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COORDINATION IN VEHICLE ROUTING

A THESIS PRESENTED IN PARTIAL FULFILMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
IN
OPERATIONS RESEARCH
AT
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CATHERINE MARGARET RIVERS
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ABSTRACT

Coordination involves the re-deployment of payload between depot and customer, and includes split deliveries, load transfers and load swapping, facilitated by the establishment of coordination sites at strategic locations. Real-world coordination includes mid-air refuelling, the use of temporary replenishment sites, trailers left for later uplift by their towing vehicles, bulk re-suppliers travelling to field operatives, fleet re-supply, and couriers swapping loads on the side of the road.

This thesis models the coordination process and investigates the basic types of coordination in single depot, pure delivery systems in both the Euclidean plane and the rectilinear grid network. Strategies are developed for dealing with dynamic situations in the rectilinear grid, which are based on the pre-processing of scenarios in order that dispatchers may select a suitable response from an existing selection at the time that dynamic values are revealed.

In addition, a procedure is suggested that reduces the number of edges and vertices of a rectilinear grid to those that may be useful within a coordination hull.
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Order of application of coordination.

Quality of initial solution.

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#### 6.1.1 Dynamic and stochastic vehicle routing

#### 6.1.2 Dynamic coordination

#### 6.2 Problem description

#### 6.3 Nomenclature

#### 6.4 DC1 Only time is known

- Time constraint
- Multiple placement
- Problem generation
- Incorporation of coordination
  - Routing scheme A
  - Routing scheme B
  - Routing scheme C
  - Routing scheme D
  - Routing scheme E
  - Routing scheme F
  - Routing strategy for DC1

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## Glossary

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<th>Term</th>
<th>Definition</th>
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<tbody>
<tr>
<td>Centroid</td>
<td>Geographical centre, weighted by population.</td>
</tr>
<tr>
<td>Close list</td>
<td>List of vehicles that approach within the near radius of each other, and the time step at which this happens.</td>
</tr>
<tr>
<td>Commodity class</td>
<td>Type of payload.</td>
</tr>
<tr>
<td>Coordinated system</td>
<td>A routing and scheduling system to which coordination has been applied.</td>
</tr>
<tr>
<td>Coordination</td>
<td>A relaxation of the standard VRP in which the load for a customer is allowed to vary its integrity or carriage before arrival at that customer.</td>
</tr>
<tr>
<td>Coordination hull</td>
<td>Area of a graph within which lie all nodes and depots and the region in which coordination can occur.</td>
</tr>
<tr>
<td>Coordination point</td>
<td>Location at which coordination may occur.</td>
</tr>
<tr>
<td>Coordination site</td>
<td>Location at which coordination does occur.</td>
</tr>
<tr>
<td>Cul de sac</td>
<td>A section of path that has an intersection at only one end.</td>
</tr>
<tr>
<td>Delivery time</td>
<td>The time required to deliver a customer's demand - a linear function of the demand and the distance travelled to reach the customer's node.</td>
</tr>
<tr>
<td>Demand</td>
<td>The requirement by a node of goods.</td>
</tr>
</tbody>
</table>
Edge
A link between two vertices in a graph.

Essential point
Node or depot. Such a point must not be eliminated from a graph.

Existing route
A route that has been identified without recourse to a change mechanism, e.g. load swapping, that may be applied.

Goods
Items that are delivered to customers. May be divided into different types.

Graph
A combination of nodes, vertices and edges representing a system of customers, depots and vehicle routes.

Grid reduction
Removal of non-essential edges and vertices from a rectilinear grid.

Horizon
Temporal limitation.

Load plan
Result of routing and scheduling. The final assignment of nodes to routes and to vehicles.

Made edge
An edge that has been created, i.e. incorporated into a tour, usually after other edges.

maxVehCap
Variable representing maximum vehicle capacity.

maxVehDist
Variable representing maximum vehicle distance.

Maximum vehicle capacity
The maximum amount of payload that can be carried at any one time by a vehicle.

Maximum vehicle distance
The maximum distance permissible by law (or by other means) that any one vehicle may travel during one work cycle.
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metastable state</td>
<td>A transition state between two stable states.</td>
</tr>
<tr>
<td>Multi-routed vehicle</td>
<td>A vehicle that services more than one route.</td>
</tr>
<tr>
<td>Node</td>
<td>Customer, represented as a location on a two-dimensional graph.</td>
</tr>
<tr>
<td>Orientation</td>
<td>An indication of the direction of travel of a vehicle around a route.</td>
</tr>
<tr>
<td>Path</td>
<td>The actual locus of a vehicle, including nodes visited and all intervening points.</td>
</tr>
<tr>
<td>Phase space</td>
<td>The phase space of a system is the space of all possible states or behaviours.</td>
</tr>
<tr>
<td>Pre-determined</td>
<td>Decided in advance.</td>
</tr>
<tr>
<td>Near radius</td>
<td>Distance from node or path that is investigated for coordination possibilities. [§4.8]</td>
</tr>
<tr>
<td>No Choice String</td>
<td>A section of a path the has no intersections except at the ends.</td>
</tr>
<tr>
<td>Nodal zone of influence</td>
<td>The area defined by a minimum distance from a node within which coordination has no significant effect.</td>
</tr>
<tr>
<td>Non-coordinated</td>
<td>Something to which coordination has not been applied.</td>
</tr>
<tr>
<td>Payload</td>
<td>A generic term for anything carried by a vehicle that can be collected and/or delivered.</td>
</tr>
<tr>
<td>Rectilinear grid</td>
<td>A mesh that consists only of straight lines intersecting at right angles.</td>
</tr>
<tr>
<td>Routing scheme</td>
<td>Result of routing and scheduling for a system of nodes, depots, demands and vehicles.</td>
</tr>
</tbody>
</table>
Route structure  The actual nodes attended (even if not serviced) by each vehicle, in the order in which they are visited.

Service  To satisfy the demand (of a node).

Spatial  Concerned with distance and physical location.

Split delivery  The demand of a node is delivered in batches, possibly by more than one vehicle (at the same or different times) or by the same vehicle at different times. Where split deliveries are permitted, the demand for a particular node could even be split en route, carried by different vehicles, re-combined and delivered intact to the node.

Surplus table  Table indicating the differences between actual payloads and/or distances and potential (capacity) values.

Temporal  Concerned with time.

Threshold  Spatial limitation.

Transfer  The movement of goods from one location to another.

Uplift  Collection by a vehicle, e.g. of goods.

Vehicle  Any object capable of carrying payload, e.g. truck, plane, bus, van.

Work cycle  The length of time in which vehicles may be away from the depot. At the beginning of the work cycle, vehicles leave the depot to service nodes; at the end of the work cycle, all vehicles must have been returned to the depot. Normally has the value of one working day.