all about?*

Madison: *I don't know, it depends.*

Interviewer: *What does it depend on?*

Madison: *Like how old you are and where you are. Sometimes it's about learning and figuring out problems and sometimes you just need to sit and write stuff down or do sheets.*

Interviewer: *And how do you feel about mathematics?*

Madison: *It's okay. I really like hands on maths and doing some problems but it just depends on what we are doing.*

Maxwell (2001) discusses how feelings and emotions are not always fixed, with students often exhibiting positive or negative dispositions depending on the tasks they engage in or the practices they are exposed to. Madison went on to describe how she felt unsuccessful in mathematics when she could not make sense of the question or was not following what the teacher said. To her, these instances told her that maths is too hard and that she's not good at it. Hunter and Anthony (2011) found similar emotions from students in a more traditional classroom setting, sharing how students' enjoyment of mathematics and perceived capability hinged on their ability to make sense of what the teacher had explained. This highlights the idea of more inquiry-based practices, where students can ask questions and interact with mathematical concepts, rather than just listening and recording them.

Madison shared a dislike of activities that required speed such as basic facts or Maths Whizz, where she kept having to complete rapid recall questions. In discussing what the perfect mathematics class would look like, Madison also provided the following responses:

Madison: *You would get to do activities that aren't fast.*

Interviewer: *What sort of activities can you think of?*

Madison: *Like getting to solve a problem and you have time to test it. Normally I don't understand what I am trying to do and I run out of time.*

During this conversation, Madison described an activity she remembered from a previous year about Fibonacci numbers:

Interviewer: *Can you tell me a bit about this activity?*

Madison: *Yeah, we had pinecones and counted the number of spirals in them and the number matched up with the Fibonacci numbers!*

Interviewer: *Sounds awesome, why do you think you liked this activity so much?*

Madison: *Cause it was hands on and made sense! I went home and watched a YouTube video about how these numbers are everywhere, like in plants and the amount of bees and in spirals.*

Throughout discussing this activity, Madison became noticeably animated, showing her enjoyment. While being identified as a struggling learner, this conversation illustrates how visual, hands on tasks provide more opportunities for all students to succeed and removes the status differences that often exist between students (Boaler et al., 2016).

In discussing testing, Madisons offered the following ideas about their actual purpose:

Interviewer: *Do you feel prepared for tests?*

Madison: *No, there's never anything we have actually done!*

Interviewer: *So why do you think we do tests?*

Madison: *To find out what level we are.*

Interviewer: *What does level mean?*

Madison: *Level means how good you are. I don't like them!*

Madison, like most students in this study, also seemed to view her ability and capability in mathematics as being aligned with her test results and group placement. As seen from previous conversations with Madison, she enjoyed exploring mathematics and looking for connections, making assessment conditions, in which her future opportunities may have be based, non-favourable.

Finally, in a conversation on mathematics outside of the classroom, Madison talked about where she sees mathematics and the differences between home and school mathematics:

Interviewer: *Where do you see mathematics outside of school?*

Madison: *It's everywhere, like when you pay for things or in nature.*

Interviewer: *Does this mathematics that you see outside of school feel the same as the mathematics you do in school?*

Madison: *Not really. At school it's like adding stuff together and timsing [Multiplication] and maths outside of school is more looking at how things work and money.*

Interviewer: *Do you get to do mathematics with anyone in your family?*

Madison: *My sister helps me with home learning and she likes maths. It sort of makes me feel better about it.*

It is clear, through these conversations, that Madison's dispositions and mindsets were influenced by the range of tasks and practices that she had engaged in. Although her views grouping and testing seemed to match a more fixed mindset and was similar to many others in this study, there were examples of positive mathematical dispositions when provided with the opportunity to engage in more rich, visual tasks. This raises questions of whether over time, and through the use of more open, rich tasks, could Madison's positive mathematical disposition and mindset become more permanent?

6.4 The power of growth mindset in mathematics

In analysing an interview with Hailey, a year 5 student who was identified as being in the top group in her class, we have the opportunity to understand how growth mindsets can be fostered. Firstly, in a conversation on what mathematics is all about and her views towards it, Hailey offered the following ideas:

Interviewer: *What do you think mathematics is all about?*

Hailey: *Finding different ways to solve our problems. There's maths all in the world, there is math everywhere.*

Interviewer: And *how do you feel about mathematics?*

Hailey: *I've always had a passion for it. I really, really enjoy it and it comforts me.*

Interviewer: *Have you always felt this way about mathematics?*

Hailey: *When I started in Room 5. I got that passion for maths and I enjoyed it more as I got older.*

Clearly, Hailey had very positive views towards mathematics which, according to her, have only increased as she progressed through school. This conversation continued with Hailey's views on what makes her feel successful and unsuccessful in mathematics:

Interviewer: *What kind of things make you feel successful in mathematics?*

Hailey: *When I make a mistake or get something right, it doesn't matter!*

Interviewer: *Is there anything that makes you feel unsuccessful?*

Hailey: Not really, I'm always making progress.

From these comments, we can establish Hailey holds a growth mindset, viewing mistakes as a part of learning and ability as changeable (Boaler, 2013). Hailey's positive view of mistakes was examined further in a conversation about her reactions to them:

Hailey: *I don't mind mistakes at all. It is just one part of succeeding.*

Interviewer: *What does it tell you about your learning?*

Hailey: *It tells me that I'm working as hard as I can.*

Interviewer: *So, what normally happens when you make mistakes?*

Hailey: *I just think of all the things I've learned and see if I can solve it. Or I ask my friends or grown up to help me.*

Like many others in this study, Hailey had the confidence to ask for help from both her friends and her teacher after persevering. While Hailey demonstrated a growth mindset and positive views towards mistakes, the common theme of learning being confined to stages could still be found:

Interviewer: *Can you describe the perfect mathematics class to me. One where you would be work hard, be successful and be confident.*

Hailey: *You could choose any maths activity as long as it is around your stage of maths.*

As this conversation continued, however, these views were overshadowed by more open views of learning mathematics:

Interviewer: *Who would you work with in your perfect classroom?*

Hailey: *It would be your choice for who you work with.*

Interviewer: *What would happen if they were in a different group normally?*

Hailey: *It doesn't matter because there's tons of ways to learn maths and you can find ways of working together.*

In analysing a discussion with Hailey around her family and mathematics, and mathematics outside the classroom, we can identify possible sources of where her mindset and positive mathematical dispositions stem from:

Interviewer: *Can you tell me a bit about your family and mathematics? Do they enjoy it?*

Hailey: *Yes, definitely! They want me to work as hard as I can. My sister loves maths too. She's doing subtraction and she's only 5!*

Interviewer: *Do you get to see mathematics at home too?*

Hailey: *Yeah we play number games in the car and I see numbers on speed signs. Mum also taught me how to count money to see if I can afford things or not.*

Interviewer: *It sounds like you have lots of fun with maths at home, do you think there is a difference between maths at school and the maths you do at home?*

Hailey: *Sort of because I get to do maths in my own way at home. I do like talking with Mum and Dad about what we are doing at school and they ask me lots of questions and get me to try stuff out with them.*

It seems that through sharing experiences in mathematics with her family, Hailey had come to see it as a connected part of her life (Boaler, 2019). The mindset that Hailey has shared in this interview provides an example of how students can come to view mistakes in a positive way that maximises their ability to learn.

6.5 Summary

Through these interviews, more insight has been given to the findings of the previous chapters. The responses and conversations with these students interviewed also seemed to reflect the differences in dispositions and mindsets that were present throughout this study.

For most students, and indeed those interviewed, grouping and assessment practices had contributed to more traditional views that mathematics is taught in stages and groups can be used to define your ability. While Anthony described why he preferred working independently and not being able to easily communicate his ideas, all students seemed to value the opportunity to receive help from others when they became stuck. This reflects previous findings surrounding the importance of being able to ask questions, reason ideas, and work collaboratively. While some students saw speed as an important aspect of performing mathematical tasks, most, including the students interviewed discussed their dislike of tasks involving speed. Madison was able to communicate how these tasks did not allow her to think things through and test her ideas while she became animated and positive when describing more open, rich tasks. It seemed that the tasks these students engaged in gave conflicting messages about what it means to do mathematics.

Family seemed to play a supporting role for these students in how they viewed mathematics and the mindsets they had developed. While all described their parents trying to help them learn and be successful, those who shared their experiences and made connections between home and school seemed to be fostering more positive mathematical dispositions in their children.

Finally, a common stakeholder of learning at school, and one that could influence the experiences of students, was the teacher. In scaffolding students' social interactions, selecting tasks that engage and motivate, and communicating the idea that mathematics is not all about speed and levels, teachers can help develop positive mathematical dispositions and growth mindsets in their students.

Chapter Seven: Conclusion and Implications

7.1 Introduction

Through analysing collected questionnaires, student self-reflections, and semi-structured interviews, the previous chapters discussed students' mathematical dispositions and mindsets towards learning, and the factors that influenced these.

Drawing on these findings, Section 7.2 discusses the implications of this research in terms of fostering positive mathematical dispositions while Section 7.3 aims to develop existing ideas on how teachers and parents can foster growth mindsets in their students and utilise mistakes to support the learning of mathematics. The limitations of this study, and opportunities for future research are then outlined in Section 7.4 and 7.5, respectively, before concluding thoughts are given in Section 7.6.

7.2 Fostering positive mathematical dispositions

In this study, the importance of students developing positive mathematical dispositions and mindsets during year 5 and 6 was established, not only for their learning at school but also their engagement throughout later life. Several themes were identified and explored in relation to the mathematical dispositions that these students had developed.

Similar to the studies of Young-Loveridge et al. (2006) and Grootenboer and Marshman (2016), the students in this study had come to view mathematics as being about aspects of number and operations rather than any strand of maths or the exploration of patterns as detailed in The New Zealand Curriculum. While the ability to recall facts and answer questions correctly was a highly important skill according to these students, relatively few held the belief of speed being important with most showing a dislike for tasks involving speed. While this view of number and accuracy seemed to stem from the closed tasks these students engaged in, there was noticeably divided views on the enjoyment of these tasks. These closed tasks, according to Sullivan et al.

(2013), contribute to more narrow perceptions of mathematics and share little in common with what real mathematicians do. Many students, however, valued the opportunities to engage in more open, hands on activities and cited closed activities as being their least favourite. These open tasks, according to Boaler (2016), help make learning more accessible to more students and provide opportunities for students to reason their ideas, ask questions, and engage in productive mathematical discourse.

It seemed that the different tasks that these students engaged in gave conflicting messages about what it means to do mathematics as well as what students' roles were within the classroom. One role, which seemed to align with the closed, more teacher lead tasks, was that of being a passive receiver of knowledge, needing to listen and follow the teacher's instructions carefully in order to be successful. This was generally accompanied by the preference of working independently. In contrast to this was a large group of students who seemed to enjoy more open tasks and who saw their role as needing to understand concepts in their entirety through asking good questions, trying hard, and helping others. In requiring students to reason their ideas, ask questions, and support others, teachers are helping students to learn mathematics in a similar way to real life mathematics, and in turn, are setting them up to be lifelong learners (Boaler, 2016).

Students beliefs on the reasons for assessment and their feelings towards it illustrate the narrow perception of mathematics that testing can give. Central reasons for testing, according to these students, was to find out their levels, how much progress they had made, and determine their groups. In being used to form groups, assessment also may have indirectly affected the opportunities and experiences that some of these students had with mathematics. It seems from these responses that assessment seemed to share the idea that getting a good score and moving up levels was an important part of mathematics.

Utilitarian views of mathematics were common for the students in this study, seeing mathematics as useful for the future such as with employment, handling money, and future education rather than any major application here and now. These views seemed to be influenced through parents' interactions with students. Those students, whose parents shared positive experiences of mathematics and found connections between home and school, came to develop more positive dispositions and were able to see mathematics as more relevant to them and their lives.

7.3 Building growth mindsets and utilising mistakes to support learning

Throughout this study, the importance of having a growth mindset for learning mathematics has been highlighted. Both growth and fixed mindsets were identified in students, and through analysing questionnaires, semi-structured interviews, and self-reflections, there was an opportunity to examine where these mindsets, and consequent views on mistakes, came from.

Most of the students in this study indicated that they were good at mathematics, with an even larger number of students believing that anyone could be good at mathematics if they worked hard. While this may imply a growth mindset, there remained a common perception that grouping was used so students could learn with others of a similar ability and be taught strategies at their level or stage. This ability grouping shared the idea that students' ability was fixed to their group or stage which raises the question of what happens to those who are placed in lower groups or those in higher groups when they begin to struggle and make mistakes. The literature reviewed discussed the advantages of mixed ability grouping for allowing struggling learners the opportunity to hear higher order thinking, and allowing all students to think about their strategies in different ways (Diachuk, 2019).

The importance of the teacher was also illustrated, with their ability to develop a classroom culture that supports the sharing and exploration of mistakes without judgement. Although many students held the belief that mistakes are just a part of learning, there were divided feelings towards making them. Positive feelings towards mistake indicated their growth mindsets and previous positive experiences with them while feelings of frustration and embarrassment when making mistakes was cited by those who held more fixed mindsets or had others react negatively towards them in the past.

The teacher also played a key role in noticing and handling mistakes, with many students being unaware of their mistakes until the teachers' intervention. The teachers' reactions to these mistakes, while being supportive, fell into two different categories. Some students described how the teacher told them the correct solution, explaining why their first solution was not correct while some students described how their teacher helped them work through their mistakes without revealing the solution. In not revealing the solution but rather posing questions and extending students thinking, the teacher took full advantage of the mistakes, and in turn, helped students develop problem solving skills and growth mindsets.

Most students, despite how they noticed their mistakes, attempted to fix them themselves first. Several strategies were identified such as listening to other's ideas and asking for help from those around them. Similarly, most students chose to help others in a positive manner, either by explaining why the solution did not work, or by taking over the solution. In examining the learning that occurred from these mistakes, a range of different learning outcomes were identified which helps demonstrate the power of mistakes in learning mathematical concepts.

7.4 Limitations of the study

When interpreting the results of this study, it is important that the context of the case study, and therefore, the complexities of teaching and mathematics education, be considered. Although Punch and Oancea (2014) believe that through providing rich, thick description, readers can assess the similarities between the study and other situations; the uniqueness of this case, along with the small sample size, requires that care be taken when generalising these results. With the researcher having an insider role, measures were taken to ensure validity and reliability. There remains the possibility however, of details being overlooked and not included due to researcher 'blindness', being seen as what normally occurs in this setting rather than being noteworthy. While the exclusion of academic data was purposeful, it did limit the understanding of why some students seemed to enjoy certain aspects of mathematics along with other questions that were raised throughout the study. Finally, in the considerations to be made when interpreting the findings of this study, is the students' past experiences prior to the classroom settings described. While the scope of this study did not allow for the tracking of students' experiences in prior years, this idea of doing so is discussed further in the following section. With these considerations, this study can only offer to add emerging insight around how mathematical dispositions and growth mindsets may be developed in year 5 and 6 students and how mistakes may be utilised to support the learning of mathematical concepts.

7.5 Opportunities for further research

This study provided examples of how mathematical dispositions and mindsets have developed within a classroom setting. The students in this study had an exposure to a range of teaching practices and tasks, which at times seemed to give contrasting views on the nature of mathematics. To gain a clearer understanding of how teaching practices and tasks fully influence students' mathematical dispositions, it would be timely to conduct similar research in a more traditional classroom or more inquiry-based classroom rather than one with aspects of both.

While the students in this study have also demonstrated an overall view of mathematics being accessible and achievable for anyone, there remains the question of whether this would continue to be the case if students' perceived themselves as being struggling learners, rather than being good at mathematics. An investigation into whether students' mathematical dispositions are affected by their perceived ability would help further teachers and parents understand the impact of grouping practices and labelling students.

Finally, for some students, these views on mathematics seemed to be more fragile and able to be changed over time. For this reason, any future studies would benefit from observations over an extended period, examining how students' dispositions and mindsets change over time given different classroom settings and progression through their schooling.

7.6 Concluding thoughts

The purpose of this study was to explore the different factors that influence Year 5 and 6 students' mathematical dispositions and mindsets towards learning. Students responses to questionnaires, semi-structured interviews, and self-reflections offered insights into how they came to view mathematics and the mindsets they held. The evidence from this study suggests that when developing a classroom practice and culture to support the development of positive mathematical dispositions and mindsets, a number of considerations need to be made.

In engaging students with group worthy tasks that are open, relevant, and allow for productive struggle, teachers' help make learning accessible for more students and help them learn mathematics in a way that is similar to real life. Assessment and grouping practices were also seen as pivotal in the development of students' mindsets through the subtle messages given about their current ability and capability to learn. By utilising mixed ability grouping and positioning students to reason their ideas, ask questions, and holding high expectations, teachers help conceptual learning and provide struggling learners opportunities to make connections. In questioning students and deflecting responsibility back onto students when they were struggling, the teachers' extended students' thinking, and in turn, supported the development of growth mindsets. In examining the different realisations and discussions that occurred from mistakes, their value in learning mathematical concepts is illustrated.

Through analysing the semi-structured interviews, the importance of family views on mathematical dispositions was identified. Through involving students in everyday mathematics, encouraging students' learning from school, and finding ways to enjoy mathematics with their children, parents too were able to support the formation of positive dispositions in students and help them value mathematics as part of their life.

These findings acknowledge the complexities of mathematics education and contribute to a number of studies concerning how students view mathematics and the many factors that influence these views. From this, teachers and parents can be better equipped to foster positive mathematical dispositions and mindsets in students and prepare them for lifelong learning.

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Appendices

Appendix A: Student Questionnaire

Views on mathematics

What words can you think of that describe mathematics?

Your answer

Why do we learn mathematics?

Your answer

I am good at mathematics

	1	2	3	4	5	
Strongly disagree	○	○	○	○	○	Strongly agree

How do we know if someone is good at mathematics?

Your answer

Anyone can be good at mathematics

	1	2	3	4	5	
Strongly disagree	○	○	○	○	○	Strongly agree

Tasks and activities

What are your favourite mathematics activities?

Your answer

What are your least favourite mathematics activities?

Your answer

What are the most important things you have learned in mathematics?

Your answer

There should be only one way to solve a mathematics problem

	1	2	3	4	5	
Strongly disagree	○	○	○	○	○	Strongly agree

Groups

Why do you think you are in your group for maths?

Your answer

What is your job as a learner during mathematics?

Your answer

Where do you feel you learn best?

- [ ] Working alone
- [ ] Working with a buddy
- [ ] Working in a group
- [ ] Working with the whole class

Why do you feel you learn best there?

Your answer

Assessment

What words can you think of that describe tests?

Your answer

Why do you think we do tests?

Your answer

To be good at mathematics you need to solve problems quickly

	1	2	3	4	5	
Strongly disagree	○	○	○	○	○	Strongly agree

Maths outside of school

Where do you see mathematics outside of school?

Your answer

Mistakes and maths

Mistakes are just a part of learning mathematics

	1	2	3	4	5	
Strongly disagree	O	O	O	O	O	Strongly agree

How do you know when you have made a mistake?

Your answer

What do you do when you have made a mistake?

Your answer

Appendix B: Semi-Structured Interview Questions

1. What do you think mathematics is all about?

2. How do you feel about mathematics? Have you always felt this way?

3. What kind of things make you feel successful in mathematics? What kind of things make you feel unsuccessful in mathematics?

4. If you could give advice to someone who is going to be your year next year about mathematics, what would it be?

5. Can you describe the perfect maths class to me. One where you would be work hard, be successful and be confident.

6. When you are stuck in a mathematics problem, what do you normally do? Who helps you?

7. Do you feel prepared when you do mathematics tests? What do you feel it tests?

8. Tell me about your family and mathematics. Do they enjoy doing mathematics? How do you know?

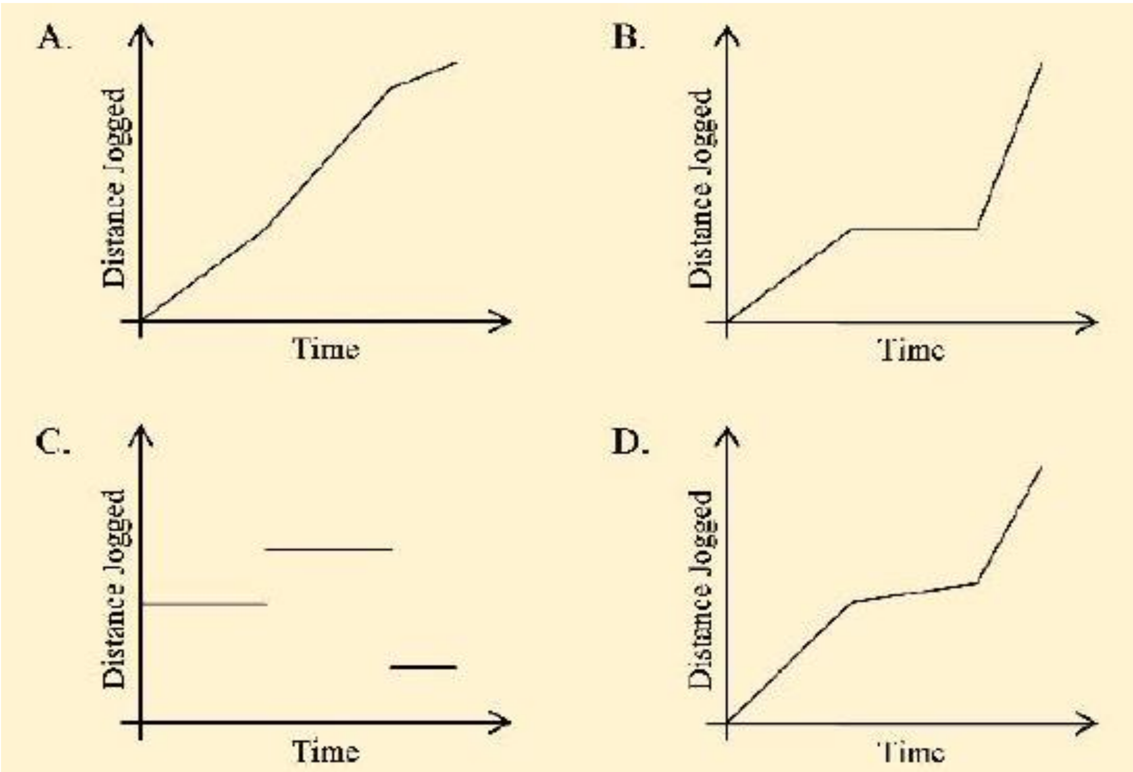
9. Is mathematics that you see and do outside of school feel the same to the mathematics you do in school?

10. How do you feel when you make a mistake in maths? What does it tell you? What happens after you make a mistake?

Appendix C: Problem Solving Task 1

Which graph represents the story?

Sally was training for cross country and was jogging along a road that went up a hill. She jogged at a comfortable, constant speed for the first 10 minutes where the road had a gentle slope. Sally then jogged at a slightly higher constant speed for the next 10 minutes because the road was flat. She jogged slowly for the last 5 minutes because this part of the road had a steep slope. Which of the following graphs could represent the distance that Zoey had jogged in relation to the number of minutes she had jogged?



Adapted from Lim (2014, p. 109).

Appendix D: Problem Solving Task 2

Ordering fractions, decimals, and percentages from smallest to largest.

- Each group member must have 3 cards which only they can touch or move.
- As a group you need to order the fractions, decimals, and percentages cards from smallest to largest.
- Each group member must explain to the group why they placed their card in that place and convince everybody that is where it should go.
- If you think there is a mistake and a card is in the wrong place, you need to justify why you think that and convince the owner of that card using mathematical language.
- You are allowed time to think of a way of convincing the group of your card's placement. This can be done with a diagram, talking, or with materials.

66.6%	0.125	2/3
0.85	90%	1
1/2	0.13	2/1
25%	40%	5/6

Appendix E: Student Self-Reflection

- Choose which questions you would like to answer from the list.
- Press record on the ipad.
- Read the question number aloud followed by your answer
- Press stop when you have finished answering your questions

1. What mistake did you make?

2. What was your thinking when you gave that solution?

3. How did you know you made a mistake?

4. How did you feel when you made a mistake?

5. What did you do when you made a mistake?

6. What did your group do when you made a mistake?

7. What did the teacher do when you made a mistake?

8. Did you learn anything from your mistake?

9. Do you think anyone else learned anything from your mistake?

10. How do mistakes make you feel about mathematics?

Appendix F: Parent and Student Information and Consent form

[LETTERHEAD]

Shaping Student Views on Mathematics:

Influences on Year 5 and 6 Students' Mathematical Dispositions and Mindset towards Learning

Parent Information Sheet [TO KEEP]

Dear Parents/Caregivers

My name is Andrew Johnson and I have been teaching at [redacted] School for five years, mostly at year 5 and 6. This year I am lucky enough to have part time study leave to complete my master's degree while still being able to teach STEM (Science, technology, engineering, and mathematics) in classrooms across the school.

As part of my study, I am doing my thesis on students' mathematical dispositions, which include their views towards mathematics, how they use mistakes to build understanding in mathematics, and how they view themselves as a learner of mathematics. The aim of this research is to help build on existing ideas around how teachers and family can build positive mathematical dispositions in students and, in turn, further support achievement. I have chosen year 5 and 6 students for this study as it is an important age for the development of these dispositions.

I would like to include your son/daughter in this research. Participation in this study requires students to complete a questionnaire which aims to elicit their perceptions of mathematics and identify possible sources of these perceptions. This questionnaire takes between 10-15 minutes and will be completed on a google form at school. In addition to the questionnaire and in order to gain more detailed information, I would also like to interview a few students about their perceptions of mathematics and how they view themselves as a learner of mathematics.

There is also the opportunity for students to voice record their thoughts on any mistakes that they make or find during a mathematical task and the learning which occurred as a result. This voice recording is voluntary and is up to the student to select questions from a list (If any) that they would like to answer.

All the information collected will only be used for this research and will be stored securely. The information will be destroyed after the completion of the research. In order to maximise confidentiality and anonymity, your child's name and school will not be used in this research, with only non-identifying information used in reporting.

Please note that you have the following rights in response to my request for your child to participate in this study:

- decline your child's participation;
- withdraw your child from the study at any time;
- you may ask any questions about the study at any time during your child's participation;

- your child provides information on the understanding that your child's name will not be used or identified;
- be given access to a summary of the project findings when it is concluded;

If you have any further questions about this study, you are welcome to contact me personally at any time:

Andrew Johnson: Phone [redacted] Email [redacted]

Or contact either of my supervisors at Massey University:

Prof Roberta Hunter: Phone (09) 414 0800 ext. 43530. Email R.Hunter@massey.ac.nz
Dr Jodie Hunter: Phone (09) 414 0800 ext. 43518. Email J.Hunter1@massey.ac.nz

This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named in this document are responsible for the ethical conduct of this research.
If you have any concerns about the conduct of this research that you want to raise with someone other than the researcher(s), please contact Professor Craig Johnson, Director (Research Ethics), email humanethics@massey.ac.nz

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[LETTERHEAD]

Shaping Student Views on Mathematics:

Influences on Year 5 and 6 Students' Mathematical Dispositions and Mindset towards Learning

Consent Form: Parent/Guardian [RETURN TO SCHOOL]
This form will be held for a period of 5 years.

I have read the information sheet and have had the opportunity to ask questions about the study. My questions have been answered to my satisfaction, and I understand I may ask further questions at any time.

I agree to ______________________________ participating in this study under the conditions outlined in the Information Sheet including being interviewed and audiotaped.

Parent/Guardian Signature: _________________________ **Date:** _________________

Parent/Guardian Name: ____________________________________