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Mathematical Modelling of Bread Dough Sheeting

A thesis presented in partial fulfilment of the requirements for the
degree of

Doctor of Philosophy in Food Engineering

at Massey University, Palmerston North, New Zealand.

Richard John Love

2003



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This is to certify that the research carried out for my Doctoral thesis entitled
“Mathematical Modelling of Bread Dough Sheeting”, in the Institute of Food,
Nutrition and Human Health, College of Sciences, Massey University, Palmerston
North, New Zealand is my own work and that the thesis material has not been used in
part or in whole for any other qualification.

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Abstract

Bread dough sheeting is an important operation in the bread making industry. The process involves the passing of a mass of dough between two, or more, rotating rollers. The function of sheeting can be: i) to shape the dough, ii) to laminate layers of product together or iii) to develop the gluten network, which gives dough many of its properties.

The model developed, in this thesis, describes the dough sheeting process using a continuum mechanics approach, solved using a perturbation technique. The bread dough rheology is described using the Criminale-Ericksen-Filbey (CEF) viscoelastic constitutive equation. It was thought that this approach may model the process better than the viscous models used elsewhere. The perturbation technique, used in solution, meant that the model remained computationally swift. The CEF equation was used as it is reasonably simple mathematically and measuring the required dough properties was quite straightforward, although, as was discovered, reproducibly measuring the properties of bread dough is never easy, not least because of the history dependent nature of bread dough. Some important assumptions made, in this model, on the basis of literature and preliminary experiments, were that: the process is two-dimensional; the process is at steady state; the process is unaffected by inertia, temperature or gravity; some parts of the process (conveyor belt speeds, for example) are unimportant; and that the dough is incompressible.

It was found that such a model can be used to predict the exit height of the dough, the forces and torques experienced by the rollers, and velocity and pressure profiles in the dough. The predictions were qualitatively consistent with validation data gathered on a pilot plant sheeter, but there were some large quantitative inaccuracies, particularly with the exit height prediction (as there is with viscous models). The inaccuracies suggest that such an approach to modelling bread dough sheeting misses some important facet of the process, possibly the compressibility of the dough. That is, a viscoelastic description of the process material will not, alone, lead to a complete model of the dough sheeting process.

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List of Symbols Used

The following symbols and their units are used in this thesis. Definitions are also given in the main text.

a, b, c, d	constants	–
C_p	specific heat capacity	$\text{kJ kg}^{-1} \text{K}^{-1}$
De	Deborah number	–
DS	digital signal	–
E	energy input during sheeting	kJ kg^{-1} or Wh kg^{-1}
F	force	N
g	acceleration due to gravity	m s^{-2}
G	modulus of elasticity	Pa
G^*	complex modulus of elasticity ($= G' + iG''$)	Pa
h	height of dough sheet	m
H	gap between viscometer plates	m
I	2 nd moment of inertia	m^4
i, j	index variables	–
K	power-law consistency index	Pa s^n
$l_{ij} \left(\dot{l}_{ij} \right)$	elongation tensor (rate of elongation tensor) (section 4.2)	(s^{-1})
m	mass	kg
m	power law flow behaviour index (1 st normal stress coefficient)	–
n	power law flow behaviour index	–
p, P	pressure	Pa
R	radius (roller)	m
R	Pearson correlation coefficient	–
Re	Reynolds number	–
s	Laplace co-ordinate variable	–
t	time co-ordinate	s
T, M	torque	N m
U, v, w	x, y, z velocity components	m s^{-1}
u_r	tangential velocity at roller surface	m s^{-1}
W	width of dough sheet	m

x, y, z	co-ordinate directions	m
\bar{X}	Laplace transform of variable X	–
Θ	reduction ratio $\left(= \frac{h_i}{\delta} \right)$	–
Ω	angular velocity of parallel plate viscometer	rad s ⁻¹
Ψ_1, Ψ_2	1 st and 2 nd Normal Stress coefficient	Pa s ⁻²
α, θ	angle	°
β, x	response coefficient for coded variable x	–
δ	nip gap between the rollers	m
δ_{ij}	Kronecker delta	–
ε	geometry ratio $\left(= \sqrt{\frac{\delta}{R}} \right)$	–
$\gamma_{ij} (\dot{\gamma}_{ij})$	strain tensor (rate of strain tensor)	(s ⁻¹)
η	viscosity	Pa s
η^*	complex viscosity ($= \eta' + i\eta''$)	
λ	time constant	s
μ	friction coefficient	–
ρ	density	kg m ⁻³
σ_{ij}	total stress tensor	Pa
$\tau_{ij} (\dot{\tau}_{ij})$	deviatoric or shear stress tensor (rate of ...)	Pa
ω	oscillatory strain frequency	Hz