STOCHASTIC PORTFOLIO PROGRAMMING,

COMPETITIVE MARKET EQUILIBRIA,

AND MARKET PORTFOLIOS AND RISK PROFILES:

A NEW ZEALAND CAPITAL MARKET ANALYSIS

A thesis presented in fulfilment of the requirements

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Dedicated to the memory of my mother Mary Isabel Young (nee Pickering)

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ABSTRACT

Mainstream modern portfolio theory has developed around the portfolio selection and asset pricing models of Markowitz' mean-variance criterion, the Capital Asset Pricing Model, Arbitrage Pricing Theory and, more recently, the models of continuous-time finance.

In the early 1960's Paul van Moeseke developed a model of asset allocation under risk conditions and, in the first instance, this thesis is a restatement of this model. To date this model has been largely overlooked by mainstream finance but it has several significant features in its favour. The model explicitly determines the risk profiles of particular financial markets by focussing on the marginal return to these markets and equating this marginal return to the investment dollar's marginal cost. As marginal cost will differ between investors the model allows for a heterogeneous investor base. Another feature of the model is that it has application across the entire risk spectrum. This thesis discusses the Moeseke model within the framework of modern portfolio theory and provides extensions to this model.

In presenting the model, attention is given to the development and major criticisms of asset pricing models and portfolio selection techniques in general. Extensions to the model incorporate the monetary policy procedures used in New Zealand since the late 1980's and consider the application of the model in times of negative real returns. This

thesis also discusses the relationship between the Moeseke model and the Arrow-Debreu model of general economic equilibrium.

A major empirical application of the model is undertaken for New Zealand's capital markets to determine the value and stability of their risk profiles. It is found that the risk profile of the New Zealand stock market is similar to that found previously for the United States and Canada with a high degree of stability. Risk profiles for the fixed interest market and the managed funds industry are also estimated. The determination of the marginal cost of the investment dollar for individual investors, institutions and international investors investing in New Zealand's capital markets is a key component for the model's application. A process for estimating these marginal costs is proposed together with these estimates.

This thesis argues that the Moeseke model and the extensions have a useful contribution to make to the modern portfolio analysis and selection process.

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In 1982 I had the privilege of meeting and studying under Professor Paul van Moeseke when I took his Advanced Economic Theory paper at Massey University. This turned out to be one of the most significant events of my academic life and the introduction to advanced economic theory and stochastic portfolio programming that I acquired from Paul at this time formed the basis for most of my subsequent academic endeavours. Paul was my supervisor for this thesis until his departure from Massey University and I owe him a considerable debt. For his support, guidance and patience I shall always be grateful.

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CHAPTER ONE

THE PROBLEM AND ITS SETTING

1.1 Introduction

In his Nobel Foundation essay (1990) H. Markowitz summarised his work on portfolio selection as being concerned with how an optimising investor should behave[1]; a normative model. The Capital Asset Pricing Model developed by Sharpe[2], Lintner[3] and Mossin[4], on the other hand, was concerned with the economic equilibrium resulting from all investors behaving as Markowitz had proposed; a positive model. These works have formed the basis of modern portfolio theory.

There have been many other contributions to modern portfolio theory but there is one specific individual contribution and one specific area of contribution that require mention in this introduction as they are fundamental to this thesis. The specific individual contribution was that of P. v. Moeseke[5] who developed a competing framework for portfolio selection to that developed by Markowitz. A crucial part of his work was the introduction of the concept that in any competitive capital market there will be an investment strategy, represented by a market portfolio, where the marginal return to the budget dollar will equal its marginal cost. The level of risk

[1] Markowitz, H. M., Foundations of Portfolio Theory. The Journal of Finance, V. 56, 1991, pp 469 - 477.

[2] Sharpe, W.F., Capital Asset Prices : A Theory of Market Equilibrium Under Conditions of Risk. The Journal of Finance , V. 19, 1964, pp 425-442.

[3] Lintner, J., The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. Review of Economics and Statistics, V.47, 1965b, pp 13-37.

[4] Mossin, J., Equilibrium in a Capital Asset Market. Econometrica, V.34, 1966, pp 768-783.

[5] Moeseke, P. v., Stochastic Linear Programming: A Study in Resource Allocation Under Risk. Yale Economic Essays, V.5, 1965a, pp 196 - 253.

aversion appropriate for investing in this particular market portfolio can be seen as the natural level of risk aversion, or the risk profile, for that particular capital market.

The specific area of contribution referred to is that of the integration of finance theory, in particular modern portfolio theory, into general economic equilibrium theory which was effectively achieved in the mid 1980s. Many theoreticians have made a contribution in this area, in particular D. Duffie[6] who has also produced a comprehensive summary of this work[7].

The major aim of this thesis is to study the risk profiles of New Zealand's capital markets after the time of this country's financial deregulation, which began in 1984, within the framework of the model developed by Moeseke. In doing so a general solution to the portfolio problem for funds invested in New Zealand's capital markets will be determined for both the private investor and the financial institution. Consideration will also be given to a portfolio solution for offshore investors. In developing a comprehensive framework for this analysis some extensions to the original model will be considered and these extensions will be detailed later in this chapter.

As part of the presentation of the Moeseke model consideration will be given to the relationship between this model and the CAPM, which itself has been integrated into general economic equilibrium theory.

6 Duffie, D., Competitive Equilibrium in General Choice Spaces. Journal of Mathematical Economics, V. 14, 1986, pp 1 - 23.

7 Duffie, D., Security Markets, Stochastic Models, Academic Press, 1988.

1.2 Statement of the Problem

This research is, foremost, an empirical study of the risk profiles of New Zealand's capital markets (debt and equity) from the viewpoint of both New Zealand investors and offshore investors within the context of modern portfolio theory. This should be of considerable interest to practitioners as the risk profile of a particular market will dictate the risk nature of the investors participating in that market. The stability of these risk profiles is clearly of interest also. Through determining these risk profiles, a general solution to the portfolio problem for funds invested in New Zealand's capital markets will be obtained. For the most part the approach will be of a generic nature though there are certain areas where the application of the model will be specific to the New Zealand situation, such as with tax regimes and monetary policy objectives and implementation.

1.3 Statement of the Subproblems

1.3.1 The First Subproblem

The first subproblem is to address the well documented criticisms and limitations of the basic portfolio selection models. In general the literature has concentrated on the Markowitz model and the Capital Asset Pricing Model (CAPM), and there have been numerous criticisms of both these models together with extensions developed to address some of these criticisms. The focus within the first subproblem is on those criticisms and limitations that also relate directly to the original model as developed by Moeseke, namely homogeneous programming by the truncated minimax criterion. These criticisms and limitations will be addressed primarily in Chapter 2 of this thesis.

The criticisms and limitations to be addressed are as follows:

- the nature of the distribution of returns,
- mean, variance and covariance estimation,
- the definition of the riskless asset and the risk free rate of return,
- nominal returns and real returns,
- the allowance of short sales,
- taxes and transaction costs,
- market segmentation,
- continuous-time trading.

1.3.2 The Second Subproblem

The second subproblem concerns the relationship between modern portfolio theory and economic theory in general and the relationship between the Moeseke model and general economic equilibrium theory in particular. As stated earlier, it has been shown by Duffie[8] and others that portfolio theory equilibria can be viewed as a subset of general economic equilibria.

[8] Duffie, D., 1986, op. cit.

Discussion of the Moeseke model within the framework of general economic equilibria will be undertaken.

While this is the main focus of the second subproblem there are three other aspects to the problem that warrant consideration. First, the relationship between the resource allocation problem in economics and the portfolio problem (money resource allocation in capital markets) in finance requires investigation, particularly in regard to the asset pricing mechanisms at work in each case. Second, the degree of competition plays an important role in general economic equilibria, but capital markets fail all three criteria of perfect competition in that financial institutions are neither infinitesimal nor perfectly informed and entry is normally subject to some form of regulation.[9] Third, as the CAPM can be incorporated into general economic equilibrium theory, the relationship between this model and the Moeseke model warrants investigation. This subproblem is addressed in Chapters 3 and 4 of this thesis.

1.3.3 The Third Subproblem

The Moeseke model defines an optimal solution, or market equilibrium, where the marginal return to the budget dollar equals its marginal cost. Marginal cost itself will differ among investors, in particular between private investors, and financial institutions.

9 Moeseke, P. v., Existence Theorems for Imperfect Capital Markets. Massey Economic Papers, V. 4, 1986, p 58.

The third subproblem relates marginal cost to the riskless asset or risk free rate of return referred to in subproblem one. In the absence of a riskless asset marginal cost may equal some minimum risk rate of return for private investors through to some margin above the minimum risk rate enabling financial institutions to raise deposits in the capital markets. Consideration of the marginal cost of the investment dollar for different types of investors is considered. A further extension is also required for instances where this minimum risk rate of return is negative, a situation which can occur for real values when inflation rates are high. This subproblem is addressed in Chapter 5 of this thesis.

1.3.4 The Fourth Subproblem

The fourth subproblem is concerned with the impacts of changing fiscal and monetary policy on solutions to the portfolio problem. Techniques have been developed for including open market operations, discount rates and reserve ratio requirements in the portfolio problem, but in the case of New Zealand there are no reserve ratio requirements and monetary policy is implemented by targeting the short-term wholesale interest rates via control of the cash required by banks to settle the daily transactions between the Government, the central bank, named the Reserve Bank, and themselves. These approaches to monetary policy require incorporation into the model.

The primary fiscal consideration relates to taxation regulation but the issue of country risk levels is also of interest. This subproblem is addressed in Chapter 5 of this thesis.

1.3.5 The Fifth Subproblem

The fifth subproblem relates to finding a solution to the offshore investor's portfolio problem. Offshore investors will generally face different risks to New Zealand investors, mainly on account of exchange rate considerations and will also face differing tax liabilities.

Also consideration needs to be given to relative real and nominal interest rates between countries and the risk premia attached to New Zealand's capital markets. This subproblem is also addressed in Chapter 5 of this thesis.

1.3.6 The Sixth Subproblem

The sixth subproblem is to incorporate the decisions and findings of the previous subproblems into the portfolio selection model in order to place objective values on the risk profiles of New Zealand's capital markets and, in doing so, obtain a general solution to the portfolio problem from the viewpoint of both New Zealand investors and international investors. The short-run and long-run stability of the risk profile values of the capital markets will also be examined in the study to determine whether changes occur. Shifts away from long-run equilibrium values, if found, will be examined in terms of their sustainability. This subproblem is addressed in Chapters 6 and 7 of the thesis.

1.4 Hypotheses

Before specifying the hypotheses for this thesis it should be noted that a number of underlying premises are required for consistency with both modern portfolio theory and economic theory.

The first of these is that New Zealand's capital markets display risk and return characteristics consistent with modern portfolio theory. That is, for risk averse investors, higher risk will be accompanied by higher expected return. Note that the entire market does not have to conform to this requirement as parts of the market may well be suited to risk takers. The quantification of risk and return is discussed in Chapter 2 of this thesis.

The second premise is that solving the money resource allocation problem under conditions of risk is consistent with solving any resource allocation problem in economics. Specifically, an optimal solution is found in competitive markets where marginal cost equals marginal return.

The third premise is that the marginal cost of investing in any risky portfolio can be determined for all investors both individually and in the aggregate.

The fourth premise is that offshore investors will differ from New Zealand investors in relation to risk and taxation considerations leading to a heterogeneous investor base. Note, however, that the Moeseke model allows for a heterogeneous investor base even in a closed market environment.

Given the above, the first hypothesis is that capital markets in general will have determinable and stable long-run risk profiles. A secondary hypothesis is that, while shifts away from the long-run risk profile of a capital market are probable, these short-run shifts will be temporary.

1.5 Delimitations

This study will not attempt to find a global solution to the portfolio problem but will separate the allocation of money resources in New Zealand's capital markets from the allocation of money resources in the capital markets of other countries for both New Zealand and offshore investors. That is, the New Zealand investment decision will be viewed separately from all other investment decisions but the conditions for both local and offshore investors to invest in New Zealand will be examined. Consideration of exchange rate risk will be required but direct investment in specific currencies other than the New Zealand dollar, hedged or unhedged, is excluded.

The New Zealand investment decision will be solved in the broader sense. It is not the purpose of the study to attempt to identify specific individual investments in which to invest nor to give detailed consideration as to whether or not this can even be done. The study will only consider investment in the major financial asset classes being the equity, bond and money markets and while specific portfolios will attach to specific risk profiles historically, it is the marginal performance of the asset classes which is of most importance, not the makeup of the specific portfolios. The risk profiles attached to investment in New Zealand's capital markets through managed funds is also considered.

1.6 Assumptions

The assumptions given here are assumptions that are required for the study to have validity for future decision making.

The first assumption is that active asset allocation decision making will continue to be an important role of the fund manager or investor as he/she invests the funds that become available to him/her.

The second assumption is that equity, bond and money markets will continue to be the major classes of financial assets in which fund managers and investors will invest.

The third assumption is that current monetary control techniques and fiscal objectives in New Zealand will continue for the foreseeable future.

The fourth assumption is that the free flow of capital between New Zealand and the international investing community will continue for the foreseeable future.

1.7 Importance of the Study

Solving the portfolio problem has become the task of a large army of fund managers and investors worldwide and is gaining in importance in New Zealand as current Government action encourages increased levels of private savings.

The concepts of risk and return are clearly understood but their magnitude and the factors affecting these magnitudes are not. By applying an appropriately modified version of the chosen portfolio selection model within the New Zealand context a clearer understanding of the procedures required for solving the portfolio problem will be obtained. In particular, objective values for the risk profiles of New Zealand's equity and fixed interest markets will be derived together with an appreciation of their stability, both short-run and long-run. Discussion of the relationship between the model used in this study and the general economic equilibrium theory framework will help to provide a sound underpinning for this study.

CHAPTER TWO

ASSET PRICING MODELS AND PORTFOLIO SELECTION TECHNIQUES : A FRAMEWORK AND CRITICISMS

2.1 Introduction

The purpose of this chapter is, foremost, to discuss the development of asset pricing models and portfolio selection techniques and to provide the framework for their analyses. By providing this framework justification for the Moeseke model used in the analysis of New Zealand's capital markets has clear reference. The model itself is presented in detail in Chapter 3. As is the case with economic models in general, models of portfolio analysis will normally belong to one of two groups: normative or positive. Normative models are concerned with norms, the rules that need to be addressed in order that satisfaction is maximized. Positive models, on the other hand, require internal consistency and set out to explain how people behave. The Moeseke model is strictly a positive model. Criticisms of asset pricing models and portfolio optimisation techniques are considerable and in general these criticisms are dealt with in this chapter. It should be noted at this point, however, that many of the criticisms in this topic area centre around degrees of precision. Capital market returns are clearly stochastic processes with their perceived likelihood of realisation largely determining asset prices. It is unlikely that any model will ever explain this process perfectly so the usefulness of any particular model becomes one of degree. The following quotation from Markowitz[10] addresses this point in a most useful way given recent advances in

[10] Markowitz, H.M., 1991, op. cit., p 471.

the theory of choice and asset pricing and its relation to general economic equilibrium theory. This latter topic is introduced in Section 2.7 of this chapter and is taken up in detail in Chapter 4.

> "...we prefer an approximate method which is computationally feasible to a precise one which cannot be computed. I believe that this is the point at which Kenneth Arrow's work on the economics of uncertainty diverges from mine. He sought a precise and general solution. I sought as good an approximation as could be implemented. I believe that both lines of enquiry are valuable."

While this thesis requires a strong theoretical basis its main objectives require a computationally feasible application.

Section 2.2 describes our investor(s) and relates their decision making processes to statistical theory with particular reference to the work of L.J. Savage.[11] Section 2.3 considers the contribution of H. M. Markowitz to modern portfolio theory and introduces the contribution of P. v. Moeseke. As noted earlier Moeseke's model (homogeneous programming by the truncated minimax criterion) is described in detail in Chapter 3. Section 2.4 is a general discussion on the CAPM and its extensions with Section 2.5 discussing Arbitrage Pricing Theory. As there is any amount of literature on these two models and their extensions the purpose of these sections is primarily to give focus to the relationship of the Moeseke model to these asset pricing models and portfolio selection techniques in general. These sections also help to identify the areas

11 Savage, L. J., The Foundations of Statistics. New York, Wiley, 1954.

of criticism which are discussed in Section 2.6. Section 2.7 is an introduction to recent advances in the theory of choice and asset pricing and its relation to general economic equilibrium theory, a topic which is covered in greater detail in Chapter 4 where the relationship of the Moeseke model to general economic equilibrium theory is also considered.

2.2 Subjective and Objective Assessment of the Probability of Likely Outcomes and Savage's Personalistic Theory of Decision Making

An investor or investors must make a choice from among a number of available options in the face of uncertainty. This is so even if the investor or investors only include benchmark products in their opportunity sets. The choices made may impact on current asset prices and current asset prices themselves will certainly impact on the choices made. As noted by Skiadas[12], the choices made, or portfolios selected, will be decided upon not only on the basis of current prices, but after the consideration of the many mutually exclusive and exhaustive scenarios that present themselves and the evaluation of each possible course of action given the occurrence of each scenario. The combination of the conditional valuations then lead to the unconditional choice. This first stage involves the formulation of conditional preferences which is then followed by the aggregation of these conditional preferences.

12 See Skiadas, C. N., Advances in the Theory of Choices and Asset Pricing. PhD Thesis, Stanford University, 1992, for a detailed discussion on conditioning and aggregation of preferences.

These are reasonable assumptions regarding the decision making processes of the investor but a number of issues are raised by this short description that require elaboration or explanation.

First there will be an adherence to the convention that uncertainty implies a lack of knowledge of the distribution of outcomes, or an approach to choice which does not take this entire distribution of outcomes into account.[13] Risk, on the other hand, implies that the distribution of outcomes, their probabilities, are known and are taken into account in the decision making process. In general this thesis considers only risk models which the Moeseke model, in particular, is, ensuring computational feasibility as will be shown in Chapter 3. While accepting that the computationally feasible method developed by Moeseke is an approximate one, it will be shown that it still possesses a very high degree of relevance in addressing the fundamental objective of this thesis, which is to evaluate the risk profiles of New Zealand's capital markets, both debt and equity.

Returning to the two stage investment process mentioned above it can be said that the formulation of conditional preferences relies on the investor's knowledge of the distribution of outcomes; their probabilities.

Savage distinguished three main classes of views in relation to the interpretation of probability; objectivistic, personalistic and necessary. His brief descriptions of these three views are as follows: [14]

13 See Chapter 3, Section 1.

14 Savage, L. J., 1954, op. cit. p3.

"Objectivistic views hold that some repetitive events, such as tosses of a coin, prove to be in reasonably close agreement with the mathematical concept of independently repeated random events, all with the same probability. According to such views, evidence for the quality of agreement between the behaviour of the repetitive event and the mathematical concept, and for the magnitude of the probability that applies (in case any does), is to be obtained by observation of some repetitions of the event, and from no other source whatsoever.

Personalistic views hold that probability measures the confidence that a particular individual has in the truth of a particular proposition, for example, the proposition that it will rain tomorrow. These views postulate that the individual concerned is in some ways "reasonable" but they do not deny the possibility that two reasonable individuals faced with the same evidence may have different degrees of confidence in the truth of the same proposition.

Necessary views hold that probability measures the extent to which one set of propositions, out of logical necessity and apart from human opinion, confirms the truth of another. They are generally regarded by their holder as extensions of logic, which tells when one set of propositions necessitates the truth of another."

The personalistic view is the appropriate one here with Savage developing a personalistic theory of decision making within a framework of states, consequences and acts. A set of states is termed an event with acts performing a mapping function assigning a consequence to every state.

The relation between acts can be written $\leq$, "is not preferred to" where this relation is a simple ordering.

Savage assumed that the ranking of two acts, given an event A, does not depend on the nature of these acts on states not belonging to A. This assumption, termed consequentialism by Hammond[15] was fundamental to Savage's definition of a conditional preference. Savage also assumed state-independence which meant that the ranking of consequences does not depend on the particular state in which they are realised.

The personalistic theory of Savage was centred around seven consistency axioms and it can be noted that Markowitz[16] presented a simpler version of the axioms of Savage in support of his approach to portfolio selection which also relied on personal probabilities where objective probabilities are not known.

Skiadas[17] suggested that the main reason for Savage introducing consequences was because probability distributions represent, in general, subjective assessments of

15 Hammond, P. J., Consequentialist Foundations of Expected Utility. Theory and Decision, V. 25, 1988, pp 25-78.

16 Markowitz, H. M., Portfolio Selection : Efficient Diversification of Investments. New York : Wiley 1959.

17 Skiadas, C. N., 1992, op. cit, p 2.

likelihood that are not directly observable. Consequences would normally have unambiguous interpretation and could therefore be viewed as objective. However, this still does not alter the fact that under the personalistic approach probabilities will always be subjective regardless of the availability of serial data. This point can be emphasized by considering an investor's decision making process within the Savage framework as follows.

An investor's preferences can be expressed, or described, over pairs of acts and states of form (f, $^{s'}A$) where f is an act and $^{s'}A$ is a particular state. A particular description (f, $^{s'}A$) represents the situation where the investor chooses act f and nature chooses state $^{s'}A$. If the investor had a conditional preference this could be written as follows: (f, $^{s'}A$) $\geq$ (g, $^{s'}A$). So long as acts and states are objectively interpreted, so are descriptions. Consequences are the result of the investor choosing f and nature choosing $^{s'}A$ and could be termed states of the person as opposed to states of nature. This can be seen as an objective approach. Consider now a particular description defined as a pair of the form (f,A) where f is an act and A is an event. An event is a set of states, all states possible, or outcomes possible for any particular act or choice decision. In order to define an event some probability distribution will be required representing, in general, subjective assessments.

A solution to this problem of subjectivity was suggested by Raiffa and Schlaifer[18] as follows.

18 Raiffa, H. and Schlaifer, R., Applied Statistical Decision Theory. Boston : Graduate School of Business Administration, Harvard University, 1961, p vii.

> "In most applied decision problems, both the preferences of a responsible decision maker and his judgements about the weights to be attached to the various possible states of nature are based on very substantial objective evidence; and quantification of his preferences and judgements enable him to arrive at a decision which is consistent with this objective evidence."

Given the fact that Savage's personalistic theory of decision was built around the behaviour of a rational person it can be said that it becomes almost immaterial if the assessment of the probability of likely outcomes is subjective or objective. What is important is the nature of that subjective assessment. This point is taken up in Section 2.3.

2.3 Markowitz Efficiency in Decision Making and an Introduction to the Moeseke Approach to Efficient Frontiers and Optimal Portfolios

Markowitz's contribution to modern portfolio theory is very well known but it is useful to note it at this point. H. M. Markowitz, regarded as the founder of modern portfolio theory, proposed that investors, being concerned with both risk and expected return, would select portfolios on the basis of mean and variance[19] where the mean (E) indicated expected return and the variance (V) indicated risk.

> "A portfolio is inefficient if it is possible to obtain higher expected (or average) return with no greater variability of return, or obtain greater certainty

[19] Markowitz, H. M., 1959, op. cit.

of return with no less average or expected return."[20]

Markowitz showed that the set of efficient portfolios could be selected from all possible portfolios where short selling was not allowed by the use of quadratic programming so long as there were estimates for the mean and variance of each investment option and estimates for the covariances between each investment option. From the set of efficient portfolios an appropriate portfolio could be selected for any individual dependent on the risk profile of the individual, but while also assuming a risk averse stance. The entire efficient frontier is generated by varying a required return, namely the interest rate. The non-negativity constraint for holdings is not essential for generating Markowitz efficient frontiers and without this constraint efficient frontiers can be easily generated using simple linear equations. It should be noted, however, that the lack of a non-negativity constraint has been the subject of considerable criticism, in particular from Markowitz himself who argued the unreality of the situation in relation to the CAPM[21] and then later put forward a version of CAPM which considered restrictions on short sales.[22]

Markowitz developed a rational framework from which risk averse investors could determine appropriate portfolios to hold and clearly showed how the risk inherent in investment decisions could be reduced on account of the fact that investment returns in general are not perfectly correlated. That is covariance values normally have a positive impact on portfolio risk. Adhering to the Markowitz approach, fund

20 Markowitz, H. M., 1959, op. cit. p 129.

21 Markowitz, H. M. , Non-Negative or not Non-Negative : A Question about CAPMs. The Journal of Finance, V. 38, 1983, pp 283-295.

22 Markowitz, H. M., Risk Adjustment. Journal of Accounting, Auditing and Finance, V. 5, 1990b, pp 213-225.

managers globally set about to hold minimum risk portfolios for the level of return that they are trying to achieve and invariably will not accept higher risk without the expectation of a higher return as compensation, while working within some asset allocation framework or set of objectives. Mean, variance, covariance estimation is central to the Markowitz approach and it is here that the major criticisms of this approach are aimed. Does history provide us with reliable mean, variance, covariance data for future predictions? Are variances and covariances good and appropriate measures of risk? Is investor utility adequately serviced by decisions based on the E.V. criterion given that quadratic utility functions are the only ones absolutely consistent with the E.V. criterion? These criticisms are taken up in Section 2.6, but it is important to emphasise a number of points in relation to these matters here. Historical data is objective and "correct" and will show clearly the climate that investors have faced in the past. Investor's future expectations will almost certainly be influenced in some way by this past performance.

While future individual financial asset performance may bear little resemblance to past performance for numerous reasons, which will be detailed in Section 2.6 also, the risk nature of the capital market being considered would logically retain some consistency between past and future periods. A highly competitive and mature equity or bond market with many years of activity behind it, is likely to have evolved into a marketplace where the expected returns from investing, given the risks involved, have some consistency over time.

It is the aim of this thesis to analyse capital market performance within the framework of market risk profiles and this approach is consistent with that developed by Moeseke[23], namely homogeneous programming by the truncated minimax criterion. While this model is described in detail in Chapter 3 the relationship between this model and the model developed by Markowitz is discussed here as well as the model's relationship to the basic linear programming model designed to solve the typical economic problem of the allocation of scarce resources.

Markowitz developed a model which generated E.V. efficient portfolios for risk averters with the constraints that no investment could be held short and the investment dollar would be fully invested. The efficient frontier covered a range from risk neutrality of the investor through to minimum variance investing. Varying the required return for the investor generates the efficient frontier. Markowitz did not define an optimal portfolio as such, but any portfolio on the efficient frontier could be optimal given the risk profile of some specific investor. A normal distribution of returns, or investors with quadratic utility functions, was required for correct application. Mean, variance and covariance estimation needed to be appropriate for the efficient portfolios to be appropriate for future investment.

Moeseke's model does not require a normal distribution of returns but applications of it have invariably assumed the normal distribution to hold based on historical returns and the intuitive logic of the approach. Moeseke set out to develop a model which could be used to solve the asset allocation problem in stochastic settings and achieved

23 Moeseke P. v., 1965a, op. cit.

this. The model determines an efficient frontier based upon a lower confidence limit with the same constraints of non-negative holdings and full investment of the investment dollar. In this case the efficient frontier can extend into risk taking situations and, for risk averters, is defined up to the point where the marginal return to the budget dollar equals zero. For the Moeseke model minimum variance means not investing at all. Moeseke also defined an optimal portfolio which, for any investor, is determined where the marginal return to the investment dollar equals its marginal cost. The marginal cost would normally vary between investor groups. For example the marginal cost to the private investor could represent the secure short-term bank deposit rate while the marginal cost to a financial institution could represent the cost of borrowing. To generate the efficient frontier a risk parameter, m, is used and varied. For m equal to zero the investor is risk neutral. For m positive increasing the investor has increasing risk aversion and for m negative the individual is a risk taker.

The Moeseke model is based on the Debreu[24] definition of the economic environment of the productive agent in terms of the values c, A and b where c refers to the net returns, c_j per unit activity level, b is the available quantities b_i of resources and A is the technical or input-output coefficients a_{ij} of production functions.

The procedure by which scarce resources can be allocated then is

$$\max_{x \varepsilon X}[f(x) = cx]$$

$$X = \{x \mid Ax \leq b, x \geq 0\}$$

where X is the set of feasible actions.

24 Debreu, G., Theory of Value. New York : Wiley, 1959.

In economics the problem is to maximise profits given the available technologies which requires minimising the resources used up in the production process. For the portfolio problem the aim is to maximise returns with minimum risk, given the available investment options, for the least amount of outlay.

2.4 The Capital Asset Pricing Model and its Extensions

The Capital Asset Pricing Model is a positive model for asset pricing developed by Sharpe (1964), Lintner (1965) and Mossin (1966) and is concerned with the economic equilibrium resulting from all investors behaving as Markowitz had proposed. In equilibrium a market portfolio is determined.

The model relies on a number of assumptions. The major underlying assumption, to coincide with the Markowitz model, is that investors are risk averse and select their portfolios by the mean-variance criterion. It is also assumed that capital markets are perfect which implies the following conditions, some of which can be relaxed without changing the conclusions of the analysis.[25]

"(1) The market comprises many buyers and sellers of securities, none of whose transactions is large enough to affect the prices in the market, and all of whom have an opportunity to invest.

[25] See Levy, H. and Sarnat, M., Portfolio and Investment Selection : Theory and Practice, Prentice/Hall, 1984, pp 396-397.

(2) There are no transaction costs or transfer taxes, nor is there an income or capital gains tax.

(3) All investors have all relevant information regarding alternative investments, and there are no costs involved in obtaining this information. All investors, therefore, have the same expectations regarding the expected returns and variances of all the alternative investment options.

(4) All investors can borrow or lend any amount in the relevant range without affecting the interest rate. The borrowing rate equals the lending rate and is the same for all investors both large and small, institutional and individual.

(5) There is a given uniform investment period for all investors; this means that all decisions are taken at a particular point in time, and all investments are held for the same period."

As summarised by Levy and Sarnat[26], the CAPM shows that when an investor is faced with n risky assets and the ability to borrow and lend at the riskless rate, the investment process can be separated into two parts. First, finding the optimum portfolio consisting only of risky assets. This is an objective task and is common to all investors. Second, finding the optimum mix of the risky portfolio with the riskless asset. This is a subjective task, dependent on the investor's individual risk preference.

As the task of finding the optimum portfolio is objective and common to all investors, the portfolio itself must, by definition, be the same for all investors. It follows therefore that the optimum portfolio must be the market portfolio as all financial

26 Levy, H. and Sarnat, M., 1984, op. cit., pp 420-422.

assets in the market place must have an owner. The market portfolio in the CAPM sense is a portfolio of financial assets which includes all financial assets in proportion to their share of the total market.

The basic equations for the CAPM are as follows:

For the CAPM risk-return relationship

$$ER_i = r + (ER_m - r)\,\beta_i$$

where ER_i = the expected rate of return on the ith asset,

r = the risk free rate of interest,

ER_m = the expected rate of return on the market portfolio,

β_i = the risk measure of the ith asset in portfolio context (beta).

The risk measure β_i is estimated by the time series regression

$$R_{it} = \hat{a}_i + \hat{\beta}_i R_{mt} + e_{it}$$

where R_{it} and R_{mt} stand for the returns in period t on the ith security and the market portfolio respectively.

For the ith security risk in portfolio context

$$\beta_i = \frac{\mathrm{Cov}(R_i, R_m)}{\sigma_m^2} \quad \text{or} \quad \beta_i = \frac{\sigma_{im}^2}{\sigma_m^2}$$

where σ_{im} = the covariance of the return to investment i with the market

σ_m^2 = the variance of returns of the market.

The equilibrium market value of the ith firm, V_{io} is given by the following certainty equivalence equation.

$$V_{io} = \frac{V_{i1} - Y\left[\bar{\sigma}_i^2 + \sum_{\substack{j=1 \\ j \neq i}}^{n} \overline{\sigma_{ij}}\right]}{1+r}$$

where V_{i1} = the expected value of the firm at period 1,

Y = the market price of risk,

$\bar{\sigma}_i^2$ = the variance of the value of firm i,

$\bar{\sigma}_{ij}$ = the covariance of the value of firm i with the value of firm j,

r = the riskless interest rate.

There have been a number of extensions to the CAPM which have arisen out of the logical need to relax some of its assumptions to make it more consistent with real market conditions.

These extensions can be grouped under a number of different subheadings following the model's development as emphasised by Sharpe[27] and Duffie[28].

27 Sharpe, W. F., Capital Asset Prices with and without Negative Holdings. The Journal of Finance, V. 46, 1991, pp 489-490.

28 Duffie, D., Dynamic Asset Pricing Theory. Princeton University Press, 1992, pp xiii-xvi.

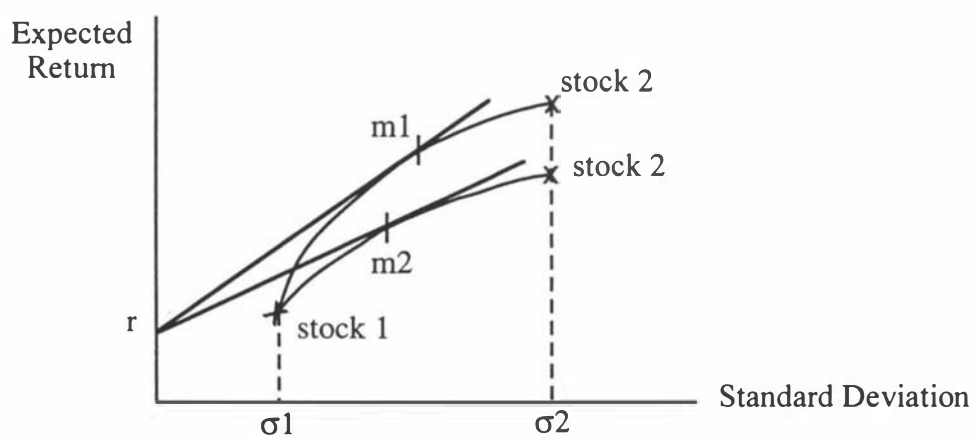
2.4.1 CAPM and Heterogeneous Expectations Regarding Means, Variances and Covariances.

Shortly after the introduction of the CAPM, Lintner[29] presented a paper which addressed the question of the homogeneity of expectations for means, variances and covariances. Naturally one would expect investors to differ in their expectations of means, variances and covariances leading to each investor facing a different subjective efficient frontier. The fact that this is the case is shown clearly in the following figure.[30]

Figure 2.4.1.1

CAPM and Heterogeneous Expectations

Regarding Expected Return



[29] Lintner, J., Security Prices, Risk, and Maximal Gains from Diversification. The Journal of Finance, V. 20, 1965a, pp 587-615.

[30] See Levy, H and Sarnat, M., 1984, op cit, p 469.

Here there are two investors, facing a two stock portfolio, who agree on all points regarding future expectations except that one investor considers stock 2 to have a higher expected return than the other investor. Consequently one investor holds portfolio m_1 and the other portfolio m_2.

This will be true for all investors with differing expectations of means, variances and covariances, however, in the aggregate Lintner showed that equilibrium for the market to clear with heterogeneous expectations is similar to when there are homogeneous expectations. The only difference being that the end-of-period prices and risk are weighted averages of the various investors' estimates.

This solution in the aggregate was a very important extension of the CAPM which was later extended upon by Rubinstein[31] who developed alternative sets of sufficient conditions under which equilibrium pricing can occur given the existence of individuals whose "resources, beliefs, and tastes are a composite of the actual individuals in the economy"[32]. He was able to do this for a more general class of utility functions and further showed that when a composite individual can be constructed, in equilibrium, rates of return are not affected by the distribution of resources among individuals or the size of the population so long as the economic characteristics of the composite individual remain unchanged.

31 Rubinstein, M., An Aggregation Theorem for Security Markets. Journal of Financial Economics, V. 1, 1974, pp 225 - 244.

32 Ibid., p 225.

2.4.2 CAPM with Real Returns

The distinction between real and nominal returns within the framework of the CAPM was first made by Lintner[33] when he considered a model in which all returns are uncertain. Lintner's model assumed that investors were mainly concerned with real returns and real purchasing power so that the utility function for each investor is defined over the real purchasing power of returns. The model further assumed that investors view both changes in the purchasing power of nominal dollars and their covariance with all other securities differently. Lintner gave consideration to the implications of such a model both with short sales allowed and with a non-negativity constraint. The very fact that real returns are considered rather than nominal returns makes the concept of a riskless asset meaningless as the risk free asset will usually exist only in the absence of inflation. Though, as Lintner points out, the shadow-price of each investor's wealth constraint will measure the marginal real riskless certainty equivalent of the investor's end of period wealth with or without the existence of a risk free asset. However, even if investors have the same expectations of ending prices and of their associated covariance matrix, the absence of a riskless asset is a sufficient condition for investors to hold different fractions of the total market supply of the different stocks in their portfolios with different optimal portfolio mixes being appropriate for different investors.

[33] Lintner, J., The Aggregation of Investor's Diverse Judgements and Preferences in Purely Competitive Security Markets. Journal of Financial and Quantitative Analysis, V. 4, 1969, pp 347-400.

With differing price and covariance expectations, coupled with differing risk preferences, these portfolio mix differences will be more pronounced. Therefore it is highly unlikely that any investor will hold the market portfolio.

Lintner further pointed out that a short selling constraint would be ineffective given the following sufficient conditions.

(a) there exists a riskless asset with an agreed return, r*,

(b) there is identical price assessment throughout the market,

and either (c_1) there exists identical covariance matrices,

or (c_2) all covariance assessments = 0 throughout the market.

Therefore if short sales are not permitted the lack of a riskless asset will affect optimal portfolios for investors. Later writers extended this work on real returns and the lack of a riskless asset. Section 2.4.3 considers Black's[34] version of the CAPM in which there is no riskless asset, and Friend, Landskroner and Losq[35] showed that the CAPM can be written in nominal terms to take account of uncertain inflation as follows:

$$ER_i = r + \sigma_{1\pi} + \frac{ER_m - r\text{-}\sigma_{m\pi}}{\sigma_m^2 - \frac{\sigma_{m\pi}}{\alpha}}\left(\sigma_{im} - \frac{\sigma_{i\pi}}{\alpha}\right)$$

where $\sigma_{i\pi}$ is the covariance of the rate of return on the ith asset and the inflation rate π,

[34] Black, F., Capital Market Equilibrium with Restricted Borrowing. The Journal of Business, V. 45, 1972, pp 444-454.

[35] Friend, I., Landskroner, Y. and Losq, E., The Demand for Risky Assets Under Uncertain Inflation. The Journal of Finance, V. 31, 1976, pp 1287-1297.

$\sigma_{m\pi}$ is the covariance of the return on the market portfolio and the inflation rate π,

α is the ratio of nominal risky assets to total nominal value of all assets in the market.

2.4.3 CAPM and Taxation

The original assumption for the CAPM model of no income or capital gains tax for investors is clearly a very restrictive assumption for real world applicability. Brennan[36] was the first to address this problem by incorporating differing investor marginal tax rates on income and capital gains into the CAPM. Brennan derived the following risk-return relationship when taxation is taken into account.

$$ER_i = r + (ER_m - r)\,\beta i + f\,(\delta i, \delta m, T)$$

where f (δi, δm, T) is a function of the dividend yield of the ith stock δi, the dividend yield of the market portfolio δm, and T, a factor which takes into account the wealth of the investor as well as the investor's marginal tax rate. The last term of the formula then reflects the impact of the dividend policy of different companies and the tax rates on dividends and on capital gains. Should the rate of tax levied on dividends be higher than that levied on capital gains, then for a given level of risk, investors will require a higher total return on securities with higher prospective dividend yields.

[36] Brennan, M. J., Taxes, Market Valuation and Corporate Financial Policy. National Tax Journal, V. 23, 1970, pp 417-427.

Brennan's model was extended by Litzenberger and Ramaswamy[37] who incorporated wealth and income related constraints on borrowing along with a progressive tax scheme. They also found evidence of a positive relationship between dividend yield and expected return on the New York Stock Exchange as well as evidence of a clientele effect. That is investors in higher tax brackets choose stocks with low dividend yields and vice versa. These extensions to the CAPM again point to differing optimal investor portfolios.

2.4.4 CAPM with no Riskless Asset (The Zero-beta model)

The CAPM with no riskless asset or the zero-beta model developed by Black[38] follows from Lintner's CAPM with real returns. In the Black model the expected return to the riskless asset is replaced by the expected return to the zero-beta portfolio. This can be shown as follows[39]

Figure 2.4.4.1
CAPM with a Riskless Asset

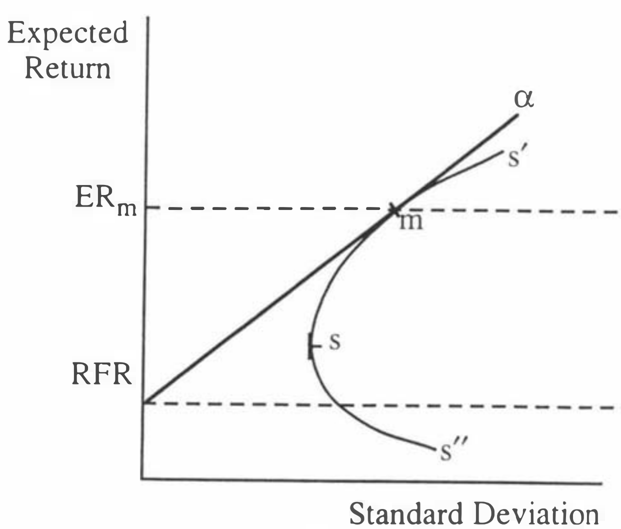
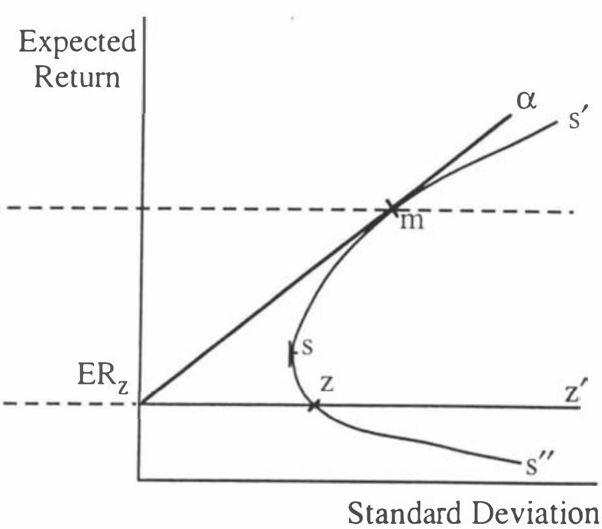


Figure 2.4.4.2
CAPM with no Riskless Asset



[37] Litzenberger, R. and Ramaswamy, K., The Effect of Personal Taxes and Dividends on Capital Asset Prices: Theory and Empirical Evidence. Journal of Financial Economics , V. 7, 1979, pp 163-195.

[38] Black, F., 1972, op. cit.

[39] See Levy, H. and Sarnat, M., 1984, op. cit., pp 464-5.

In both figures the line SS' represents the efficient frontier with S representing the global minimum variance portfolio. All points along the lines S'S" represent minimum variance portfolios for the given level of expected return but those portfolios from S to S" are inefficient. The market portfolio is represented by m which, by definition, has a beta of 1.

For CAPM with a riskless asset the amount invested in the riskless asset or borrowed at the riskless rate for investing in the market portfolio is given by the point the investor wishes to be along the line RFR to α. Any investor can be satisfied with a combination of the market portfolio and an investment in, or borrowing at, the risk free rate. The risk return relationship is given by

$$ER_i = RFR + (ER_m - RFR)\ \beta i$$

Black showed that in the absence of the risk free rate the following risk return relationship holds.

$$ER_i = ER_z + (ER_m - ER_z)\ \beta i \qquad (2.4.4.1)$$

From equation (2.4.4.1) for ER_i equal to ER_z , βi must equal zero as ER_m is clearly greater than ER_z . Therefore portfolio z is a zero beta portfolio. Any minimum variance portfolio can be obtained as a linear combination of two other minimum variance portfolios, so by investing in a combination of the market portfolio and a positive or negative holding in the zero beta portfolio an investor can achieve their optimal point on the SS' efficient frontier. The efficient frontier represents the investor's opportunity set in this case. These two portfolios could be represented by two mutual funds for example. Note that for any m there will be a corresponding zero beta portfolio.

2.4.5 CAPM and Continuous-Time Trading

The original CAPM is a single period model. Extending CAPM to continuous-time trading was first considered by Leroy[40] and Merton[41] in 1973 although Fama[42] in 1970 had proved that risk averters maximising expected utility through consumption over a lifetime would act in a manner per period that was indistinguishable from someone with a single period horizon. That is, a single period model could be applied to successive periods in a multiperiod setting.

Since the work of Fama all continuous-time models have been based on the random character of returns or Brownian motion, that is they are consistent with weak form market efficiency. This causes some difficulty with the reconciliation of the single period models' logical requirement of investors being risk averse.

If the expected rate of return for a financial asset is dependent on the relationship between the riskiness of the asset and the risk averse nature of the investor, both return and risk are obviously necessary considerations. As Leroy[43] points out, however, the random character of returns relates only to the first moment of returns. Within the context of a single period model this is not

40 Leroy, S. F., Risk Aversion and the Martingale Property of Stock Prices. International Economic Review, V. 14, 1973, pp 436-446.
41 Merton, R. C., An Intertemporal Capital Asset Pricing Model. Econometrica, V. 41, 1973, pp 867-888.
42 Fama, E.F., Multiperiod Consumption - Investment Decisions. The American Economic Review, V. 60, 1970, pp 163-174.
43 Leroy, S. F., ibid.

a problem as the expected value and variance of the next period price is taken as given. With continuous-time trading, however, markets must be cleared for any realisation of past returns. A continuous-time trading model, therefore, generates an intertemporal probability distribution for rates of return on assets and, in general, successive rates of return may be either positively or negatively correlated depending on the relationship between past returns and the expectation and variance of the next period returns.

Leroy concluded, however, that while risk neutrality was easier to reconcile with the random character of returns in continuous-time models, the presence of risk aversion was not necessarily inconsistent with randomness, just difficult to justify on a rigorous theoretical basis.

Merton's[44] intertemporal CAPM showed that the expected excess return on any asset is given by a multi-beta version of the original CAPM. The number of betas required to give the expected excess return being one plus the number of state variables required to describe the characteristics of the investment opportunity set. The approach was one of putting a single period valuation formula into a multiperiod setting.

Rubinstein[45] added to the continuous-time models by developing a simple multiperiod formula for determining the present value of a series of cash flows received over many future dates. This valuation technique is consistent with

[44] Merton, R. C., 1973, op. cit.

[45] Rubenstein, M., The Valuation of Uncertain Income Streams and the Pricing of Options. The Bell Journal of Economics, V. 7, 1976, pp 407-425.

rational risk averse investor behaviour and equilibrium in financial markets. Lucus[46] developed a model to examine the stochastic behaviour of equilibrium asset prices in an economy with identical consumers and a single consumption good produced by a number of different processes. Assets are defined as claims on the output of these processes with equilibrium determining the asset prices. The model was intertemporal in that it was concerned with market determined movements in asset prices.

Breedon[47] further developed the intertemporal CAPM with the development of a model based on Merton's intertemporal CAPM but which derived a single beta asset pricing model in a multi-good, continuous-time frame with uncertain consumption goods prices and investment alternatives. In achieving this, asset betas are measured relative to changes in the aggregate real consumption rate rather than relative to the market. At approximately the same time as Breedon's contribution, Harrison and Kreps[48] supplied "an almost definitive conceptual structure to the whole theory of dynamic security prices"[49]. In particular they considered the determination of prices of contingent claims on particular securities through arbitrage considerations alone. In an extension of the work of Black and Scholes[50] it was shown that there exists a single price for a specified contingent claim which, together with the given securities price, will not permit arbitrage profits.

46 Lucus, R. E., Asset Prices in an Exchange Economy. Econometrica, V. 46, 1978, pp 1429-1445.

47 Breedon, D. T., An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities. Journal of Financial Economics, V. 7, 1979, pp 265-296.

48 Harrison, J. J. and Kreps, D. M., Martingales and Arbitrage in Multiperiod Securities Markets. Journal of Economic Theory, V. 20, 1979, pp 381-408.

49 Duffie, D., 1992, op. cit., p xiv.

50 Black, F. and Scholes, M., The Pricing of Options and Corporate Liabilities. Journal of Political Economy, V. 81, 1973, pp 637-654.

Finally, one should note the contribution of Cox, Ingersoll and Ross[51] who developed a continuous-time equilibrium model of a simple but complete economy which could be used to examine the behaviour of asset prices. The model determines the equilibrium price of any asset in terms of the underlying real variables in the economy. That is both asset prices and their stochastic properties are determined endogenously. This model drew mainly on the work of Merton[52] and Lucus[53] taking the continuous-time aspects of the Merton model into a model with an economic structure similar to that of Lucus. This model was used by Cox, Ingersoll and Ross[54] to develop a model for explaining the term structure of interest rates which includes anticipations, risk aversion and investment alternatives.

This series of extensions to the original CAPM are of substantial importance as they have been fundamental in bringing together finance and economic theory. This is a topic which will be introduced in Section 2.7.

51 Cox, J. C., Ingersoll, J. E. and Ross, S. A., An Intertemporal General Equilibrium Model of Asset Prices. Econometrica, V. 53, 1985a, pp 363-384.

52 Merton, R. C., 1973, op. cit.

53 Lucus, R. E., 1978, op. cit.

54 Cox, J. C., Ingersoll, J. E. and Ross, S. A., A Theory of the Term Structure of Interest Rates. Econometrica, V. 53, 1985b, pp 385-407.

2.4.6 The Three Moment CAPM

The three moment CAPM was first introduced by Kraus and Litzenberger[55] to take skewness or asymmetry into account in asset pricing. Prior to that only total skewness present in a distribution had been examined. In keeping with CAPM Kraus and Litzenberger showed that systematic skewness rather than total skewness was the appropriate approach in market equilibrium valuation.

The Kraus and Litzenberger model can be written as follows[56].

$$R_i - R_F = b_0 + b_1\, \beta_{im} + b_2\, \gamma_{im}$$

where R_i = the expected return from security i,

R_F = the risk-free rate,

$R_i - R_F$ = the expected risk premium of security i,

β_{im} = the systematic risk (beta) of security i,

γ_{im} = the systematic skewness (gamma) of security i,

b_0 = the mean intercept term,

b_1 = the estimated market price of beta,

b_2 = the estimated market price of gamma.

55 Kraus, A. and Litzenberger, R. H., Skewness Preference and the Valuation of Risk Assets. The Journal of Finance, V. 31, 1976, pp 1085 - 1100.

56 See Tan, K-J., Risk, Return and the Three-Moment Capital Asset Pricing Model: Another Look. Journal of Banking and Finance, V. 15, 1991, p 451.

This equation shows the linear relationship between the expected return on a security and its systematic risk and systematic skewness, given the risk-free rate, the market return, the systematic risk and the systematic skewness of the security.

Tests on the three-moment CAPM have been mixed. The Tan[57] study found no significant relationship between the first three statistical moments for a study of US. mutual funds returns for the period 1970 - 1986.

The importance of the third moment in portfolio selection and equilibrium models will be discussed further in Section 2.6.

2.4.7 CAPM with Transactions Costs

Transactions costs are generally incurred when buying and selling securities and the original CAPM assumption of zero transactions costs is unrealistic. This is particularly true given the CAPM result of investors holding the entire securities market. New issues are common and an investor's wealth will keep changing at regular intervals through income and expenditure activities.

Levy[58] showed that, when transactions costs are included, the optimal position for a particular investor might be to hold only a small number of securities

57 Tan, K-J., 1991, op. cit., pp 449 - 460.

58 Levy, H., Equilibrium in an Imperfect Market: A Constraint on the Number of Securities in the Portfolio. The American Economic Review, V. 68, 1978, pp 643 - 658.

which is the situation generally observed in the market place. The Levy model is as follows:

$$R_i = R_F + \frac{\sum_k T_k(R_k - R_F)}{\sum_k T_k} \beta_{ki}$$

where R_i = expected rate of return on security i,

R_F = the risk-free rate,

R_k = the mean rate of return on the portfolio held by investor k,

T_k = the wealth invested by investor k,

β_{ki} = the beta of asset i with respect to the portfolio held by investor k (which is not necessarily the market portfolio).

That is the expected return on security i is equal to the risk-free rate plus a weighted average of the risk premium required by all investors. Should all investors hold the market portfolio then $R_k = R_m$ and $\beta_{ki} = \beta_i$. In this case

$$\frac{\sum_k T_k(R_m - R_F)}{\sum_k T_k} \beta_i = (R_m - R_F)\, \beta_i$$

so $R_i = R_F + (R_m - R_F)\, \beta_i$, the original CAPM equation.

2.4.8 CAPM with Market Segmentation through Incomplete Information

As in the case of the Levy model described in 2.4.7, the observed fact that many investors hold few securities was the prime motivation behind Merton's[59] model of capital market equilibrium with incomplete information. The Merton model started from the premise that an investor will only consider investing in a security if he/she is informed about that security. In keeping with the original CAPM, being informed about security k was viewed as knowing $\bar{R}k$, βk and σ_k^2, that is the expected return, beta and variance of security k. Therefore if all investors are informed about all securities the original Sharpe-Lintner-Mossin CAPM applies.

Merton suggested that information costs were a main contributor to the existence of uninformed investors. These information costs could be put into two categories, first the cost of gathering and processing data and second, the cost of transmitting information from one party to another. In general company reporting requirements should ensure that fundamental information about a company is available to all investors but many investors will limit their investment horizon to a manageable number of securities which may well be a very small percentage of all investment options in a large market place.

Also investment analysis companies will, in general, be unable to cover all investment options in depth with larger companies tending to be serviced by more analysis for investor guidance than smaller companies. A further

59 Merton, R. C., A Simple Model of Capital Market Equilibrium with Incomplete Information. The Journal of Finance, V. 42, 1987, pp 483 - 510.

contributor to market segmentation relates to the difficulty in analysing some information. For example what will the future cost of energy be given an environment of depleting fossil fuels but improving energy generating technology?

Merton showed that with incomplete information the value of security k will always be lower than with complete information and that this under-valuation will be greater the smaller the "informed" investor base.

2.4.9 CAPM with Restrictions on Short Sales

The main critic of the allowance of short sales in the original CAPM has been Markowitz[60], a problem which he finally addressed himself[61] within the CAPM framework.

Markowitz showed that the CAPM with a non-negativity restriction leads to investors selecting one of a number of mean-variance efficient portfolios. That is there would exist distinct clientele sets which makes the Markowitz CAPM similar in nature to the Levy and Merton models discussed in Sections 2.4.7 and 2.4.8. In the Levy model the clientele sets are the result of transactions costs while in the Merton model they are the result of each investor knowing only about a subset of the available securities. In the Markowitz model

60 In particular see Markowitz, H. M., 1983, op. cit. and Markowitz, H. M., Normative Portfolio Analysis: Past, Present and Future. Journal of Economics and Business, V. 42, 1990a pp 99 - 103.

61 Markowitz, H. M., 1990b, op. cit.

investors hold different sets of securities because they choose their portfolios from different segments of the linear set of efficient portfolios.

2.5 Arbitrage Pricing Theory

In relation to capital market equilibrium models, the major competitor to the CAPM is the Arbitrage Pricing Theory (APT) model. This model was developed by Ross[62] and deviates strongly from CAPM in that it does not assume risk aversion, nor does it assume that investors make their decisions in the mean-variance framework. Instead it assumes that the security's rate of return is generated by the following process:

$$R_i = ER_i + \beta_i (I - EI) + e_i$$

where R_i = the rate of return on security i,

ER_i = the expected return (or mean) of security i,

I = the value of the factor generating the security returns,

EI = the mean of the factor generating the security returns,

β_i = a coefficient measuring the effect of changes in the factor I on the rate of return R_i,

e_i = a random deviation (noise).

62 Ross, S., The Arbitrage Theory of Capital Asset Pricing. Journal of Economic Theory, V.13, 1976, pp 341-360.

I is a common factor to all securities, say GDP growth for example, and the model can include any number of such factors seen as being appropriate for the generation of security rates of return.

The equilibrium relationship for this model can be given as follows:

$$ER_i = ER_z + (EI - ER_z)\,\beta_i$$

where ER_z = the expected rate of return to the zero-beta portfolio such that $Cov(I.R_z) = 0$

A special case of the APT model is where the returns generating factor I is taken to be that of the market portfolio with rate of return R_m. In this case the following equation is obtained:

$$ER_i = ER_z + (ER_m - ER_z)\,\beta_i$$

This is the classical CAPM position so here the two models coincide. Three basic assumptions of the APT model are as follows. First, the average portfolio noise is zero ($\sum_{i=1}^{n} x_i e_i = 0$) and this can only hold for a very large number of assets in a portfolio. Second, investors will hold a zero-beta portfolio ($\sum_{i=1}^{n} x_i \beta_i = 0$). Third, a zero amount is invested in the zero-beta portfolio ($\sum_{i=1}^{n} x_i = 0$). This requires some short selling with the investor receiving the proceeds to reinvest.

The assumptions that investors hold a very large number of assets in their portfolios and that short sales are allowed with the proceeds available for reinvestment are characteristic of the classical CAPM.

An important extension to the work of Ross was given by Huberman[63], who clearly defined arbitrage and presented a simple proof that no arbitrage implies the mean return on an asset is approximately linearly related to the covariances of the assets returns with economy wide common factors.

Later Wei[64] presented an asset pricing theory that unified APT with CAPM. As Wei emphasised, APT stresses the role of the covariance between asset returns and exogenous factors, while CAPM stresses the role of the covariance between asset returns and the endogenous market portfolio. By adding the market portfolio as an extra factor to the APT Wei showed that an exact asset pricing relation could be obtained. The market portfolio could be added to all other factors in the APT to give the APT in an infinite economy or the unified asset pricing theory in a finite economy. Conversely the market portfolio could replace some factors or, in the extreme case, all factors reducing the model to the original CAPM.

63 Huberman, G., A Simple Approach to Arbitrage Pricing Theory. Journal of Economic Theory, V. 28, 1982, pp 183 - 191.

64 Wei, K. C. J., An Asset-Pricing Theory Unifying the CAPM and APT. The Journal of Finance, V. 43, 1988, pp 881 - 892.

2.6 Criticisms and Limitations of Portfolio Selection Models and Asset Pricing Models

The normative and positive models of portfolio selection and asset pricing are, and have since inception, been subject to a range of justifiable criticisms. The extensions to CAPM and the introduction of APT which were discussed in Section 2.5 invariably were developed to help address valid criticisms. The normative models of the Markowitz type have also seen a considerable research effort aimed at developing better ways of estimating efficient portfolios through the study of returns and risk associated with the whole range of financial assets.

In relation to CAPM, the pricing of assets and appropriate portfolio decisions rely upon a linear relationship between the expected return of each security and its covariance with, or regression coefficient against, the market portfolio.

Investors seek mean-variance efficiency and in the original model have identical beliefs and identical constraint sets. Arbitrage pricing theory dropped the mean-variance assumption but added an assumption concerning the joint distribution of security returns. It also introduced unidentified factors as price determinants making testing extremely difficult.

Critics of the Markowitz normative model attack primarily the appropriateness of the mean-variance assumption which requires either a normal distribution of returns or investors with quadratic utility functions to be appropriate. A second strong line of attack relates to mean, variance, covariance estimation with attacks also aimed at single time frames and the allowance or otherwise of transaction costs and taxes.

Critics of CAPM also attack the appropriateness of the mean-variance assumption as well as the existence and use of the risk free asset, the allowance of short sales, market segmentation, single time frames and transaction costs and taxes.

This section considers the major criticisms of the normative and positive models considered previously and, in particular, considers how these criticisms relate to the Moeseke model.

2.6.1 The Nature of the Distribution of Returns

Risk averse investors require compensation for taking on extra risk. If risk is defined as being the level of variability of returns, mean and variance become the logical parameters for decision making. They are also appropriate if returns are normally distributed or the investor's utility function is a quadratic.

Consider first the normal or Gaussian distribution. This is characterised by its first two moments, being the mean (E) and the variance or standard deviation (σ). It therefore follows that mean-variance analysis is appropriate for situations where returns are normally distributed. Considerable study has been undertaken to determine if returns from capital markets are normally distributed with varying results. The results can be summarised as showing that the distributions are not perfectly normal, sometimes including a skewness component and often being leptokurtic, that is having too many observations in the tails and centre of the distribution. In general though, the hypothesis that the rates of return on capital markets are normally distributed cannot be easily rejected. The possibility that the underlying distributions belong to a more general class of so-called stable paretian distributions which includes the normal distribution as a special case was proposed by Fama[65]. The advantage of stable paretian distributions or the symmetric stable distribution was that kurtosis could be explicitly accounted for. There is, however, another family of symmetric distributions that can account for the observed "fat tails", being the Student or t distribution. Blattberg and Gonedes[66] analysed these two families of distributions and concluded that the student model had greater descriptive validity than the symmetric-stable model.

Going almost full circle, Frankfurter and Lamoureux[67] compared the robustness in application of the normal, or Gaussian, assumption of security return distributions to that of the stable paretian distributions and concluded that the Gaussian assumption was preferable to the general stable assumption.

The debate on the appropriate distribution to describe security returns is clearly an important one though it should be noted that this debate focuses on

65 Fama, E. F., The Behaviour of Stock Market Prices. Journal of Business, V. 39, 1965, pp 34 - 105.

66 Blattberg, R. C. and Gonedes, N. J., A Comparison of the Stable and Student Distributions as Statistical Models for Stock Prices. The Journal of Business, V. 47, 1974, pp 244 - 280.

67 Frankfurter, G.M. and Lamoureux, C.G., The Relevance of the Distributional Form of Common Stock Returns to the Construction of Optimal Portfolios. Journal of Financial and Quantitative Analysis, V.22, 1987, pp 505-511.

observed returns data rather than the probability beliefs of the investors. In relation to the kurtosis problem, Granger and Morgenstern[68], among others, observed "fat tails" and central concentration for U.S. equity data but pointed out that the numerous difficulties associated with time series data typical of that generated in equity markets could go some way to explaining the observed kurtosis. Stock market data does not conform well to a true time series in that the time period between each observation or "new price" will vary, sometimes greatly. Also once a price is set at any point in time many trades may occur at that price until the market has cleared. Granger and Morgenstern suggested that an appropriate statistic for analysis in such markets, particularly if all observed trades are included, was

$$\text{ratio (open price to close price)} / \sqrt{\text{volume}}$$

and showed this distribution to be approximately normal. It has also been shown by Morgenstern and Granger [69] and others that over long time spans the use of log prices rather than raw prices leads to more stability in the distribution as well as solving the problem of the data being bounded from below. They showed that analysis of returns data generated in this way did, in general, confirm to the normal distribution.

The problem of observed skewness has also evoked considerable academic interest as it makes intuitive sense that investors might be prepared to forego

68 Granger, C. W. J. and Morgenstern, O., Predictability of Stock Market Prices, D. C. Heath and Company, Lexington, Mass. 1970.

69 Ibid.

some return for the small chance of a large return, or wish to be compensated for the small chance of a large loss. That is, investors might have a liking for positive skewness and a disliking for negative skewness. Levy and Sarnat[70] provided some empirical evidence of this and Arditti[71] and Levy[72] discussed utility functions which would be appropriate for this kind of investor behaviour. However the whole issue of whether investors do consider skewness when making investment decisions was not answered by these three articles. Francis[73], however, analysed quarterly returns from 113 mutual funds over nine years and concluded that investors do not generally consider skewness when making investment decisions. At a more fundamental level Fogler and Radcliffe[74] showed differing intervals and starting points can have a dramatic effect on observed skewness. They pointed out that yearly data for the Standard and Poors Composite Index from 1948 to 1969 showed positive skewness whereas semiannual and quarterly intervals for the same time frame showed negative skewness. An analytical investigation of the intervaling effect on skewness was carried out by Hawawini[75] who demonstrated that the higher order moments were more sensitive to the length of the differencing interval than lower order moments. It was further noted by Beedles and Simkowitz [76] that extreme observations have greater impacts on higher moments and such

70 Levy H. and Sarnat, M., 1984, op. cit., pp 162 - 166.

71 Arditti, F. D., Risk and the Required Return on Equity. The Journal of Finance, V. 22, 1967, pp 19 - 36.

72 Levy, H., A Utility Function Depending on the First Three Moments. The Journal of Finance, V. 24, 1969, pp 715 - 720.

73 Francis, J. C., Skewness and Investors' Decisions. Journal of Financial and Quantitative Analysis, V. 10, 1975, pp 163 - 172.

74 Fogler, H. R. and Radcliffe, R. C., A Note on Measurement of Skewness. Journal of Financial and Quantitative Analysis, V. 9, 1974, pp 485 - 489.

75 Hawawini, G. A., An Analytical Examination of the Intervaling Effect on Skewness and Other Moments. Journal of Financial and Quantitative Analysis, V. 15, 1980, pp 1121 - 1127.

76 Beedles, W. L. and Simkowtiz, M. A., A Note on Skewness and Data Errors. The Journal of Finance, V. 33, 1978, pp 288 - 292.

observations should be examined carefully in relation to their usefulness in a data set.

Empirical evidence has also shown that increased diversification tends to result in a progressive loss of skewness questioning the validity of the skewness consideration in well diversified portfolio construction[77]. Further evidence against the usefulness of the third moment in stock returns was presented by Singleton and Wingender[78] who showed that while positive skewness in stock returns continues to be observed with almost constant frequency in cross section analysis, neither individual stocks nor portfolios show any persistent skewness over time. This result has been supported by Sengupta and Sfeir[79].

In contrast to this, however, Badrinath and Chatterjee[80] contend that both daily and monthly returns on the US. CRSP equally weighted and value-weighted indices from July 1962 to December 1985 are adequately explained as a skewed, elongated (g x h) distribution[81].

When one considers the nature of financial asset markets, particularly equity markets, it should be expected that skewness will be observed in security or

77 See Kane, A., Skewness Preference and Portfolio Choice. Journal of Financial and Quantitative Analysis, V. 17, 1982, pp 15 - 25.

78 Singleton, J. C. and Wingender, J., Skewness Persistence in Common Stock Returns. Journal of Financial and Quantitative Analysis, V. 21, 1986, pp 335 - 341.

79 Senguptal, J. K. and Sfeir, R. E., Market Volatility and Skewness Persistence. Applied Economics Letters, V. 1, 1994, pp 215 - 218.

80 Badrinath, S. G. and Chatterjee, S., On Measuring Skewness and Elongation in Common Stock Return Distributions: The Case of the Market Index. Journal of Business, V. 61, 1988, pp 451 - 472.

81 See Tukey, J. W., Exploratory Data Analysis, Reading, Mass : Addison-Wesley, 1977.

bond returns from time to time. A series of unexpectedly good or bad reports on a company could well lead to a positively or negatively skewed distribution. There may well be some stocks which are more susceptible to this outcome because of the nature of their business. Also there are numerous products available in the market place designed specifically to attract investors with a liking for positive skewness, consider for example the options markets. It can be concluded, however, that the usefulness of a three moment model for standard equity (and bond) investments is at best minimal.

As well as being appropriate where returns conform to a Gaussian or normal distribution mean-variance analysis is also appropriate when investors have quadratic utility functions of the type:

$$U(R) = a + bR + cR^2$$

where $U(R)$ = the utility of returns,

R = returns.

The expected utility of R is given by

$$EU(R) = a + bER + cER^2$$

where ER = expected return.

However by definition variance $\sigma^2 = ER^2 - (ER)^2$

so $EU(R) = a + bER + c(ER)^2 + c\sigma^2$

That is the quadratic utility function can be expressed explicitly as a function of the mean and variance of returns[82].

Further, mean-variance analysis can be applied as an approximation of many concave utility functions (utility functions appropriate for risk averters) with negligible error as long as the range of returns is not too widely spread[83] adding support for the use of mean-variance analysis even if returns are not normally distributed.

This last result is very useful as there are two severe criticisms of quadratic utility functions. First, they only conform with the von Neumann-Morgenstern[84] axioms for expected utility if

$$R \leq \frac{1}{-2\beta}$$

where R = return,

β = c/b for any $U(R) = a + bR + CR^2$

and $b > 0$. For risk aversion $c < 0$ must also hold.

Second, they do not display decreasing absolute risk aversion which is a generally accepted behavioural trait of investors. This is because the third derivative of a quadratic function is always equal to zero whereas decreasing

82 See Levy, H. and Sarnat, M., 1984, op. cit., p244.

83 See Levy, H. and Markowitz, H. M., Approximating Expected Utility by a Function of Mean and Variance. The American Economic Review, V. 69, 1979, pp 308-317.

84 Neumann, v. J. and Morgenstern, O., Theory of Games and Economic Behaviour, Princeton, NJ: Princeton University Press, 3rd ed., 1953.

absolute risk aversion requires the third derivative of the utility function to be positive.

It can be noted at this point that Markowitz[85] did not use the normal distribution of returns or any other two-parameter family of probability distributions to justify the mean-variance criterion he proposed. Instead he rationalised the mean-variance criterion solely on an investor utility basis. In particular he used the quadratic approximation approach as previously mentioned. The argument for taking this approach is detailed in Markowitz (1987)[86].

Moeseke takes a substantially different view of portfolio efficiency for investors from that of Markowitz. Moeseke views the homogeneous programming by the truncated minimax criterion as being a model where the only subjective component is in relation to the weights attached to outcomes. These relative weights are attached to the expected outcome and the standard deviation of that outcome being 1 and m respectively, m being a risk parameter, the greater its value the more weight there is being placed on the risk component of the outcome. According to Moeseke, for any particular market there will be an objective value of m where the marginal cost of the investment dollar will equal its marginal return for investors in the aggregate. This value of m then is the objective risk profile of the particular market being investigated[87]. There will be investors who are too risk averse to participate in

85 Markowitz, H. M. , 1959, op. cit., pp 205 - 274.
86 Markowitz, H. M., 1987, op. cit., pp 52 - 56.
87 The Moeseke model is explained in detail in Chapter 3 of this thesis.

this or any other market. There will also be investors who will use this or any other market but will take a risk stance greater than that required at the margin for entry. For the market as a whole, however, the only important behavioural characteristic is the objective summation of the investors' weightings attached to the standard deviation of outcomes. The investors could just as easily be risk takers as a group, but as would be expected, results from the application of the Moeseke model show risk aversion as standard for capital markets. Unlike the Markowitz justification of mean-variance being quadratic-like utility functions for investors, the Moeseke justification lies in the intuitive sense of investors choosing their portfolios dependent on the linear weighting they wish to place on risk, represented by the standard deviation of returns; a straightforward linear weighting that can be viewed as a personal probability approach in the Savage[88] sense. The Moeseke model, which bases its efficiency criterion upon a lower confidence limit, assumes a normal or Gaussian distribution but this assumption is not critical and can be relaxed[89]. With the exclusion of products specifically designed to exhibit skewness, however, the normal distribution of returns remains a good approximation for equity returns in particular, and therefore an appropriate personal probability approach.

A final point on the rationale for the mean-variance or mean-standard deviation approach was elaborated by Tsiang[90] who showed that as well as this

88 Savage, L. J., 1954, op. cit.

89 See v. Moeseke 1965a, op. cit.

90 Tsiang, S. C., The Rationale of the Mean-Standard Deviation Analysis, Skewness Preference, and the Demand for Money. The American Economic Review, V. 62, 1972, pp 354 - 371.

approach being applicable for investors with quadratic utility functions, or where returns are normally distributed,it is also applicable where the aggregate risk taken by the investor is small compared with his/her total wealth.

2.6.2 Mean, Variance and Covariance Estimation

For the Markowitz and Moeseke models estimates of mean, variance and covariance are essential for generating efficient frontiers. In the CAPM covariances are replaced by betas though the CAPM itself was not designed to generate efficient frontiers. It can be noted, however, that the use of beta in the single index model of portfolio selection does align this model with the CAPM[91].

Invariably the major criticism of mean, variance and covariance estimation is the reliability of the estimates. In particular can historical data give reliable estimates of future performance? Covariance estimation has been subjected to criticism purely on account of the enormity of the task for large numbers of securities. Note however that Markowitz and Perold[92] suggested a scenario approach to simplify the covariance calculations.

It should be noted that Markowitz never considered historical means, variances and covariances to be necessarily appropriate for generating efficient

91 For a detailed discussion of the single index model of portfolio selection see Levy, H. and Sarnat, M., 1984. op. cit., pp 356-385 or Elton, E.J. and Gruber, M.J., Modern Portfolio Theory and Investment Analysis. Fifth Edition, New York, Wiley, 1995, pp 128-206.

92 Markowitz, H. M. and Perold, A. F., Portfolio Analysis with Factors and Scenarios. The Journal of Finance, V. 36, 1981, pp 871 - 877.

frontiers for portfolio selection despite the fact that much criticism of the Markowitz approach is focused on the unreliability of historical data. Economic theory would suggest that excess economic profits for a particular company or group of companies would not be long-run sustainable in a competitive environment and favourable historical means may be the result of excess economic profits. Also, change is a fact of life for many companies through management change, change of direction, change in the economic environment and so on. This important consideration relating to the stationarity of the means, variances and covariances of returns was analysed by Kryzanowski[93] with the stationarity hypothesis being rejected for monthly US data.

Numerous other studies have questioned the relevance of historical data for estimating means, variances and covariances. Jorion[94] found very poor predictive ability in international monthly historical stock market returns. A result supported by Green[95] who also showed that shrinkage estimates of means and variances did little to help solve the problem.

Many researchers have attempted to find better ways of estimating expected return and variance but as Black[96] has pointed out their attempts have often relied on data mining rather than theory. Considering expected return in the

93 Kryzanowski, L., The E-V Stationarity of Secure Returns. Journal of Banking and Finance, V. 11, 1987, pp 117 - 135.

94 Jorion, P., International Portfolio Diversification with Estimation Risk. Journal of Business, V. 58, 1985, pp 259 - 278.

95 Green, N. R., A Test of Historical and Shrinkage Estimates of Expected Returns in International Portfolio Selection. Massey University Unpublished Thesis, 1993.

96 Black, F., Estimating Expected Return. Financial Analysts Journal, V. 49, 1993, pp 36 - 38.

first instance, Black emphasises the fact that estimates based on past data are inaccurate and while, with a long enough time series, average historical returns can be accurately estimated, this does little to help in the estimation of expected return.

Theory can help by considering reasons for the mispricing of securities. Fundamentally though, for risk averse investor, a lower return should be expected if risk is lower. In relation to this point Ballie and De Gennaro[97] have shown that a positive relationship between mean returns and standard deviation of US equity data from 1970 to 1987 is weak. While this result is interesting in itself it should be noted that equity markets will be catering to the needs of a wide range of investors, many of whom may not be risk averse. If segments of the market are dominated by such investors results as stated above are to be expected.

Variances tend not to be stationary over time as has been shown by Christie[98] and Schwert[99] although French, Schwert and Stambaugh[100] did show evidence of a positive relationship between expected risk premiums and volatility.

If one is attempting to construct portfolios for investment via the Markowitz, Moeseke or single index models, clearly the integrity of mean, variance and

97 Ballie, R. T. and De Gennaro, R. P., Stock Returns and Volatility. Journal of Financial and Quantitative Analysis, V. 25, 1990, pp 203 - 214.

98 Christie, A. A., The Stochastic Behaviour of Common Stock Variances: Value, Leverage and Interest Rate Effects. Journal of Financial Economics, V. 10, 1982, pp 407 - 432.

99 Schwert, G. W., Why Does Stock Market Volatility Change Over Time? The Journal of Finance, V. 44, 1989, pp 1115 - 1153.

100 French, K. R., Schwert, G. W. and Stambaugh, R. F., Expected Stock Returns and Volatility. Journal of Financial Economics, V. 19, 1987, pp 3 - 29.

covariance, or beta, estimates are paramount. However, if one is concerned with the risk profile of a particular market based on its historical performance, and therefore the risk profile one might expect investors to accept via a personal probability approach, then historical data clearly has a role to play. This said, the data used in this study has been tested for normality with these results being reported in Chapter 7 .

2.6.3 The Risk Free Asset, Nominal Returns and Real Returns, both Positive and Negative

As pointed out by Lintner[101] inflation negates the concept of a totally risk free asset and being able to borrow and lend at the risk free rate is of course unrealistic.

It should be remembered, however, that portfolio selection models were developed through the fifties and sixties when inflationary pressures were minimal. The seventies and eighties were different though the nineties to date has seen a move towards the lower inflation rates of earlier years.

Inflation adds a trend to returns data that leads to an overstatement of variation around the mean. Also nominal means, variances and covariances will not show stability across time periods with differing inflationary trends. A logical solution is to convert all data to real values but this creates its own problems.

101 Lintner, J., 1969, op. cit.

Real values can only be approximated in hindsight and estimating real values into the future is a problem of a magnitude that is dependent on the accuracy of inflationary expectations.

In December 1974, Lintner's presidential address to the American Finance Association Annual meeting focused on the impact of inflation on accepted models of security returns[102]. His opening words were as follows:

> "We are meeting at a time when few matters are of more serious concern to students of Finance and to members of the financial community than the impact of inflation on our financial institutions and markets and its implications for investment policy."

Rising inflation can have both positive and negative implications for asset prices, equity prices in particular. Rising inflation through the early to mid seventies bought about by the first oil shock of 1973 had a devastating impact on equity prices globally as rising interest rates could not be matched by company dividend increases as costs escalated. There have, however, been numerous examples of a major economic expansion having a very positive impact on equity prices at the same time as increasing inflationary pressures as was the case with Germany after reunification.

[102] Lintner, J., Inflation and Security Returns. The Journal of Finance, V. 30, 1975, pp 259 - 280.

There have been numerous papers written on the relationship between asset prices and inflation such as that by Fama and Schwert[103]. This particular study for the US from 1953 - 1971 showed that common stock returns were negatively related to expected inflation as well as unexpected inflation. Other researchers have concentrated on the impact of progressive tax rates in inflationary times, in particular where there are capital gains taxes as well as income taxes; see for example Yaari, Palmon and Marcus[104] and Palmon and Yaari[105].

In relation to the Moeseke model a particular problem arises in times of negative real returns which can occur when inflation rises sharply. In general current practice is for central banks to ensure this does not happen by raising interest rates but this has not always been the case in the past. Negative real returns occur when inflation is running above the rate of interest and this has occurred in New Zealand during the seventies and eighties. Under the Moeseke model an optimal portfolio is defined as being that portfolio where the marginal return to the investment dollar equals its marginal cost. As the marginal return approaches zero, however, the optimal solution becomes $x^* = 0$ where x^* is the portfolio of risky assets. That is the investor simply does not invest because he/she is too cautious to do so. If real returns are negative,

103 Fama, E. F. and Schwert, G. W., Asset Returns and Inflation. Journal of Financial Economics, V. 5, 1977, pp 115 - 146.

104 Yaari, U., Palmon, D. and Marcus, M., Stock Prices Under Inflation with Taxation of Nominal Gains. The Financial Review, V. 15, 1980, pp 38 - 54.

105 Palmon, D. and Yaari, U., Share Values - Inflation and Escalating Tax Rates. Journal of Banking and Finance, V. 5, 1981, pp 395 - 403.

however, not investing may be inappropriate. A solution to this problem is suggested in Chapter 5.

While the definition of the risk free rate may cause problems for the CAPM or the Markowitz approach in cases where the investor is concerned with comparison between risky and riskless assets, no such difficulties exist for the Moeseke model. This is because the appropriate interest rate, or opportunity cost to the investor is set as being simply that opportunity cost which is appropriate in the market place at that time. For the private investor the rate may be the rate on bank deposits while for the institution it would logically be that institution's marginal cost of capital. Each group or investor will face an optimal portfolio, or optimal position, relative to the appropriate prevailing rate in the marketplace.

2.6.4 The Allowance of Short Sales

Short sales are not allowed in the Moeseke model as is the preferred case with the Markowitz model. Short sales are an essential part of the CAPM, though as Lintner[106] pointed out an inoperative one in a number of cases, in particular when equilibrium is achieved. Markowitz[107] proposed an extension to the CAPM which did not allow short sales as has been discussed. For New Zealand the reality is that short sales are not allowed in the physical market, except within very short time frames. Like most countries with reasonably

[106] Lintner J., 1969, op. cit.
[107] Markowitz, H. M., 1990b, op cit.

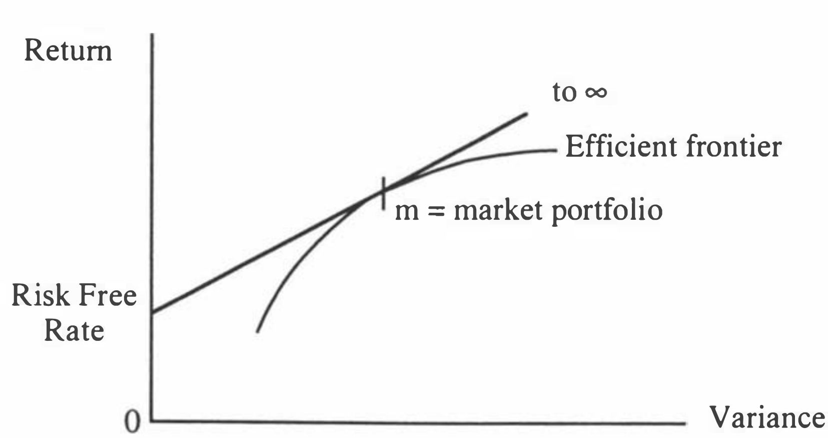
well developed financial markets, however, New Zealand has a futures market giving the opportunity for the short selling of stocks and bonds for future delivery.

Sharpe[108], in his essay in The Journal of Finance to commemorate his Noble prize, put forward justification of the allowance of short sales to enable selection of fully optimal portfolios in the CAPM sense but did also stress the point that futures and options markets have allowed for a greatly increased efficiency of capital markets, bringing them closer to the idealised world assumed by the CAPM. Probably the most important point in this debate regarding negative or non-negative holdings is the realism of the situation.

It can be noted at this point that there are other unrealistic situations with both CAPM and the Markowitz model as limits are approached.

Figure 2.6.4.1

CAPM with Limitless Lending or Borrowing



[108] Sharpe, W.F., 1991, op. cit.

Figure 2.6.4.1 is the traditional CAPM result with limitless lending or borrowing of the riskless asset. All investors buy the market portfolio and have some portion of borrowing or lending of the risk free asset. (Here it is the risk free asset that can be shorted limitlessly.) The investor too conservative to invest in the market will only hold the risk free asset. The holders of only the market, unleveraged, will be risk averse, but the risk neutral investor, wishing only for the highest expected return will borrow an infinite amount of the riskless asset. This is clearly an unrealistic position to take.

Figure 2.6.4.2

The Markowitz Model and the Minimum Variance Portfolio

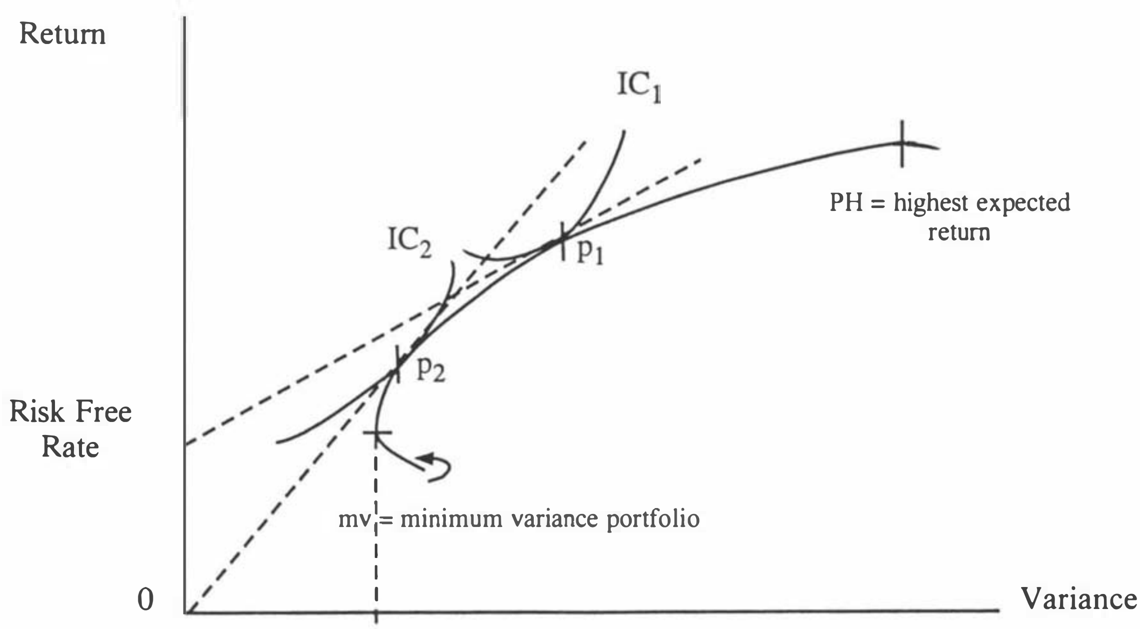


Figure 2.6.4.2 is the traditional Markowitz model. An efficient frontier is generated, short sales are not allowed, and the investor will invest in that portfolio which best suits his or her risk preference. In this diagram a tangent is drawn to the efficient frontier from two points, one depicting the risk free rate and one from the origin. Under quadratic programming the efficient

frontier is generated by varying the interest rate. The portfolio shown as P_1 would be regarded as appropriate for all investors if there existed a risk free asset with limitless borrowing or lending. In fact at this point the model looks very similar to the CAPM. Without being able to borrow at the risk free rate, however, portfolio PH is clearly appropriate for risk neutral investors. (This portfolio will almost certainly be a one stock investment). The minimum variance portfolio, however, has an expected return higher than the risk free asset, is very close in terms of risk and return to portfolio P_2, but is for consideration only if the interest rate or the required return is $-\infty$, clearly an unrealistic situation. While the approaching of this limit does not cause as serious a problem as with the CAPM limit problem it is difficult to see intuitively how the investor would relate the minimum variance portfolio to his or her investment portfolio decision. Indifference curve analysis is the appropriate method in determining an appropriate portfolio for the Markowitz model but it is not possible for an optimal indifference curve to pass through the point of the minimum variance portfolio.

In contrast the Moeseke model is clearly defined over a much wider range of risk attitudes, starting from the investor who is too risk averse to make any risky investment through to risk takers or risk lovers.

2.6.5 Other areas of criticism

Positive models continue to be "perfected" in an attempt to explain precisely the process of asset pricing and the appropriate makeup of risky portfolios. Normative models continue to be "perfected" in an attempt to select the most appropriate portfolio for an investor or managed fund. Taxes and transactions costs will always play a part in both approaches. Pogue[109] extended the Markowitz model to include variable transactions costs and taxes among other things. The treatment of taxes and transactions costs in the Moeseke model is detailed in Chapter 6.

Market segmentation is a fact of life. There are investors who deliberately endeavour to concentrate investments on certain sectors of the market. They may be more or less risk averse or simply have more specific interests. Plan sponsors in the US often deliberately select mangers to manage funds in certain sectors of the market with some managed funds selling themselves to the market place by focusing on this specialisation. More simply, the market place is just too large to be carefully analysed by other than the very largest financial institutions.

This certainly can cause problems for the positive theorist seeking out that one internally consistent model to explain all asset pricing, but is of little

[109] Pogue, G. A., An Extension of the Markowitz Portfolio Selection Model to Include Variable Transactions Costs, Short Sales, Leverage Policies and Taxes. The Journal of Finance, V. 25, 1970, pp 1005 - 1027.

importance within the normative model of Markowitz and the positive model of Moeseke which focuses on marginal costs and marginal returns.

Continuous time models are another area of importance with considerable progress having been made in recent times by the positive theorists. For a solution, however, a time frame is specified and that is effectively approaching instantaneity in a continuous time model. The static approach can, in reality, be fairly similar. A single time frame may be anything from a year to a day or points either side.

This concludes the discussion on the major criticisms and limitations of capital asset pricing and portfolio selection models though some of these issues will be considered again when considering the Moeseke model in detail.

The last section of this chapter is an introduction to the relationship between modern portfolio theory and general economic equilibrium theory.

2.7 The Relationship between Portfolio Theory and Economic Theory

Analysis of the relationship between portfolio theory and economic theory has been a major area of research, particularly during the 1980's. An overview of this work has been presented by Gross[110] on which the following discussion is based. In economics the basic general equilibrium model is normally regarded as being the Arrow-Debreu

110 Gross, E., On the General Equilibrium Foundations of Finance Theory. Macquarie University Graduate School of Management Monograph, 1992.

model (1954). Integrating finance theory into this general equilibrium model has been achieved via contributions from a number of theoreticians with the integration requiring substantial extensions to the Arrow-Debreu model. A comprehensive assessment of the literature relating to this integration was carried out by Duffie (1988)[111]. To incorporate portfolio theory into a general economic equilibrium model it is important first to analyse the asserted differences between economics and finance theory.

Markowitz[112] noted that portfolio theory differed from the theory of the firm and the theory of the consumer in that it was concerned with investors rather than with manufacturing firms or consumers, and also that it was concerned with economic agents that act under uncertainty. As Gross[113] pointed out, S.A. Ross held the view that the essential difference between economics and finance theory was that the former was concerned with barter economies where real commodities were exchanged, while the later was concerned with trading intertemporal contracts (financial securities). Moeseke took a more fundamental view stating that portfolio theory dealt with the money economy rather than the real economy.

In the Arrow-Debreu model there are two types of agents, consumers and producers, and the objects of choice are commodities. A commodity can be defined by its physical characteristics as follows:

111 Duffie, D., 1988, op. cit.
112 Markowitz, H. M., 1991, op. cit.
113 Ibid.

(i) time of availability,

(ii) location of availability,

(iii) state of nature conditional upon which it is made available.

The third characteristic of a commodity does take into account the uncertainty aspect of decisions involving the future.

In relation to the Ross position, the general Arrow-Debreu model does not include the trading of intertemporal contracts but Arrow[114] himself introduced securities into a general equilibrium model though this was at the level of allocation of securities rather than the trading of securities.

Radner[115] however firmly placed the trading of securities into general economic equilibrium theory when he considered a sequence of spot markets for both goods and securities and showed the existence of an equilibrium given certain conditions.

From this point it was now a matter of integrating the major theoretical results of finance theory into general economic equilibrium theory. These results of finance theory included the CAPM and Arbitrage Pricing Theory in particular. Again as noted by Gross[116] this involved a reinterpretation of the objects of choice which were now to be securities instead of commodities and the selection of a mathematical object to represent the choice space for returns on securities. This problem was solved by

114 Arrow, K.J., The Role of Securities in the Allocation of Risk Bearing. Review of Economic Studies, V. 31, 1964, pp 91 - 96.

115 Radner, R., Existence of Equilibrium of Plans, Prices and Price Expectations in a Sequence of Markets. Econometrica, V. 40, 1972, pp 289 - 303.

116 Gross, E., 1992, op. cit.

Duffie[117] allowing the Arrow-Debreu model to clearly be extended to the case of stochastic economies.

It is interesting, within the context of this thesis, to discuss how the Moeseke model might fit within the framework of general economic equilibrium theory. While the Moeseke model is not an asset pricing model as such, the model does determine the "price" of entering any particular capital market. It is important to relate this to the price setting mechanisms for commodities, as defined by Arrow-Debreu, and also to the price setting mechanisms for securities. These matters are addressed in Chapter 4.

The main purpose of the preceding discussion has been to discuss the development of portfolio selection and asset pricing models and to discuss the Moeseke model within this context. The Moeseke model is detailed in Chapter 3, extended in Chapter 5, and applied in Chapters 6 and 7.

The application of the Moeseke model given in Chapters 6 and 7 focuses on examining the risk profiles of New Zealand's capital markets rather than generating portfolios for investors, although portfolios are given as an output of the analysis. It can be noted that shifts in the risk profile of markets would relate to shifts in the systematic risk of markets. Another important focus of the application relates to marginal costs for investors and these are examined in detail in Chapters 5 and 7.

117 Duffie, D., 1986, op. cit.

CHAPTER THREE

HOMOGENEOUS PROGRAMMING BY THE TRUNCATED MINIMAX CRITERION

3.1 Introduction to the Truncated Minimax Criterion

The description of the basic homogeneous programming by the truncated minimax criterion model which follows in Sections 3.1 to 3.4 is taken directly from Moeseke (1965a, 1965b, 1968, 1977), Moeseke and Hohenbalken (1974) and Young (1985). More precise references are given where appropriate. In general proofs are not given but can be found in the original articles. As this model is applied in this thesis it is described in detail with the format followed being that presented by Moeseke for the most part.

As stated in Chapter 2, Section 3, the Moeseke model was developed from the linear programming model designed to solve the typical economic problem of the allocation of scarce resources. To follow Moeseke's approach[118] let "risk" be characterised by a model which formally takes into account the entire probability distribution of outcomes, either in a subjective or objective manner[119]. Expected value or expected utility would then be regarded as "risk" methods or approaches as they take into account the entire distribution of outcome. Uncertainty, on the other hand, applies where the approach limits its view to one or a few specific points in the domain of the

118 Moeseke, P. v., 1965a, op. cit. p 199.

119 A definition suggested by Knight, F. H., 1921, Risk, Uncertainty and Profit (New York: Houghton Mifflin) Ch 7.

distribution of outcomes. An example of this later position would be the maximin criterion for decision making. By this definition homogeneous programming by the truncated minimax criterion is a "risk" model.

As is fundamental to any portfolio selection model, homogeneous programming by the truncated minimax criterion is a procedure for allocating the investment dollar, a scarce resource, in an economic environment where there is imperfect knowledge of that environment. That is, some of the parameters are stochastic rather than deterministic.

Consider then the following linear programming model for the allocation of scarce resources[120]:

$$\max_{x \varepsilon X} [f(x) = cx] \tag{3.1.1}$$

$$X = \{x | Ax \leq b, x \geq 0\} \tag{3.1.2}$$

The symbols c, A, b are parameter matrices over the reals with dimensions 1 x n, m x n and m x 1 respectively, x is a real n-tuple of variables. In a typical economic problem where X is the set of feasible actions (a given subset of R^n where R is a non-negative real line); c refers to net returns c_j per unit activity level; b to available quantities b_i of resources r (where r is a real n-tuple); A to the technical or input-output coefficients a_{ij} of the vector $x = x(r)$ of production functions.

120 See Moeseke, P.v., 1965a, op.cit., pp 199-200.

Debreu[121] defined the economic environment of the production agent in terms of the values c, A and b. The interpretation of imperfect knowledge within this environment then is that some or all of the parameters are stochastic rather than deterministic.

The vector [$c_1,...c_n, a_{11},...a_{mn}, b_1,...b_m$] can be denoted by p, and $P = \{p\}$ refers to parameter space where p is the state of nature. In the deterministic case there will normally be a solution which will be unique (ordinary linear programming).

In the stochastic case however p will be some subset in R^{n+mn+m} (stochastic linear programming).

The approach here, as mentioned earlier, is a risk approach where the distribution of outcomes are formally taken into account. Therefore it is assumed that the distribution of the components of p is known or at least explicitly stated. The description of the distribution may be subjectively or objectively based[122].

If p is stochastic then the maximand of (3.1.1) is

$$f(x,p);\ f: X \times P \rightarrow R \qquad (3.1.3)$$

In a situation of imperfect knowledge the entrepreneur, or investor, will select an x, nature will deliver a $p \varepsilon P$ and the resulting outcome $f(x|p)$ will be observed. Note though that by (3.1.2) $X = X(p)$ so that an arbitrary x is feasible only if

121 Debreu, G., 1959, op. cit.

122 For a discussion on subjective and objective probability approaches see Chapter 2, Section 2.

$$x \varepsilon X(p) \tag{3.1.4}$$

In other words not only is there an imprecise relationship between actions and outcomes but also the set of actions that limited resources will permit cannot be accurately determined.

Given the above framework, the Moeseke model was developed as follows[123].

Initially two assumptions are made, both of which can be relaxed somewhat as shown by Moeseke[124]. The first assumption is that only c is random and therefore the stochastic-feasibility issue mentioned above is not a constraint. Next it is assumed that the components of c are jointly normal. This being the case the distributions of (x) of the maximand will also be normal and therefore completely characterised by their first two moments being expected return and the standard deviation of returns.

For any entrepreneurial decision there will be a set of possible consequences. These sets of consequences require ranking which in turn requires a subjective weighting of all consequences to each decision to be carried out. As the model is a "risk" model, the criterion for setting the weights are defined in terms of the complete probability distribution of the outcomes.

The criterion proposed by Moeseke is referred to as the truncated minimax criterion[125] and is based on the Expected Return - Variance criterion proposed by Markowitz in

123 See Moeseke, P.v., 1965a, op.cit., pp 201-216.
124 Ibid.
125 Baumol, W., 1963, later proposed the same criterion independently under the name "expected gain-confidence limit criterion.

his pioneering work on portfolio selection[126]. The E-V criterion states that only efficient decisions should be considered. Efficient decisions being those which give the maximum expected return for any level of risk or which are the least risk decisions for any level of expected return. Choice among efficient decision is left to one's relative valuation of risk versus return. Note, however, that while the minimax criterion is based on the E-V criterion it is not the same. The E-V criterion maximizes expected return for a given level of risk whereas the minimax criterion generates an efficient frontier made up solely of lower confidence limits as will be shown.

Under the normality assumption the following risk preference functional can be constructed in terms of $Ef(x)$ and $\sigma f(x)$ possessing a confidence limit interpretation.

$$\phi f(x) = Ef(x) - m\sigma f(x), (m \varepsilon R) \qquad (3.1.5)$$

For any distribution having the entire real line as its domain, such as the normal distribution, min $f(x,p) = -\infty$ (all x in X) and therefore

$$\max_{x \varepsilon X} \min_{p \varepsilon P} f(x,p)$$

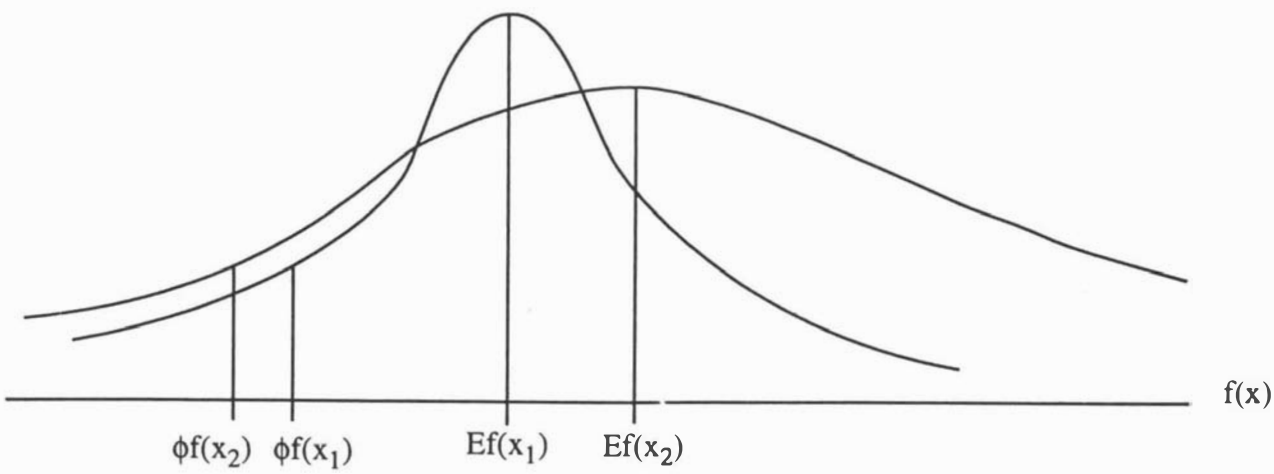
is indeterminate.

The truncated minimax rule adapts the minimax rule to normally distributed outcomes as shown in figure 3.1.1 which considers the graph of two competing distributions X_1 and X_2.

126 Markowitz, H. M., Portfolio Selection. Journal of Finance, V.7, 1952, pp 77-91.

Figure 3.1.1

The Truncated Minimax Rule for Two Competing Distributions



Source: Moeseke, P.v., 1965a, p 208.

For the normal distribution for example, the table of the normal distribution gives

$$(2\pi)^{-\frac{1}{2}} \int_{-\infty}^{m} \exp(-t^2/2)dt = \alpha \qquad (3.1.6)$$

where α=0.05 is the confidence limit corresponding to m=1.65. By substituting m=1.65 into equation (3.1.5) we are effectively comparing the lower 0.05 confidence limits of competing distributions. In other words the maximisation corresponds to applying the minimax criterion to the distributions after truncating them at their 0.05 confidence limits.

$$\min_{p \varepsilon P} \; f(x,p) = \phi f(x) \qquad (3.1.7)$$

$$\max_{x \varepsilon X} \; \phi f(x) = \max_{x \varepsilon X} \; \min_{p \varepsilon P} \; f(x,p) \qquad (3.1.8)$$

$Max \quad \phi f(x)$ is a truncated minimax in that the entrepreneur or risk taker considers nature's most antagonistic response to be limited to say the 95 percent level and ignores the most antagonistic 5 percent of cases.

Moeseke then defined the parameter m in (3.1.5) and (3.1.6) as the risk preference parameter of the truncated minimax criterion. Given the normality assumption, the risk preference could be read directly from (3.1.6), or interpreted as a relative weighting between expectations and standard deviations of outcomes. For example in the later case an m value of 1.0 means the entrepreneur places as much weighting on risk as he/she does on return. The interpretation would hold even without the normality assumption. This is because, regardless of the normality assumption, the probability statement expressed by the Bienaymé - Tchebyshev inequality[127]

$$prob[|f(x) - Ef(x)| \geq |m|\sigma f(x)] \leq \frac{1}{m^2} \tag{3.1.9}$$

can always be made. This probability statement is a general statement applicable for all distributions allowing one to determine the maximum probability of obtaining an outcome less than some value.

Different values of the risk parameter m therefore correspond to different attitudes of the decision maker towards risk as shown in the following table.

127 Cramer, C.F.H., Mathematical Methods of Statistics, Princeton : Princeton University Press, 1946, pp 182 - 183.

Table 3.1.1

The Truncated Minimax (and Maximax) Criterion

Value of Risk Preference m	Character of $\max_{x \varepsilon X} \phi f(x)$	Risk Attitude
> 0	Truncated Minimax Criterion	Risk Averting
$= 0$	Borderline Case	Risk Neutral
< 0	Truncated Maximax Criterion	Risk Seeking

Source: Moeseke, P.v., 1965a, p210

The truncated minimax criterion now allows for efficient decision making when having to account for a stochastic parameter. Aligning the portfolio problem to the typical production problem in economics where c, A and b stand for per unit returns, input coefficients and resources, respectively; in the portfolio case there exists stochastic returns on securities from the set of all security options, given a specified budget restriction.

The following is a summary of the principal characteristics of the truncated minimax criterion[128].

1. It represents a linear weighting of expected return and standard deviation of return (considered as a scalar risk measure). That is m expresses the investor's risk preference.

128 See Moeseke, P. v. and Hohenbalken, B. v., Efficent and Optimal Portfolios by Homogeneous Programming. Zeitschrift fur Operations Research, V. 18, 1974, pp 207-208.

2. It can be interpreted in terms of confidence limits.

3. The criterion allows the minimax to be determined (min $h(x) = -\infty$ for all $x \neq 0$) by truncating the left tail of the distribution of $h(x)$.

3.2 Efficient Portfolios

As shown by Moeseke[129], the yield of a portfolio can be denoted by

$$\bar{c}x, \quad x\varepsilon B \qquad (3.2.1)$$
$$B = \{x \geq 0 | rx \leq 1\}, \quad r > 0$$

where B is the budget set; x is a n-tuple representing a portfolio containing x_i units of the ith security; c is the n-tuple of expected yields, and r is the n-tuple of security prices.

The investor's budget can be set at unity without loss of generality.

Now define $\phi(x, m) = (\bar{c}x - m(xVx)^{\frac{1}{2}})$

where $(xVx)^{\frac{1}{2}}$ is the standard deviation of the portfolio

and then, for a given m, the following mathematical programme can be applied.

$$\max_B \phi(x, m) = \max_B(\bar{c}x - m(xVx)^{\frac{1}{2}}) \qquad (3.2.2)$$

[129] See Moeseke, P.v., Stochastic Portfolio Programming: The Game Solution. In Stochastic Programming: Proceedings of the International Oxford Conference, M. Dempster, ed, Academic Press, London 1977, pp498-499.

representing a linear weighting of expectation and standard deviation. A solution to programme (3.2.2) will be denoted by $x(m)$ and for any solution the value of ϕ will be denoted by $\phi(x(m), m)$.

On account of the assumption that randomness is limited to the components of c, by the truncated minimax criterion, (3.2.2) becomes the new maximand superseding (3.1.1).

Note now that criterion (3.2.2) converts a stochastic linear programming problem into a linear homogeneous one, as shown by Moeseke[130].

By definition a function $F : R^n \to R$ is homogeneous of degree one (linear homogeneous) if

$$F(\lambda x) = \lambda F(x), \quad (x \varepsilon R^n, \lambda \varepsilon R) \tag{3.2.3}$$

If λ is restricted to $R>0$ then it is positively homogeneous of degree one. This being the case all homogeneous functions are positively homogeneous, in particular $\phi(x, m)$ is positively homogeneous of degree one.

Further, the programming problem is a problem in convex programming as X is convex by definition and in (3.1.2) $F(x)$ is either a concave maximand or a convex minimand.

130 See Moeseke, P. v., 1965a, op. cit., pp 211-214.

Both points have economic significance. A convex constraint set represents non-increasing marginal rates of transformation (or opportunity cost) if all co-ordinates $x_j (j=1...n)$ denote outputs, as is the case here. A concave maximand indicates non-increasing average returns for situations of both monopoly and risk aversion. The opposite applies for a convex maximand.

Note that the risk functional $\phi(x) = \bar{c}x - m(xVx)^{\frac{1}{2}}$ will be convex for $m \leq 0$, risk taking, and concave for $m \geq 0$, risk aversion.

Since X is convex and $F(x)$ is concave the following propositions hold[131].

1. For all $m \geq 0$ there will be a solution x^* of (3.2.2) which is an efficient portfolio.

2. Varying $m \geq 0$ will give a complete set of efficient portfolios which are solutions to (3.2.2).

3. The standard deviation of a solution to (3.2.2) is a non-increasing function of the relative weight, m, attached to it. That is $\Delta m \Delta (x^* V x^*)^{\frac{1}{2}} \leq 0$.

The fundamental Kuhn-Tucker duality theorem applies to convex programming[132]. Importantly though it has been shown that an extramand that is homogeneous of degree one with linear constraints will have a dual solution which is totally independent of any primal variables[133]. In contrast, non-linear programming problems

131 See Moeseke, P. v. and Hohenballan, B. v., 1974, op. cit., pp 206-207.
132 Kuhn, H. W. and Tucker, A. W., Nonlinear Programming. Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, Berkeley: University of California Press, 1951, pp 481-492.
133 Eisenberg, E., Duality in Homogeneous Programming. Proceedings of the American Mathematical Society, V. 12, 1963, pp 783-787.

do not, in general, have a dual solution that contains only the dual variables[134].

Given the above, the standard results of homogeneous programming hold; see Moeseke (1965a, 1965c) and LeBlanc and Moeseke (1976). In particular this is true of the duality theorem of homogeneous programming which states that the primal convex homogeneous programming problem,

$$\max_{x \varepsilon X} \mathrm{f}(x) \quad X = \{h(x) \leq b, x \geq 0\} \tag{3.2.4}$$

has a solution x^* if and only if, a certain dual problem has a solution v^* such that

$$\mathrm{f}(x^*) = V^* b \tag{3.2.5}$$

Stating the dual problem explicitly

$$\min_{v \varepsilon V} vb \tag{3.2.6}$$

$$V = \underset{x \varepsilon X}{U} \; V(x), V(x) = \{v | vh_x \geq F_x, V \geq 0\} \tag{3.2.7}$$

For the typical production programme with revenue $f(x^*)$, resources b_i, and resource prices (or rents) V_i^*, this means that in convex homogeneous programming revenue is exactly distributed over factor rewards or, for the portfolio problem, there exists a factor price system to guide allocation under risk.

134 Dorn, W. S., Duality in Quadratic Programming. Quarterly of Applied Mathematics, V. 18, 1960, pp 155-162.

Restating the portfolio problem; $\max \phi(x) = \bar{c}x - m(xVx)^{\frac{1}{2}}$

Then by the duality theorem $\bar{c}x^* - m(x^*Vx^*)^{\frac{1}{2}} = V^*b$ (3.2.8)

Rearranging (3.2.8) gives $\bar{c}x^* = V^*b + m(x^*Vx^*)^{\frac{1}{2}}$ (3.2.9)

which can be written as $Ex^* = V^*b + m\sigma x^*$ (3.2.10)

That is, the expected value equals the original factor payments plus the risk taker's take.

From (3.2.8) $$m = \frac{Ex^* - V^*b}{\sigma x^*} \qquad (3.2.11)$$

which is the expected net profit divided by the standard deviation of net profit or the inverse of the coefficient of variation of net profit. For the normal distribution, $z = (x - \mu)/\sigma$.

For the portfolio problem, V^*b can be written as λ^* as in the portfolio problem b is the budget, set to unity.

Rewrite (3.2.8) as follows.

$$\bar{c}x^* - m(x^*Vx^*)^{\frac{1}{2}} = \lambda^* \qquad (3.2.12)$$

where λ^* is the marginal return to the unit of investment, or the investment dollar in terms of utility function ϕ. That is the dollar's marginal value to the investor using criterion ϕ [135].

[135] For a fonnal proof see Moeseke, P.v., A General Duality Theorem of Convex Programming. Metroeconomica, V.17, 1965c, p 164.

3.3 Optimal Portfolios

As shown by Moeseke and Hobenbalken[136] a portfolio can now be defined as being optimal if the following optimality criterion is satisfied.

$$\max_x \phi(x|m^+) \tag{3.3.1}$$

where $m^+ = \max\{m \geq 0 \mid \max \phi(x|m) \geq r\}$ (3.3.2)

and r is an appropriate rate of interest or cost of capital in the capital market. It is important to be clear as to the flexible interpretation of r at this point. For the financial intermediary this will be the cost of attracting the marginal dollar on the capital market, for example the cost of term deposits or the return required on the financial intemediary's own shares. For the private investor, the cost could be seen as an opportunity cost measured by the rate of interest on deposits.

As seen by the duality theorem of homogeneous programming, the primal problem

max $\phi(x|m)$ on X implies the dual problem

min λ on $L = \{\lambda \geq 0 | \lambda_q \geq \phi x; x \varepsilon X\}$ (3.3.3)

where q is the n-tuple of security prices

and ϕ_x is the gradient vector $[\delta\phi|\delta x ..., \delta\phi|\delta x_n]$

136 Moeseke, P.v. and Hohenbalken, B.v., 1974, op.cit. pp 208-210.

Also for any dual solution λ^*

$$\lambda^* = \phi(x^*|m) = \bar{c}x^* - m(x^*Vx^*)^{\frac{1}{2}} \tag{3.3.4}$$

This optimality criterion, therefore, selects that portfolio which allocates the budget with maximum caution under the additional criterion that the marginal value of the investment dollar is not exceeded by its marginal cost.

The optimal portfolio x^* satisfies

$$\max_{x \varepsilon X} \quad \phi(x|m^*) = \phi(x^+|m^+) = \lambda^+ = r \tag{3.3.5}$$

The uniqueness of m^+ is shown by Moeseke and Hohenbalken as follows.[137]

It can be argued that a value of m, say m_0, can be selected that is small enough, and another value of m, say m_1, $m_1 \geq m_0$ that is big enough so that

$$\max_x \phi(x|m_0) > r > \max_x \phi(x|m_1) \tag{3.3.6}$$

If there exists an m^+ such that

$$\max_x \phi(x|m^+) = r \tag{3.3.7}$$

137 See Moeseke P. v. and Hohenbalken B. v., 1974, op. cit., pp 209-210.

and if x^+ denotes a solution to (3.3.6), (x^+, m^+) clearly solves (3.3.1, 3.3.2) and x^+ is an optimal portfolio (usually unique but not necessarily so).

If there are values m_0,m_1 satisfying (3.3.5) then there exists a unique m^+ satisfying (3.3.6).

This value m^+ can be viewed as giving the objective risk profile of the market from which the optimal portfolio is derived.

Note that given the existence of a unique m^+ where $\lambda^+=r$ from (3.2.9)

$$m^+ = \frac{\bar{c}x^+ - r}{\sigma x^+} \tag{3.3.8}$$

That is the solution to the portfolio problem is found where $m = m^+$ equates to the inverse of the coefficient of variation of net profits. The expected value of net profit is $(\bar{c}x^+ - r)$. This definition of an optimal portfolio was later proposed by Sharpe within the context of the CAPM as will be shown in Section 3.7. A further point that should be noted here is that economically trivial cases can exist but should be excluded. This can be done by assuming the following[138].

$\phi(x(m), m) = V(m) > r$ for m=0 (assumption 1)

$\phi(x(m^0), m^0) = V(m^0) < r$ for m^0 large enough (assumption 2)

[138] See Moeseke, P. v., 1977, op.cit. p502.

Assumption 1 states $Ex>r$ for some $x \varepsilon \beta$. If this was not the case no portfolio would have a higher expected yield than the market rate of interest and therefore no conservative investor or financial institution would invest in the particular financial market concerned.

Also unless assumption 2 is satisfied the marginal value $v(m)$ of the extra budget dollar would exceed its cost (r) for all investors, even those with the most conservative of attitudes. The rate of interest, or cost of capital, would then be bid up in the marketplace until it became attractive for some conversative investors. Note that, for many investors, entering a financial market such as a stock market is not seen as a viable option. These investors are so conservative that $v(m)<r$ at all times.

3.4 An Algorithm for Identifying Efficient and Optimal Portfolios

There are a number of algorithms that can be applied to identify the set of efficient portfolios or an optimal portfolio, in particular see Moeseke (1965a), Moeseke and Hohenbalken (1971) and Goldsmith and Stachurski (1988).

The algorithm used for the analysis in this thesis is the original algorithm proposed by Moeseke (1965a) and is applied as follows.[139]

139 See Young, M., Portfolio Selection by Homogeneous Programming. M.A. Thesis, Massey University, 1985, pp 28-31.

Iteration 1

Evaluate $\phi(x) = cx - m(xVx)^{\frac{1}{2}}$ for $\quad x_i = 1 \quad c = 1,2,4....n$
$\quad x_k = 0 \quad k \neq i$

i.e. $\quad \phi(x_i) = c_i - m\sigma_{ii}^{\frac{1}{2}}$

From all risk adjusted minimums select the investment option with the maximum value (call it option k). At this point this investment option will make up 100% of the portfolio.

Put $x^1 = [x_1^1 \quad x_2^1 \quad ....x_k^1 \quad ....x_n^1]$
where $x_k^1 = 1$
$x_i^1 = 0 \quad$ all $i \neq k$

Iteration 2

Step 1:

Differentiate ϕx_i with respect to $x_i \quad i = 1,2....n$

$$\frac{d\phi}{dx_i} = \phi_{x_i} = c_i - \frac{m \sum_{j=1}^{n} \sigma_{ij} x_j}{(xVx)^{\frac{1}{2}}}$$

Evaluate ϕ_{x_i} at $x^1 \quad i = 1,2....n$

i.e. $\quad \phi x_i(x^1) = c_i - \frac{m\sigma_{ik}}{\sigma_{kk}^{\frac{1}{2}}}$

Select the investment option with the highest value (call it option l) as the second entrant.

Put $\bar{x}^1 = [\bar{x}_1^1, \bar{x}_2^1, \quad ....\bar{x}_l^1 \quad ....\bar{x}_n^1]$

where $\quad \bar{x}_l^1 = 1$
$\qquad\quad \bar{x}_i^1 = 0 \qquad\qquad$ all $i \neq l$

Step 2:

Apply the following formula to calculate λ

$$\lambda = \frac{\gamma u^{-1} q - [(\gamma U^{-1} q)^2 - (q U^{-1} q)(\gamma U^{-1} \gamma - m^2)]^{\frac{1}{2}}}{(q U^{-1} q)}$$

where

$$\gamma = \begin{bmatrix} \bar{c} x^1 \\ \overline{cx}^1 \end{bmatrix}$$

for x^1 and $\bar{x}^1$ as previously calculated.

$$U = \begin{bmatrix} x^1 V x^1 & x^1 V \bar{x}^1 \\ \bar{x}^1 V x^1 & \bar{x}^1 V \bar{x}^1 \end{bmatrix}$$

Note $x^1 V \bar{x}^1 = \bar{x}^1 V x^1$

$q = [1 \quad 1]$ $\qquad$ or $\qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\qquad$ as appropriate.

Step 3:

Having obtained the value λ as at the second iteration apply the following formula,

$W^1 = U^{-1}(\gamma - \lambda q)$

Normalisation gives $W_1 = \dfrac{W_1^1}{W_1^1 + W_2^1}$ $\qquad$ and $W_2 = \dfrac{W_2^1}{W_1^1 + W_2^1}$

Then put $x^2 = W_1 x^1 + W_2 \bar{x}^1$

GENERAL ITERATION $t + 1$

Step 1:

Evaluate ϕ_{x_i} at x^t i=1,2....n

Select the investment option with the highest value as the next entrant (call it option y).

Put $\bar{x}^t = [\bar{x}_1^t \quad \bar{x}_2^t \quad ....\bar{x}_y^t \quad ....\bar{x}_n^t]$

where $\bar{x}_y^t = 1$
$\bar{x}_i^t = 0$ all $i \neq y$

Step 2:

Evaluate λ as before

where $\gamma = \begin{bmatrix} \bar{c}\, x^t \\ \bar{c}\bar{x}^t \end{bmatrix}$

x^t and $\bar{x}^t$ as previously calculated

and $U = \begin{bmatrix} x^t V x^t & x^t V \bar{x}^t \\ \bar{x}^t V x^t & \bar{x}^t V \bar{x}^t \end{bmatrix}$

Step 3:

$W^T = U^{-1}(\gamma - \lambda q)$

Normalise then put $x^t = W_1 x^{t-1} + W_2 \bar{x}^{t-1}$

This process is repeated until an investment option that is already part of the portfolio is selected a second time. As soon as that occurs the previous iteration is deemed to be the last and a final calculation is carried out as follows

$$\lambda = \frac{\{\bar{c}V^{-1}p - [(\bar{c}v^{-1}p)^2 - (pV^{-1}p)(\bar{c}v^{-1}\bar{c} - m^2)]^{\frac{1}{2}}\}}{(PV^{-1}P)}$$

As all returns are in terms of one investment dollar

$$p \quad = \quad \begin{bmatrix} 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}$$

V = covariance matrix for those investment options selected for the portfolio only.

$\bar{c}$ = Matrix of means for those same investment options.

Having calculated a final value for λ the formula $X = SV^{-1}(\bar{c} - \lambda p)$ can be applied to give the final percentages for the allocation of funds between investment options where S is the normalisation factor. As λ gives the marginal value of the budget dollar, under the condition that it should not be exceeded by its marginal cost, the optimal portfolio can now be selected.

It is important to note at this point that from (3.2.7) a dual solution $V^*<0$ is not feasible, hence for any feasible solution $V \geq 0$ and so too the optimal dual solution $V^* \geq 0$.

For the portfolio problem the dual variable is $\lambda \geq 0$. With the budget set at unity $\lambda^* = \phi(x^*)$. If $\phi(x) \leq 0 \quad \forall x$ then $x^* = 0$ and $\phi(x^*) = 0 = \lambda^*$, thus satisfying $\lambda^* \geq 0$.

Computationally note the following remark from Moeseke.[140]

Let $x^* = \{x \mid Px = 1\}$ where x is the n-tuple of security amounts and $p>0$ is the n-tuple of security prices.

If there exists a positive value of $\phi(x)$ on X then necessarily for all x^M, $x^M \varepsilon X^*$.

Proof: Let there be an x^M such that $px^M<1$. Define $k=1/px^M>1$. Clearly $kx^M \varepsilon X^*$. By the premise $\phi(x^M) > 0$. Hence $\phi(kx^M) = k\phi(x^M) > \phi(x^M)$. By contradiction this establishes the desired result.

By this remark the feasible set is restricted to the *(n-1)* simplex X^* with vertices $\hat{x}^i$ satisfying

$$\hat{x}^i_j \begin{bmatrix} > 0 \quad \text{if} \quad j = i \\ \\ = 0 \quad \text{if} \quad j \neq i \quad (i,j = 1 \ldots n) \end{bmatrix}$$

The remark deals only with the case where $\phi(x)$ attains a positive value on X. In the opposite case the algorithm detailed above will necessarily lead to a maximizer $\hat{x}$ of ϕ on $X^* \cap X$ such that $\phi(\hat{x}) \leq 0$. Then by homogeneity, $0 = x^M$.

Homogeneity $\Rightarrow \quad \phi(0) = 0$.

140 Moeseke, P. v., 1965a, op. cit., pp 248-250.

Note also that as $M \to \infty$ clearly $\phi(x) = \bar{c}x - m\sigma x$ becomes $\leq 0 \quad \forall x$.

The optimal solution is then $x^{*}=0$.

This can be seen as stating that the minimum-variance choice is always not to invest in anything other than the riskless asset. Discussion on the existence and makeup of the riskless asset has been given in Chapter 2.

The inadmissibility of $\lambda<0$ does raise an interesting point in relation to the application of this model. This is because it may be desirable to consider real returns as opposed to nominal returns and financial market data has from time to time included periods of negative real returns. A solution to this problem is suggested in Chapter 5.

3.5 Previous Extensions to the Homogeneous Programming by the Truncated Minimax Criterion Model

3.5.1 Portfolios with Reserve Coefficient

Extensions to the model detailed in this section and Section 3.5.2 are taken from LeBlanc and Moeseke[141] with the reader again being referred to the original work for proofs where these are not given. In the previous sections of the chapter consideration has been given to explaining a model of portfolio optimisation in the general case where the investor wishes to maximise utility

141 LeBlanc, G. and Moeseke, P. v., Portfolios with Reserve Coefficient. Metroeconomica, V. 31, 1979, pp 103 - 118.

with maximum caution. An important consideration was the marginal cost of the investment dollar. For the private investor that cost could be viewed as an opportunity cost equal to the rate of interest available either on government securities (for maximum security) or general bank term deposits. The situation would be different for the financial intermediary. Here the cost could best be viewed as equating to the cost of the marginal dollar which it is able to attract in the capital market which it wishes to access.[142] The cost of this dollar will almost invariably be influenced by the actions of the monetary authority of the country in which the financial intermediary is operating. The three common tools of monetary policy are open market operations, reserve coefficients and discount or bank rates for borrowing which will generally determine the rate of interest on deposits.[143] This section will consider open market operations, or more particularly the interest rate on government securities which is determined by these, and reserve coefficients.

Consider then the financial intermediary's portfolio programme (*P*)

$$\max\ c_0 x_0 + cx - m(xVx)^{\frac{1}{2}} \tag{3.5.1.1}$$

$$\text{on } X = \{(x_0, x) \geq 0 \mid x_0 + ux \leq 1, x_0 \geq \beta\}$$

where the extensions to the model are c_0, the rate of interest paid on government securities or reserve assets, and x_o, which is that portion of the total portfolio invested in the government securities by compulsion. Denote

142 See Section 3.3.

143 New Zealand Monetary Policy differs substantially from this more usual model and will be discussed in detail in relation to the portfolio selection model in Chapter 5.

the reserve coefficient giving this proportion as β. The terms C_0 and β then are instruments of monetary policy. As well as having the budget constraints there is now a liquidity constraint also as set by the monetary authority. A solution to *(P)* can be denoted (x_0^*, x^*) and the dual variables corresponding to the budget and liquidity constraints can be denoted as λ_1 and λ_2 respectively with optimal values denoted by λ_1^* and λ_2^* .

Given $\phi x = cx - m(xVx)^{\frac{1}{2}}$ the maximal in *(P)* can be written

$$G(x_0, x) = c_0 x_0 + \phi(x) \tag{3.5.1.2}$$

and *(P)* reduces to

$$\underset{X}{Max} \quad G(x_0, x)$$

Further $\underset{X}{Max} \quad G(x_0, x) = c_0 x_0 + \phi(x)$

such that $x_0 + ux \leq 1 \quad : \quad \lambda_1$, the budget constraint

and $-\ x_0 \leq -\beta \quad : \quad \lambda_2$ the liquidity constraint

As by the duality theorem of homogeneous programming[144] the primal and dual solutions x^*, v^*, satisfy

$$f(x^*) = V^* b = V^* h(x^*)$$

in the case of *(P)*

$$G(x_0^*, x^*) = c_0 x_0^* + \phi(x^*) = \lambda_1^* - \lambda_2^* \beta$$ (3.5.1.3)[145]

144 Moeseke, P. v., 1965b, op. cit., pp 161 - 170.

145 Note this formula is a corrected version of (2.2) from LeBlanc, G and Moeseke, P.v., 1979, op. cit., p 106.

A number of separation properties derived by LeBlanc and Moeseke are restated here directly from the original article[146]. In the cases where proofs have also been given these proofs have been taken directly from LeBlanc and Moeseke (1979).

Proposition 3.5.1.

If *(P)* possesses a solution (x_0^*, x^*) then it has a solution $(\bar{x}_0, \bar{x})$ such that

$$\bar{x}_0 \quad \varepsilon\{\beta, 1\}$$

where $0 < \beta < 1$

For $x_0^* = 1$ the prospects in the security market must be so poor that the financial intermediary will voluntarily place all funds in government securities. Therefore ignoring this economically trivial case,

$$x_0^* = \beta \qquad (3.5.1.4)$$

Proposition 3.5.2

Next to *(P)* consider a subsidiary programme *(P')* relating to the security market only. Then x^* is part of a solution (x_0^*, x^*) to *(P)* if and only if it solves *(P')*.

By homogeneity this proposition states that the makeup of the risk portfolio will be independent of the value of β. That is the ratios x_i^*/x_j^* will not alter with changes to the budget fraction allocated to that portfolio.

146 See LeBlanc, G and Moeseke, P.v., 1979, op. cit. pp 106-111.

Lemma 3.5.3

$$\lambda_1^* = c_0 + \lambda_2^* \tag{3.5.1.5}$$

Proof: The Lagrangian associated with *(P)* is

$$L(x_0, x; \lambda_1, \lambda_2) = c_0 x_0 + \phi(x) + \lambda_1(1 - x_0 - ux) + \lambda_2(x_0 - \beta)$$

By the Kuhn-Tucker conditions with respect to x_0^*

$$c_0 x_0 + (\lambda_2^* - \lambda_1^*)x_0^* = 0$$

As $x_0^* = \beta > 0$ the Lemma clearly holds.

Lemma 3.5.4

$$\phi(x^*) = \lambda_1^*(1 - \beta)$$

Proof: Substituting (3.5.1.4) into (3.5.1.3) gives

$$c_0\beta + \phi(x^*) = \lambda_1^* - \lambda_2^*\beta$$

$$c_0\beta + \lambda_2^*\beta + \phi(x^*) = \lambda_1^*$$

From (3.5.1.5)

$$\beta\lambda_1^* + \phi(x^*) = \lambda_1^*$$

$$\phi(x^*) = \lambda_1^* - \beta\lambda_1^*$$

$$\phi(x^*) = \lambda_1^*(1 - \beta)$$

As dual variables can be interpreted as implicit prices, or rents, in terms of the objective function, λ_1^* can be seen as the marginal return to the budget dollar that the financial intermediary with objective G receives. That amount received over and above c_0, namely λ_2^* can be seen as the excess marginal return or the risk premium that the intermediary obtains by investing in risky securities. From the monetary authority's viewpoint λ_2^* can be seen as a saving which it is able to obtain through its power to impose a liquidity constraint.

Proposition 3.5.5

The value of the dual λ_1^* is independent of the values of c_0 and β.

That is $\qquad \lambda_1^* = \bar{\lambda}_1 \qquad$ (3.5.1.6)

This separation result says that the value of λ_1^* depends solely on the security market.

Corollary 3.5.6

The values of the duals λ_1^*, λ_2^* are independent of the value of β.

Proof: $\qquad \lambda_1^* = \bar{\lambda}_1 \qquad$ by (3.5.1.6) and since c_0 does not change

$\lambda_2^* = \bar{\lambda}_2 \qquad$ by (3.5.1.5)

Corollary 3.5.7

The optimal value of the objective G is linear in β.

Proof: As λ_1^* and λ_2^* are independent of changes of β the result follows by homogeneous duality (3.5.1.3) : $G(x_0^*, x^*) = \lambda_1^* - \lambda_2^*\beta$

From here a substitution and elasticity rule can be given.

Proposition 3.5.8

For a given G the elasticity of the risk premium λ_2 with respect to coefficient β is negative 1.

Proof: By assumption $G(x_0^*, x^*) = G(\bar{x}_0, \bar{x})$ so that by (3.5.1.3)

$$\lambda_1^* - \lambda_2^*\beta = \bar{\lambda}_1 - \bar{\lambda}_2\bar{\beta}$$

Since by proposition (3.5.5) $\lambda_1^* = \bar{\lambda}_1$ this yields

$$\lambda_2^*\beta = \bar{\lambda}_2\bar{\beta}$$

so that $$(\beta | \lambda_2^*)(d\lambda_2 | d\beta) = -1$$

That is, raising the reserve coefficient by one-tenth can, in the intermediary's estimation, be compensated for by a one-tenth lowering of the risk premium λ_2^*. As λ_1^* remains constant this means increasing the yield c_0 on government securities by an amount equivalent to one-tenth of λ_2^*.

If this occurs the intermediary will recoup the loss it suffers through having to hold a smaller fraction of risky assets, on the increased return on government securities.

Corollary 3.5.9

$$\Delta C_0/\Delta\beta = \bar{\lambda}_2/\beta = \lambda_2^*/\bar{\beta}$$

where β, λ_2^* are the original values and $\bar{\beta}, \bar{\lambda}_2$ are the changed values.

This corollary states that λ_2^* properly normalised per unit of $\bar{\beta}$ can be seen as the marginal rate of substitution between the two instruments of monetary policy considered in this section, namely β and c_0.

3.5.2 The Maximal-Caution Criterion and the Rate of Interest

The third instrument of monetary policy considered by LeBlanc and Moeseke was the rate of interest paid on deposits, a rate determined to a large extent by the discount rate policy of the central bank. The interest rate paid on deposits is clearly the marginal cost of capital for financial intermediaries with some adjustment necessary to take into account the cost to the financial intermediary of reserve requirements policy.

In the earlier sections of this Chapter it was shown how an optimal portfolio could be determined where the marginal value of the investment dollar equalled its marginal cost.

For the extended model define the programme $\underset{X}{Max} \quad G(x_0, x, m) = \Gamma(m)$

where m is the maximal permissible m.

For any given m $\quad \Gamma(m) = \lambda_1^* - \lambda_2^* \beta = \lambda_1^*(1 - \beta) + c_0 \beta.$

For an optimal solution make m as large as possible under the condition that

$$\Gamma(m^+) \geq r$$

where r is the rate of interest on deposits.

m^+ exists, is unique and satisfies $\quad Max \quad G(x_0^+, x^+, m^+) = \Gamma(m^+) = r$

Therefore from (3.5.1.3) $\quad r = Max \quad G = c_0 x^+ + \phi(x^{+)} = \lambda_1^+ - \lambda_2^+ \beta$

and from (3.5.1.5) $\quad r = c_0 + \lambda_2^+ - \lambda_2^+ \beta$

$$r - c_0 = \lambda_2^+(1 - \beta) \tag{3.5.2.1}$$

The above formula shows how the three instruments of monetary policy are linked from the viewpoint of the monetary authority. The interest differential that the financial intermediary will be prepared to pay as a premium to the yield on government securities will be equal to the risk premium earned on that portion of the budget invested in risky securities.

Proposition 3.5.10

If the financial intermediary conforms to the maximal-caution criterion then

$$\Delta c_0/(r - c_0) = \Delta\beta/\overline{\beta}(1 - \beta) \tag{3.5.2.2}$$

Formula (3.5.2.2) shows the quantitative link between the interest bearing instruments and the reserve coefficient if the intermediary or investor is to remain on the same indifference curve.

This section concludes with a simple numerical example.[147]

Consider a programme *(P)* for a portfolio of three assets x_0, x_1, x_2 , where

$$c_1 = 1, c_2 = 1 + 1/\sqrt{5}, m = 1$$

$$V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The programme becomes:

$$\text{Maximise} \quad c_0x_0 + x_1 + (1 + 1/\sqrt{5})x_2 - (x_1^2 + x_2^2)^{\frac{1}{2}}$$

$$\text{Subject to} \quad \begin{aligned} & x_0 + x_1 + x_2 \leq 1 \\ & x_0 \geq \beta > 0 \\ & x_1, x_2 \geq 0 \end{aligned}$$

The table lists primal and dual solutions for 4 alternative pairs of values of c_0 and β giving rise to 4 programmes P_1, P_2, P_3, P_4.

147 Source: LeBlanc, G. and Moeseke, P. v., 1979, op. cit., pp 116-117.

Programme No	P_1	P_2	P_3	P_4
c_0	.2	.2	.2	.4117
β	.1	.4	.25	.25
x_0^*	.1	.4	.25	.25
x_1^*	.3	.2	.25	.25
x_2^*	.6	.4	.50	.50
$\phi(x^*)$	0.4975	0.3317	0.4146	0.4146
$G(x_0^*,x^*) = i$	0.5175	0.4117	0.4646	0.5175
λ_1^*	0.5528	0.5528	0.5528	0.5528
λ_2^*	0.3528	0.3528	0.3528	0.1411

The above table confirms a number of results previously stated.

i Homogeneous duality: $G(x_0^*,x^*) = \lambda_1^* - \lambda_2^*\beta$

ii Proposition 3.6.1: $x_0^* = \beta$

iii Proposition 3.6.2: $x_1^*/x_2^* = \frac{1}{2}$ in all 4 programmes

iv Lemma 3.6.3: $\lambda_1^* = c_0 + \lambda_2^*$

v Lemma 3.6.4 $\phi(x^*) = \lambda_1^*(1-\beta)$

vi Proposition 3.5.5: The value of the dual λ_1^* is independent of the values of c_0 and β and takes the same value in all four programmes.

vii Corollary 3.5.6: The values of the duals λ_1^*, λ_2^* are independent of the values of β and take the same values in P_1, P_2 and P_3 for a constant c_0.

viii Corollary 3.5.7: G is linear in β; its value in P_3 is midway between its values in P_1 and P_2.

The remaining checks apply to the programmes P_1 and P_4 where G has the same value.

ix Proposition 3.5.8: For given G the elasticity of risk premium λ_2 with respect to coefficient β is minus 1.

That is $\beta\lambda_2^*$ from P_1 equals $\beta\lambda_2^*$ from P_4 (0.3528 in both cases).

x Corollary 3.5.9: $\Delta c_0/\Delta\beta = \bar{\lambda}_2/\beta = \lambda_2^*/\bar{\beta}$

That is (.2117/.15) = (.1411/.1) = (.3528/.25)

xi Proposition 3.5.10: $\Delta c_0/(r - c_0) = \Delta\beta/\bar{\beta}\,(1 - \beta)$

{.2117/(0.5175-.2)} = .15/{.25(1-.1)}

In this example the value of m was stable at 1. If r remains stable and there is an increase in the reserve coefficient then either the financial intermediary has to take a higher risk, that is reduce the value of m, to compensate, or else there has to be a compensatory increased in the value of c_0. If the reserve coefficient is increased then λ_2^* must decrease by the same proportion in order to leave the value of G or r unaltered (full compensation for the financial intermediary). Therefore c_0 must increase by the amount λ_2^* has decreased.

For example say $\lambda_1^*=10\%$, $C_0=5\%$ and $\beta=20\%$. Then λ_2^* must equal 5% by (3.5.1.5). If β is increased to 40% then λ_2^* must decrease by 2.5% in order to compensate the financial intermediary which requires C_0 increasing to 7.5%.

3.5.3 Existence Theorems for Imperfect Capital Markets

A final extension of the portfolio model put forward by Moeseke[148] considered the existence of solutions to the portfolio problem given the fact that capital markets have imperfections. Moeseke points out that Capital Markets fail all three criteria of perfect competition since financial institutions are not infinitesimal and their investment decisions measurably interact; the institutions do not have perfect information and their subjective distributions and risk attitudes differ; finally, entry into any category of finance companies is subject to legal and institutional restrictions. This final contribution describes the selection of E.V. efficient and E.V. optimal portfolios given the interdependence of financial intermediaries and their dependence on monetary policy. The existence of non-cooperative equilibria is established by adapting fixed point theorems to three models of increased generality. The models derived by Moeseke (1986) are restated here without proof.

The subjective distributions of investors, hence their respective mini-maximands ϕ_i now depend not only on their own, but also on their competitor's investment decisions.

148 Moeseke, P. v., 1986, op. cit.

In the first model interest rates payable on deposits are given for every financial institution, directly or indirectly by the discount policy of the monetary authority. Every investor selects an optimal portfolio but the subjective distribution for each investor is now conditional upon decisions x_{-i} (the investment decision of every other investor).

The maximisation problem changes from $G(m) = \max_{X} \phi(x, m)$ to

$$G_i(m_i | x_{-i}) = \max_{X_i} \phi_i(x_i, m_i | x_{-i}) \qquad (3.5.3.1)$$

where $\quad X_i = \{x_i \geq 0 | ux \leq 1\}$

and $\quad G_i(\overline{m}_i | x_{-i}) < r$ for $\overline{m}$ large enough

where $\quad M_i = [0, \sup_i \overline{m}_i]$

Then there exists sequences x^*, m^* such that all investors hold optimal portfolios, ie, portfolios satisfying.

$$\phi_i(x_i^*, m_i^* | x_{-i}^*) = \min_{M_i} ; \max_{X_i} \phi_i(x_i, m_i | x_{-i}^*) \text{ for all } i \qquad (3.5.3.2)$$

In the second model discount strategies r of the monetary authority, as well as investors' decisions x, now codetermine equilibrium. Subjective distributions are influenced by other investors as well as by the interest rates r : two equilibrium series are to be determined, viz x^* and r^*.

The strategy sets R_i are defined as closed segments.

$$R_i = [G_i(o), G_i(\overline{m}_i)]$$

Then there exists sequences r^*, m^*, x^* such that all investors hold optimal portfolios, that is portfolios satisfying

$$\phi(x_i^*, m_i^* | r^*, x_{-i}^*) = \min_{M_i} \max_{X_i} \phi_i(x_i, m_i | r^*, x_{-i}^*) \quad \text{for all } i \tag{3.5.3.3}$$

The third model incorporates the reserve coefficient into the equilibrium model by specifying that i has to invest a fraction β_i of the budget in government bonds with return c_i^0. This fraction is the reserve coefficient and the new maximand for i is

$$\phi_i(x_i^0, x_i, m_i) = c_i^0 x_i^0 + Ex_i - m_i(x_i Vx_i)^{\frac{1}{2}} \tag{3.5.3.4}$$

and the new budget set is

$$X_i = \{(x_i^0, x_i) \geq 0 | x_i^0 + ux_i \leq 1, x_i^0 \geq \beta_i\} \tag{3.5.3.5}$$

where x_i^0 is the budget fraction invested by i in government bonds.

Then there exists sequences $x_0^*, x^*, m^*, r^*, \beta$ such that all investors hold optimal portfolios, that is portfolios satisfying

$$\phi_i(x_i^{0*}, x_i^*, m_i^* | r^*, \beta^*, x^*) = \min_{M_i} \max_{X_i} \phi_i(x_i^0, x_i, m_i | r^*, \beta^*, x^*)$$
for all i (3.5.3.6)

While these extensions to the Moeseke model are not applied in this thesis it is worthwhile to note that solutions to the portfolio problem will still exist in a dynamic setting where the actions of the central monetary authority and competitors must be taken into account.

3.6 The Relationship Between the Homogeneous Programming by the Truncated Minimax Criterion Model and the Capital Asset Pricing Model

It was shown in section 3.3. that, by the truncated minimax criterion, an optimal portfolio will be found when the portfolio problem $\max_{X} \phi(x|m)$ is solved with $m = m^{+}$ set equal to the inverse of the coefficient of variation of net profits. That is, as given by (3.3.7).

$$m^{+} = \frac{\bar{c}x^{+} - r}{\sigma\, x^{+}}$$

The three basic claims that allow for the derivation of the CAPM are:

1. All investors agree with respect to the expected returns, the variance of returns and the covariances of returns of all risky assets and make their investment decisions based on the Markowitz expected return - variance criterion.

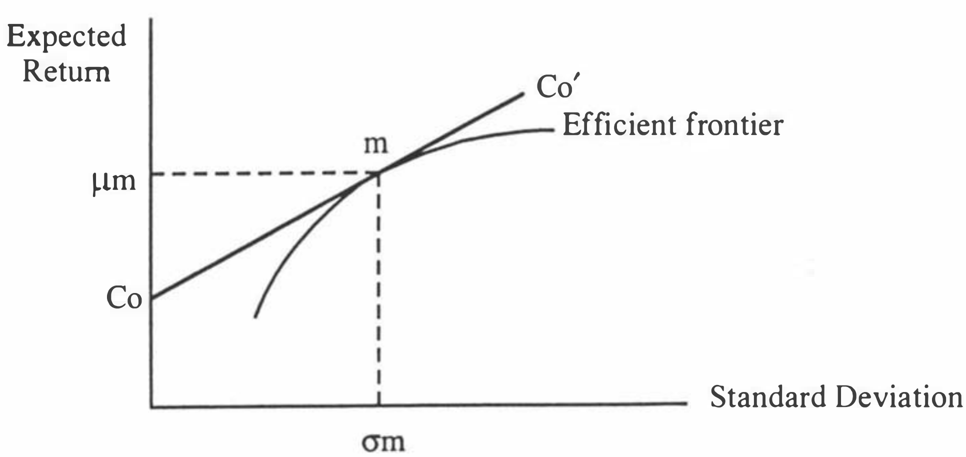
2. There exists a riskless asset and any amount of borrowing and lending can occur at the riskless interest rate. Then by the separation theorem of

investment and financing[149] all investors will hold the same risky portfolio m, the investment decision.

3. The portfolio m is determined by the tangency point of the efficient frontier and the straight line $y=a+bx$ where a equals the risk free rate of interest and $b>0$. This second condition equates to assumption 1 in Section 3.3.

Figure 3.6.1

Optimal Portfolios by the Homogeneous Programming by the Truncated Minimax Criterion Model and by the CAPM



The point of tangency is where the slope of the efficient frontier equals the slope of interest rate line $c_0c_0^{'}$ defined as b.

Clearly $$b = \frac{\mu_m - c_o}{\sigma_m} \tag{3.6.1}$$

149 See Tobin, J., Liquidity Preference as Behaviour Towards Risk. Review of Economics Studies, V.25, 1958, pp 58-86.

Therefore the optimal portfolio (1) derived from the homogeneous programming by the truncated minimax model will equate to the optimal portfolio (2) derived from the CAPM when

$$\frac{\bar{c}x^+ - r}{\sigma x^+} = \frac{\mu_m - c_0}{\sigma_m}$$

Recall for the optimal portfolio (1) we take the largest m compatible with the condition that the marginal yield should not fall below the marginal cost of the dollar budget, that is the interest rate paid on deposits or the cost of raising deposits.

This criterion is

$$(P) \qquad \max_X G(x, m^+) = \Gamma(m^+)$$

$$m^+ = \sup\{m \geq 0 | \Gamma(m) \geq r\}$$

Now a portfolio optimal under criterion *(P)* can be obtained by solving the pseudo-concave programme.[150]

$$(Q) \qquad \max_X (cx - r)/\sigma x$$

Proposition 3.6.1. Portfolio (x^+) solves *(P)* if and only if it solves *(Q)*.[151]

150 See Mangasarian, O., Nonlinear Programming. McGraw-Hill, New York, 1969.
151 For a related programme with proof see LeBlanc, G. and Moeseke, P. v., 1979, op. cit.

This proposition relates the maximal caution criterion of Moeseke to the so called market portfolio of the CAPM.

For an investor who sees the marginal cost of the investment dollar as equating to the risk free rate c_0,*(Q)* becomes

$$\max_{X}(cx - c_0)/\sigma x$$

As our x relates to a particular portfolio rewrite as

$$\max_{X}(\bar{c}x^{+} - c_0)/\sigma x^{+}$$

The first basic claim that allowed for the derivation of the CAPM is also required for the Moeseke model to obtain a market portfolio equivalence. That is there must be complete agreement among investors (homogeneous expectations) with respect to expected returns, the variance of returns and the covariance of returns of all risky assets and investment decisions must be based on the Markowitz expected return - variance criterion. The unrealistic assumption of unlimited borrowing of the riskless asset is not required for the Moeseke model.

For market portfolio equivalence there is instead a requirement that all investors must make their investment decisions based on the optimality criterion (3.3.1). This being the case all investors will hold the same portfolio which, by definition, must be a market weighted percentage of all risky assets. Then

$\bar{c}x^{+}$ will equal to μ_m and σx^{+} will equal to σm.

Note that the risk free asset is not being directly included in the market portfolio here. This asset is held by those investors too risk averse to hold just the market portfolio or by those investors (financial institutions) required or desiring to hold the risk free asset. In general the marginal cost of the investment dollar for individual investors and financial institutions would not be the same. That is $co \neq r$ for at least some investors. Also for at least some investors the optimality criterion (3.3.1) would not be viewed as appropriate. These two points conform with the observation that investors hold substantially different portfolios in general.

3.7 The Rationale for Selecting the Homogeneous Programming by the Truncated Minimax Criterion Model

The preceding discussion has described the basic homogeneous programming by the truncated minimax criterion model and extensions to it. This model was selected for application in this thesis for two main reasons. First the model explicitly describes the risk profile of particular capital markets by the use of the risk parameter, m. A major focus of the study is to analyse the value of m for particular capital markets as well as considering its stability.

The second major reason for selecting the model is because it acknowledges that appropriate portfolios for investors are investor specific but that for a particular market an optimal, or market portfolio can be derived where the marginal return to the investment dollar equals its marginal cost in the aggregate. While such a portfolio is in itself independent of the actions of the monetary authority, these actions may, and generally will, lead those financial institutions affected by them to alter their portfolios to accommodate the restrictions. In the special case where investors have homogeneous expectations, the marginal cost of the investment dollar equates to the risk free rate for all investors and all investors make their investment decision based on the optimality criterion (3.3.1), the optimal or market portfolio of the Moeseke model equates to the market portfolio of the CAPM.

Chapter 4 discusses the incorporation of the major theoretical results of finance theory into the general economic equilibrium model and discusses the relationship between the homogeneous programming by the truncated minimax criterion model and the CAPM within this framework. Chapter 5 discusses further extensions to the Moeseke model to extend its usefulness both in a general sense and also in relation to the New Zealand capital market environment of the mid to late 1980's and early to mid 1990's.

CHAPTER FOUR

PORTFOLIO THEORY AND THE GENERAL ECONOMIC EQUILIBRIUM MODEL

4.1 Introduction

The purpose of this chapter is to discuss at what level and under what conditions the homogeneous programming by the truncated minimax model might be incorporated into a model of general economic equilibrium.

General equilibrium analysis began with the work of Walras[152], the first appointee to the Chair of Economics in the Faculty of Law at Lausanne, 1870. In a general equilibrium system the interdependency of all economic markets are considered with general equilibrium existing when supply equals demand in all markets. Walras constructed a mathematical model of general equilibrium being a system of simultaneous equations by which all prices and quantities in an economy could be uniquely determined. The work of Walras was very much just the beginning of the development of general economic equilibrium theory. His model implied a static world with no uncertainty.

Von Neumann[153] addressed the question of the mathematical assumptions that would be required to ensure a solution to the general economic equilibrium problem and

152 Walras, L., Élements d'economie politique pure, 4th ed. Lausanne: L Corbas, 1874-77. English Translation by W. Jaffé, Elements of Pure Economics. London: Allen & Unwin, 1954.

153 Neumann, J. v., A Model of General Economic Equilibrium. Review of Economic Studies, V. 13, 1945-46, pp 1-9.

showed that mathematical proof of the existence of a solution required a generalisation of Brouwer's Fixed-Point Theorem, that is the use of very fundamental topological facts. A summary of approaches to proving the existence of competitive equilibrium was later given by Debreu[154].

Arrow and Debreu[155] later developed a general equilibrium model which is now normally regarded as the basic model from which all recent advances in this area have come. As noted in Chapter 2, Section 7 the Arrow-Debreu model has two types of agents, consumers and producers with the objects of choice being commodities. Commodities are defined by their physical characteristics being;

(i) time of availability,

(ii) location of availability,

(iii) state of nature conditional upon which they are made available.

The basic Arrow-Debreu model moved from the static world of Walras into a dynamic setting but even though characteristic (iii) implies uncertainty, this basic model did not include uncertainty in an explicit sense. Also the trading of intertemporal contracts was not included in the basic Arrow-Debreu model.

One fundamental but unrealistic condition in the Arrow-Debreu model for the existence of competitive equilibrium was the requirement that all agents must hold

154 Debreu, G., Existence of Competitive Equilibrium, in K. Arrow and M. Intiligator, Handbook of Mathematical Economics, V. ii, Amsterdam, North Holland, 1982, pp 697-743.

155 Arrow, K. J. and Debreu, G., Existence of an Equilibrium for a Competitive Economy. Econometrica, V. 22, 1954, pp 265-290.

identical beliefs regarding the prices which would exist in each potential state of the world at any point in the future.

This assumption was relaxed in a paper presented by Radner[156] where he proved that competitive equilibrium could still exist even if different people had different beliefs about the future state of the world. A requirement for this existence, however, was for economic decision makers to have unlimited computational capacity for choice among strategies. Radner further pointed out that there are two fundamentally different types of uncertainty in any economic environment. First is the uncertainty about the environment and second, the uncertainty about the behaviour of others. The Arrow-Debreu model deals implicitly with uncertainty about the environment. Radner argued that the second type of uncertainty can lead to a liquidity requirement so agents have the capacity to react to the actions of others. In theory there is no role for money or liquidity in the basic Arrow-Debreu model.

In addition, a major economic interpretation that came from the mathematical proof of the existence of equilibrium in the Arrow-Debreu model was for the necessity of complete markets. The notion of the existence of complete markets is that there exists, and is available, a sufficient range of goods and services to satisfy all consumers. That is there are very few to no gaps in the availability of markets. For the most part this would be satisfied in the present but there are clearly substantial gaps in the provision of markets which are concerned with future events. Under the basic Arrow-Debreu

156 Radner, R., Competitive Equilibrium under Uncertainty. Econometrica, V. 36, 1968, pp 31-58.

model the existence of competitive equilibrium only holds under a situation of complete markets.

The issue of incomplete markets has been analysed by a number of writers with two main approaches originally being considered. Arrow and Hahn[157] and Grandmont[158] considered a temporary equilibrium approach which assumes that economic agents have given expectations of prices in the future and consider whether prices exist that will clear current markets. Radner[159], on the other hand, adopted the rational expectations approach, regarding expectations as variables and investigating whether or not a set of current prices and expected prices exists such that all markets, both current and future, clear. Hart[160] considered the optimality of equilibrium when the market structure is incomplete and concluded that any economy with incomplete markets is like a typical second best situation. The opening of new markets would possibly make matters worse rather than better until markets are complete.

Duffie[161], however, demonstrated the existence of equilibria with incomplete financial markets given that the information structure is given by a finite event tree and provided that securities are purely financial. Others to contribute to this area of study

157 Arrow, K. J. and Hahn, F. H., General Competitive Analysis. Holden Day, San Francisco, 1971.

158 Grandmont, J-M., On the Short-Run Equilibrium in a Monetary Economy. Allocation under Uncertainty : Equilibrium and Optimality, J Dreze, Ed., Chapter 12, MacMillan, New York, 1974.

159 Radner, R., 1972, op. cit.

160 Hart, O. D., On the Optimality of Equilibrium when the Market Structure is Incomplete. Journal of Economic Theory, V. 11, 1975, pp 418-443.

161 Duffie, D., Stochastic Equilibria with Incomplete Financial Markets. Journal of Economic Theory, V. 41, 1987, pp 405-416.

include Chae[162], Geanakopos[163] and Mas-Colell[164].

Another important assumption of the basic Arrow-Debreu model was the existence of an environment in which no agent, producer or consumer, has a degree of control over prices charged in relation to either the product cost or labour cost. Lipsey and Lancaster[165] showed that suboptimality would exist if there was just one violation of this assumption but more violations may or may not make matters worse. Conversely if many violations exist removing one or some may or may not improve the level of efficiency of the economy. A summary of the literature on economies in which there exist agents with market power was carried out by Silvestre[166].

The problems that have been noted here in relation to general economic equilibrium are clearly very important. Similar problems exist in relation to capital asset pricing and allocation models and are often dealt with in the assumptions of these models. Consideration is now given to the problem of the incorporation of securities into the general economic equilibrium model.

162 Chae, S., Existence of Competitive Equilibrium with Incomplete Markets. Journal of Economic Theory, V. 44, 1988, pp 179-188.

163 Geanakoplos, J., An Introduction to General Equilibrium with Incomplete Asset Markets. Journal of Mathematical Economics, V. 19, 1990, pp 1-38.

164 Mas-Colell, A., Indeterminacy in Incomplete Market Economies. Economic Theory, V. 1, 1991, pp 45 - 61.

165 Lipsey, R. G. and Lancaster, K., The General Theory of Second Best. Review of Economic Studies, V. 24, 1956, pp 11-32.

166 Silvestre, J., The Market-Power Foundations of Macroeconomic Policy. Journal of Economic Literature, V. 31, 1993, pp 105 - 141.

4.2 The Incorporation of Securities into the General Economic Equilibrium Model

The incorporation of securities into the general economic equilibrium model took place in two distinct steps. In the first instance securities were introduced into the Arrow-Debreu Model by Arrow[167] in an extension which he described as being one which allowed for conditions of subjective uncertainty. In this extension to the basic Arrow-Debreu Model, Arrow proved the following theorem:

"If $\sum_{s-1}^{S}; \pi_{in} U_i (X_{is}, ..., X_{isc})$ is quasi-concave, in all its variables,

then any optimal allocation of risk-bearing can be achieved by perfect competition on the securities and commodities markets, where securities are payable in money".

Arrow supposed I individuals and S possible states of nature where in the Sth state, amount x_{sc} of commodity c (c=1, ...c) is produced. Further he assumed that each individual acts on the basis of subjective probabilities as to the states of nature where π_{is} is the subjective probability of state s according to individual i. x_{isc} is the amount of commodity c claimed by individual i if state s should occur. U_i is the utility achieved for individual i.

The quasi-concave requirement, proven by Arrow, has a very important implication in this model as a concave function is always quasi-concave (though not conversely). This being the case, the individual utility function U_i must be concave meaning that to

167 Arrow, K., 1964, op. cit.

insure the viability of the competitive allocation of all possible assignments of probabilities π_{is} all individuals must be risk averse.

The next step was taken by Radner[168] when he firmly placed the trading of securities into general economic equilibrium theory by introducing a sequence of stock markets where the ownership rights to the profits of firms could be traded. Radner's approach was to state that each producer in an economy would receive revenue to be distributed to shareholders (consumers) on a specified date. There were some substantial limitations to Radner's model however. First each consumer starts with an initial share holding with the first market taking place before the activities of production and consumption begin. Also shareholders in the Radner model have unlimited liability making them more like partners than shareholders in the traditional sense. Given these limitations, Radner showed the existence of equilibrium.

4.3 The Integration of the Major Theoretical Results of Finance Theory into General Economic Equilibrium Theory

With securities and the trading of securities incorporated into general economic equilibrium theory the next logical step was to integrate the major results of finance theory as well. As noted by Gross[169], not only did the objects of choice, as given in the basic Arrow-Debreu model, require reinterpretation to include securities as well as commodities but a suitable choice of a mathematical object to represent the choice space for returns on securities was also required.

168 Radner, R., 1972, op. cit.
169 Gross, E., 1992, op. cit.

Duffie[170] proposed a solution to this problem through the study of infinite - dimensional spaces. As stated by Duffie[171] the study of infinite - dimensional spaces in general economic equilibrium theory has application where there exists blends of commodity characteristics or uncertain states of the world as exists with returns on securities. Duffie showed that Arrow-Debreu equilibria existed in a general class of topological vector spaces of commodity bundles allowing the Arrow-Debreu model to be extended to the case of stochastic economies. Working along a similar path Mas-Collel[172] also showed the existence of price equilibrium in topological vector lattices.

From here the major theoretical results of finance theory were incorporated into general economic equilibrium theory. The incorporation of the CAPM into general economic equilibrium theory follows, as shown by Duffie[173], together with a description of the framework proposed by Duffie for the analysis of the major theoretical results of finance theory.

Competitive or market equilibrium occurs when there exists a system of prices by which supply and demand equates in every market given firms' profit maximising production decisions and individuals' preferred affordable consumption choices.

The basic structure of this model as taken directly from Duffie is as follows[174].

170 Duffie, D., 1986, op. cit.
171 Ibid, p. 2.
172 Mas-Colell, A., The Price of Equilibrium Existence Problem in Topological Vector Lattices. Econometrica, V. 54, 1986, pp 1039 - 1053.
173 Duffie, D., 1988, op. cit.
174 See Duffie, D., ibid, pp 39 - 41.

In an economy there exist firms and individual agents. Let the finite set of firms be denoted by $J = \{1, ..., J\}$ and the finite set of individual agents be denoted by $I = \{1, ..., I\}$. The production set for each firm can be defined as $Y_j \subset L$ and the choice set for each individual as $X_i \subset L$, where L is a vector space of choices. Each individual also has a preference relation $\geq i$ on X_i, an endowment vector $w_i \varepsilon L$, and a share $\theta_{ij} \varepsilon [0, 1]$ of the production vector $y_j \varepsilon Y_j$ chosen by firm j for each firm $j \varepsilon J$. As the individual agents will own all the firms

$$\sum_{i=1}^{I} \theta_{ij} = 1 \text{ for all } j \varepsilon J.$$

An economy, E, is made up of all the firms and individual agents and can be denoted as follows;

$$E = ((x_i, \geq_i, w_i) ; (Y_j) ; (\theta_{ij})), \; i \varepsilon I, j \varepsilon J \tag{4.3.1}$$

This type of economy is termed a production - exchange economy.

Within any particular economy firms and individual agents will allocate resources to production and consumption respectively. For firms the production allocation will be a J-tuple $y = (y_1, ..., y_J)$ with $y_i \varepsilon Y_J$ for all $j \varepsilon J$. For individual agents the consumption allocation will be an I-tuple $x = (x_1, ..., x_I)$ with $x_i \varepsilon X_i$ for all $i \varepsilon I$. Combining both the production and consumption components, an allocation can be defined as an $(I + J)$-tuple (x,y).

As stated by Duffie, an allocation (x, y) will be feasible if

$$\sum_{i=1}^{I} (x_i - w_i) = \sum_{j=1}^{J} y_j \tag{4.3.2}$$

Also an allocation (x, y) will be strictly supported by a price vector $p \, \varepsilon \, L'$ if $p \neq 0$,

$$z >_i , x_i \Rightarrow p.z > p.x_i \quad \forall z \, \varepsilon \, X_i, \ \forall i \, \varepsilon \, I \tag{4.3.3}$$

and

$$p.y_j \geq p.z \quad \forall z \, \varepsilon \, Y_j, \ \forall_j \varepsilon J \tag{4.3.4}$$

Finally an allocation (x,y) is budget - constrained by a price vector p if, for each i ε I

$$p.x_i \leq p \, . \left[w_i + \sum_{j=1}^{J} \theta_{ij} y_j \right] \tag{4.3.5}$$

Given some price vector p, conditions (4.3.3) and (4.3.5) can be seen as optimality conditions for individual agents and (4.3.4) is market value maximisation by firms, given p.

If (x, y) is some feasible allocation, is strictly supported by a price vector and is budget-constrained then a triple $(x, y, p) \, \varepsilon \, L^I \times L^J \times L'$ is an equilibrium for an economy, E. This is the Arrow-Debreu or Walrasian competitive equilibrium.

As noted earlier, defining a correct choice space for the incorporation of the major results of finance theory into general economic equilibrium theory was required.

Duffie[175] showed that the appropriate choice space for the CAPM is $L^2(p)$ sometimes denoted $L^2(\Omega, F, P)$ where the solution given states of nature, Ω, events, F, and prices P, is in the vector space L^2. L^2 is a Banach space belonging to the L^q class of vector spaces.

Restating the approach derived by Duffie, given any two elements x and y of L, the product of x and y pointwise can be stated as follows.

$[x\ y]\ (m) = x\ (m)\ y\ (m),\ m \in M$

where M is a given measure space.

Further, for any $q \in (1,\infty)$ the conjugate of q is the scalar $q^* \in (1,\infty)$ defined by $1/q + 1/q^* = 1$. Define 1^* to be ∞.

Then from Hölder's Inequality : for any $q \in (1,\infty)$, if $x \in L^q\ (\mu)$ and $y \in L^{q^*}\ (\mu)$, then $xy \in L^1\ (\mu)$, and

$$\| x y \|_1 \leq \| x \|_q \, \| y \|_{q*} \tag{4.3.6}$$

If $q = q^*$ then the relation (4.3.6) is the Cauchy-Schwarz or Buniakovski's inequality. This can only be so for $q = q^* = 2$ which is a special case and therefore $L^2\ (\mu)$ is self-dual.

An important theorem to note at this point is the Riesz Representation Theorem which states that for $q \in (1,\infty)$, let p be a continuous linear functional on $L^q\ (\mu)$. Then there

175 See Duffie, D., 1988, op. cit, pp 61 - 65.

exists a unique $\pi \varepsilon L^{q*}(\mu)$ such that

$$p.x = \int_m x(m)\pi(m)d\mu(m) \qquad \forall_x \varepsilon L^q(\mu) \tag{4.3.7}$$

Duffie then defined the appropriate choice space for any economy as $L^2(p)$, and noted that within an economy there will exist a vector x in $L^2(p)$ which will be a random variable describing the amount x(w) of consumption received in a state of the world $w \varepsilon \Omega$. Given the integrability condition $\int \Omega x^2 dP < \infty$, the variance of x will be finite and the Cauchy-Schwarz inequality implies that the expected value of any *x* in $L^2(p)$ is also finite.

Duffie further noted that if p is a continuous price functional on $L^2(p)$, then by the Riesz Representation Theorem there will exist a unique random variable π in $L^2(p)$ such that $p.x = E(\pi x)$ for all x in $L^2(p)$. The covariance of two elements x and y of $L^2(p)$, denoted cov(x, y), will be the scalar

$$\text{cov}(x, y) = E(xy) = E(x)E(y)$$

and the variance of any x in $L^2(p)$, denoted var(x) will be cov(x,x). An important result that can now be stated which directly relates to the CAPM[176] is that π of the price functional p implies that

$$p.x = E(\pi)E(x) + \text{cov}(x,\pi), \quad x \varepsilon L^2(p) \tag{4.3.8}$$

176 See Duffie, D., 1988, op. cit., p 94.

That is the market value of any asset x is the covariance between x and a fixed asset π, plus a fixed multiple of E(x).

The asset π then becomes a pricing asset and any asset x in M which has a positive value will have a return defined by the random variable

$$R_x = \frac{x}{p.x} \tag{4.3.9}$$

and an expected return defined as $\mu_x = E(R_x)$. Given the asset x and any other asset in $L^2(p)$ that also has a positive variance and price, say the asset z, then the beta of any asset x in $L^2(p)$ relative to z, β_{xz}, is $cov(R_x, R_z)/var\ (R_z)$.

Consider now the pricing asset π of relation (4.3.8) where $E(\pi) \neq 0$ and $var(\pi) \neq 0$ and $r = 1/E(\pi)$. Any marketed asset x with a positive market value satisfies

$$(\mu_x - r) = \beta\ x\pi\ (\mu_\pi - r) \tag{4.3.10}$$

where the scalar r is the return on the riskless asset 1Ω.

Proof:

From (4.3.9) $\qquad Ex = E(Rx\ p.x) = \mu x\ p.x \qquad$ (4.3.11)

and as π is an asset like x

$$E\pi = E(R\pi\ p.x) = \mu\pi\ p.\pi \tag{4.3.12}$$

Applying (4.3.8) to π

$$p.\pi = E(\pi^2) = E(\pi)^2 + Var(\pi)$$

Given $\beta x\pi = cov(Rx.\ R\pi) / Var(R\pi)$

$$\beta x\pi = \frac{cov(x\pi)/(p.x p.\pi)}{Var(\pi)/(p.\pi)^2}$$

$$= \frac{cov(x\pi)/p.x}{Var(\pi)/p.\pi}$$

Substituting for cov(x.π) and var (π)

$$\beta x\pi = \frac{p.x - E(\pi)\ E(x) / p.x}{(p.\pi - E(\pi)^2) / p.\pi}$$

From (4.3.11) and (4.3.12)

$$= \frac{p.x - E(\pi)(\mu x\ p.x) / p.x}{(p.\pi - E(\pi)^2)/ p.\pi}$$

$$= \frac{1 - E\pi\ \mu x}{(p.\pi - E(\pi))\ (\mu\pi\ p.\pi) / p.\pi}$$

$$\beta x\pi = \frac{1 - E\pi\ \mu x}{1 - E\pi\ \mu\pi}$$

Given $r = \dfrac{1}{E\pi}$

so $\quad \beta x\pi = \dfrac{\mu x - r}{\mu\pi - r}$

giving the required result.

The economy described at the beginning of this section was termed a production-exchange economy. This can be simplified to just an exchange economy as follows:

$$E = (x_i \geq_i , w_i),\ i \varepsilon I \qquad (4.3.13)$$

$(x, p)\ \varepsilon\ L^1 \times L^{\prime}$ is an equilibrium if x is a feasible consumption allocation, is strictly supported by a price vector and is budget-constrained.

Taking now the exchange economy described above for the choice space $L^2(p)$.

Let $A \subset L^2(p)$ denote a security set for E. The total endowment

$m = \sum_{j=1}^{J} Y_j$ can be described as the market portfolio. The marketed subspace M is the span of A.

From Duffie[177] the Capital Asset Pricing Model can be stated as follows.

> "Given an equilibrium for the security market exchange economy (ε A) for the choice space $L^2(p)$ suppose:

177 Duffie, D., 1988, op. cit, p 96.

(a) the preference relation of each agent is strictly variance-averse,

(b) the endowment w_i of each agent i is in the marketed subspace M,

(c) the marketed subspace M is finite-dimensional,

(d) each agent's choice set is $L^2(p)$,

(e) the riskless asset 1Ω is marketed and has non-zero market value,

(f) the market portfolio m has a non-zero variance."

Given the above any asset x in M will satisfy:

$$(\mu_x - r) = \beta_{xm} (\mu_m - r) \tag{4.3.14}$$

which is the Capital Asset Pricing Model.

For proof see Duffie as referenced.

4.4 General Economic Equilibrium Theory and the Homogeneous Programming by the Truncated Minimax Criterion Model

It was suggested in Chapter 3, Section 6 that under certain strict assumptions the optimal portfolios of the CAPM and the Moeseke model equate. Further, the CAPM can be incorporated into general economic equilibrium theory as shown by Duffie and detailed above. Given the assumptions that $A \subset L^2(p)$ denotes a security set for the economy, M is the total market endowment and the marketed subspace M is the span of A, the homogeneous programming by the truncated minimax model can be stated as for the CAPM with the following adaptations. Supposition (b) requires every agent i to make his/her investment decisions based on some common optimality criterion, say (2.22) and supposition (e) requires restating as follows:

(e) the marginal cost of the investment dollar is a riskless asset 1Ω which is marketed and has non-zero market value.

These suppositions are then consistent with that special case of the Moeseke model where the optimal portfolio derived from it equates with the optimal portfolio of the CAPM.

These observations are useful in helping to position the truncated minimax model within the context of modern portfolio theory.

As discussed in Chapter 2, observations show that investors or agents do not, in general, hold the CAPM market or optimal portfolio. For the truncated minimax model the market or optimal portfolio is that portfolio which ensures that the marginal return to the investment dollar equals its marginal cost with the marginal cost normally differing between investors or agents. Further, investors or agents holding these portfolios will only be those for whom maximal caution is appropriate. While the model defines optimality it does, however, have a very useful degree of flexibility in line with the observations in the marketplace. The model's focus on a risk parameter for determining the equilibrium risk profile of any particular market also has useful application.

A final point to note is that the existence of differing marginal costs and differing investment objectives in a marketplace requires an interdependence of choices.

Moeseke has shown previously[178] the existence of noncooperative equilibria within the framework of the truncated minimax model.

178 Moeseke, P.v., 1986, op. cit.

CHAPTER FIVE

FURTHER EXTENSIONS TO THE HOMOGENEOUS PROGRAMMING BY THE TRUNCATED MINIMAX CRITERION MODEL

5.1 Introduction

The purpose of this chapter is to detail the considerations and extensions to the Moeseke model that will aid in the application of this model to New Zealand's capital markets with the main aim being to analyse the risk profile of these markets. The topic areas considered are as follows:

(i) As the model is concerned with the marginal cost of the investment dollar rather than the risk-free rate of return specifically this marginal cost needs to be clearly defined in the New Zealand context. Consideration is also given as to whether a range of values can be applied rather than one specific value for an investor group as has been the approach used to date in applying this model.

(ii) There have been times in New Zealand's recent past when the real marginal cost of the investment dollar or the real return to the minimum risk asset equivalent has been negative. While this is not an issue within the time frame considered in this thesis, it is important to give consideration to this situation and the necessary adaptation to the Moeseke model to take this into account.

(iii) In Chapter 3, Section 6 extensions to the model to include monetary policy tools were discussed. Monetary policy in New Zealand is conducted in a way which differs significantly from the original environment for which these extensions were developed and they need modification for New Zealand application.

(iv) An important extension to both the CAPM and the Markowitz model was the incorporation of taxation into the models. This is required for the Moeseke model in the New Zealand context since this has not been done to date. A particular difficulty here, as is often the case in other countries, is that there exists a heterogeneous investor base in relation to tax issues. For example some investors face capital gains taxes while others do not and overseas investors face differing tax regimes in general depending on what country they reside in.

(v) A final area of consideration relates to another cause of a heterogeneous investor base, namely the risks New Zealand investors face compared to offshore investors in relation to exchange rate risk and country risk.

5.2 The Marginal Cost of Investing or the Minimum Risk Rate of Return

As shown in Chapter 3, Section 3 the optimal portfolio x^+ satisfies

$$\max \phi(x|m^*) = \phi(x^+|m^+) = \lambda^+ = r$$

where r is the marginal cost of the investment dollar. This marginal cost will differ for different investors, for example between private investors and financial institutions investing on their own account. Within the managed funds industry the financial institutions are investing on behalf of private investors so the marginal cost for them should be the same as the marginal cost for the private investor though in the New Zealand case fund managers would have access to wholesale priced Government stock or Treasury Bills that, in general, would not be available to the private investor. Even retail government securities may be difficult for private investors to obtain as these are normally traded with a $10,000 minimum and a more appropriate marginal cost for these investors could be bank deposit rates.

Another matter for consideration is that of time frame. The lambda values are calculated for $m \geq 0$, $\lambda \geq 0$ for feasible solutions for risk averse investors. Using historical data to calculate mean-variance requires a time frame (number of observations) with statistical integrity. No matter how r is defined it is almost certain to alter in value over the time frame being considered. This would not be of much concern if the shifts were around a stable mean but the problem would be a serious one if the mean value of r was non-stationary.

In fact a possible explanation for non-stationary means for historical stock returns would be non-stationary mean values of r. These problems are often present in nominal returns data but are not as likely to be present in real returns data. Both nominal and real returns are considered in this thesis.

Nominal and real values of r used in this thesis cover a range of values as appropriate to the investor class under consideration. The fact that these values, or marginal costs, are not riskless does not cause any specific problems in the application of the Moeseke model as the purpose of these values is to define a point at which optimality is achieved with maximum caution in the market place. The products represented by r values are an investment option or actual costs and are viable alternatives to the "market" portfolio, or costs that can be avoided in the case of the financial intermediary. The risk embedded in these products is of no significance to the model itself however. These are minimum risk positions, therefore there would be no less risky alternative available other than doing nothing.

5.3 Negative Real Returns

In Chapter 3, Section 4 the point was made that a dual solution $V^{*}<0$ is not feasible. Repeating the position, for the portfolio problem the dual variable is $\lambda \geq 0$. With the budget set at unity $\lambda^* = \phi(x^*)$. If $\phi(x) \leq 0 \quad \forall x$ then $x^* = 0$ and $\phi(x^*) = 0 = \lambda^*$, thus satisfying $\lambda^* \geq 0$.

Given that $\phi\,(x^+ \mid m^+) = \lambda^+ = r$ for $\lambda^+ \geq 0$ then $r \geq 0$ must also hold.

There are, however, numerous examples of negative real returns having occurred in different countries at different times.

Going back to the oil shocks of the 1970s, the failure of governments or central banks to use the interest rate structure within the economy effectively to control inflation led to many examples of this situation. In the case of New Zealand, interest rates had been very stable for many years prior to the first oil shock of the early 1970s and when inflation rose sharply in the mid 1970s, interest rates, which were to a large extent government controlled, were not allowed to rise enough to compensate investors for the effects of the inflation. Foreign exchange controls that were in place meant investors were unable to move funds offshore to achieve better returns. The deregulation of the 1980s in New Zealand coupled with the 1989 Reserve Bank Act, giving the central bank the independent responsibility to control inflation, has made a repeat of this situation unlikely although, in hindsight, in the early part of the deregulation period high volatility in both interest rates and inflation levels did lead to short periods of negative real interest rates.

Hong Kong faced the same problem in the early 1990s though for quite different reasons. With the Hong Kong dollar being fixed at 7.8 Hong Kong dollars for $1 US with a high level of certainty, interest rates in Hong Kong tend to track closely the US rates as the currencies are close to perfect substitutes. Strong increases in property prices in Hong Kong in the early 1990s proved inflationary with inflation going above 10% in 1991. At the same time, however, interest rates in the US were low as the Federal Reserve was actively using a policy of a low discount rate to stimulate economic growth. The result was a period of high negative real interest rates in Hong Kong.

It can be noted that these situations tend to be very destabilising as the purchasing of real assets on borrowed finance becomes a "good investment" leading to real asset prices being bid up strongly with the subsequent bidding up of equity prices for those equities which represent real asset holdings.

In relation to the Moeseke model

for $\phi(x^+|m^+) = \lambda^+ = r < 0$ the solution is not to invest.

As this may be inappropriate the following approach is suggested.

For $r = -\eta$; $\eta > 0$

$$\max_{x} \phi\,(x|m^+) = \lambda^+$$

where $m^+ = \max\,\{m \geq 0 |\, \max \phi(x|m) \geq r + \eta\}$

and $\lambda^+ = \phi(x^+|m) = (\bar{c} + \eta)x^+ - m(x^+Vx^+)^{\frac{1}{2}}$

The optimal portfolio x^+ will then be found where

$$\max_{x \varepsilon X} \phi\,(x|m^*) = \phi\,(x^+|m^+) = \lambda^+ = 0$$

That is the real interest rate, being < 0 is reset to a zero base in order that a solution can be obtained when applying the programme.

5.4 The Moeseke Model and New Zealand Monetary Policy

Monetary policy implementation in New Zealand differs substantially from the monetary policy implementation described in the extension to the Moeseke model as analysed by LeBlanc and Moeseke[179]. In New Zealand there are no reserve ratio requirements and no discount rate. The central bank (Reserve Bank of New Zealand) implements monetary policy by targeting, indirectly, the short-term wholesale interest rates with the primary objective of maintaining inflation in a 0-2% band.

The argument in support of this is that it is the short-term wholesale interest rates which best influence those economic variables which impact on inflation. Also the Reserve Bank can influence short-term wholesale interest rates reasonably easily. The Reserve Bank does not set interest rates directly, that is left to the market to do, but it can certainly influence the level of interest rates.

Adjusting short-term wholesale interest rates impacts on inflation in a number of ways. In particular there will be an impact through changes in the exchange rate. New Zealand has a floating exchange rate and higher real short-term interest rates encourage an inward flow of funds putting upward pressure on the exchange rate, leading to a deflationary impact on traded goods prices. Higher short-term interest rates also impact on retail interest rates, put downward pressure on asset prices such as equities, and signal the possibility of some credit rationing. All of these effects impact on nominal domestic demand and therefore real economic activity. Downward

179 LeBlanc, G. and Moeseke, P. v., 1979, op. cit.

pressure on real economic activity will squeeze wholesale and retail margins as well as unit labour costs. Both of these factors have a deflationary impact on prices.

The Reserve Bank of New Zealand targets short-term wholesale interest rates through its management of the daily settlement process between itself, the Government and the settlement banks that operate in New Zealand. Central to this process is the control of a very small part of the money supply called primary liquidity. Primary liquidity is made up of the settlement account balances that the settlement banks hold at the Reserve Bank and the total of discountable Reserve Bank Bills on issue. Reserve Bank Bills are normally issued for a period of 63 days and can be discounted (sold back to the Reserve Bank for settlement cash) when they have 28 days or less to run till maturity.

The settlement banks have to hold at least part of their precautionary balances in primary liquidity assets, mainly because the Reserve Bank will only accept cash in settlement for its own transactions and those of the government. Also banks tend to use Reserve Bank cash to settle transactions among themselves to reduce default or credit risk and the public often requires cash from banks to settle their own transactions.

The exact amount of primary liquidity that any particular bank will require will depend on the size and volatility of the daily flows between itself, the Reserve Bank and other private sector financial institutions as well as the relative difficulty in predicting such flows in advance. If a particular bank believes it may be short of settlement cash on a particular day that bank will have to bid for deposits in the money market.

As there are no reserve ratio requirements in New Zealand, the availability of primary liquidity along with the size of a particular bank's capital dictates the extent to which the bank can increase its loans to the public or its level of business in general.

Given the importance of primary liquidity and the control that the Reserve Bank has over it, there are four ways in which the Reserve Bank can take action to influence short-term wholesale interest rates. These are as follows:

(i) The daily settlement cash target can be altered. This target is the total amount of settlement cash left in the system after the day's transfers have been made, that is the total of the balances of the settlement accounts held at the Reserve Bank. No bank is permitted to have a negative balance in their account so the lower the daily cash target the more likelihood that banks will need to discount Reserve Bank Bills in order to keep their settlement accounts with the Reserve Bank in credit. The Reserve Bank likes to keep the daily settlement cash target reasonably stable and as at June 1996 it stood at $20 million.

(ii) The amount of Reserve Bank Bills on issue can be altered. The Reserve Bank tends to issue the bills twice weekly with the total volume of discountable securities standing at approximately $500 million as at June 1996. If the Reserve Bank wished to send a signal to the market that it was moving to a tighter monetary policy position, reducing the amount of Reserve Bank Bills on issue would be one way to do this.

(iii) When banks find themselves in a position of having to discount Reserve Bank Bills to maintain a credit balance in their settlement accounts they do so at a discount rate set by the Reserve Bank. There is a penalty for doing this and as at June 1996 the penalty discount margin for Reserve Bank Bills was 0.9 percent above the seven day money market rates.

(iv) Credit balances held by banks in their settlement accounts earn an interest rate below the seven day bank bill yield so banks must trade off the risk of being short of settlement cash against the costs of holding too much settlement cash. Should the Reserve Bank raise the interest rate on settlement cash the demand for these settlement balances would increase which in turn would increase demand for deposits on the money market. This would be seen as a tightening of monetary conditions.

On any particular day the Reserve Bank will have estimated the likely settlement cash flows for that day. The cash flows that are important are those between the government and the Reserve Bank and the settlement banks. On that day there will be expectations of payments to and from government including such things as benefit payments, government stock interest payments and redemptions and the like. If there is a net outflow of funds expected on a certain day the Reserve Bank will have to inject funds back into the system. If a net inflow of funds is expected the opposite will be required. These actions by the Reserve Bank are open market operations or OMOs.

Given the above method for the implementation of monetary policy one can now give consideration to this approach in terms of the Moeseke model. The financial intermediary who faces a marginal cost to the investment dollar equal to the marginal cost at which that dollar can be attracted in the market place is faced with the following situation.

There is no reserve asset ratio but financial intermediaries will hold some portion of their assets in reserve type securities, that is government securities. That portion held in reserve type securities will be determined by the actions of other financial intermediaries, which in turn will be determined by the position they wish to hold in the market place (high risk, low risk for example). The market place as a whole will have a view as to what is appropriate. With no reserve requirement, however, the central bank is in no position to pay less on government securities so the interest rate paid on these securities will be a market rate determined by the interest rate on all other securities and the risk premium the market requires on them as compared to those of the government. The interest rate on the Reserve Bank Bills mentioned earlier can be strongly influenced by the actions of the central bank however with a flow on effect into the short-term money market.

Consider then the financial intermediary's portfolio programme (Q)

$$\max c_\alpha x_\alpha + cx - m\,(xVx)^{1/2} \qquad (5.4.1)$$

on $\quad X = \{x_\alpha, x) \geq 0 | x_\alpha + ux \leq 1, x_\alpha \geq \gamma\}$

With reference to formula (3.5.1.1) in Chapter 3, Section 5.1, c_0 is replaced by c_α, the rate of interest paid on government securities in the market place. This interest rate will be a function of their overall demand and the discount deemed acceptable compared to other generally more risky securities both internally and offshore. Instead of the reserve coefficient β we now have that proportion held in government securities by choice, say γ. A study of the current balance sheets of New Zealand's major financial intermediaries would suggest that a value of 15% as an average γ would be a reasonable estimate. Note, however, that each financial intermediary will select a γ appropriate to their own needs.

C_α and γ are not directly instruments of monetary policy in this case but as with the LeBlanc, Moeseke extension to the Moeseke model there is now a liquidity constraint as well as a budget constraint. While the liquidity constraint is self imposed the monetary authority is in a position to force reconsideration of this self imposed constraint should it so desire via its control of primary liquidity.

Given $\phi x = c_x - m(xVx)^{\frac{1}{2}}$ the maximal (Q) can be rewritten as:

$$G(x_\alpha, x) = c_\alpha x_\alpha + \phi(x) \qquad (5.4.2)$$

and (Q) reduces to

$$\underset{X}{Max} \quad G(x_\alpha, x)$$

and $$\underset{X}{Max} \quad G(x_\alpha, x) = c_\alpha x_\alpha + \phi(x)$$

such that $x_\alpha + u_x \le 1 \quad : \quad \lambda_1,$ being the budget constraint

and $-x_\alpha \le -\gamma \quad : \quad \lambda_2,$ being the self imposed liquidity restraint

Also $G(x_\alpha^*, x^*) = c_\alpha x_\alpha^* + \phi(x^*) = \lambda_1^* - \lambda_2^* \gamma$

The same separation properties as for the LeBlanc, Moeseke extension to the Moeseke model, Chapter 3, Section 5.1, apply in this case with qualifications to be noted.

In particular

(i) $x_\alpha^* = \gamma$ (5.4.3)

(ii) The ratios x_i^*/x_i^* do not vary with the budget fraction 1-γ allocated to that portfolio.

(iii) $\lambda_1^* = c_\alpha + \lambda_2^*$

λ_2^* is still the excess over and above c_α of the marginal return of the budget fraction invested in risky securities, the risk premium that the intermediary obtains by investing in those risky securities. From the monetary authority's viewpoint, however, λ_2^*, while still a saving, exists on account of the self imposed liquidity constraint and will almost certainly be a substantially smaller value than it might have been had the monetary authority itself imposed the liquidity constraint.

(iv) The value of the dual λ_1^* will still be independent of c_α and γ, its value depending solely on the security market, however c_α and γ will be dependent on the value of λ^*_1 to some extent as the risk premium λ^*_2 will be market determined and influenced by the value of λ^*_1.

(v) The value of the duals λ^*_1, λ^*_2 will be independent of the value of γ if the assumption that equilibrium exists in the market place is made. In particular, it is assumed that financial institutions will only adjust the percentage they invest in government securities out of choice if they are compensated for the change by an appropriate shift in c_α.

(vi) The optimal value of the objective G is linear in γ.

(vii) For a given G the elasticity of the risk premium λ_2 with respect to coefficient γ is minus 1. That is if λ^*_1 remains constant, then the financial intermediary will only be prepared to increase the proportion of total assets held in government securities by one-tenth if the risk premium λ^*_2 decreases by one-tenth and this can only happen if the yield on government securities increases by an amount equivalent to one-tenth of λ^*_2.

(viii) $\Delta c_\alpha/\Delta\gamma = \bar{\lambda}_2/\gamma = \lambda_2^*/\bar{\gamma}$

where γ, λ^{*}_{2} are the original values and $\bar{\gamma}, \bar{\lambda}_2$ are the changed values.

That is λ^{*}_{2} properly normalised per unit of $\bar{\gamma}$ can be interpreted as the marginal rate of substitution between the chosen government securities holding, γ, and the interest rate set in the market place for those government securities, c_{α}.

While the interest rate on government securities is not an instrument of monetary policy under this model and reserve requirements are nonexistent, the third instrument of monetary policy considered by LeBlanc and Moeseke, namely the rate of interest paid on deposits, is very much a tool of monetary policy. This rate is determined in this case, however, not by a discount rate but by the central bank's control of primary liquidity.

The interest rate paid on deposits is the marginal cost of capital to the financial intermediary with some adjustment necessary to account for the cost to the financial intermediary of holding their chosen level of government securities.

From (3.5.2.1)

$$r - c_{\alpha} = \lambda_2^{*}(1 - \gamma) \tag{5.4.4}$$

As with the LeBlanc, Moeseke extension the interest rate differential the financial intermediary can afford to pay, over and above the yield on government securities, equals the risk premium received on the budget fraction invested in risky securities.

For the financial intermediary conforming to the maximal-caution criterion

$$\Delta c_\alpha/(r - c_\alpha) = \Delta\gamma/\bar{\gamma}(1 - \gamma) \tag{5.4.5}$$

5.5 The Moeseke Model and New Zealand Taxation Law

The tax regime faced by New Zealand residents in relation to income and capital gains from financial assets as at June 1996 was as follows.

5.5.1 The Fixed Interest Markets

Financial institutions, managed funds and individuals are all treated equally. As at June 1996 there was a 24% withholding tax on income from which an exemption can be obtained for charities. Exemptions can also be obtained in special circumstances such as for nominee companies passing on income to other parties. The final tax rate on this income will be that which is appropriate for the recipient, normally the top rate of 33%. Any realised capital gains or losses are assessable for tax purposes, also at the appropriate tax rate of the recipient.

A regime called the Accruals Regime is also in place which, as at June 1996, comes into effect for all bond holders with holdings in excess of NZ$600,000 or interest income in excess of NZ$70,000 per annum. For all bond holders in this category unrealised capital gains or losses must be accounted for every year with tax paid or refunded accordingly.

Offshore investors earning interest income in New Zealand are subject to a 10% withholding tax on income earned only, with no requirements or ability to declare capital gains or losses for tax purposes in New Zealand.

5.5.2 The Equity Market

Financial institutions, managed funds and individuals are all treated equally in relation to dividend income on equities. As at June 1996 there was a 33% withholding tax which, in general, would be the tax rate faced by equity investors. There was also an imputation credits system in operation which, if a dividend is fully imputed, effectively negates the 33% tax imposed. The level of the imputation credit attached to a dividend payment is dependent on the effective tax rate of the company paying the dividend.

To see the impact of imputation credits consider three separate companies paying a ten cents per share dividend.

1. Company with no imputation credits.

   Dividend = 10.00 cps Imputation credit = 0.00 cps

   Withholding Tax = 3.30 cps Payment = 6.70 cps

2. Company on an effective tax rate of 20% (partial credit)

   Dividend = 10.00 cps Imputation credit = 2.9851 cps

   Withholding tax = 4.2851 cps Payment = 8.70 cps

3. Company on an effective tax rate of 33% (Full Credit)

   Dividend = 10.00 cps Imputation credit = 4.92537 cps

   Withholding tax= 4.92537 cps Payment = 10.00 cps

As at June 1996 offshore investors faced a 10% withholding tax on dividend payments and, in many cases, are able to take advantage of the imputation credits attached to these payments.

In relation to capital gains there is a clear distinction made between institutions or individuals who are "in the business" of actively investing for the purpose of making capital gains and institutions or individuals who are only investing on a long-term passive basis. This distinction is vague but has effectively led to a situation where, in general, financial institutions and managed funds pay tax at 33% on capital gains and receive tax credits on capital losses. Individuals and totally passive funds such as index funds, in general, have no requirement to declare capital gains or losses for tax purposes. This gives individuals investing directly into the New Zealand equities market or passive index funds a distinct tax advantage over the active managed funds industry.

Offshore investors are treated in the same way as individuals normally are with no requirement or ability to declare capital gains or losses for tax purposes in New Zealand.

In relation to both the fixed interest and equity markets offshore investors will face their own individual tax regimes in their different resident countries.

5.5.3 Incorporating the New Zealand Tax Law into the Moeseke Model

For the optimal solution to the Moeseke Model

$$\phi(x^* | m) = \bar{c}x^* - m(x^* Vx^*)^{\frac{1}{2}} = \lambda^* = r$$

Taxation considerations affect $\bar{c}$ and r explicitly. That is they affect the expected return to each risky asset and they affect the marginal cost of the investment dollar. The impact of taxation on $\bar{c}$ is clear and can be taken into account readily. The impact of taxation on r is also clear for individuals and can be taken into account by simply considering the after tax return on bank deposits for example. For financial institutions where the marginal cost of the investment dollar is the marginal cost of raising funds in the market place taxation can be accounted for in the same way as for individuals or managed funds as the cost of raising funds is itself a tax deductible item.

5.6 A Comparison of New Zealand and Offshore Investor Risks

The first most obvious risk that may well be viewed differently by offshore investors, as compared to New Zealand based investors, will be exchange rate risk. It could be argued that in a truly global market the exchange rate risk for New Zealand dollars against a market weighted basket of all other currencies should be the same for a New Zealand resident as it is for a US resident, for example. The reality, however, is that New Zealand residents normally require New Zealand dollars to transact their business while US residents require US dollars. The exchange rate risk of New Zealand dollars is going to be seen as being higher for the US resident than for the New Zealand resident and therefore the US resident will require compensation for taking on this risk when investing in New Zealand money market products, bonds or equities. This compensation can be viewed as an extra component of the marginal cost faced by the offshore investor. A procedure for incorporating exchange rate risk into the marginal cost of the investment dollar for the offshore investor is proposed and applied in Chapter 7 Section 3.2.

The other risks that require mention here are the country risk, including political risk and inflation risk. As New Zealand is a small open economy which allows and encourages the free flow of capital, these risks are best exposed through the risk premium placed on New Zealand government securities by investors generally and would normally be a function of the country's credit rating. In this case, however, New Zealand and offshore investors would be affected in similar ways as the marginal cost of the investment dollar would be increased for all investors if a credit rating was to drop for example.

The important issue for offshore investors would be the risk embodied in interest rate differentials between countries and a procedure for incorporating the cost of carrying this risk into the marginal cost of the investment dollar for offshore investors is also presented in Chapter 7 Section 3.2.

The issues and extensions to the Moeseke model discussed in this chapter are implemented where necessary and are therefore incorporated into the data analysis section of this thesis which follows.

CHAPTER SIX

THE COMPILATION OF THE DATA BASE AND METHODOLOGY FOR ITS USE

6.1 Introduction

This chapter sets out the data used in this thesis, its sources and application and is structured as follows. Section 2 of this chapter lists the data used with Section 3 detailing the procedures employed to have the data in an appropriate format for analysis. Finally Section 4 discusses the factors and methodology used in determining the marginal cost of the investment dollar, a value which determines points of optimality as has been shown earlier.

The first decision to make relates to the time frame to be used for the study. It was considered important that the time frame was reasonably homogeneous in terms of government and monetary policy. As deregulation of the New Zealand economy began in earnest in 1985 data before this time was deemed inappropriate. In terms of the deregulation itself a chronology of the important milestones in this process is worth noting. This can be found in Appendix one.

From this chronology it can be seen that 1985 and 1986, in particular, saw considerable change in the New Zealand economy. These changes continued on through the late eighties and into the nineties but it is reasonable to state that by 1987 the direction of change was clearly defined and well advanced. In particular the floating of the currency, the free movement of funds in and out of the economy, the new monetary policy regime, and the deregulation of the banking sector had all been implemented. The beginning of 1987 also saw stock exchange computerisation that lead to a stock market database with a very high level of integrity. It was therefore decided that the period of analysis would begin from January 1, 1987 and go through as far as practicable into 1995.

Two further points that should be noted in relation to this time frame are that managed funds data which is used in this study become more readily available from January, 1987 on and also that the period does include the stock market crash of October 20, 1987. It was considered of value to have this market shock as part of the period under investigation in order to note the impact of such a shock on the risk profile of the stock market and capital markets in general.

6.2 The Data Sets Used for the Analysis

When deciding on the investment alternatives to include in the analysis from New Zealand's capital markets, liquidity and general investment acceptability were the primary considerations. In the later case trustee investment in New Zealand has been guided by the Prudent Person rule since 1988 so any reasonably sound investment is acceptable and available to all investors. In general investments considered prior to this time carried trustee status. The Moeseke model is applied to equity market company and sector data, managed funds data and interest rate data for both nominal and real returns and the relevant data used for the analysis are now detailed.

6.2.1 Equity Market Data

To estimate risk and return on New Zealand equities two data blocks were used. First, fortnightly company data was obtained from the New Zealand Stock Exchange data base for the major listings from January 1, 1987 to October 31, 1995. The companies selected were those which were included in the top 40 company grouping for a minimum of three years of the study period. Selection was restricted to the top 40 companies as these companies are the most liquid and make up the bulk of the market capitalisation. The major index used to gauge stock market performance in New Zealand is the NZSE40, taking into account just the top 40 companies. A minimum three year observation period was required for each company as this was the shortest time frame considered in calculating historical returns and risk levels. Table 6.2.1.1 lists the companies analysed and their respective periods of inclusion together with the percentage of the total market that they made up as at the beginning of each year of the study period.

Table 6.2.1.1
Selected Companies' Inclusion Periods And Their Percentage Of The Total Market

		87	88	89	90	91	92	93	94	95
1	Air New Zealand				2.74	1.72	2.32	3.33	3.45	3.82
2	Bank of New Zealand		4.03	4.85	6.05	4.28	3.54			
3	BNZ Finance	1.08	0.66	0.85	0.81	0.78	0.56	0.60	0.48	0.45
4	Brierley Investments	11.56	9.55	9.27	15.15	11.11	9.58	8.37	8.04	7.08
5	Carter Holt Harvey	2.78	2.37	3.66	5.74	9.02	10.26	13.08	12.46	12.13
6	Cavalier Corporation	0.09	0.06	0.10	0.24	0.21	0.32	0.40	0.40	0.26
7	Ceramco Corporation	0.82	1.03	0.50	0.77	0.47	0.80	0.99	0.84	0.34
8	Colonial Motors	0.16	0.25	0.21	0.21	0.18	0.12	0.17	0.13	0.13
9	Corporate Investments	0.33	0.52	0.73	1.46	1.07	0.71	0.30	0.26	0.22
10	DB Group	1.11	2.08	4.18	3.89	4.20	1.38	1.30	0.68	0.79
11	Donaghys	0.06	0.09	0.13	0.24	0.31	0.30	0.41	0.35	0.29
12	Ernest Adams	0.02	0.06	0.09	0.13	0.20	0.14	0.14	0.09	0.07
13	Elders Resources NZFP	3.99	4.80	8.96	6.51	5.84				
14	Enerco New Zealand							0.27	0.51	0.49
15	Fay Richwhite	2.32	0.63	0.78	0.87	0.94	0.71	0.77	0.43	
16	Fernz Corporation	0.13	0.32	0.71	1.22	1.13	1.36	2.00	2.06	1.52
17	Fisher & Paykel	0.79	1.26	1.36	1.61	1.02	0.77	1.32	0.98	1.10
18	Fletcher Challenge	8.27	13.65	16.86	17.79	21.29	15.35	11.00	10.30	11.94
19	Goodman Fielder	5.02	8.53	10.69	9.23	11.12	8.11	7.66	4.66	3.63
20	Guinness Peat Group						0.61	0.44	0.58	0.59
21	Hallenstein Glasson	0.11	0.11	0.07	0.09	0.09	0.17	0.49	0.48	0.37
22	Independent Newspapers	0.40	0.82	1.28	1.98	2.37	1.68	2.39	1.87	1.54
23	Jarden Corporation	0.86	0.45	0.65	0.45					
24	Lion Nathan	1.86	2.87	4.25	5.45	6.46	6.22	6.18	4.44	3.55
25	Macraes Mining					0.42	0.76	0.58	1.13	0.91
26	Mair Astley	0.14	0.22	0.37	0.22	0.12	0.17	0.39	0.12	0.06
27	Milburn New Zealand	0.06	0.11	0.15	0.23	0.29	0.22	0.37	0.64	0.58
28	Natural Gas Corporation							1.18	1.70	1.48
29	New Zealand Oil & Gas	0.07	0.10	0.36	0.36	0.50	0.34	0.39	0.19	0.10
30	New Zealand Refining	0.06	0.13	0.29	0.35	0.86	1.11	1.73	1.75	1.45
31	Owens Group	0.14	0.07	0.09	0.12	0.15	0.23	0.33	0.25	0.27

Table 6.2.1.1 Cont....

		87	88	89	90	91	92	93	94	95
32	PDL Holdings	0.07	0.10	0.07	0.10	0.08	0.09	0.54	0.38	0.22
33	Progressive Enterprises							0.85	0.84	0.67
34	Salmond Smith Biolab	0.15	0.10	0.10	0.10	0.10	0.18	0.27	0.32	0.12
35	Sanford	0.22	0.34	0.33	0.50	0.74	0.81	1.41	0.96	0.67
36	Southern Petroleum	0.09	0.09	0.22	0.21	0.33	0.56	0.51	0.59	0.40
37	Steel & Tube	0.30	0.32	0.43	0.49	0.31	0.28	0.69	0.59	0.56
38	Telecom Corporation						21.53	17.99	20.27	21.67
39	Trans Tasman Properties	2.30	2.26	2.32	3.05	1.88	1.18	0.34	0.06	0.11
40	Whitcoulls Group						0.26	0.78	1.34	0.94
41	Wilson Neill	0.69	0.74	0.68	0.84	0.78	0.11	0.08	0.07	0.00
42	Wilson & Horton	0.91	1.28	1.61	2.43	2.50	1.69	2.19	1.83	1.90
	Percentage of Market Share	46.96	60.00	77.20	91.63	92.87	94.53	92.23	86.52	82.42
	Total Companies Listed	288	234	195	139	111	110	125	138	149

The second set of equity market data used was sector indices data. This data is monthly returns data covering the same time period as the companies data and was also obtained from the New Zealand Stock Exchange database. A listing of these sectors is given in Table 6.2.1.2.

Table 6.2.1.2

New Zealand Stock Market Sectors

1	Agriculture
2	Automotive
3	Building
4	Chemicals
5	Electrical
6	Energy & Fuel
7	Engineering
8	Finance & Banks
9	Food
10	Forestry
11	Investment
12	Liquor & Tobacco
13	Meat & By-products
14	Media & Communication
15	Medical Supplies
16	Miscellaneous
17	Property
18	Retailers
19	Textiles & Apparel
20	Transport & Tourism

6.2.2 Managed Funds Data

Managed funds, in theory, give an "efficient" portfolio mix for investors given a particular risk preference. These funds can be asset class specific or invested across a number of asset classes. As this study is only concerned with risk profiles in New Zealand capital markets the only funds considered here were New Zealand fixed interest, New Zealand equity, and New Zealand balanced funds. The balanced funds tend to hold a mix of equity and fixed interest investments. Tables 6.2.2.1 to 6.2.2.3 list the funds analysed. The period for which data was available is a shorter period than for the other data sets being from January 1989 to December 1995. Managed funds in New Zealand report their results on a monthly basis and again the individual time frames considered were three year minimums, to ensure statistical integrity.

Table 6.2.2.1

Managed Funds: New Zealand Fixed Interest

1	AMP Fixed Interest Security
2	ANZ New Zealand Fixed Interest
3	BNZ New Zealand Strategic Bonds
4	Joseph Banks New Zealand Bonds
5	National Bank Income
6	National Mutual Life Fixed Interest
7	New Zealand Funds Management Fixed Interest
8	Prudential Income Trust
9	Sovereign New Zealand Fixed Interest

Table 6.2.2.2

Managed Funds : New Zealand Equities

1	AMP Share Fund
2	BNZ New Zealand Blue Chip
3	Guardian Assurance Equity
4	Guardian New Zealand Equity
5	Joseph Banks New Zealand Equity
6	Joseph Banks New Zealand Equity Imputation
7	National Mutual Life Share
8	New Zealand Funds Management Equity
9	Prudential New Zealand Equity
10	Sovereign New Zealand Equity
11	Tower New Zealand Equity

Table 6.2.2.3

Managed Funds : New Zealand Balanced

1	AMP Managed Fund Balanced (B)
2	AMP Managed Fund Balanced (C)
3	AMP Managed Fund Balanced (M)
4	ANZ Growth Trust
5	ANZ Life Managed Fund
6	BNZ Balanced
7	Colonial Mutual Life Market Linked
8	Countrywide Bank Kiwi Trust
9	Countrywide Bank Life Multi Fund
10	Guardian Balanced
11	Guardian Assurance Balanced
12	Joseph Banks Asset Growth
13	Joseph Banks Capital
14	Joseph Banks Growth
15	National Bank Fund of Funds Balanced
16	National Mutual Life Balanced
17	Norwich Life Global
18	New Zealand Funds Management Balanced
19	Oceanic Managed
20	Prudential Balanced Growth
21	Prudential Beaver
22	Prudential Stag
23	Southpac Balanced
24	Sovereign Balanced Growth
25	Sovereign Conservative
26	Sovereign High Growth
27	Sun Alliance Bond Managed
28	Tower Multi Sector
29	Westpac Balanced
30	Westpac Life Investment
31	Westpac Retirement Balanced

6.2.3 Interest Rate Data

The fixed interest data used in this thesis is as follows. First the 90 day bank bill rate and the three month bank deposit rate were considered as the interest rates most appropriate in determining marginal costs. The interest rate data analysed within the Moeseke model was the six month, one and three year bank deposit rates and five and ten year government bond rates. Again the period under consideration is January 1987 to October 1995, for all but the ten year government bonds for which data was only available from January 1988. Monthly data was again used as for the stock market sector data and the managed funds data. This interest rate data can be found in Appendix two of this thesis.

6.2.4 Exchange Rate Data

The exchange rate data required for this thesis is as follows. Country cross rates are utilized being those of New Zealand's three major offshore investors, namely the United States dollar, the pound Sterling and the Australian dollar. This exchange rate data can be found in Appendix three of this thesis. The period again was January 1987 to October 1995 and monthly data was considered appropriate.

6.2.5 Inflation Rate Data

The inflation rate data used in this thesis is the consumer price index (CPI) as published by Statistics New Zealand. This inflation data can be found in Appendix four of this thesis for the period January 1987 to September 1995 and is only available on a quarterly basis.

6.3 Data Adjustment Requirements

The data for analysis by the Moeseke model was the equity, managed funds and interest rate data. The major analysis was carried out with nominal returns data though an analysis of real returns data was also carried out for the same asset classes by adjusting returns for actual inflation. The exchange rate data was employed to estimate the marginal cost of the investment dollar for offshore investors.

Returns data required adjustments for tax and all benefits, where applicable, and for inflationary impacts in those instances where real returns were applied. As the investment horizon is considered to be longer term, transaction costs have not been included in the analysis. These costs would be similar across the different asset classes and this point should be remembered when analysing the results presented.

(i) Equity Market Data

The companies data obtained was raw price data which needed to be adjusted for taxation, bonus issues and cash issues before calculating the first differences or the percentage change in value per period. For the equity market company data and the sector data taxation was applied as for individuals, that is dividends being taxable with imputation credits and realised capital gains being non-taxable. Realised capital gains are taxable in some instances as noted in Chapter 5, Section 5.

(a) Dividend Analysis

As noted in Chapter 5, Section 5 dividend imputation currently operates in New Zealand as has been the case since 1988. All dividends have been taxable over the period of study with tax credits applying to offset the individual company's effective tax rate. For convenience it was decided to assume a fixed tax rate being the top tax rate at the time of the dividend payment. This rate was 45% from January 1, 1987 until September 30, 1988 and 33% from October 1, 1988 on. These rates were deemed appropriate as most New Zealand investors would be subject to the top tax rates in New Zealand as these rates impact at low levels of individual income. As at June 1996 the top tax rate was operative on income above $30,875 for individuals and from the first dollar of taxable income for companies. Investors facing lower tax rates would logically choose different optimal portfolios to take advantage of this difference, namely individuals on lower gross incomes or organisations with tax free status and offshore investors.

The dividend adjustment is applied as follows:

$$P^d = P - t\,(d + m)$$

where P^d = price in cents per share adjusted for dividend

P = price in cents per share at time dividend ex date T-1

t = the current tax rate

d = dividend in cents per share

m = imputation credit

(b) Bonus Issue Adjustments

In general bonus issues are tax free in the hands of shareholders so no tax adjustments are required. Adjusting for bonus issues is applied as follows:

$$P^b = \frac{Pb}{b+1}$$

where P^b = price in cents per share adjusted for the bonus issue

P = price in cents per share at time bonus ex date -1

b = number of shares required to be held in order to be issued one new share : $\frac{1}{b}$ = bonus ratio.

(c) Cash Issue Adjustments

Cash issues are new issues of shares on which a cash payment is required and can be for extra ordinary shares in the company giving the issue or for some different class of share or shares in another company. These alternatives require different treatment. In the case of the issuing of further ordinary shares the adjustment is applied as follows:

$$P^{1} = \frac{Pb + c}{b + 1}$$

where P^{1} = price in cents per share adjusted for the cash issue

P = price in cents per share at time cash issue ex date -1

b = number of shares required to be held in order to be entitled one new share on receipt of a specified cash payment

c = cost per share to accept the issue

In the case of the issue being for shares other than ordinary shares as did occur, albeit infrequently, over the period under study, the above formula cannot be used as the newly issued share will almost certainly be priced quite differently from the original shares held. In these instances it has been assumed that the initial price adjustment on going ex the issue relates solely to the issue's value to its holder and the adjustment is made on this basis.

Should a share have more than one benefit owing at any one time, the price adjustments are made to include all benefits simultaneously for identical ex dates or in the order in which each benefit accrues to the shareholder for differing ex dates. In the cases where new shares issued differed from existing shares in some way, not being entitled to the next declared dividend of the company for example, adjustments to take these differences into account were also required.

After all adjustments had been made first differences were calculated for analysis.

For the sector data all bonus and cash issue adjustments had already been incorporated along with gross dividend payments as the indices used were gross indices. To allow for taxation the extra returns to the gross indices, as compared to the capital indices, were calculated, and taxation was then applied in the same way as for the equity market company data.

(ii) Managed Funds Data

The managed funds data used in this thesis was supplied by IPAC Securities Limited[180] and was actual returns data net of New Zealand resident tax and management fees. Tax is applied on both dividend income and capital gains in this case, a difference in tax treatment to that for the equity market company data and the equity market sector data as noted in Chapter 5, Section 5.

(iii) Interest Rate Data

The data used here is monthly returns data being a function of the coupon rate and the impact on capital value of shifts in the market interest rate. Tax is applied on the total return, both interest and capital gains, at the top tax rate for the period under consideration.

(iv) Real Returns Data

The last set of adjustments made to all data were those required to take into account inflationary impacts. Real returns data was generated by allowing for the inflationary impact over the time period considered.

The adjustments were made in the following manner using the CPI inflation data[181]

180 IPAC Securities (NZ) Limited, P O Box 4022, Auckland, New Zealand.
181 See Levy, H. and Sarnat, M., 1984, op. cit., pp 34 - 36.

$$R_r = \left(\frac{1 + R_n}{1 + h}\right) - 1$$

where R_r = the real rate of return to the security,

R_n = the nominal rate of return to the security,

h = the inflation rate.

For real returns data, quarterly data was used in line with the CPI figures available. On account of the smaller number of observations, whole period data and January 1988 to September 1995 data only was considered. A more detailed analysis of real returns was considered to be outside the scope of this thesis.

6.4 Determining the Marginal Cost of the Investment Dollar

A crucial part of the analysis of the risk profile of New Zealand's capital markets is in determining realistic values for the marginal cost of the investment dollar. As has been noted on a number of occasions throughout this thesis, this marginal cost will differ for different investors be they financial institutions or individuals, New Zealand based or offshore. It may also differ over time depending on shifts in the cost of capital in the capital markets.

For the New Zealand investor it is assumed that the appropriate marginal costs are those set directly in New Zealand's capital markets. This is a reasonable assumption as in an environment of free flowing capital between countries, as exists in New Zealand, these marginal costs will be determined in relation to marginal costs of capital in other countries. Feasible sets of marginal costs for different investors are determined by observation and are presented in Chapter 7.

For offshore investors the determination of marginal costs is more complex as has been discussed in Chapter 5, Section 6. Marginal costs for offshore investors from the United States, the United Kingdom and Australia are estimated by considering the exchange rate risk and the interest rate differentials. These estimates are also presented in Chapter 7.

Once the data required for analysis was prepared it was run through a computer programme which carried out the portfolio selection for a range of values of the risk parameter, m. The results obtained from this programme also included the appropriate lambda values or marginal return values. The computer programme used was developed by the writer for carrying out this analysis and applies the algorithm detailed in Chapter 3, Section 4.

The results of the analysis are detailed in Chapter 7 together with the estimates of the marginal cost of the investment dollar for a variety of investors. The impact of differing marginal costs on portfolio construction is analysed and the stability of the risk profiles for different markets is also considered.

CHAPTER SEVEN

RESULTS AND ANALYSIS

7.1 Introduction

This chapter presents the results from the study of the risk profiles of New Zealand's capital markets in the following format.

Section 2 details the returns data sets used in the Moeseke model and presents the results of tests for normality together with a discussion on the significance of these results. Section 3 considers the issue of appropriate marginal costs for the different types of investors for each time frame considered. Country risk premia and exchange rate risk for offshore investors are detailed and discussed.

Sections 4 and 5 give the results of the stock market, managed funds and fixed interest market analysis for nominal and real values respectively with Section 6 being an analysis of the value and stability of the resulting risk profiles of New Zealand's capital markets.

7.2 Normality Tests for Returns Data Sets

As discussed in Chapter 2, Section 6.1 the application of portfolio selection and asset pricing models which focus on the first two moments of the distribution of returns only require normality of returns or investors with quadratic utility functions to be strictly correct in a mathematical sense. However, it has also been shown, as reported in Chapter 2, Section 6.1, that capital market returns, in general, either approximate the normal distribution or belong to a more general class of stable paretian distributions. In any event for the current application the Bienaymé- Tchebyshev inequality holds[182].

It is, however, of general interest to analyse the nature of the distribution of returns for the data used in this application of the Moeseke model. A full analysis is presented in Appendix five of this thesis with the results summarised here. Unit root tests were also carried out on all data sets and these results are presented in Appendix six of this thesis with the results being summarised in Section 7.2.9. Note that the data presented in Appendix five also includes mean and standard deviation data for all data sets.

7.2.1 New Zealand Stock Market Company Data: Nominal Returns

Data set: Data used is fortnightly returns data from January 1987 to October 1995 being 227 observations for 31 companies. Eleven companies with shorter time frames were also considered.

In general this data did not conform to the normal distribution with most data sets having too many points in the central region and in the tails of the distributions. That is the distributions were of the leptokurtic type having a positive excess kurtosis coefficient. Some evidential skewness was present in some of the distributions, both positive and negative. The distribution were,

182 See p 78.

however, reasonably symmetric as can be seen from the graphs in Appendix five.

Table 7.2.1

Distribution of Returns for New Zealand Stock Market Company Data: Nominal Returns

Distribution	Number	Positive Skewness	Negative Skewness	Kurtosis Leptokurtic
Approximately Normal	4	-	-	-
Significantly Different from Normal: 5% Level	38	20	2	38

7.22 New Zealand Stock Market Company Data: Real Returns

Data set: Data used is quarterly returns data from January 1987 to September 1995 being 35 observations for 31 companies.

In this case the distribution of returns were normal in approximately sixty percent of cases with the kurtosis problem again being the main contributor to a lack of normality. The longer time period between observations would be a reasonable explanation as to why many of the companies considered show normality here when they did not for the nominal data. This is because a shorter time period between observations tends to give an excess weighting to observations at the centre of the distribution of returns.

Table 7.2.2

Distribution of Returns for New Zealand Stock Market Company Data:

Real Returns

Distribution	Number	Positive Skewness	Negative Skewness	Kurtosis Leptokurtic
Approximately Normal	19	-	-	-
Significantly Different from Normal: 5% Level	12	4	4	12

7.2.3 New Zealand Stock Market Sector Data: Nominal Returns

Data set: Data used is monthly returns data from January 1987 to October 1995 being 105 observations for 20 sectors.

As for the company nominal data the majority of distributions were significantly different from normal but, again, the main reason was because these distributions were leptokurtic. Four of the distributions showed evidence of positive skewness.

Table 7.2.3.

Distribution of Returns for New Zealand Stock Market Sector Data:

Nominal Returns

Distribution	Number	Positive Skewness	Negative Skewness	Kurtosis Leptokurtic
Approximately Normal	7	-	-	-
Significantly Different from Normal: 5% Level	13	4	0	13

7.2.4 New Zealand Stock Market Sector Data: Real Returns

Data Set: Data used is quarterly returns data from January 1987 to September 1995 being 35 observations for 20 sectors.

With the longer time period between observations only three distributions were significantly different from normal. All were leptokurtic distributions and one of these showed evidence of negative skewness.

Table 7.2.4

Distribution of Returns for New Zealand Stock Market Sector Data: Real Returns

Distribution	Number	Positive Skewness	Negative Skewness	Kurtosis Leptokurtic
Approximately Normal	17	-	-	-
Significantly Different from Normal: 5% Level	3	0	1	3

7.2.5 Managed Funds Data: Nominal Returns

Data Set: Data used is monthly returns data from January 1989 to December 1995 being 84 observations for 51 funds of which there are 9 fixed interest funds, 11 equity funds and 31 balanced funds.

The observed pattern for the managed funds nominal returns data was again similar to the stock market company and sector data, though with a higher percentage of distributions approximating the normal distribution. Again all distributions that were significantly different from normal were leptokurtic distributions. Of the 15 distributions that displayed evidence of skewness 7 were from the 11 equity funds (64%) with the balance coming from the 31 balanced funds (26%).

Table 7.2.5

Distribution of Returns for Managed Funds Data: Nominal Returns

Distribution	Number	Positive Skewness	Negative Skewness	Kurtosis Leptokurtic
Approximately Normal	20	-	-	-
Significantly Different from Normal: 5% Level	31	15	3	31

7.2.6 Managed Funds Data: Real Returns

Data Set: Data used is quarterly returns data from January 1989 to December 1995 being 28 observations for 51 funds of which there are 9 fixed interest funds, 11 equity funds and 31 balanced funds.

Again the longer the time period between observations appears to have led to a substantial reduction in the impact of kurtosis (leptokurtic) on the distributions with only 5 of the 51 distributions being significantly different from normal.

Table 7.2.6

Distribution of Returns for Managed Funds Data: Real Returns

Distribution	Number	Positive Skewness	Negative Skewness	Kurtosis Leptokurtic
Approximately Normal	46	-	-	-
Significantly Different from Normal: 5% Level	5	0	2	5

7.2.7 Interest Rate Data: Nominal Returns

Data Set: Data used is monthly returns data from January 1987 to October 1995 being 105 observations. This is the case for all interest rates except 10 year government stock. Data for this was only available from January 1988 on giving 93 observations. In total 5 different interest rates were considered.

Of the five data sets considered here it was the shorter term interest rate returns distributions that were significantly different from normal. The main problem again was with kurtosis (leptokurtic).

Table 7.2.7

Distribution of Returns for Interest Rate Data: Nominal Returns

Distribution	Number	Positive Skewness	Negative Skewness	Kurtosis Leptokurtic
Approximately Normal	3	-	-	-
Significantly Different from Normal: 5% Level	2	1	0	2

7.2.8 Interest Rate Data: Real Returns

Data Set: Data used is quarterly returns data from January 1987 to October 1995 for all except 10 year government stock data being 35 and 31 observations respectively for 5 different interest rates.

In this case the longer time period between observations had no positive impact on the kurtosis present with two of the distributions being leptokurtic. These same two leptokurtic distributions also displayed evidence of negative skewness.

Table 7.2.8

Distribution of Returns for Interest Rate Data: Real Returns

Distribution	Number	Positive Skewness	Negative Skewness	Kurtosis Leptokurtic
Approximately Normal	3	-	-	-
Significantly Different from Normal: 5% Level	2	0	2	2

Overall the data employed for the application of the Moeseke model is approximately normal in many instances though there is often significant kurtosis of the leptokurtic type. This is particularly so for the stock market nominal data. As the distributions, in general, are approximately symmetrical, however, the leptokurtic nature of some distributions is not seen as a significant problem in the current application. It should be noted that where evidence of skewness was found this skewness, both positive and negative, was minimal.

7.2.9 Unit Root Test for Returns Data Sets

Autocorrelation should not be present in a distribution that resembles a normal distribution. If a first-order autoregressive process is not present, then for $y_t = \alpha + \beta y_{t-1} + \varepsilon_t, t = 1 \ldots T$ where the coefficient α is the intercept, β will equal zero and y_t will be constant with a random disturbance. It is assumed that the error term will have mean equal to zero and a variance which is constant over time. In implementing a unit root test a significant test statistic suggests stationarity of y_t. If a series is a random walk the first differences of the series will be stationary.[183] As all data sets being analysed are first differences a significant test statistic will support randomness which is consistent with normality. Unit root tests were carried out using PC Give software and in all but two cases the returns data displayed stationarity of y_t with no lags. A complete set of the unit root test results is given in Appendix six.

183 See Holden, K. and Thompson, J., Co-Integration: An Introductory Survey. British Review of Economic Issues, V. 14, 1992, p49.

7.3 The Marginal Cost of the Investment Dollar

In determining the marginal cost of the investment dollar for investors the approach taken is as follows. Investors are divided into three categories: i) domestic individual investors ii) domestic fund managers and financial institutions; and iii) offshore investors. The time frames considered coincide with the time frames used in the capital market analysis. These are shown in Table 7.3.1.

The risk profiles with their resulting levels of risk and return are calculated on the basis of past performance giving an "objective" view of the sum of all investors' understanding of the market environment. Marginal costs can be objectively observed at any point in time for local investors with funds to allocate to domestic investment, but for offshore investors, facing exchange rate risk and possible shifts in relative interest rate levels, marginal cost becomes a stochastic variable. Even for local investors, however, marginal cost will almost certainly alter over time and some consideration needs to be given to the impact of changing marginal costs over any particular time horizon. These issues are addressed in the following sections.

Table 7.3.1

Time Periods for Risk Profile and Portfolio Optimization Analysis

Time Period	Company Nominal	Company Real	Sector Nominal	Sector Real	Managed Funds Nominal*	Managed Funds Real*	Interest Rates Nominal	Interest Rates Real
Jan 87 - Oct 95	*	*	*	*			*	*
Jan 88 - Oct 95	*	*	*	*			*	*
Jan 89 - Oct 95	*		*		*	*	*	
Jan 90 - Oct 95	*		*		*		*	
Jan 91 - Oct 95	*		*		*		*	
Jan 92 - Oct 95	*		*		*		*	
Jan 93 - Oct 95	*		*		*		*	
Jan 87 - Dec 91	*		*				*	
Jan 88 - Dec 92	*		*				*	
Jan 89 - Dec 93	*		*		*		*	
Jan 90 - Dec 94	*		*		*		*	
Jan 87 - Dec 89	*		*				*	
Jan 88 - Dec 90	*		*				*	
Jan 89 - Dec 91	*		*		*		*	
Jan 90 - Dec 92	*		*		*		*	
Jan 91 - Dec 93	*		*		*		*	
Dec 92 - Dec 94	*		*		*		*	

* Note for Managed Funds data time frames are to Dec 95 not Oct 95

7.3.1 The Marginal Cost of the Investment Dollar for Domestic Individual Investors, Domestic Fund Managers and Financial Institutions

A straight-forward and justifiable approach in determining the marginal cost of the investment dollar for domestic individual investors, domestic fund managers, and financial institutions, would be to take a short-term interest rate at the end of the estimation time period, say the three month bank deposit rate for domestic individual investors and the 90 day bank bill rate for domestic fund managers and financial institutions. Note that the bank bill market in New Zealand is strictly a wholesale market. If the interest rates at the end of the estimation period have been consistent over the whole period under consideration, these values would be reasonable estimates of marginal cost. However, if the interest rates had been changing significantly over the estimation period, this objective determination of marginal cost would become questionable, even for local investors. Table 7.3.2 gives the above mentioned interest rates as at the end of each estimation period together with the mean and standard deviation for each period.

Table 7.3.2

End of Period Interest Rates, Mean and Standard Deviation

for Three Month Bank Deposits and 90 Day Bank Bills

January 1987- October 1995

Time Period	Three Month Bank Deposit Rate % P.A.			90 Day Bank Bill Rate % P.A.		
	End of Period	Mean for Period	S.D. for Period	End of Period	Mean for Period	S.D for Period
Jan 87 - Oct 95	8.45	10.5	3.87	8.28	11.53	4.99
Jan 88 - Oct 95	8.45	9.64	3.18	8.28	10.3	3.64
Jan 89 - Oct 95	8.45	8.99	2.86	8.28	9.51	3.19
Jan 90 - Oct 95	8.45	8.37	2.63	8.28	8.77	2.86
Jan 91 - Oct 95	8.45	7.39	1.64	8.28	7.71	1.8
Jan 92 - Oct 95	8.45	6.89	1.26	8.28	7.14	1.4
Jan 93 - Oct 95	8.45	7.04	1.41	8.28	7.28	1.57
Jan 87 - Dec 91	7.5	13.27	2.74	7.4	14.9	4.05
Jan 88 - Dec 92	7.15	11.11	2.96	7.6	12.01	3.36
Jan 89 - Dec 93	5.3	9.51	3.08	5.18	10.13	3.4
Jan 90 - Dec 94	8.6	8.29	2.83	9.57	8.72	3.08
Jan 87 - Dec 89	12.87	14.64	2.13	14.28	16.72	3.6
Jan 88 - Dec 90	12	13.27	0.87	13.03	14.47	1.1
Jan 89 - Dec 91	7.5	11.67	1.95	7.4	12.54	2.09
Jan 90 - Dec 92	7.15	9.62	2.9	7.6	10.18	3.07
Jan 91 - Dec 93	5.3	7.26	1.76	5.18	7.64	1.93
Jan 92 - Dec 94	8.6	6.35	0.84	9.57	6.61	1.07

As can be seen from the above table three month bank deposit rates and 90 day bank bill rates have generally not been stable for the periods under consideration. One could therefore adopt the approach that investors, particularly long-term investors, would not necessarily view current marginal costs as appropriate for long-term decision making. It is therefore proposed that an appropriate marginal cost would be a risk adjusted one with the risk adjustment being appropriate in this context following the same format as Moeseke proposed for risky assets within his model.

That is $r = \overline{r} + m\sigma$ (7.3.1.1)

where r = the interest rate or marginal cost,

$\overline{r}$ = the mean interest rate over the estimation period,

m = the risk parameter,

σ = the standard deviation of the interest rate.

Table 7.3.3 gives risk adjusted marginal costs for all time periods. As a major focus of this analysis is on stock returns and the risk preference of investors could, generally, be deemed to be consistent, a value of m equal to 0.5 is used here in line with previous findings of Moeseke and others[184]. The value of m in this instance could be altered as appropriate when determining the m value for the market under consideration. An advantage of this approach is that cautious investors are less likely to invest in risky assets in a volatile interest rate environment where interest rates rises would discourage such investment. In an environment of very stable interest rates $m\sigma$ will approach zero and r will be approximately equal to $\overline{r}$ in line with the original Moeseke model.

184 See Moeseke, P.v., 1965a, Moeseke, P.v. and Hohenbalken, B.v., 1973 and Young, M., 1985.

Table 7.3.3

Risk Adjusted Marginal Costs of the Investment Dollar

Time Period	Three Month Bank Deposit Rate % P.A.			90 Day Bank Bill Rate % P.A.		
	Mean for Period	S.D. for Period	Risk Adjusted Marginal Cost	Mean for Period	S.D. for Period	Risk Adjusted Marginal Cost
Jan 87 - Oct 95	10.5	3.87	12.43	11.53	4.99	14.02
Jan 88 - Oct 95	9.64	3.18	11.23	10.3	3.64	12.12
Jan 89 - Oct 95	8.99	2.86	10.42	9.51	3.19	11.1
Jan 90 - Oct 95	8.37	2.63	9.68	8.77	2.86	10.2
Jan 91 - Oct 95	7.39	1.64	8.21	7.71	1.8	8.61
Jan 92 - Oct 95	6.89	1.26	7.52	7.14	1.4	7.84
Jan 93 - Oct 95	7.04	1.41	7.74	7.28	1.57	8.06
Jan 87 - Dec 91	13.27	2.74	14.64	14.9	4.05	16.92
Jan 88 - Dec 92	11.11	2.96	12.59	12.01	3.36	13.69
Jan 89 - Dec 93	9.51	3.08	11.05	10.13	3.4	11.83
Jan 90 - Dec 94	8.29	2.83	9.7	8.72	3.08	10.26
Jan 87 - Dec 89	14.64	2.13	15.7	16.72	3.6	18.52
Jan 88 - Dec 90	13.27	0.87	13.7	14.47	1.1	15.02
Jan 89 - Dec 91	11.67	1.95	12.64	12.54	2.09	13.58
Jan 90 - Dec 92	9.62	2.9	8.07	10.18	3.07	11.71
Jan 91 - Dec 93	7.26	1.76	8.14	7.64	1.93	8.6
Jan 92 - Dec 94	6.35	0.84	6.77	6.61	1.07	7.14

7.3.2 The Marginal Cost of the Investment Dollar for Offshore Investors

For offshore investors the marginal cost of investing in a country such as New Zealand may well be different to that for domestic investors. One could take the CAPM view and argue that a perfectly diversified portfolio will be one invested over all global assets and therefore the only risk borne by the investor is global systematic risk and the investor is a "global citizen". The offshore investor could hedge all New Zealand dollar based investments against exchange rate risk but for an investor with a global perspective the cost of hedging should be fully compensated for by the risk premium the global investor will receive from New Zealand dollar based assets. New Zealand is a small open economy which allows and encourages the free flow of capital, so interest rates should be set at levels which fully compensate investors for any perceived risks such as country risk, including political risk and exchange rate and inflation risk. Relative differences in inflation between countries would normally be reflected explicitly in the exchange rate.

It could be argued then that the marginal cost of investing in New Zealand's capital markets is the same for offshore investors as for domestic investors. In reality, however, measuring country risk is an important and fundamental function for all international investors. It is not the purpose of this thesis to carry out an exhaustive analysis for the determination of marginal costs for offshore investors but consideration of the offshore investor's viewpoint will be based on the premise that the choice he or she faces is simply between his or her domestic market and the New Zealand market. Given this assumption equation (7.3.2.1) gives a basic model for the possible determination of such marginal costs. The basis of the analysis is similar to that for domestic investors but includes the risk embodied in interest rate differentials and exchange rate shifts. It could be argued that including a marginal cost adjustment for both interest rate differentials and exchange rate shifts may include some double counting on account of the fact that some exchange rate risk may be embodied in interest rate differentials. For offshore investors, however, the interest rate levels and the exchange rate movement exposure will be accounted for separately and are both costs or benefits to the investor.

Note that the marginal cost of the investment dollar may well be higher in the offshore investor's home country than in New Zealand.

The suggested model for determining the marginal cost of the investment dollar for offshore investors is as follows:

$$mc = \overline{r}_h + m\sigma_1 + (\overline{r}_t - \overline{r}_h) + m\sigma_2 + \overline{e} + m\sigma_3 \qquad (7.3.2.1)$$

where mc = marginal cost to the investment dollar for the offshore investor,

$\overline{r}_h$ = the mean home interest rate for the offshore investor over the estimation period,

σ_1 = the standard deviation of the home interest rate over the estimation period,

$\overline{r}_t$ = the mean interest rate of the target country over the estimation period,

σ_2 = the standard deviation of the target country interest rate over the estimation period,

$\overline{e}$ = the mean exchange rate movement for the offshore investor in relation to the target country over the estimation period,

σ_3 = the standard deviation of the exchange rate movement for the offshore investor in relation to the target country over the estimation period,

m = the risk parameter.

Table 7.3.4 gives a series of marginal costs for offshore investors based on the above model. Optimal portfolio results are not considered for offshore investors. The results, based on the model, indicate that the marginal cost of the investment dollar is higher for offshore investors from the United Kingdom, United States and Australia than it is for local investors. United States investors, however, are shown to have the lowest marginal costs of the three countries considered in this application. It is of interest to note that United States investors are also the most active in New Zealand's equity market.

Table 7.3.4

Risk Adjusted Marginal Costs of the Investment Dollar

for Offshore Investors as Compared to Domestic Investors

Time Period	United Kingdom		United States		Australia		New Zealand
	90 Day Bank Bills - Mean Rate	Risk Adjusted Marginal Cost	90 Day Bank Bills - Mean Rate	Risk Adjusted Marginal Cost	90 Day Bank Bills - Mean Rate	Risk Adjusted Marginal Cost	Risk Adjusted Marginal Cost
Jan 87 - Oct 95	9.86	15.6	6.1	14.63	10.42	15.43	14.02
Jan 88 - Oct 95	9.88	13.56	6.03	12.48	10.02	13.75	12.12
Jan 89 - Oct 95	9.79	12.42	5.78	11.36	9.59	12.84	11.1
Jan 90 - Oct 95	9.07	11.65	5.23	10.41	8.22	11.09	10.2
Jan 91 - Oct 95	7.9	10.15	4.66	8.9	6.95	9.18	8.61
Jan 92 - Oct 95	6.96	9.22	4.36	8.19	6.16	8.16	7.84
Jan 93 - Oct 95	6.06	8.27	4.57	8.38	6.09	8.32	8.06
Jan 87 - Dec 91	12.09	18.97	7.44	17.82	13.69	18.57	16.92
Jan 88 - Dec 92	12.05	14.99	6.85	14.16	12.24	15.47	13.69
Jan 89 - Dec 93	11.13	12.7	5.96	12.09	10.67	13.63	11.83
Jan 90 - Dec 94	9.45	11.55	5.11	10.45	8.3	11.15	10.26
Jan 87 - Dec 89	11.42	20.95	7.8	20.02	14.7	20.87	18.52
Jan 88 - Dec 90	13.09	17.42	8.24	15.95	14.96	17.47	15.02
Jan 89 - Dec 91	13.4	14.09	7.59	13.95	13.98	15.37	13.58
Jan 90 - Dec 92	11.9	12.3	5.85	11.91	10.22	12.56	11.71
Jan 91 - Dec 93	8.97	9.89	4.27	8.83	7.12	9.2	8.6
Jan 92 - Dec 94	7.01	8.71	3.91	7.5	5.73	7.48	7.14

7.4 Optimal Portfolios for Nominal Returns Data

Having suggested a procedure for determining marginal cost values for the range of time periods analysed, attention can now be given to the results obtained from the implementation of the Moeseke model for the capital markets under consideration. This section deals specifically with the stock market, managed funds and fixed interest market analysis for nominal data. A complete set of results for all returns data and for all time periods considered is given in Appendix seven.

This Section is divided into two parts. First, six "optimal" results are given for company, sector, the three classes of managed funds and fixed interest data. These results were calculated for the period January 1989 to October 1995 in all cases. A before tax marginal cost of 10.4% was used in line with the results given in Table 7.3.3 for three month bank deposit rates. Justification for this marginal cost value is given in the preceding section and would logically apply to domestic individual investors. The time period chosen for these particular "optimal" examples is the longest period considered excluding the 1987 stock market crash and for which the same time period can be used for all data sets. These results are shown in tables 7.4.1 to 7.4.6. Graph 7.4.1 shows the efficient frontier for the company data for this particular time period.

The second part of this section considers optimality for domestic fund managers and financial institutions who hold some of their funds in reserve type assets. The final part of the section considers the stability of the risk profiles estimated for the New Zealand stock market, managed funds and the fixed interest market.

Table 7.4.1

New Zealand Stock Market Optimal Stock Portfolio

Nominal Returns

Period : January 1989 - October 1995 for Fortnightly Data

m=0.48 Lambda = 0.27				
Company Number *	Company Name	% of Portfolio	Expected Net Return per Fortnight %	Standard Deviation of Expected Net Return per Fortnight %
27	Milburn New Zealand	25.04	1.5708	5.4138
30	New Zealand Refining	19.53	1.7669	7.0741
32	PDL Holdings	8.33	1.4347	8.3723
37	Steel & Tube	7.92	1.5136	8.5315
36	Southern Petroleum	4.78	0.8738	10.583
11	Donaghys	19.92	0.9348	4.9233
31	Owens Group	4.74	1.0859	6.2365
8	Colonial Motors	9.74	0.5704	6.329
Optimal Portfolio		100	1.3128	2.1626
Coefficient of Variation				1.6473

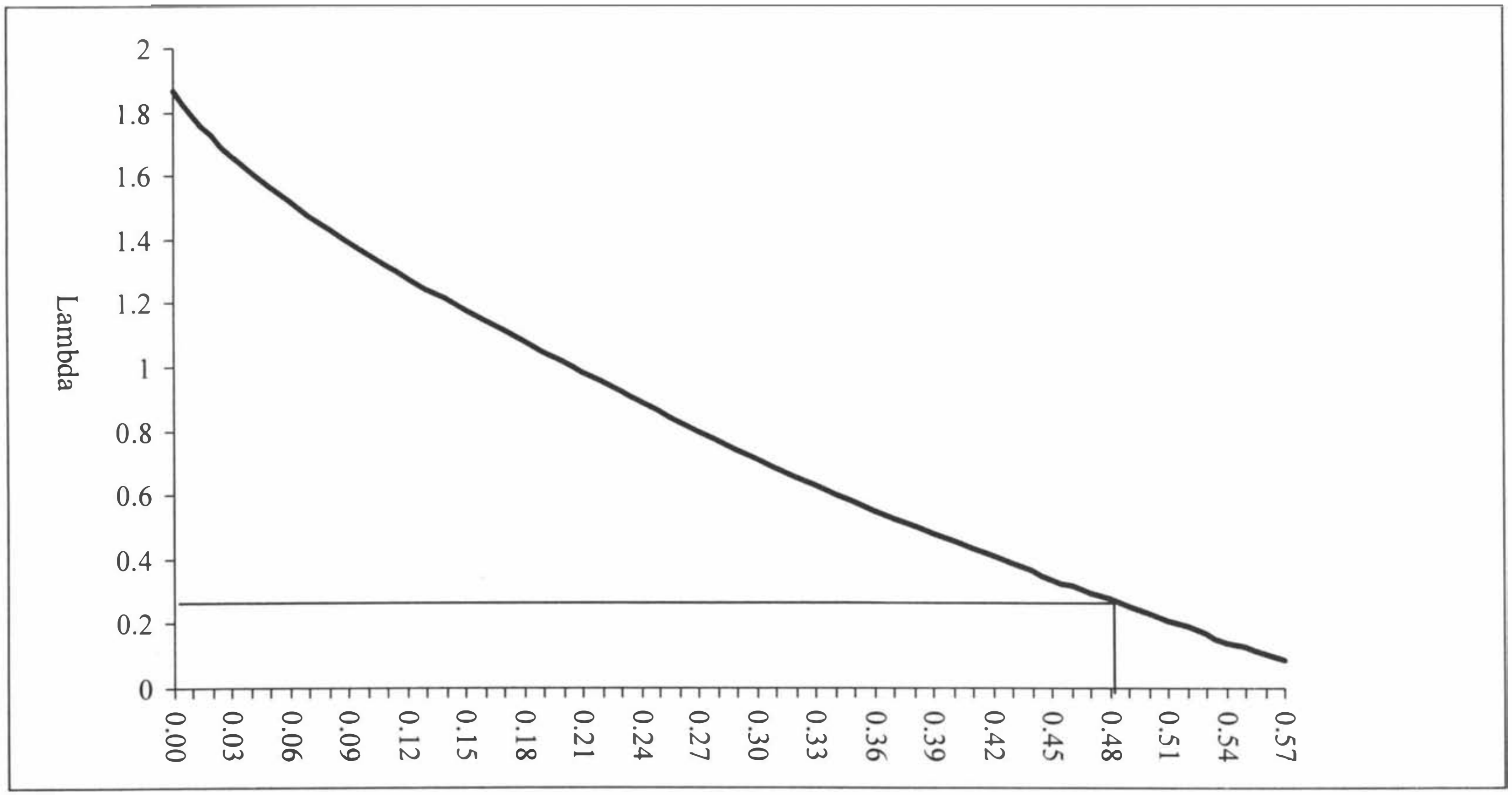
*Note: The ordering of the companies is that of their selection into the portfolio.

Graph 7.4.1

Marginal Return to the Investment Dollar

New Zealand Stock Market Stock Portfolios - Nominal Returns

Period January 1989 - October 1995 for Fortnightly Data



Risk Factor m

The two most significant aspects of the results shown in Table 7.4.1 and Graph 7.4.1 are, first, that a reasonable marginal cost of 0.27% per fortnightly period gives a risk profile value, m, of .48, a value that has previously been shown to be a good approximation of the risk profile of equity markets in both the United States and Canada[185] as well as New Zealand.[186] Also worth noting is that the risk of the optimal portfolio as shown by the standard deviation is less than half that of the least risky stock in the portfolio.

Table 7.4.2

New Zealand Stock Market Optimal Sector Portfolio

Nominal Returns

Period : January 1989 - October 1995 for Monthly Data

m=0.46 Lambda = 0.58				
Sector Number*	Sector Name	% of Portfolio	Expected Net Return per month %	Standard Deviation of Expected Net Return per Month %
6	Energy & Fuel	66.32	3.8767	7.528
3	Building	24.14	3.0552	8.0008
4	Chemicals	5.14	1.9918	8.4237
7	Engineering	4.4	2.2352	8.8228
Optimal Portfolio		100	3.5093	6.3374
Coefficient of Variation				1.8059

*Note: The ordering of sectors is that of their selection into the portfolio

185 See Moeseke, P.v., 1985a and Moeseke, P.v. and Hohenbalken, B.v., 1973.

186 See Young, M., 1985.

Table 7.4.2 gives an "optimal" result for the New Zealand stock market sector data. The risk profile given by m is slightly more aggressive than for the company data but again in line with the previously documented value for m of 0.5 for stock markets.

Table 7.4.3

New Zealand Fixed Interest Funds Optimal Portfolio

Nominal Returns

Period : January 1989 - October 1995 for Monthly Data

m= 0.14 Lambda = 0.58				
Fixed Interest Fund Number*	Fixed Interest Fund Name	% of Portfolio	Expected Net Return per Month %	Standard Deviation of Expected Net Return per Month
9	Sovereign New Zealand Fixed Interest	77.05	0.798	1.5817
8	Prudential Income Trust	22.95	0.6648	1.049
Optimal Portfolio		100	0.7674	1.3426
Coefficient of Variation				1.7495

* Note: The ordering of the funds is that of their selection into the portfolio.

Table 7.4.4

New Zealand Equity Funds Optimal Portfolio

Nominal Returns

Period : January 1989 - October 1995 for Monthly Data

m= 0.17 Lambda = 0.58				
Equity Fund Number	Equity Fund Name	% of Portfolio	Expected Net Return per Month %	Standard Deviation of Expected Return per Month %
3	Guardian Assurance Equity	100	1.1652	3.3464
Optimal Portfolio		100	1.1652	3.3464
Coefficient of Variation				2.8720

Table 7.4.5

New Zealand Balanced Funds Optimal Portfolio

Nominal Returns

Period : January 1989 - October 1995 for Monthly Data

m= 0.11 Lambda = 0.58				
Balanced Fund Number	Balanced Fund Name	% of Portfolio	Expected Net Return per Month %	Standard Deviation of Expected Return per Month %
11	Guardian Assurance Balanced	100	0.7761	1.8023
Optimal Portfolio		100	0.7761	1.8023
Coefficient of Variation				2.3223

The results from the managed funds analysis are interesting in that the risk portfolios of managed funds investment in New Zealand have lower m values than for the New Zealand stock market. That is, the returns for the risk taken are such that conservative private investors may well prefer direct investment into the stock market or short-term bank deposits given the recent performance history of the funds. It is also of interest to note that the coefficient of variation for both the equity and balanced fund portfolios are higher than for the stock market company and sector portfolios. Also the coefficient of variation for the fixed interest fund portfolio is higher than that of the New Zealand fixed interest portfolio.

Table 7.4.6

New Zealand Fixed Interest Optimal Portfolio

Nominal Returns

Period: January 1989 - October 1995 for Monthly Data

m= 0.20 Lambda = 0.58				
Fixed Interest Number	Fixed Interest Name	% of Portfolio	Expected Net Return per Month %	Standard Deviation of Expected Return per Month %
5	10 Year Government Stock	100	0.851	1.3716
Optimal Portfolio		100	0.851	1.3716
Coefficient of Variation				1.6118

The results from the fixed interest analysis show an m value of 0.2 with the entire investment dollar invested in 10 year government stock. The m value of 0.2 appears lower than might have been expected but it should be noted that New Zealand has seen negative yield curves over much of the period under consideration together with high and volatile short-term interest rates as the Reserve Bank targets the 90 day bank bill rate in its efforts to keep inflation in a 0 to 2% band. The fact that 10 year government stock is selected for the conservative portfolio means that the more stable longer term interest rates have more than compensated for the impact of an often negative yield curve.

Optimality for domestic fund managers and financial institutions can also be calculated by applying marginal cost values such as those given for 90 day bank bills in Table 7.3.3. In general, 90 day bank bills should be at a higher effective yield than the three month bank deposit rate. Large investors such as fund managers can lend into the market if appropriate and financial institutions can lend and borrow into this market for wholesale funds. The added overhead cost for a financial institution, such as a bank, to access the retail bank deposit rate would normally mean that the interest rate on three month bank deposits would need to be below the wholesale 90 day bank bill rate for these borrowings to be economically viable. Interestingly, however, there have been periods between January 1987 and October 1995 where the three month bank deposit rate has actually been above the 90 day bank bill rate as can be seen from Table 7.3.2. A possible explanation for this would again be the aggressive action taken by the Reserve Bank in targeting the 90 day bank bill rate in its price stability maintenance role.

For the financial institution though, there is the added consideration of maintaining a market acceptable level of "reserve type assets" as discussed in Chapter 5 Section 4. An increase in the percentage holding of reserve type assets for a financial institution, either by choice or through market necessity could be compensated for either by an increase in yields on reserve type assets or a shift to a more risky portfolio for that portion not held in reserve type assets. An example of this trade-off is given in Table 7.4.7 assuming a stable yield on reserve type assets. Note that this example is based on an equity investment strategy only which is considerably more restrictive than would normally not be the case for a financial institution.

Table 7.4.7

Optimal Stock Portfolios for Different Reserve Type Asset Holdings

Nominal Returns

Period: January 1989 - October 1995 for Fortnightly Data

Lambda = r = 0.2900		$c_\alpha = 0.24$		
Company No.	Company Name	Reserve Type Assets 0%	Reserve Type Assets 10%	Reserve Type Assets 20%
8	Colonial Motors	9.3964	8.3472	7.2875
11	Donaghys	19.746	17.7177	15.6843
27	Milburn	25.2252	22.7615	20.3036
30	New Zealand Refining	19.768	17.8655	15.9701
31	Owens Group	4.7525	4.2799	3.8075
32	PDL Holdings	8.4128	7.5965	6.7826
36	Southern Petroleum	4.7283	4.2405	3.7512
37	Steel & Tube	7.9708	7.1912	6.4132
		100	90	80
$c_\alpha x_\alpha$		0	0.024	0.048
$c\bar{x}$		1.318	1.1878	1.0578
$(xVx)^{\frac{1}{2}}$		2.1733	1.9594	1.7459
m		0.473	0.4706	0.4674
$\Gamma(m)^*$		0.29	0.29	0.29
$\lambda_{\bar{1}}$		0.29	0.2952	0.3022
$\lambda_{\bar{2}}$		0	0.0552	0.0622

* Note: $\Gamma(m) = c_\alpha x_\alpha + cx - m(xVx)^{\frac{1}{2}}$. For all other definitions see Chapter 5 Section 4.

As can be seen from the above table, shifts in the makeup of the risky portfolio given shifts in the requested level of reserve type assets are minor here. This is mainly due to the closeness in value between r and c_α. In an economy with enforced reserve requirements and a wider gap between r and, in this case, c_0, adjustments to the risky portfolio given changes in the reserve requirements would be more substantial.

7.5 Optimal Portfolios for Real Returns Data

As real returns data is quarterly data adjusted for inflation, only two real returns time periods were considered for the stock market company and sector data and interest rate data with only one time period being considered for the managed funds data. The "optimal" results shown here are for a similar time period as considered for the nominal data in the previous section, that is January 1989 to September 1995. In all cases the m values found were much higher than for the nominal results. In general the managed funds had the lowest m values though the fixed interest managed funds had a m value of 0.25 compared to 0.22 for the fixed interest market result. Coefficients of variation were also higher for the managed funds in general though again the fixed interest managed funds did better than the fixed interest market itself.

There appears to be two possible explanations for the higher m values in the case of real returns. First the removal of the inflationary impact from the data series might well have reduced overall variability. In all cases, except for the fixed interest market data, the coefficient of variation decreased for the real returns data. It is also possible that variability was reduced on account of the longer time frame between observations. Volatile periods within particular quarters would have influenced the results as was the case on a few occasions.

The real returns results are given in Tables 7.5.1 to 7.5.6.

Table 7.5.1

New Zealand Stock Market Optimal Stock Portfolio

Real Returns

Period: January 1989 - September 1995 for Quarterly Data

m= 0.50 Lambda = 0.21				
Company Number*	Company Name	% of Portfolio	Expected Net Return per Fortnight %	Standard Deviation of Expected Net Return per Fortnightly %
30	New Zealand Refining	38.78	10.7094	16.98
27	Milburn New Zealand	29.94	10.1593	17.3956
31	Owens Group	5.91	6.2494	14.938
37	Steel & Tube	6.59	10.1043	24.6781
35	Sanford	18.78	4.9904	14.8138
Optimal Portfolio		100	9.1456	11.8311
Coefficient of Variation				1.2936

* Note the ordering of the companies is that of their selection into the portfolio.

Table 7.5.2

New Zealand Stock Market Optimal Sector Portfolio

Real Returns

Period: January 1989 - September 1995 for Quarterly Data

m= 0.85 Lambda = 1.10				
Sector Number*	Sector Name	% of Portfolio	Expected Net Return per Month %	Standard Deviation of Expected Net Return per Month %
6	Energy & Fuel	65.79	11.1085	12.7179
3	Building	21.1	9.0278	15.6418
9	Food	13.11	3.8364	12.9623
Optimal Portfolio		100	9.7162	10.1578
Coefficient of Variation				1.0455

* Note the ordering of the sectors is that of their selection into the portfolio.

Table 7.5.3

New Zealand Fixed Interest Funds Optimal Portfolio

Real Returns

Period: January 1989 - September 1995 for Quarterly Data

m= 0.25 Lambda = 1.10				
Fixed Interest Fund Number	Fixed Interest Fund Name	% of Portfolio	Expected Net Return per Month %	Standard Deviation of Expected Net Return per Month %
9	Sovereign NZ Fixed Interest	100	1.5279	1.8775
Optimal Portfolio		100	1.5279	1.8775
Coefficient of Variation				1.2288

Table 7.5.4

New Zealand Equity Funds Optimal Portfolio

Real Returns

Period: January 1989 - September 1995 for Quarterly Data

m= 0.26 Lambda = 1.10				
Equity Fund Number	Equity Fund Name	% of Portfolio	Expected Net Return per Month %	Standard Deviation of Expected Return per Month %
3	Guardian Assurance Equity	100	2.6711	6.1974
Optimal Portfolio		100	2.6711	6.1974
Coefficient of Variation				2.3202

Table 7.5.5

New Zealand Balanced Funds Optimal Portfolio

Real Returns

Period: January 1989 - September 1995 for Quarterly Data

m= 0.24 Lambda = 1.10				
Balanced Fund Number	Balanced Fund Name	% of Portfolio	Expected Net Return per Month %	Standard Deviation of Expected Return per Month %
11	Guardian Assurance Balance	100	1.9779	3.5239
Optimal Portfolio		100	1.9779	3.5239
Coefficient of Variation				1.7816

Table 7.5.6

New Zealand Fixed Interest Optimal Portfolio

Real Returns

Period: January 1989 - September 1995 for Quarterly Data

m= 0.22 Lambda = 1.10				
Fixed Interest Number	Fixed Interest Name	% of Portfolio	Expected Net Return per Month %	Standard Deviation of Expected Return per Month %
5	10 Year Government Stock	100	1.7834	3.0187
Optimal Portfolio		100	1.7834	3.0187
Coefficient of Variation				1.6927

7.6 The Value and Stability of the Risk Profiles of the Data Sets Analysed

The stability of the m value is now considered for the nominal returns data with stability results given in Tables 7.6.1a and 7.6.1b.

Table 7.6.1a

Risk Profile Stability: Optimal m Values for each Time Period

Nominal Returns

Time Period	Marginal Cost Before Tax P.A.	Stock Market Company Data	Stock Market Sector Data	Fixed Interest Investment
Jan 87 - Oct 95	12.43	0.3 - 0.4	0.3 - 0.4	0.0 - 0.2
Jan 88 - Oct 95	11.23	0.4 - 0.5	0.4 - 0.5	0.0 - 0.2
Jan 89 - Oct 95	10.42	0.4 - 0.5	0.4 - 0.5	0.0 - 0.2
Jan 90 - Oct 95	9.68	0.3 - 0.4	0.5 - 0.6	0.0 - 0.2
Jan 91 - Oct 95	8.21	0.5 - 0.6	0.5 - 0.6	0.0 - 0.2
Jan 92 - Oct 95	7.52	0.4 - 0.5	0.5 - 0.6	0.0 - 0.2
Jan 93 - Oct 95	7.74	0.3 - 0.4	0.4 - 0.5	0.0 - 0.2
Jan 87 - Dec 91	14.64	0.3 - 0.4	0.3 - 0.4	0.2 - 0.4
Jan 88 - Dec 92	12.59	0.4 - 0.5	0.6 - 0.7	0.2 - 0.4
Jan 89 - Dec 93	11.05	0.6 - 0.7	0.6 - 0.7	0.4 - 0.6
Jan 90 - Dec 94	9.7	0.4 - 0.5	0.5 - 0.6	0.0 - 0.2
Jan 87 - Dec 89	15.7	0.3 - 0.4	0.2 - 0.3	0.2 - 0.4
Jan 88 - Dec 90	13.7	0.3 - 0.4	0.4 - 0.5	0.0 - 0.2
Jan 89 - Dec 91	12.64	0.3 - 0.4	0.6 - 0.7	0.2 - 0.4
Jan 90 - Dec 92	8.07	0.5 - 0.6	0.7 - 0.8	0.6 - 0.8
Jan 91 - Dec 93	8.14	0.9 - 1.0	1.2 - 1.3	0.6 - 0.8
Jan 92 - Dec 94	6.77	0.5 - 0.6	0.5 - 0.6	0.0 - 0.2

Table 7.6.1b

Risk Profile Stability: Optimal m Values for each Time Period

Nominal Returns

Time Period	Marginal Cost Before Tax P.A.	Fixed Interest Managed Funds	Equity Managed Funds	Balanced Managed Funds
Jan 89 - Dec 95	10.42	0.1 - 0.2	0.1 - 0.2	0.1 - 0.2
Jan 90 - Dec 95	9.68	0.1 - 0.2	0.1 - 0.2	0.1 - 0.2
Jan 91 - Dec 95	8.21	0.2 - 0.3	0.3 - 0.4	0.0 - 0.1
Jan 92 - Dec 95	7.52	0.1 - 0.2	0.2 - 0.3	0.1 - 0.2
Jan 93 - Dec 95	7.74	0.0 - 0.1	0.1 - 0.2	0.0 - 0.1
Jan 89 - Dec 93	11.05	0.2 - 0.3	0.2 - 0.3	0.1 - 0.2
Jan 90 - Dec 94	9.7	0.1 - 0.2	0.1 - 0.2	0.2 - 0.3
Jan 89 - Dec 91	12.64	0.3 -0.4	0.0 - 0.1	0.1 - 0.2
Jan 90 - Dec 92	8.07	0.5 - 0.6	0.1 - 0.2	0.6 - 0.7
Jan 91 - Dec 93	8.14	0.6 - 0.7	0.6 - 0.7	0.2 - 0.3
Jan 92 - Dec 94	6.77	0.1 - 0.2	0.2 - 0.3	0.1 - 0.2

The New Zealand share market was analysed through the use of two data sets, top forty company returns and sector returns. These data sets displayed some differences in relation to average m values, or risk profiles, with the company data giving an average m value of between 0.4 and 0.5 and the sector data giving an average m value of between 0.5 and 0.6. The main outlier for both data sets came in the January 1991 to December 1993 time frame with the company m value rising to between 0.9 and 1.0 and the sector m rising to between 1.2 and 1.3. Given the previous findings of an m value of 0.5 for equity markets it is interesting to note that sixteen of the seventeen observations for the company data shown in Table 7.6.1a fell between 0.3 and 0.7 with fourteen of the seventeen observations for the sector data falling within the same range.

When considering the time frames of five years and over all observations for both the company and sector data fell within this range.

Given that the average m value was just under 0.5 for the company data and just over 0.5 for the sector data, consideration needs to be given to the differences between these two data sets. For the company data any particular company had to be listed over the entire period of any particular period analysed whereas the sector data simply included all companies listed within that sector for all or any part of a particular period. The sector data therefore gave a more comprehensive analysis of the market overall. It should be noted, however, that the sector data used was gross index data after adjusting for tax on dividends. The dividends themselves effectively compound in a gross index unlike for the company data where dividends are simply part of returns on their ex date. This dividend compounding effect would help to explain the higher average m value for the sector data as compared to the company data. Overall it can be said that the evidence here strongly supports the view that an m value of 0.5 is appropriate for the New Zealand equity market and that this value is relatively stable, particularly for longer period data sets of five years or more. Shifts away from this value can be expected to be followed by shifts back to this value.

The fixed interest results were surprising as a value for m higher than 0.5 had been expected. This was the case for the period of January 1990 through to December 1993 but on average the m value was between 0.2 and 0.4, falling below 0.2 on many occasions. As has been mentioned before, the fact that the Reserve Bank has actively influenced the short-term wholesale interest rates in recent times, in particular the ninety day bank bill rate, in its efforts to keep inflation in a 0 - 2% band has probably influenced this result. As a result New Zealand has seen negative yield curves for a large part of the period under consideration. In fact the period between January 1991 through to October 1995 was the only substantial period when a positive yield curve was evident and even then the curve was almost flat on many occasions. Given this fact, the observed risk profile for the fixed interest market may not be typical, and further studies over a time period where the more normal positive yield curve dominates would be desirable. It is interesting to note that in 1995 to 1996 a number of fund managers have argued that fully hedged offshore bond portfolios are better to hold than New Zealand bond portfolios on account of the risky nature of the later.

Also surprising were the managed funds results. Apart from the periods January 1990 to December 1992 and January 1991 to December 1993 the m values were around 0.1 to 0.3 in almost all cases. The fixed interest managed funds had an m value average of just over 0.2 while the equity and balanced funds had a m value average of between 0.1 and 0.2. Overall the values were reasonably stable with twenty-three of the thirty-three observations falling between 0.1 and 0.3.

When comparing the fixed interest funds with the fixed interest investments the average m value is lower for the fixed interest funds. This would be consistent with fund managers not being able to add value coupled with the impact of fund management fees. The equity funds have an average m value well below the stock market company and sector results. Again the failure to add value coupled with fund management fees would be an explanation but it should also be remembered that capital gains on equity investments are taxable for managed funds in New Zealand but generally not for direct equity investors. This fact would have had some impact on these results. Balanced funds tend to be a combination of fixed interest and equity investments and again these funds showed low average values of m, much in line with the equity funds. A number of studies have shown that New Zealand managed funds do not add value[187], in fact they often take value away. This finding is supported by this analysis.

A final point worth considering is the m values for the New Zealand capital markets as a whole, and this is looked at in relation to the equity market. Table 7.6.2 gives the values for the periods under consideration, where available, for the entire New Zealand stock market (NZSE), the top forty companies (Top 40), New Zealand small companies, being all stocks not included in the top forty (SCI), and for the top ten companies (Top 10). All indices have been adjusted for tax in the same manner as for the New Zealand Stock Exchange sector data. A full comparison can only be made for time frames from 1992 on given the length of time some of the indices have been in existence, but overall the m values for the market indices are much lower than for the optimal portfolios as would be expected.

187 In particular see Boustridge, P. and Young, M., An Appraisal of Managed Funds Performance for New Zealand Registered Funds Using Sharpe's Style Analysis. Massey University Finance Department Working Paper Series, 96/2, 1996.

Table 7.6.2

Risk Profiles for the New Zealand Stock Market

Monthly Nominal Indices Data

Time Period	NZSE Indices m values	Top 40 Indices m values	SCI Indices m values	Top 10 Indices m values
Jan 87 - Oct 95	-0.0829			
Jan 88 - Oct 95	-0.003			
Jan 89 - Oct 95	0.0317			0.0297
Jan 90 - Oct 95	0.0186			0.0149
Jan 91 - Oct 95	0.2174			0.1879
Jan 92 - Oct 95	0.186	0.1657	0.2597	0.1683
Jan 93 - Oct 95	0.2149	0.202	0.063	0.2317
Jan 87 - Dec 91	-0.2115			
Jan 88 - Dec 92	-0.0901			
Jan 89 - Dec 93	0.0463			0.031
Jan 90 - Dec 94	-0.0106			-0.0177
Jan 87 - Dec 89	-0.1953			
Jan 88 - Dec 90	-0.227			
Jan 89 - Dec 91	-0.1085			-0.1022
Jan 90 - Dec 92	-0.1087			-0.1377
Jan 91 - Dec 93	0.346			0.2755
Jan 92 - Dec 94	0.1675	0.1406	0.3028	0.1422
Mean - All periods	0.1123	0.1694	0.2085	0.0748
Mean - Jan 92 - Oct 95 periods	0.1895	0.1694	0.2085	0.1807

A relevant question to ask is whether investors are driven by mean-variance efficient risk and return or overall market risk and return. The difference between these two approaches has been addressed in a theoretical context in this thesis but the gap between the m values for each of the groups is certainly of interest. The most relevant comparisons are between the Top 40 results and the stock market company results and between the NZSE results and the stock market sector and equity managed funds results.

Graph 7.4.1 shows the Moeseke efficient frontier and by definition the market as a whole is always going to sit somewhere below this efficient frontier. Investors and fund managers who are trying to add value, that is trying to beat the market, are actively attemping to push out as far as possible to the future expected efficient frontier given their particular risk appetite. Note that the position of the market may also be influenced by investments which do not conform to the standard risk return tradeoff. While active investment or active funds management has its critics, the very process of investors aiming for a mean-variance efficient portfolio, in much the same way as greyhounds chase the metal rabbit on the race track, both aids market efficiency and makes the mean-variance efficient frontier an appropriate driver of investment decision making. The fact that investors and fund managers may fail to add value, that is fail to make any progress away from the market and towards the mean-variance efficient frontier, does not detract from the importance of the efficient frontier's existence to the portfolio selection process.

The significance and implications of the results reported in this chapter are discussed in Chapter 8.

CHAPTER EIGHT

CONCLUSION

The major aim of this thesis has been to examine the usefulness of the Moeseke model in the process of portfolio analysis and construction. A second aim has been to discuss the place of the Moeseke model within the context of modern portfolio theory. This first aim has been addressed with the aid of an empirical study of New Zealand's capital markets using the Moeseke model. This model is a positive model with its main focus being that an optimal investment decision under uncertainty is determined where the marginal cost of the investment dollar equals its marginal return after allowing for an appropriate level of risk. The point at which aggregate marginal cost and marginal return equates will determine the risk profile of the particular market under consideration.

Moeseke proposed that the marginal return to the investment dollar be adjusted for risk by adjusting expected returns by an appropriate weighting of the standard deviation of returns. The size of the weighting required, in order for aggregate marginal cost to equal marginal return determines the risk profile of any particular market. The marginal cost of the investment dollar can be viewed as being the return to a substantially riskless investment such as a government bond or bank deposit for individual investors, and the cost of raising deposits in the marketplace for financial institutions. It has been proposed in this thesis that marginal cost is a stochastic variable and requires a risk adjustment. This is true of marginal cost for all investors but particularly for offshore investors investing into foreign markets.

The Moeseke model is not an asset pricing model as such but the model does determine a "market" portfolio and under certain strict assumptions the "market" portfolio of the Moeseke model will equate to the "market" portfolio of the CAPM. It is well known that, in equilibrium, the market asset becomes the pricing asset within the CAPM. It is important to note, however, that the Moeseke model does not require or expect all assets in any particular market to be priced in the same manner. As with the Markowitz model, investors in any particular market will have a range of risk preferences and each asset will be priced where demand and supply equate at any point in time. The Moeseke model can also be applied in risk taking situations so is clearly a more robust model than both the positive CAPM and the normative Markowitz model. Further, the Moeseke model expects investors in any particular market to have differing marginal costs of capital and therefore differing optimal portfolios. The optimal or "market" portfolio for any particular market will be that portfolio determined where the weighted average of all marginal costs equal the portfolio's marginal return. Note also that reserve type asset holdings, compulsory or voluntary, will impact on the optimal portfolios of financial institutions.

Within the Moeseke model optimal solutions can be determined for investors given estimates of return and risk. In line with much of the work in portfolio determination, means, variances and covariances are applied in this thesis with justification for this approach having been given. In the absence of good expectations measures, historical data has been used. The justification for this approach is based on the premise that historical risk and return for any particular market are the best estimates of future risk and return and that historical values are known and understood by investors in general and therefore become objective estimates in the sense proposed by Savage. It is important to keep firmly in mind that the particular assets that make up a historically determined optimal portfolio is not the important consideration. What is important is the risk and return of that portfolio and the risk profile of the market that the optimal portfolio represents. This is particularly true with equities where individual companies change in their structure and focus over time. It might then be argued that the important historical information is the risk and return of the market as a whole calculated from some market index, total or otherwise. This would assume, however, that all investors have identical objectives within that market and all assets represented in that market are efficiently priced under the same criteria. It could certainly be argued that optimal portfolio selection based on historical risk and return will give an upward bias to return estimates and a downward bias to risk estimates but when the focus is on maximal caution for the investor this approach is considered preferable to a total market approach where maximal caution is not necessarily a high priority. Further, active investment or active funds management is fundamentally the process of attempting to beat the market by aiming to invest on the mean-variance efficient frontier in most cases. The mean-variance efficient frontier, then, is the appropriate driver of investment decision making as stated in chapter 7.

The empirical analysis of New Zealand's capital markets reported in this thesis has shown, in particular, that the stock market has a long-run risk profile much in line with that found in other stock markets, in particular the United States and Canada. A value for the risk parameter, m, of 0.5 appears to be appropriate for markets of this type. It is not surprising that the long-run risk profiles of stock markets are similar across open market economies given the competitive environment faced by the companies represented in these stock markets. Not surprisingly, evidence was found of short-run shifts away from this long-run market risk profile and further study focussing on the causes of short-run shifts away from an m value of 0.5 and the market mechanism for returning the risk profile to its long-run equilibrium would certainly be of interest, both from the academic and professional investor perspective. Periods where the risk profile of a particular stock market alters, thereby attracting investors that normally would not invest in such markets or discouraging investors who would normally participate would not logically be long-run sustainable and could therefore give useful timing signals to the marketplace.

The same analysis has been applied to the fixed interest market and to managed funds but it should be noted that the fixed interest market results obtained in this thesis may well be peculiar to the period. One would expect the long-run risk profile of the fixed interest market to show more risk aversion than the stock market, but this was not the situation found here. The results obtained for the managed funds industry in New Zealand were also of interest with the appropriate risk profiles for the different classes of funds showing less risk aversion than for the stock market. Risk profile m values for the three classes of managed funds considered were similar to those of the fixed interest market and for the entire stock market as calculated from the indices data. The failure of fund managers to add value or reduce risk would explain these results and while funds, large funds in particular, might not be expected to add value or reduce risk it is important that this fact is understood by investors purchasing these funds.

A fundamental argument of this thesis is that the most valuable information we can have for any market where risky assets are traded is that market's long-run and current risk profiles. The long-run risk profile can be estimated with relative ease as has been shown in this thesis. Estimation of the current risk profile is clearly more difficult and largely outside the scope of this thesis but worthy of further study.

Another important area of further study would be to place the Moeseke model precisely within the context of modern portfolio theory. This would require proofs of the exact relationship between this model and the CAPM within a clearly specified conceptual framework.

This thesis has examined the Moeseke model, the theory supporting it and its extensions and has also given further extensions to the model. A reasonably extensive application has also been undertaken. Optimal and market portfolios have been discussed within the context of this model and the importance of market risk profiles and their stability has been emphasised. The model has a clear and important role to play in modern portfolio theory, particularly to individual, institutional and offshore investors as they continue to develop their understanding of the risk-return trade-off within financial markets. The model also has an important role to play for the monetary policy makers as they consider the manner in which they influence marginal costs and ultimately the investment decisions of investors.

APPENDIX ONE

A Chronology of the Deregulation Process of the New Zealand Economy 1984 - 1994

November 8, 1984	Presentation of the New Zealand Budget in which a wide range of subsidies and incentives, particularly to the rural sector were either abolished or given a timetable for being phased out.
December 21, 1984	Exchange control regulations were relaxed.
January 25, 1985	The New Zealand Futures Exchange began trading, its first contract being a United States dollar contract.
February 7, 1985	The compulsory ratios system which required financial institutions to hold a portion of their total assets in government and public securities was abolished.
March 4, 1985	The New Zealand dollar was floated.
March 6, 1985	Limits on foreign ownership in New Zealand financial institutions, advertising agencies and fishing processors were abolished.

May 20, 1985	The New Zealand Futures Exchange began trading a 90 day bill contract.
July 18, 1985	The government announced that requests from foreign investors for up to 100% ownership in all areas of the economy except rural land and air services would be considered.
January 27, 1986	The Stock Exchange began testing its computerisation programme which became fully operational for confirmed trade recording in June 1986. The system was later extended to include broker settlements, screen trading and share registration.
March 24, 1986	The Reserve Bank began targeting the levels of daily settlement cash balances in its implementation of monetary policy.
April 3, 1986	The Reserve Bank made explicit the definition of primary liquidity, being cash plus government stock with 30 days or less until maturity. Later, on September 1 1988, Reserve Bank Bills were introduced as the stock instrument for settlement institution as government stock proved too lumpy in terms of the quantity

	qualifying as primary liquidity. The time frame for discounting was also reduced to 28 days or less till maturity.
October 1, 1986	A Goods and Services Tax was introduced being a flat 10% consumption tax on goods and services with very few exceptions. A reduction in income tax rates occurred at the same time. This tax increased to 12.5% on July 1, 1989.
December 24, 1986	The Reserve Bank Amendment Act was passed into law, effective from 1 April 1987. This Act gave the Reserve Bank the right to issue banking licences to any qualifying party, being parties of sound financial condition and with expertise in banking. This Act lead to most major financial intermediaries in New Zealand moving to bank status and also attracted some further offshore participants into the market place.
December 17, 1987	The government announced further plans for economic reform including an increase in the Goods and Services Tax, a programme of asset sales to repay $14 billion or one-third of the national debt by 1992, a single personal income tax rate, a full dividend imputation scheme, the

abolition of tax incentives on savings schemes. The package lead to a rift between the then Finance Minister and the Prime Minister and while most of the package proceeded the single personal income tax rate did not.

October 1, 1988	The top tax rate was reduced from 45% to 33%.
December 15, 1989	The Reserve Bank of New Zealand Act was passed into law giving the Reserve Bank sole responsibility for the implementation of monetary policy in New Zealand with the sole aim of maintaining stable prices. Stable prices are currently deemed to be an inflation rate in a range of 0-2%. It should be noted, however, that since 1985 the Reserve Bank had been operating under an understanding with the government that the Reserve Bank would implement monetary policy with stable prices as the top priority.
May 15, 1992	The Employment Contracts Act took effect which deregulated the labour market by doing away with compulsory unionism and allowing personal contracts of employment between employees and employers to be negotiated.

June 27, 1994	The Fiscal Responsibility Act was passed into law which set down guidelines for responsible fiscal management. In particular the intention of the act is for the government to avoid budget deficits and move to being debt free. Success in this area has been substantial with the strong probability that offshore debt will be fully repaid by the end of 1996.

This chronology has been sourced from Reserve Bank Bulletins 1985 - 1995.

APPENDIX TWO

Interest Rate Data

Date	6 Month Deposit	1 Year Deposit	3 Year Deposit	5 Year Government Stock	10 Year Government Stock
Jan 87	16.00	14.50	14.00	17.29	
Feb 87	16.00	14.50	14.00	17.76	
Mar 87	16.75	15.25	15.25	18.22	
Apr 87	16.32	16.75	15.00	17.47	
May 87	17.15	17.19	14.12	16.67	
Jun 87	17.87	17.46	16.90	16.11	
Jul 87	18.06	17.60	16.22	16.34	
Aug 87	17.77	16.62	14.66	16.38	
Sep 87	17.62	17.76	14.53	16.34	
Oct 87	16.73	16.20	15.05	16.40	
Nov 87	16.58	16.08	14.59	15.92	
Dec 87	15.79	15.16	14.59	15.32	
Jan 88	16.13	15.63	14.26	15.06	14.13
Feb 88	14.13	13.88	13.57	14.43	13.66
Mar 88	13.88	13.75	13.22	13.42	13.05
Apr 88	14.88	14.75	13.14	13.18	12.95
May 88	14.38	14.25	13.04	13.11	12.78
June 88	14.63	13.50	12.58	13.09	12.81
Jul 88	14.38	14.13	12.47	12.91	12.69
Aug 88	13.50	13.63	12.34	13.03	12.74
Sep 88	13.75	13.25	12.66	12.95	12.76
Oct 88	13.06	13.13	12.00	12.96	12.82
Nov 88	13.63	13.63	12.00	13.33	13.20
Dec 88	13.38	13.88	13.51	13.98	13.80
Jan 89	12.13	12.38	12.00	13.54	13.38
Feb 89	12.63	13.13	12.00	13.11	13.10
Mar 89	12.25	12.38	11.50	13.19	13.23

Interest Rate Data Cont....

Date	6 Month Deposit	1 Year Deposit	3 Year Deposit	5 Year Government Stock	10 Year Government Stock
Apr 89	12.63	12.75	11.50	13.17	13.30
May 89	12.50	12.63	11.50	13.11	13.22
Jun 89	12.63	12.88	11.50	13.11	13.10
Jul 89	12.50	12.56	12.50	13.02	12.95
Aug 89	12.25	12.38	11.90	12.13	12.23
Sep 89	12.75	12.75	11.33	12.08	12.23
Oct 89	13.13	13.25	11.48	12.24	12.46
Nov 89	13.25	13.31	10.50	12.23	12.31
Dec 89	12.88	12.88	11.50	12.40	12.44
Jan 90	13.13	13.25	10.25	12.19	12.26
Feb 90	12.94	13.13	10.24	12.06	12.10
Mar 90	12.88	13.06	11.60	12.09	12.21
Apr 90	12.88	13.06	11.26	12.26	12.38
May 90	13.06	13.25	10.34	12.31	12.34
Jun 90	13.06	13.13	10.18	12.31	12.16
Jul 90	13.25	13.38	10.31	12.27	12.08
Aug 90	14.13	14.13	10.17	12.88	12.71
Sep 90	13.70	13.70	10.90	12.88	12.87
Oct 90	13.60	13.55	10.90	13.01	12.92
Nov 90	13.20	13.00	11.50	12.95	12.96
Dec 90	12.10	12.00	11.25	12.35	12.35
Jan 91	11.80	11.70	11.00	12.16	12.14
Feb 91	11.60	11.55	11.00	11.53	11.46
Mar 91	11.55	11.45	10.75	11.39	11.29
Apr 91	10.10	10.10	10.40	10.88	10.79
May 91	9.15	9.25	9.63	9.96	9.99
Jun 91	9.20	9.30	9.40	9.64	9.80
Jul 91	9.00	9.00	9.50	9.64	9.76
Aug 91	8.80	8.85	9.25	9.74	9.93
Sep 91	7.45	7.50	9.25	9.25	9.44
Oct 91	7.60	7.60	8.35	8.63	8.89
Nov 91	7.80	7.90	8.35	8.58	8.86

Interest Rate Data Cont....

Date	6 Month Deposit	1 Year Deposit	3 Year Deposit	5 Year Government Stock	10 Year Government Stock
Dec 91	7.60	7.50	8.02	8.55	8.97
Jan 92	7.50	7.40	7.68	8.57	8.98
Feb 92	7.20	7.30	7.68	8.84	9.32
Mar 92	7.10	7.30	7.50	8.61	9.06
Apr 92	6.95	7.00	7.40	8.43	8.96
May 92	6.50	6.60	7.30	8.27	8.78
Jun 92	6.30	6.25	7.30	8.15	8.61
Jul 92	5.75	5.70	6.75	7.37	7.97
Aug 92	5.90	5.90	6.75	7.15	7.87
Sep 92	6.15	6.25	6.60	7.31	7.93
Oct 92	5.95	5.95	6.60	7.18	7.83
Nov 92	5.95	5.95	6.50	7.07	7.69
Dec 92	7.05	7.05	6.50	7.45	7.82
Jan 92	7.05	6.90	6.55	7.63	7.86
Feb 93	6.90	6.80	6.55	7.53	7.79
Mar 93	6.90	6.90	6.75	7.25	7.45
Apr 93	6.85	6.70	6.90	7.22	7.32
May 93	6.40	6.45	6.60	7.05	7.27
Jun 93	6.25	6.40	6.90	7.13	7.33
Jul 93	5.55	6.65	6.75	6.68	6.94
Aug 93	5.45	5.55	6.63	6.29	6.55
Sep 93	5.45	5.70	6.50	5.94	6.27
Oct 93	5.35	5.35	6.00	5.86	6.12
Nov 93	5.75	5.70	6.00	5.93	6.22
Dec 93	5.40	5.50	6.15	5.77	6.08
Jan 94	5.20	5.25	6.00	5.31	5.69
Feb 94	5.10	5.10	5.75	5.40	5.81
Mar 94	6.00	5.80	5.75	5.99	6.29
Apr 94	6.00	5.95	5.83	6.87	7.01
May 94	5.60	6.05	5.90	7.00	7.19
Jun 94	6.25	6.60	6.50	7.27	7.48
Jul 94	6.65	6.85	7.00	8.02	8.09

Interest Rate Data Cont....

Date	6 Month Deposit	1 Year Deposit	3 Year Deposit	5 Year Government Stock	10 Year Government Stock
Aug 94	6.80	7.10	7.75	8.11	8.20
Sep 94	7.40	7.75	8.00	8.92	9.02
Oct 94	8.00	8.15	8.50	8.93	9.00
Nov 94	8.65	8.90	8.75	9.05	9.10
Dec 94	9.15	9.40	8.75	8.86	8.73
Jan 95	9.15	9.25	8.75	8.83	8.59
Feb 95	9.20	9.10	8.75	8.80	8.58
Mar 95	9.05	8.85	8.75	8.52	8.38
Apr 95	8.70	8.55	8.50	8.01	7.87
May 95	8.75	8.55	8.25	7.65	7.44
Jun 95	8.80	8.55	7.75	7.70	7.48
Jul 95	8.50	8.10	7.75	7.73	7.61
Aug 95	8.70	8.25	7.75	7.98	7.94
Sep 95	8.70	8.30	7.50	8.00	7.78
Oct 95	8.20	7.80	7.50	7.42	7.31

APPENDIX THREE

Exchange Rate Data
To New Zealand Dollars

Date	Pound Sterling	U.S. Dollar	Australian Dollar
Jan 87	0.3521	0.5415	0.8194
Feb 87	0.3625	0.5588	0.8294
Mar 87	0.3520	0.5637	0.8116
Apr 87	0.3489	0.5797	0.8253
May 87	0.3527	0.5749	0.8063
Jun 87	0.3640	0.5872	0.8158
Jul 87	0.3534	0.5636	0.8100
Aug 87	0.3732	0.6085	0.8500
Sep 87	0.3938	0.6466	0.8817
Oct 87	0.3430	0.5887	0.8832
Nov 87	0.3523	0.6318	0.9098
Dec 87	0.3514	0.6631	0.9136
Jan 88	0.3759	0.6700	0.9374
Feb 88	0.3759	0.6644	0.9246
Mar 88	0.3475	0.6572	0.8827
Apr 88	0.3589	0.6728	0.8864
May 88	0.3748	0.6980	0.8750
June 88	0.3935	0.6738	0.8505
Jul 88	0.3871	0.6669	0.8308
Aug 88	0.3782	0.6425	0.7833
Sep 88	0.3654	0.6152	0.7853
Oct 88	0.3547	0.6269	0.7637
Nov 88	0.3554	0.6530	0.7510
Dec 88	0.3512	0.6290	0.7359
Jan 89	0.3444	0.6100	0.6948
Feb 89	0.3573	0.6298	0.7562
Mar 89	0.3644	0.6108	0.7511
Apr 89	0.3632	0.6143	0.7712
May 89	0.3777	0.5987	0.7862

Exchange Rate Data Cont....

Date	Pound Sterling	U.S. Dollar	Australian Dollar
Jun 89	0.3692	0.5710	0.7618
Jul 89	0.3531	0.5857	0.7730
Aug 89	0.3785	0.5915	0.7784
Sep 89	0.3681	0.5933	0.7619
Oct 89	0.3684	0.5821	0.7608
Nov 89	0.3762	0.5911	0.7536
Dec 89	0.3713	0.5944	0.7520
Jan 90	0.3596	0.5961	0.7830
Feb 90	0.3455	0.5910	0.7724
Mar 90	0.3542	0.5794	0.7689
Apr 90	0.3538	0.5779	0.7634
May 90	0.3437	0.5765	0.7496
Jun 90	0.3380	0.5876	0.7493
Jul 90	0.3234	0.5867	0.7507
Aug 90	0.3213	0.6163	0.7550
Sep 90	0.3296	0.6195	0.7449
Oct 90	0.3113	0.6077	0.7742
Nov 90	0.3106	0.6017	0.7902
Dec 90	0.3102	0.5853	0.7621
Jan 91	0.3054	0.6008	0.7670
Feb 91	0.3136	0.5992	0.7643
Mar 91	0.3378	0.5873	0.7561
Apr 91	0.3467	0.5865	0.7529
May 91	0.3382	0.5796	0.7698
Jun 91	0.3536	0.5774	0.7521
Jul 91	0.3393	0.5670	0.7352
Aug 91	0.3396	0.5720	0.7341
Sep 91	0.3254	0.5626	0.7101
Oct 91	0.3197	0.5561	0.7148
Nov 91	0.3169	0.5603	0.7150
Dec 91	0.2904	0.5476	0.7159
Jan 92	0.3003	0.5348	0.7231
Feb 92	0.3104	0.5473	0.7264
Mar 92	0.3189	0.5496	0.7203

Exchange Rate Data Cont....

Date	Pound Sterling	U.S. Dollar	Australian Dollar
Apr 92	0.3038	0.5396	0.7102
May 92	0.2977	0.5377	0.7076
Jun 92	0.2883	0.5456	0.7235
Jul 92	0.2853	0.5479	0.7333
Aug 92	0.2743	0.5430	0.7555
Sep 92	0.3152	0.5384	0.7429
Oct 92	0.3353	0.5271	0.7612
Nov 92	0.3384	0.5177	0.7489
Dec 92	0.3392	0.5136	0.7473
Jan 92	0.3435	0.5211	0.7629
Feb 93	0.3653	0.5225	0.7536
Mar 93	0.3622	0.5340	0.7530
Apr 93	0.3463	0.5451	0.7651
May 93	0.3524	0.5488	0.7912
Jun 93	0.3662	0.5379	0.8019
Jul 93	0.3687	0.5459	0.8154
Aug 93	0.3761	0.5688	0.8360
Sep 93	0.3657	0.5472	0.8545
Oct 93	0.3755	0.5585	0.8303
Nov 93	0.3691	0.5492	0.8241
Dec 93	0.3777	0.5578	0.8255
Jan 94	0.3810	0.5744	0.8015
Feb 94	0.3914	0.5814	0.7972
Mar 94	0.3786	0.5622	0.8018
Apr 94	0.3806	0.5759	0.8060
May 94	0.3895	0.5878	0.8027
Jun 94	0.3857	0.5953	0.8147
Jul 94	0.3919	0.5983	0.8150
Aug 94	0.3884	0.6046	0.8115
Sep 94	0.3817	0.6029	0.8151
Oct 94	0.3752	0.6139	0.8266
Nov 94	0.3973	0.6239	0.8169
Dec 94	0.4120	0.6425	0.8271
Jan 95	0.4030	0.6407	0.8383

Exchange Rate Data Cont....

Date	Pound Sterling	U.S. Dollar	Australian Dollar
Feb 95	0.3977	0.6343	0.8588
Mar 95	0.4069	0.6525	0.8919
Apr 95	0.4166	0.6735	0.9276
May 95	0.4140	0.6651	0.9236
Jun 95	0.4200	0.6716	0.9333
Jul 95	0.4197	0.6702	0.9109
Aug 95	0.4195	0.6504	0.8632
Sep 95	0.4139	0.6536	0.8695
Oct 95	0.4174	0.6621	0.8696

APPENDIX FOUR

Inflation Rate Data

Date	Index
Dec 86	871
Mar 87	892
Jun 87	921
Sep 87	935
Dec 87	955
Mar 88	972
Jun 88	979
Sep 88	988
Dec 88	1,000
Mar 89	1,011
Jun 89	1,023
Sep 89	1,059
Dec 89	1,072
Mar 90	1,082
Jun 90	1,101
Sep 90	1,112
Dec 90	1,124
Mar 91	1,131
Jun 91	1,132
Sep 91	1,136
Dec 91	1,135
Mar 92	1,140
Jun 92	1,143
Sep 92	1,147
Dec 92	1,150
Mar 93	1,151
Jun 93	1,158
Sep 93	1,164
Dec 93	1,166

Inflation Rate Data Cont....

Date	Index
Mar 94	1,166
Jun 94	1,171
Sep 94	1,185
Dec 94	1,199
Mar 95	1,213
Jun 95	1,225
Sep 95	1,227

APPENDIX FIVE

Normality Test Results

Selected Companies

Nominal Data

		Mean	Standard Deviation	Skewness	Kurtosis	Normality χ^2	
1	Air New Zealand	0.005788	0.058807	0.140049	0.094232	0.85805	
2	Bank of New Zealand	-0.000314	0.085185	-0.407469	1.377554	10.948	**
3	BNZ Finance	0.000453	0.070916	-0.26895	4.127198	81.289	**
4	Brierley Investments	-0.00066	0.065201	-0.099901	2.518464	42.085	**
5	Carter Holt Harvey	0.003775	0.070076	0.333255	4.436554	87.364	**
6	Cavalier Corporation	0.008278	0.078675	0.854196	3.445142	36.323	**
7	Ceramco Corporation	-0.001443	0.074523	0.055456	2.268594	36.319	**
8	Colonial Motors	0.003328	0.058568	0.485879	5.650109	112.93	**
9	Corporate Investments	-0.00153	0.110665	0.286102	1.278939	13.97	**
10	DB Group	0.002245	0.067834	0.735961	4.715018	68.224	**
11	Donaghys	0.008116	0.050521	0.199239	1.403769	16.761	**
12	Ernest Adams	0.005885	0.071933	2.258891	20.213365	150.46	**
13	Elders Resources NZFP	-0.006644	0.065131	-0.667141	1.093706	7.7891	*
14	Enerco New Zealand	0.015928	0.046609	0.156221	1.014295	6.0563	*
15	Fay Richwhite	-0.002216	0.122359	3.751891	33.432219	157.53	**
16	Fernz Corporation	0.009416	0.078472	-0.912771	10.550045	206.13	**
17	Fisher & Paykel	0.003624	0.062123	0.675654	3.976128	55.491	**
18	Fletcher Challenge	0.003641	0.061238	0.141304	2.376835	38.188	**
19	Goodman Fielder	-0.002772	0.061037	0.456625	2.508377	33.824	**
20	Guinness Peat Group	0.008268	0.061356	0.553809	0.621431	5.3368	
21	Hallenstein Glasson	0.01174	0.09771	0.874364	4.503923	53.574	**
22	Independent Newspapers	0.004324	0.057325	0.806339	5.178528	73.305	**
23	Jarden Corporation	-0.01058	0.10597	0.830853	5.3439	49.496	**
24	Lion Nathan	0.00023	0.061492	-1.891258	19.346919	219.49	**
25	Macraes Mining	0.008682	0.08196	0.620519	0.803145	9.0045	*
26	Mair Astley	0.001435	0.103608	0.803824	2.058958	21.954	**

Normality Test Results, Nominal Data Cont....

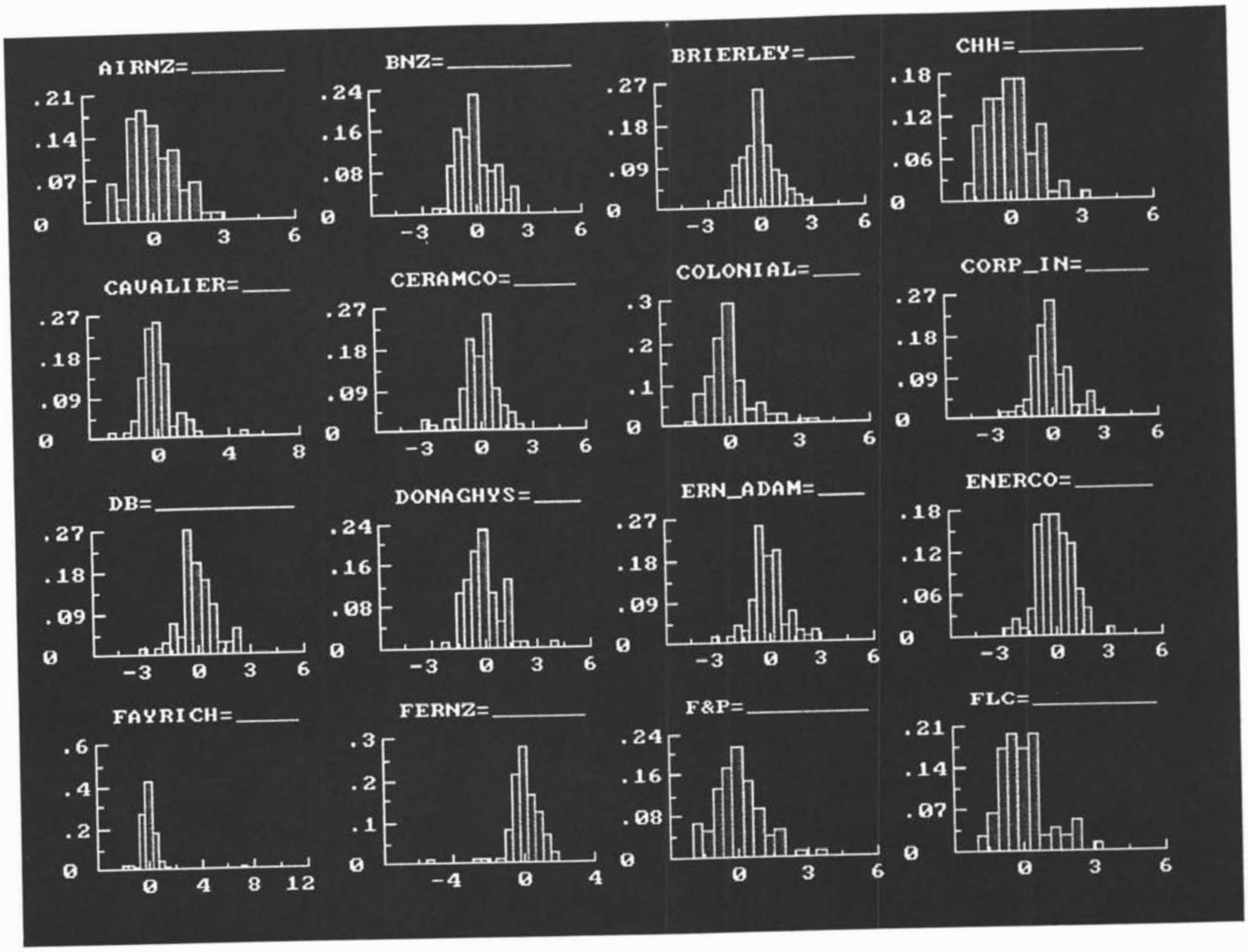
		Mean	Standard Deviation	Skewness	Kurtosis	Normality χ^2	
27	Milburn New Zealand	0.014285	0.058668	1.710448	9.696681	71.795	**
28	Natural Gas Corporation	0.019401	0.069662	-0.253252	0.085507	1.1548	
29	New Zealand Oil & Gas	0.003858	0.108367	2.140606	10.326479	88.961	**
30	New Zealand Refining	0.019591	0.074985	0.240833	4.926193	105.39	**
31	Owens Group	0.005486	0.084643	1.366814	9.864212	111.64	**
32	PDL Holdings	0.009591	0.084675	0.677425	4.031392	56.619	**
33	Progressive Enterprises	-0.002132	0.04568	0.380348	0.978826	5.6159	
34	Salmond Smith Biolab	0.004177	0.090724	0.403327	2.898503	44.257	**
35	Sanford	0.006725	0.065023	0.658184	3.17347	39.586	**
36	Southern Petroleum	0.011529	0.146535	3.552613	24.263441	252.15	**
37	Steel & Tube	0.01195	0.088067	1.941053	13.493009	97.076	**
38	Telecom Corporation	0.010855	0.042885	0.383964	1.047259	6.6124	*
39	Trans Tasman Properties	-0.005271	0.163254	1.461814	8.039022	68.372	**
40	Whitcoulls Group	0.015324	0.093534	0.811627	1.501447	10.409	**
41	Wilson Neill	-0.010476	0.164439	0.933622	9.134155	161.6	**
42	Wilson & Horton	0.00489	0.054074	-0.394605	4.83468	95.503	**

* Significantly different from normal 5% level.

** Significantly different from normal 1% level.

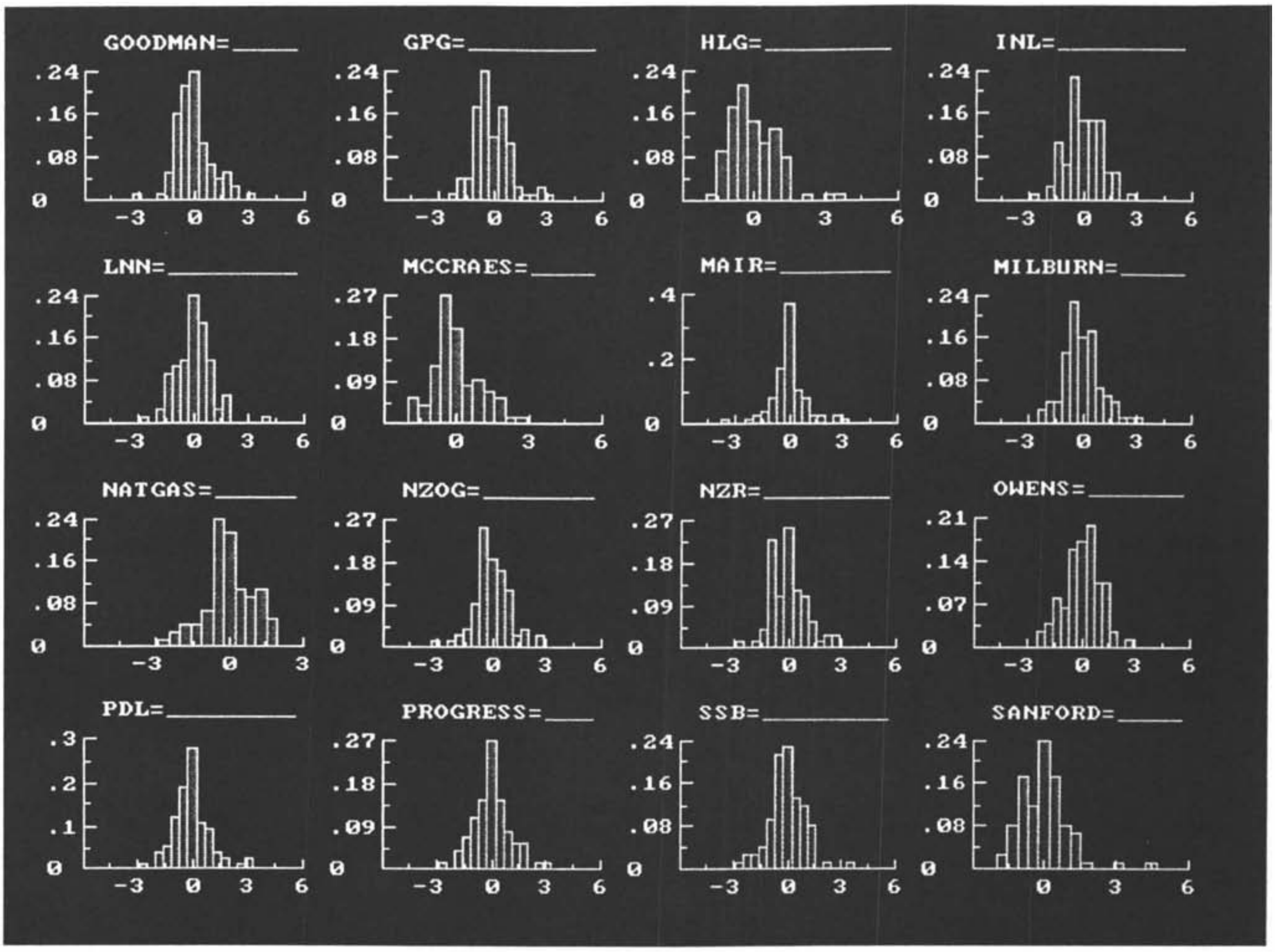
Distribution of Returns - Selected Companies

Nominal Data



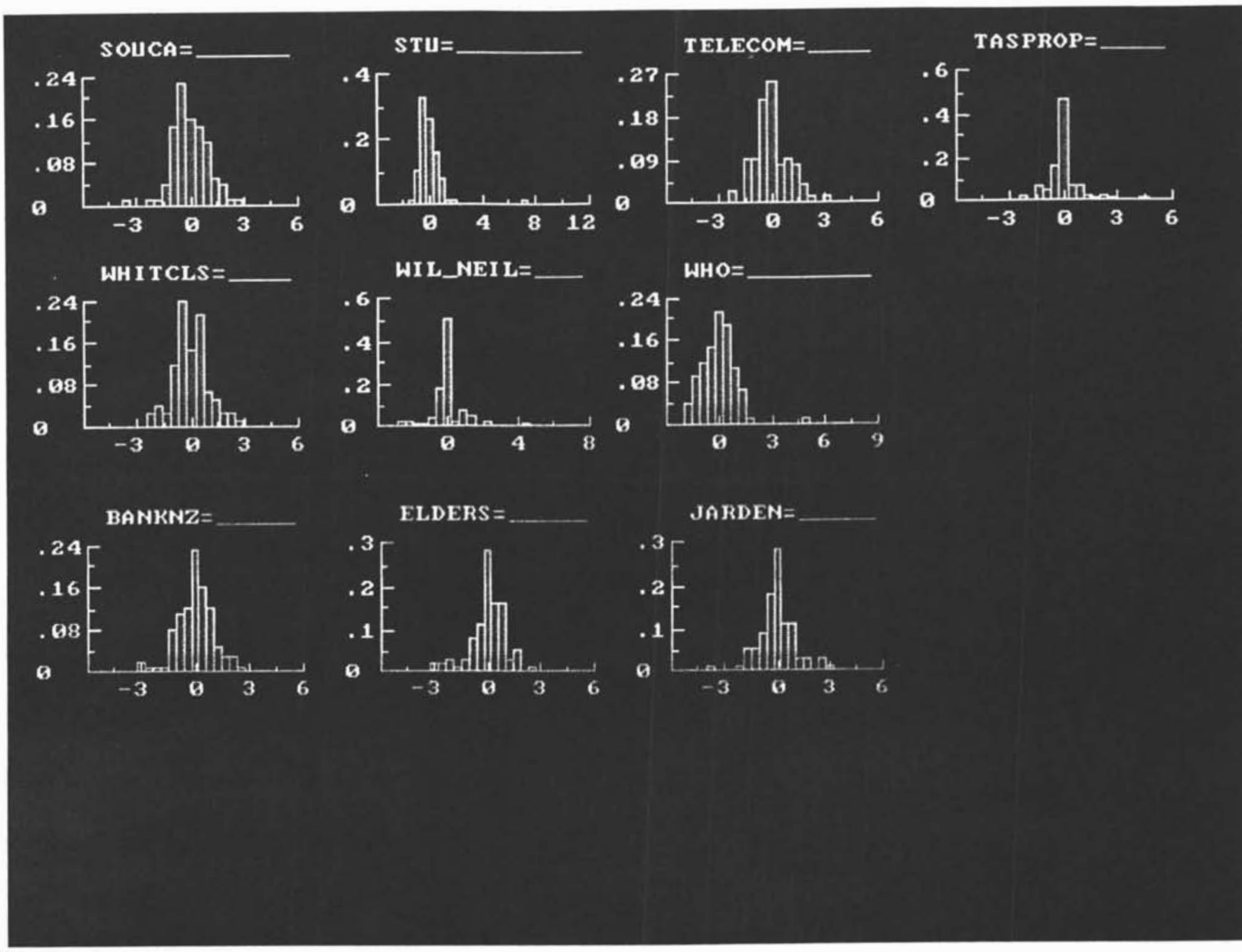
Distribution of Returns - Selected Companies

Nominal Data



Distribution of Returns - Selected Companies

Nominal Data



Normality Test Results

Selected Companies

Real Data

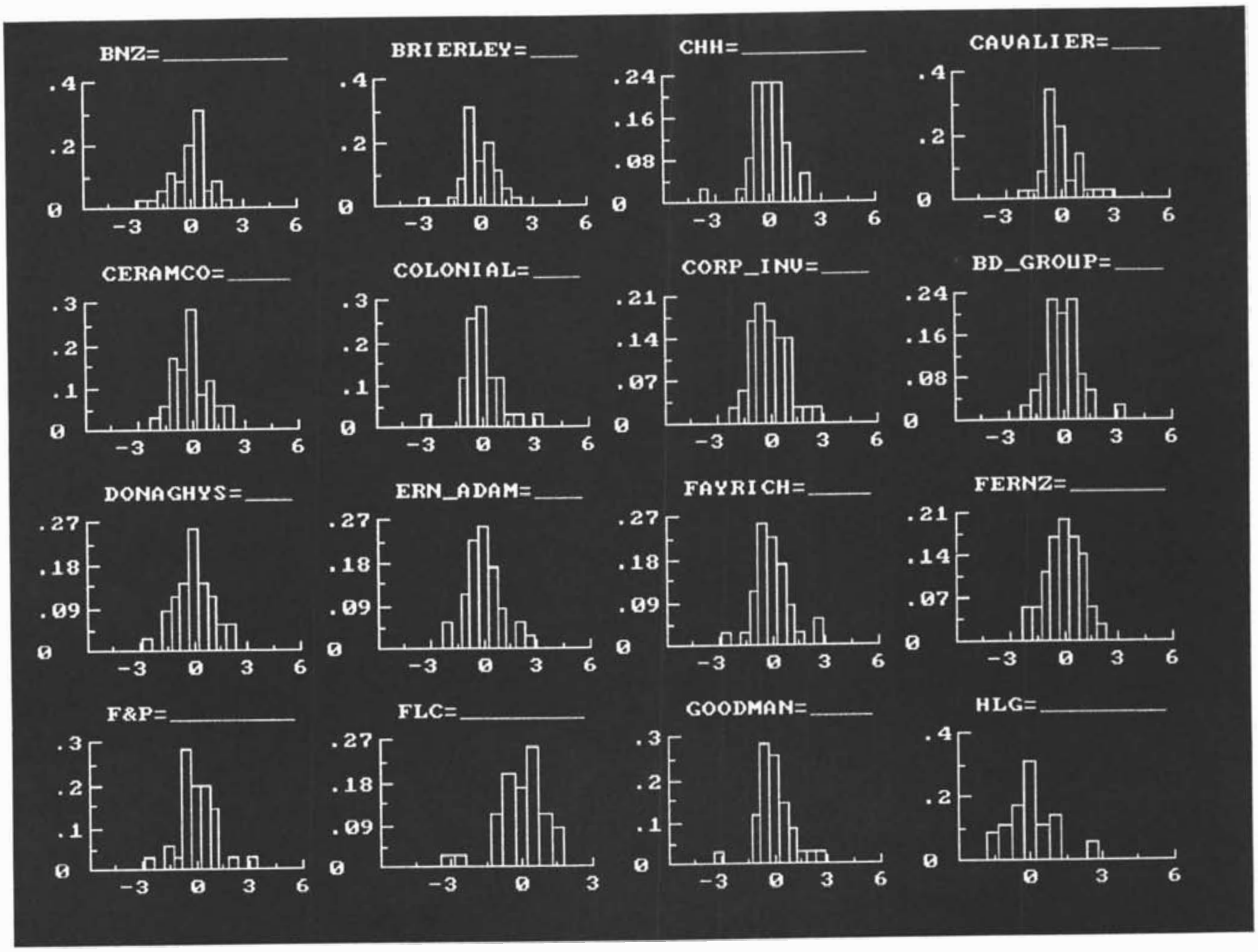
		Mean	Standard Deviation	Skewness	Kurtosis	Normality χ^2
3	BNZ Finance	-0.006594	0.181883	-0.290281	0.144368	1.3748
4	Brierley Investments	-0.006778	0.189442	-0.529666	0.989537	4.4407
5	Carter Holt Harvey	0.015336	0.169513	-0.949572	3.087638	12.271 **
6	Cavalier Corporation	0.052975	0.257773	0.500471	0.443022	2.4916
7	Ceramco Corporation	-0.007444	0.239559	0.294617	-0.428999	0.69958
8	Colonial Motors	0.004542	0.142889	0.345365	2.257673	13.158 **
9	Corporate Investments	-0.029236	0.247611	0.453067	-0.210868	1.5421
10	DB Group	0.003215	0.18276	0.621958	1.617034	7.1325 *
11	Donaghys	0.044842	0.15072	-0.026091	-0.181087	0.33381
12	Ernest Adams	0.026289	0.168829	0.542023	0.475677	2.6548
15	Fay Richwhite	-0.028451	0.254098	0.464751	0.942905	4.3815
16	Fernz Corporation	0.048134	0.202433	-0.029964	-0.415836	0.024302
17	Fisher & Paykel	0.014539	0.161674	0.244995	2.146681	12.892 **
18	Fletcher Challenge	0.016642	0.165802	-0.91356	1.431868	5.7179
19	Goodman Fielder	-0.026275	0.152218	0.014722	2.26587	14.372 **
21	Hallenstein Glasson	0.063969	0.255506	0.85504	0.980399	4.8064
22	Independent Newspapers	0.017337	0.151598	-0.258325	0.083198	1.1682
24	Lion Nathan	-0.009877	0.146029	-1.020166	3.164233	11.826 **
26	Mair Astley	0.011707	0.314263	0.876735	0.672952	5.1442
27	Milburn New Zealand	0.089193	0.180264	0.249494	-0.435466	0.49088
29	New Zealand Oil & Gas	0.004542	0.22075	0.526632	0.20373	2.1061
30	New Zealand Refining	0.117323	0.183846	-0.088874	-0.067039	0.62311
31	Owens Group	0.03026	0.203479	-0.91073	3.615847	16.066 **
32	PDL Holdings	0.074131	0.338054	1.302629	1.19619	15.697 **
34	Salmond Smith Biolab	0.026185	0.259877	0.036118	-0.151849	0.39854
35	Sanford	0.033226	0.167519	-0.223379	-0.31948	0.40171
36	Southern Petroleum	0.053369	0.324763	1.317325	2.218341	10.013 **
37	Steel & Tube	0.071419	0.239844	0.610746	1.390333	6.0346 *
39	Trans Tasman Properties	-0.060064	0.351707	1.36105	3.177673	10.17 **
41	Wilson Neill	-0.088813	0.385065	2.351139	9.820972	20.845 **
42	Wilson & Horton	0.024108	0.16169	0.176805	0.81477	4.3227

* Significantly different from normal 5% level.

** Significantly different from normal 1% level.

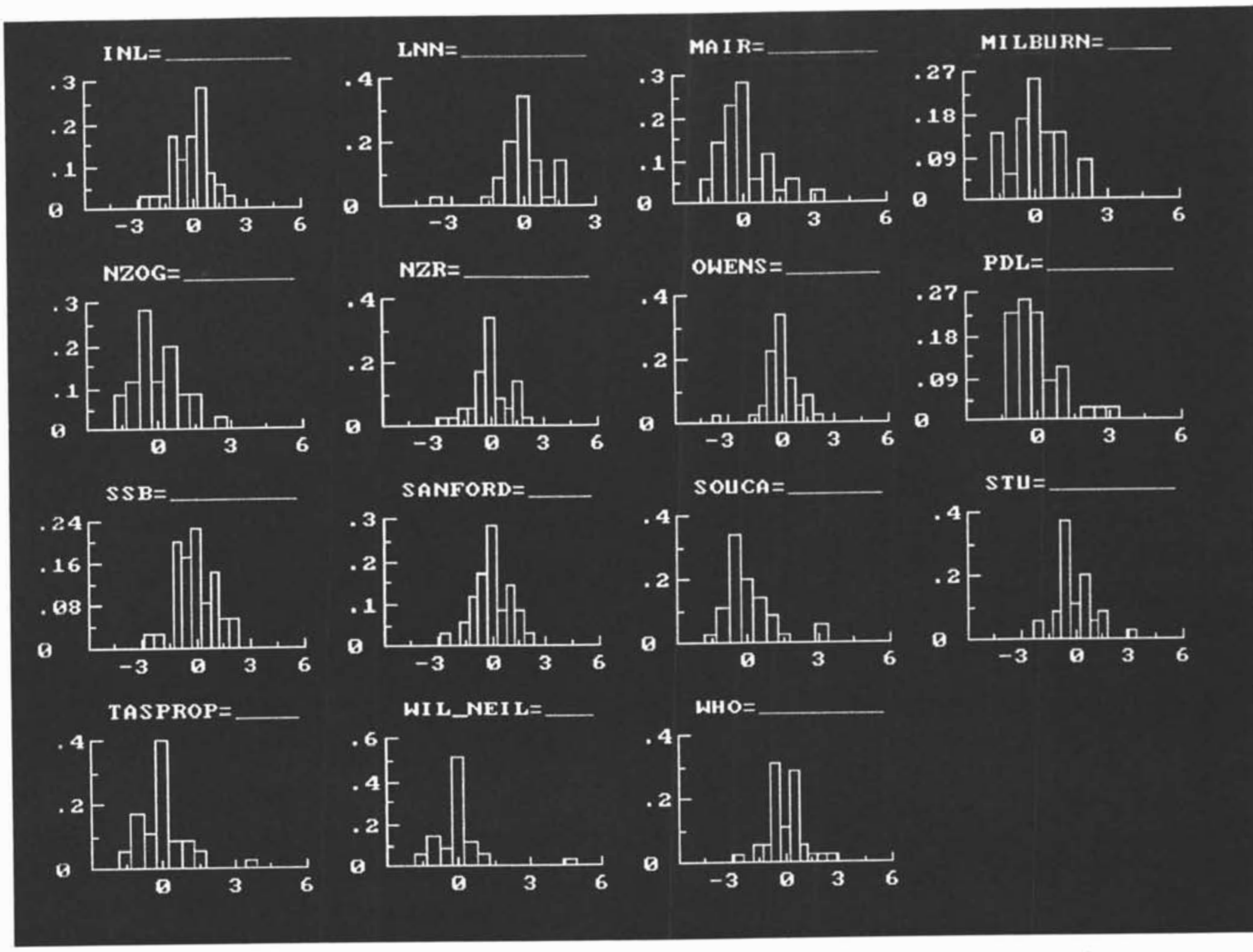
Distribution of Returns - Selected Companies

Real Data



Distribution of Returns - Selected Companies

Real Data



Normality Test Results

Stock Market Sectors

Nominal Data

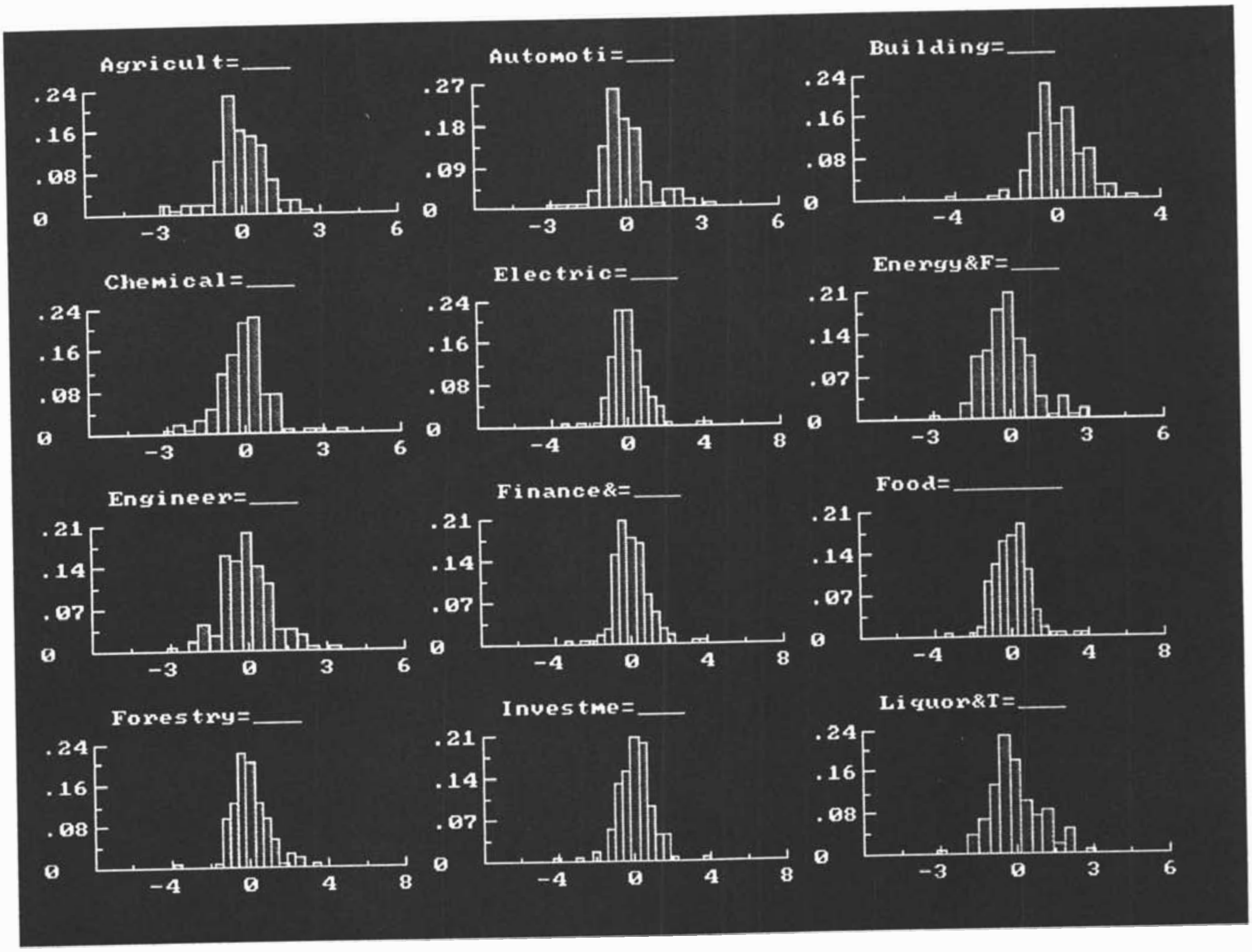
		Mean	Standard Deviation	Skewness	Kurtosis	Normality χ^2	
1	Agriculture	-0.004665	0.085704	-0.414639	0.93797	5.7559	
2	Automotive	0.007313	0.084064	0.68568	1.639103	10.014	**
3	Building	0.021797	0.085315	-0.275336	2.053325	17.411	**
4	Chemicals	0.020928	0.081805	0.122678	2.136346	19.615	**
5	Electrical	0.008395	0.09051	0.749471	4.220395	33.931	**
6	Energy & Fuel	0.033011	0.07757	0.56893	0.914976	6.2737	*
7	Engineering	0.017844	0.084872	0.288159	0.841005	5.1157	
8	Finance & Banks	-0.002063	0.09693	0.472206	2.563737	20.878	**
9	Food	0.010231	0.073861	0.648187	2.524606	17.289	**
10	Forestry	0.006739	0.083557	0.392437	2.110368	16.755	**
11	Investment	-0.005847	0.094001	-0.353653	3.180245	31.191	**
12	Liquor & Tobacco	-0.000102	0.066161	0.339804	0.400779	2.8005	
13	Meat & By-products	-0.010791	0.104649	-0.000565	1.911851	17.03	**
14	Media & Communication	0.011562	0.070284	-0.328876	0.843402	5.1155	
15	Medical Supplies	0.00476	0.099143	0.164121	-0.124334	0.52975	
16	Miscellaneous	0.005306	0.085672	0.075913	0.061584	0.52841	
17	Property	-0.02237	0.107017	0.55895	3.970886	37.146	**
18	Retailers	0.005885	0.091344	1.020333	1.974396	15.528	**
19	Textile & Apparel	0.01451	0.08967	0.496574	1.135425	7.0248	*
20	Transport & Tourism	0.005614	0.083836	0.06801	0.326477	1.6481	

* Significantly different from normal 5% level.

** Significantly different from normal 1% level.

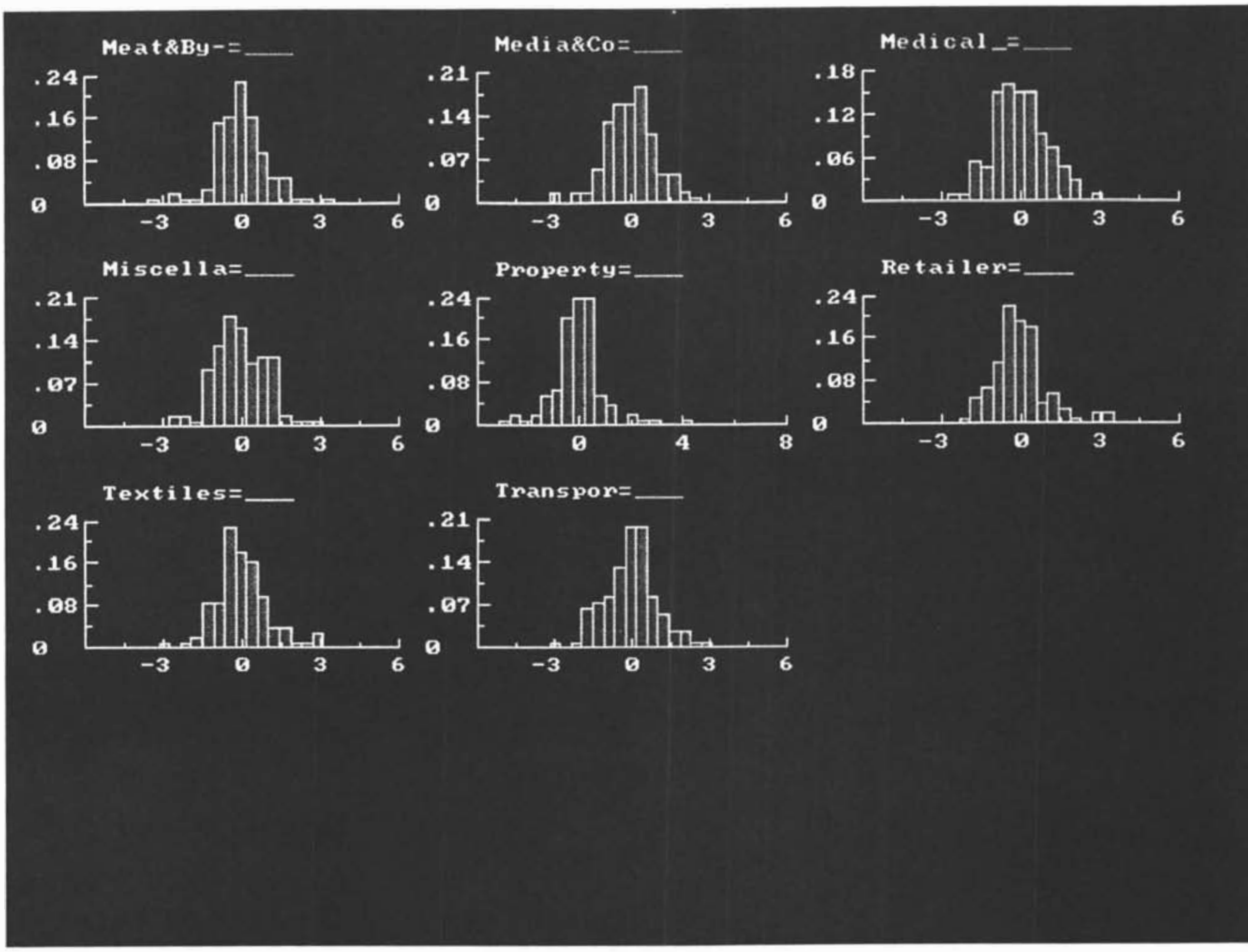
Distribution of Returns - Stock Market Sectors

Nominal Data



Distribution of Returns - Stock Market Sectors

Nominal Data



Normality Test Results

Stock Market Sectors

Real Data

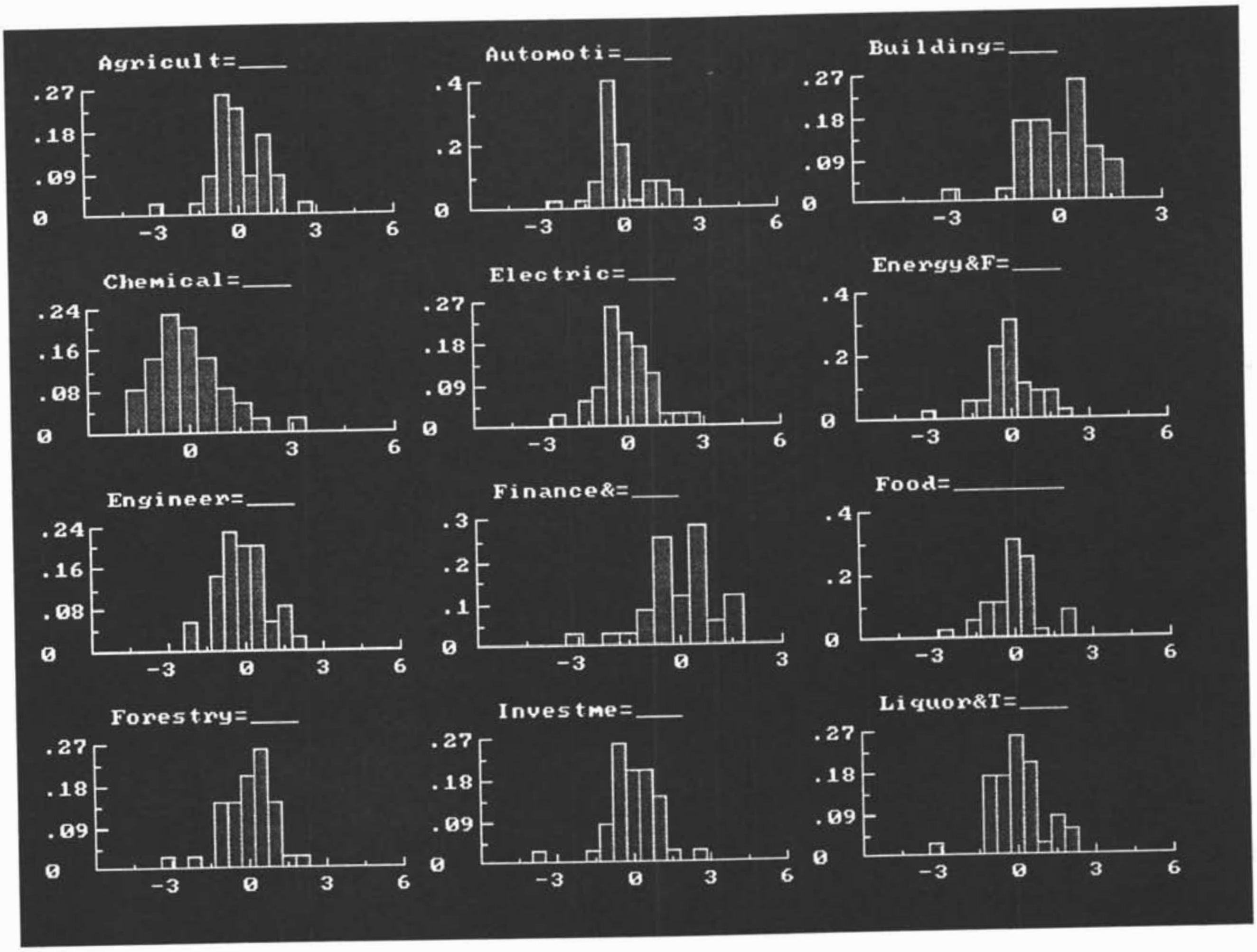
		Mean	Standard Deviation	Skewness	Kurtosis	Normality χ^2
1	Agriculture	-0.021694	0.170855	-0.312079	1.288057	6.7524 *
2	Automotive	0.00361	0.148087	0.423763	0.456461	2.4551
3	Building	0.061109	0.175326	-0.600099	0.703362	3.3517
4	Chemicals	0.055036	0.188445	0.691222	0.515754	3.3316
5	Electrical	0.014136	0.168513	0.253103	0.679465	3.513
6	Energy & Fuel	0.089253	0.143763	-0.295461	0.872617	4.4273
7	Engineering	0.044645	0.170448	0.040212	0.300551	1.8829
8	Finance & Banks	-0.016471	0.165924	-0.601146	0.617571	3.1281
9	Food	0.022411	0.152382	-0.086935	0.943289	5.155
10	Forestry	0.008847	0.141054	-0.834417	1.099412	4.8319
11	Investment	-0.022607	0.177771	-0.873665	2.959298	12.48 **
12	Liquor & Tobacco	-0.015781	0.094449	-0.110881	0.847987	4.5834
13	Meat & By-products	-0.039423	0.18993	-0.075153	0.893093	4.8706
14	Media & Communication	0.023642	0.135781	-0.796881	0.924155	4.3924
15	Medical Supplies	0.004505	0.205678	0.489939	-0.057266	1.7428
16	Miscellaneous	0.005854	0.170324	0.025424	0.060426	0.98248
17	Property	-0.073441	0.176223	-0.388729	2.402995	13.926 **
18	Retailers	0.007267	0.174174	0.36377	-0.627839	1.6624
19	Textile & Apparel	0.032133	0.163231	-0.034012	-0.428694	0.020068
20	Transport & Tourism	0.008621	0.172112	-0.155827	-0.490217	0.19743

* Significantly different from normal 5% level.

** Significantly different from normal 1% level.

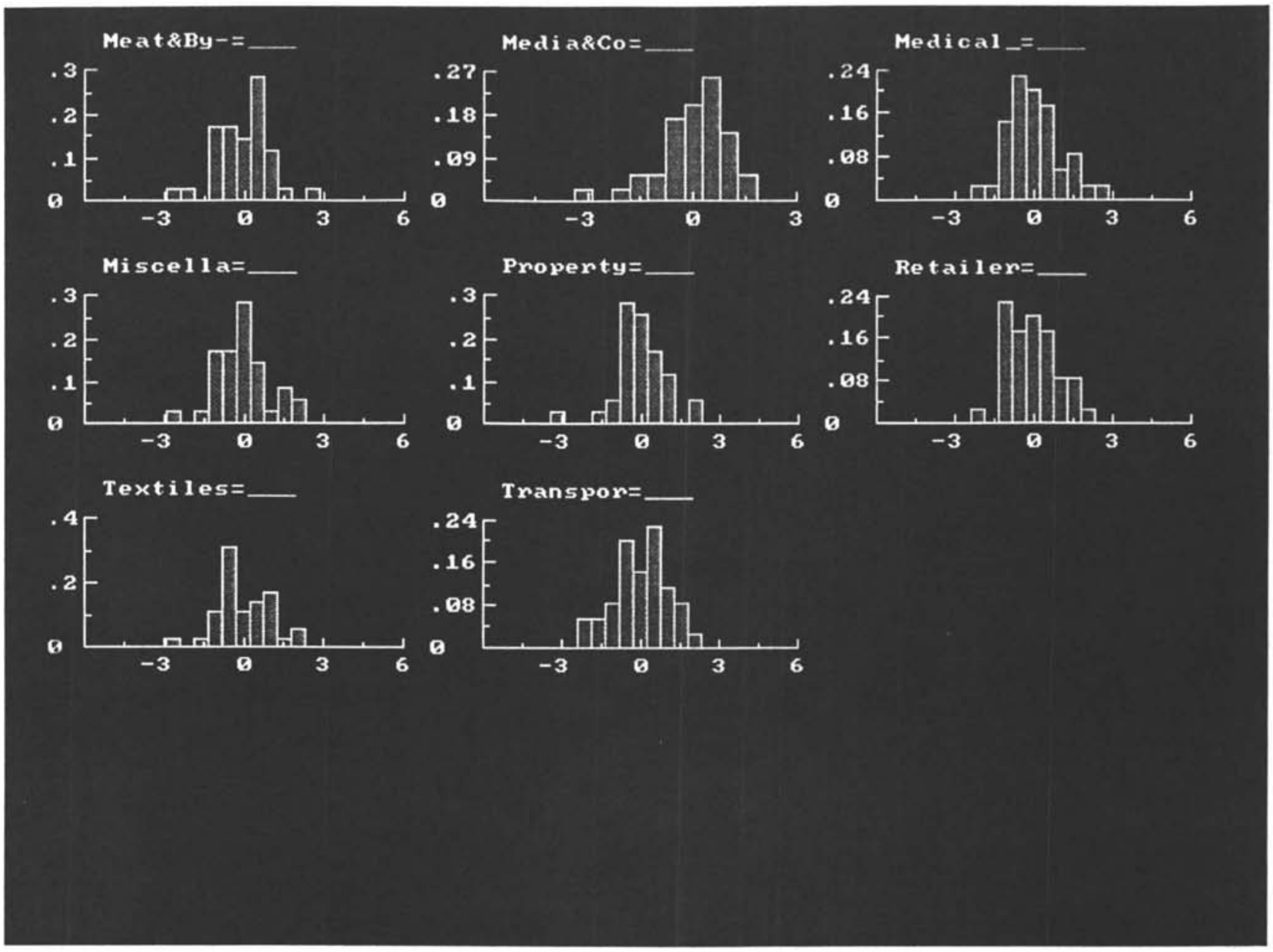
Distribution of Returns - Stock Market Sectors

Real Data



Distribution of Returns - Stock Market Sectors

Real Data



Normality Tests Results

Managed Funds: New Zealand Fixed Interest

Nominal Data

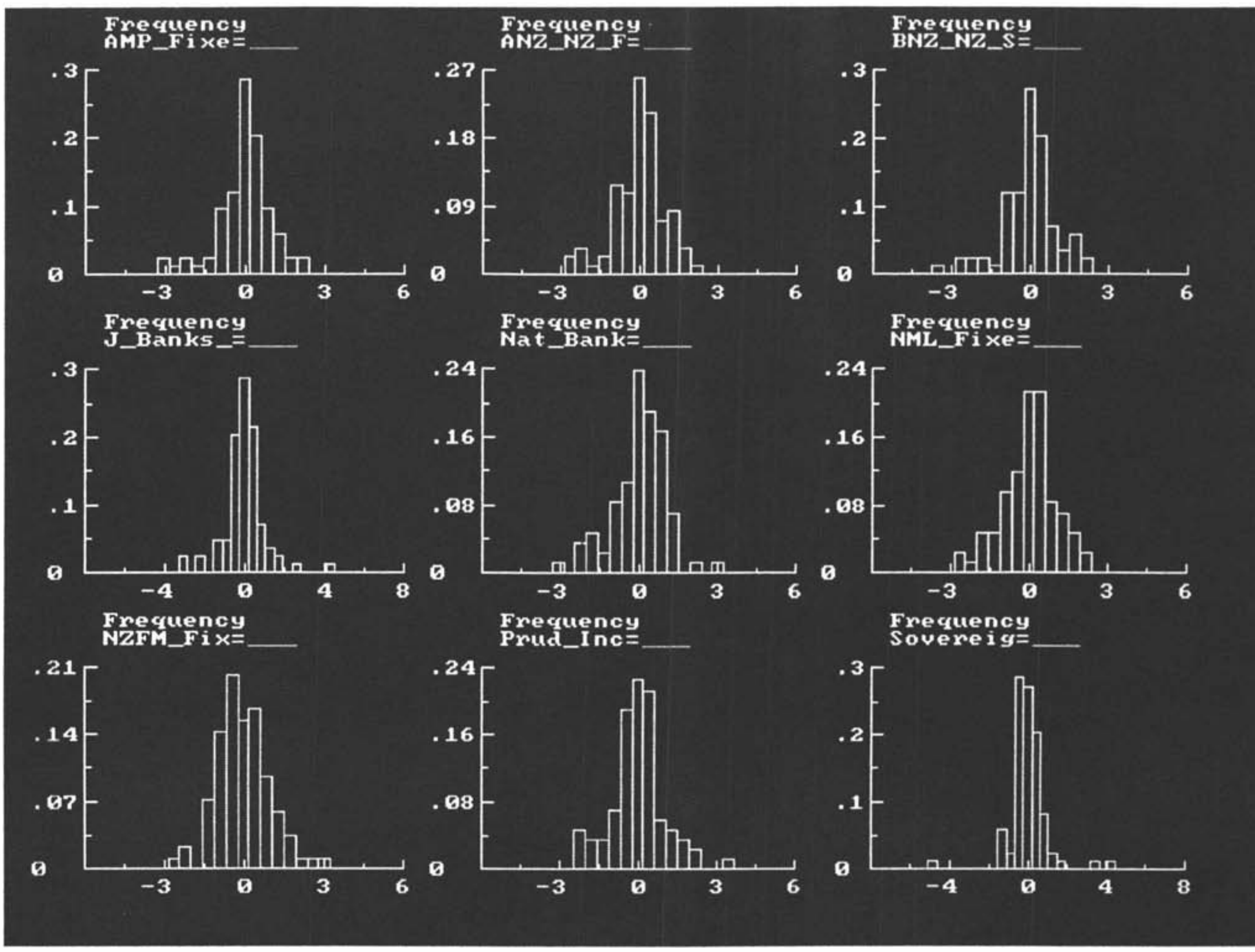
		Mean	Standard Deviation	Skewness	Kurtosis	Normality χ^2
1	AMP Fixed Interest Security	0.006143	0.008521	-0.80095	1.51011	8.7954 *
2	ANZ New Zealand Fixed Interest	0.006014	0.008104	-0.525886	0.552801	4.1958
3	BNZ New Zealand Strategic Bonds	0.006267	0.008777	-0.583355	1.230941	6.7939 *
4	Joseph Banks New Zealand Bonds	0.005924	0.013406	0.441869	4.903722	48.03 **
5	National Bank Income	0.005484	0.009911	-0.435056	0.742796	4.2229
6	National Mutual Life Fixed Interest	0.005953	0.0121	-0.201713	0.093505	0.98697
7	New Zealand Funds Management Fixed Interest	0.006313	0.011841	0.357015	0.695201	3.8429
8	Prudential Income Trust	0.006522	0.010376	0.185603	1.17885	7.5401 *
9	Sovereign New Zealand Fixed Interest	0.00779	0.01558	0.391408	9.418755	114.76 **

* Statistically different from normal 5% level.

** Statistically different from normal 1% level.

Distribution of Returns - Managed Funds: New Zealand Fixed Interest

Nominal Data



Normality Test Results

Managed Funds: New Zealand Fixed Interest

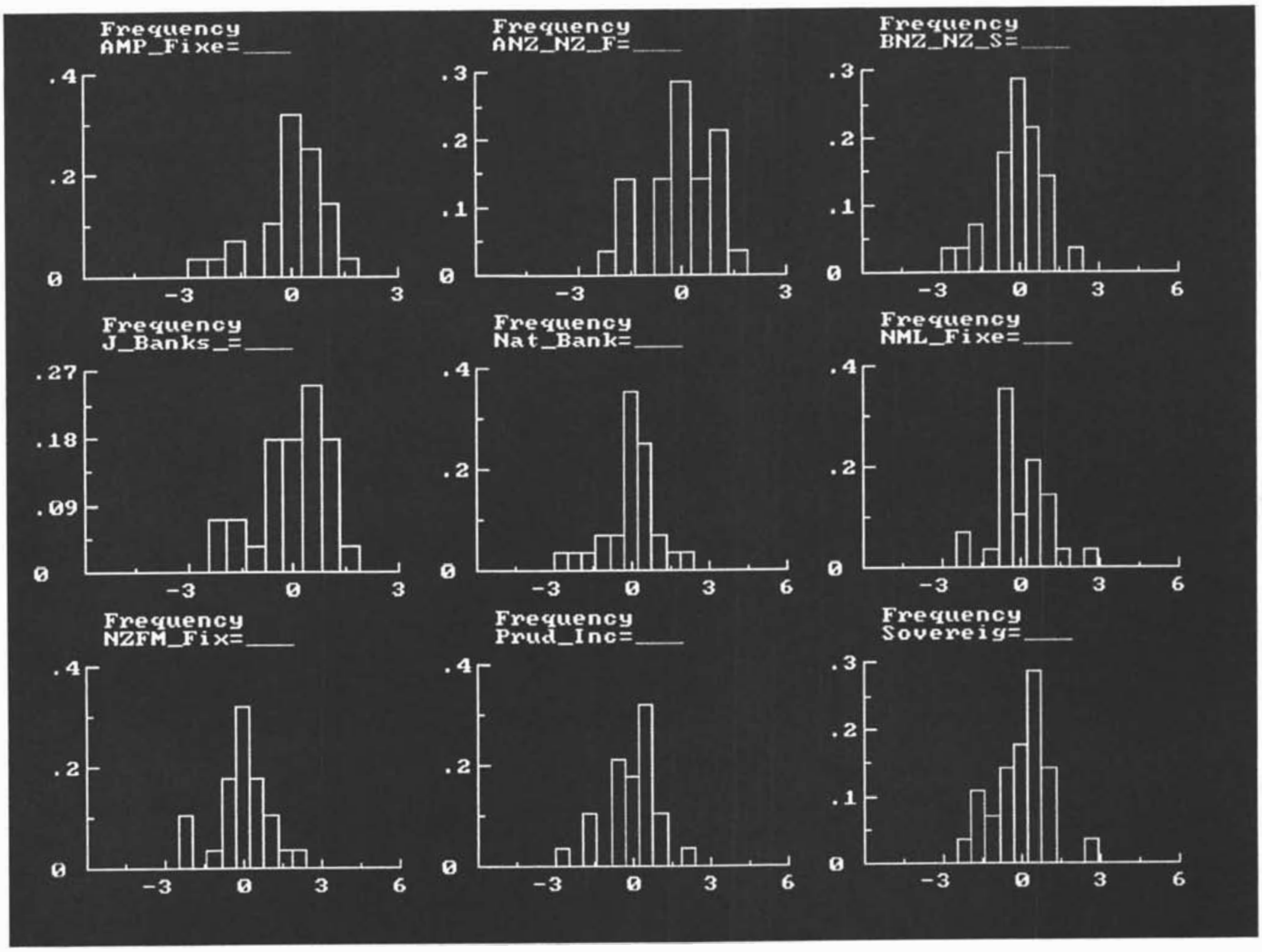
Real Data

		Mean	Standard Deviation	Skewness	Kurtosis	Normality χ^2
1	AMP Fixed Interest Security	0.010718	0.01586	-1.075686	1.476071	5.9928 *
2	ANZ New Zealand Fixed Interest	0.010347	0.016359	-0.592421	-0.27001	2.4586
3	BNZ New Zealand Strategic Bonds	0.011106	0.017168	-0.84192	1.201676	4.7387
4	Joseph Banks New Zealand Bonds	0.009957	0.019182	-0.585643	-0.296717	2.4499
5	National Bank Income	0.008745	0.018797	-0.537073	0.911525	4.1674
6	National Mutual Life Fixed Interest	0.010099	0.01965	-0.066138	0.857661	4.8439
7	New Zealand Funds Management Fixed Interest	0.011192	0.019453	-0.303868	0.37868	2.315
8	Prudential Income Trust	0.011816	0.016379	-0.494108	0.39789	2.355
9	Sovereign New Zealand Fixed Interest	0.015478	0.018466	-0.030112	0.119564	1.3922

* Statistically different from normal 5% level.

Distribution of Returns - Managed Funds: New Zealand Fixed Interest

Real Data



Normality Test Results

Managed Funds: New Zealand Equities

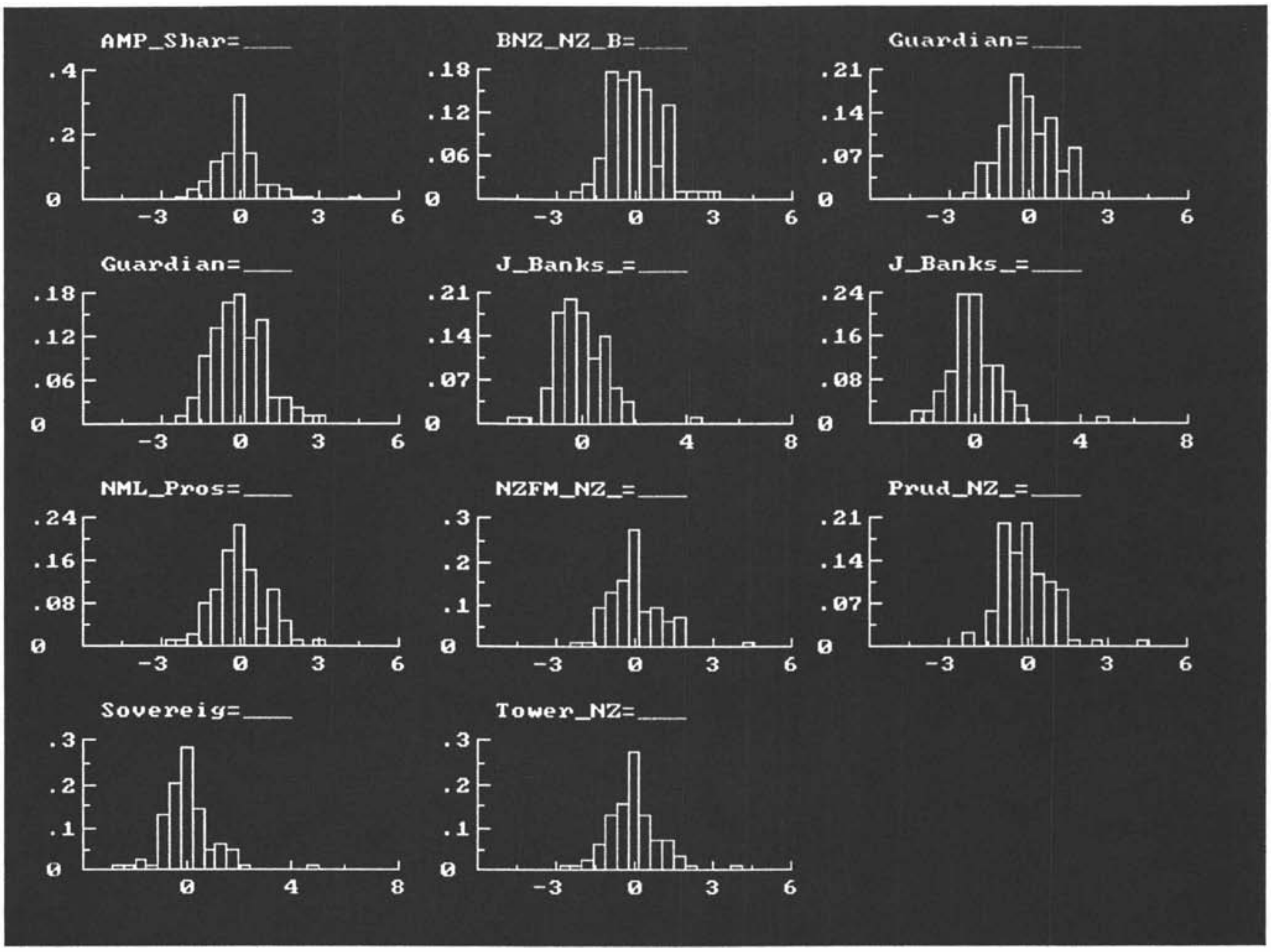
Nominal Data

		Mean	Standard Deviation	Skewness	Kurtosis	Normality χ^2	
1	AMP Share Fund	0.007721	0.041895	0.998845	2.955513	14.466	**
2	BNZ New Zealand Blue Chip	0.006637	0.051682	0.516572	0.437455	3.959	
3	Guardian Assurance Equity	0.011375	0.032922	0.218384	-0.193467	0.76984	
4	Guardian New Zealand Equity	0.008831	0.055264	0.416845	0.145364	2.638	
5	Joseph Banks New Zealand Equity	0.007645	0.049675	0.90475	3.435657	18.674	**
6	Joseph Banks New Zealand Equity Imputation	0.011169	0.053619	1.279486	4.984896	22.14	**
7	National Mutual Life Share	0.000092	0.045377	0.21237	0.272334	1.5968	
8	New Zealand Funds Management Equity	0.008452	0.046665	0.908324	2.501077	12.643	**
9	Prudential New Zealand Equity	0.00718	0.048557	0.992556	2.887153	14.174	**
10	Sovereign New Zealand Equity	0.006442	0.043334	1.223468	4.998842	23.297	**
11	Tower New Zealand Equity	0.006054	0.046722	0.6699	2.294133	13.295	**

** Significantly different from normal 1% level.

Distribution of Returns - Managed Funds: New Zealand Equities

Nominal Data



Normality Test Results

Managed Funds: New Zealand Equities

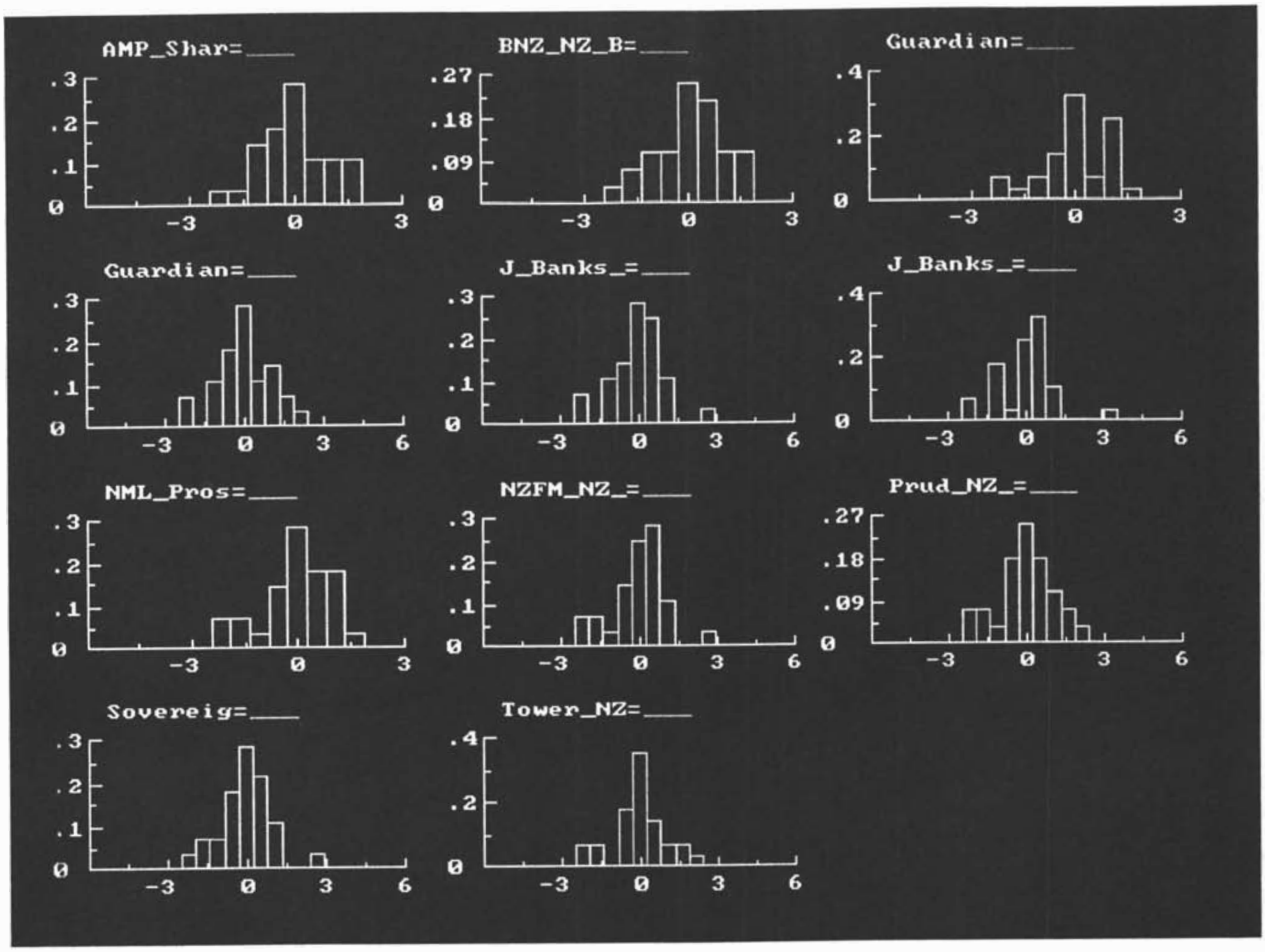
Real Data

		Mean	Standard Deviation	Skewness	Kurtosis	Normality χ^2
1	AMP Share Fund	0.015414	0.071433	0.033999	-0.500811	0.019382
2	BNZ New Zealand Blue Chip	0.011726	0.082433	-0.50734	-0.240109	1.6234
3	Guardian Assurance Equity	0.026772	0.060857	-0.536981	-0.239413	1.8556
4	Guardian New Zealand Equity	0.0177	0.0824	-0.237837	-0.195457	0.60862
5	Joseph Banks New Zealand Equity	0.015365	0.086837	0.088254	1.147181	6.504 *
6	Joseph Banks New Zealand Equity Imputation	0.026719	0.100976	0.281807	1.494607	8.2138 *
7	National Mutual Life Share	-0.006925	0.082076	-0.683757	-0.044534	3.0063
8	New Zealand Funds Management Equity	0.017754	0.081219	0.038044	0.998462	5.6571
9	Prudential New Zealand Equity	0.013411	0.077297	-0.338778	0.02809	1.2195
10	Sovereign New Zealand Equity	0.011542	0.073624	0.258153	1.108723	5.9696
11	Tower New Zealand Equity	0.010788	0.083698	-0.370955	0.045127	1.3059

* Statistically different from normal 5% level.

Distribution of Returns - Managed Funds: New Zealand Equities

Real Data



Normality Test Results

Managed Funds: New Zealand Balanced

Nominal Data

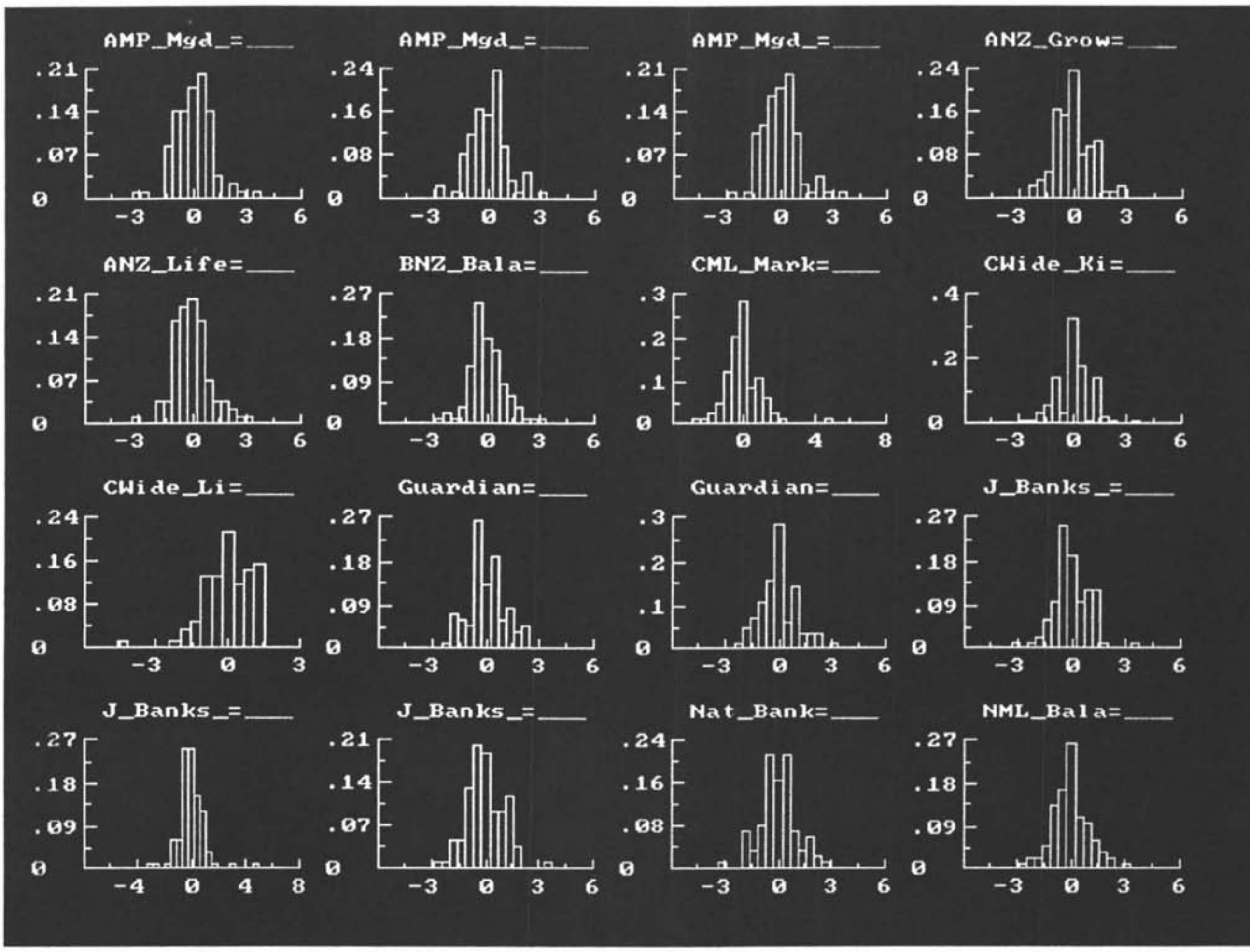
		Mean	Standard Deviation	Skewness	Kurtosis	Normality χ^2	
1	AMP Managed Fund Balanced (B)	0.004979	0.016906	0.174995	1.767464	13.215	**
2	AMP Managed Fund Balanced (C)	0.00477	0.017744	0.149419	1.205954	7.8735	*
3	AMP Managed Fund Balanced (M)	0.005154	0.017609	0.588307	1.292206	7.1044	*
4	ANZ Growth Trust	0.006844	0.025563	0.343052	0.05603	1.831	
5	ANZ Life Managed Fund	0.006321	0.016476	0.437495	1.246624	7.1495	*
6	BNZ Balanced	0.005026	0.015594	0.250762	0.744705	4.1114	
7	Colonial Mutual Life Market Linked	0.005195	0.020211	1.124964	4.889371	24.985	**
8	Countrywide Bank Kiwi Trust	0.004738	0.024455	0.312249	0.52217	2.9141	
9	Countrywide Bank Life Multi Fund	0.003857	0.014006	-0.903381	2.098442	11.081	**
10	Guardian Balanced	0.005459	0.01916	0.138457	-0.052432	0.44626	
11	Guardian Assurance Balanced	0.009167	0.018604	0.460081	0.410004	3.3256	
12	Joseph Banks Asset Growth	0.005567	0.018037	0.194255	1.118359	7.0312	*
13	Joseph Banks Capital	0.005084	0.015717	1.072435	6.592346	43.861	**
14	Joseph Banks Growth	0.00561	0.022837	0.320957	0.548335	3.0676	
15	National Bank Fund of Funds Balanced	0.005516	0.01541	0.05458	0.319699	1.5971	
16	National Mutual Life Balanced	0.006375	0.024193	0.248948	0.832861	4.7004	
17	Norwich Life Global	0.006781	0.014886	0.225103	1.888554	14.198	**
18	New Zealand Funds Management Balanced	0.005058	0.018496	0.375872	1.32919	8.0326	*
19	Oceanic Managed	0.006988	0.013939	-0.063769	0.815082	4.7548	
20	Prudential Balanced Growth	0.005622	0.020758	0.585355	0.949008	5.6419	
21	Prudential Beaver	0.005901	0.019606	1.311568	5.32074	23.488	**
22	Prudential Stag	0.007219	0.022552	1.408926	6.40477	27.938	**
23	Southpac Balanced	0.005361	0.015145	0.042096	0.518127	2.7049	
24	Sovereign Balanced Growth	0.006203	0.021267	0.58109	4.21293	35.283	**
25	Sovereign Conservative	0.005654	0.015511	1.222643	5.579966	27.546	**
26	Sovereign High Growth	0.006278	0.024574	0.336528	3.759657	35.151	**
27	Sun Alliance Bond Managed	0.003457	0.017533	-0.518213	1.915391	11.841	**
28	Tower Multi Sector	0.005357	0.016428	-0.297078	1.079101	6.3898	*
29	Westpac Balanced	0.005434	0.010744	0.916077	4.559201	28.509	**
30	Westpac Life Investment	0.005469	0.012723	0.070462	0.529712	2.7777	
31	Westpac Retirement Balanced	0.007076	0.015689	1.667002	7.618083	28.746	**

* Statistically different from normal 5% level.

** Statistically different from normal 1% level.

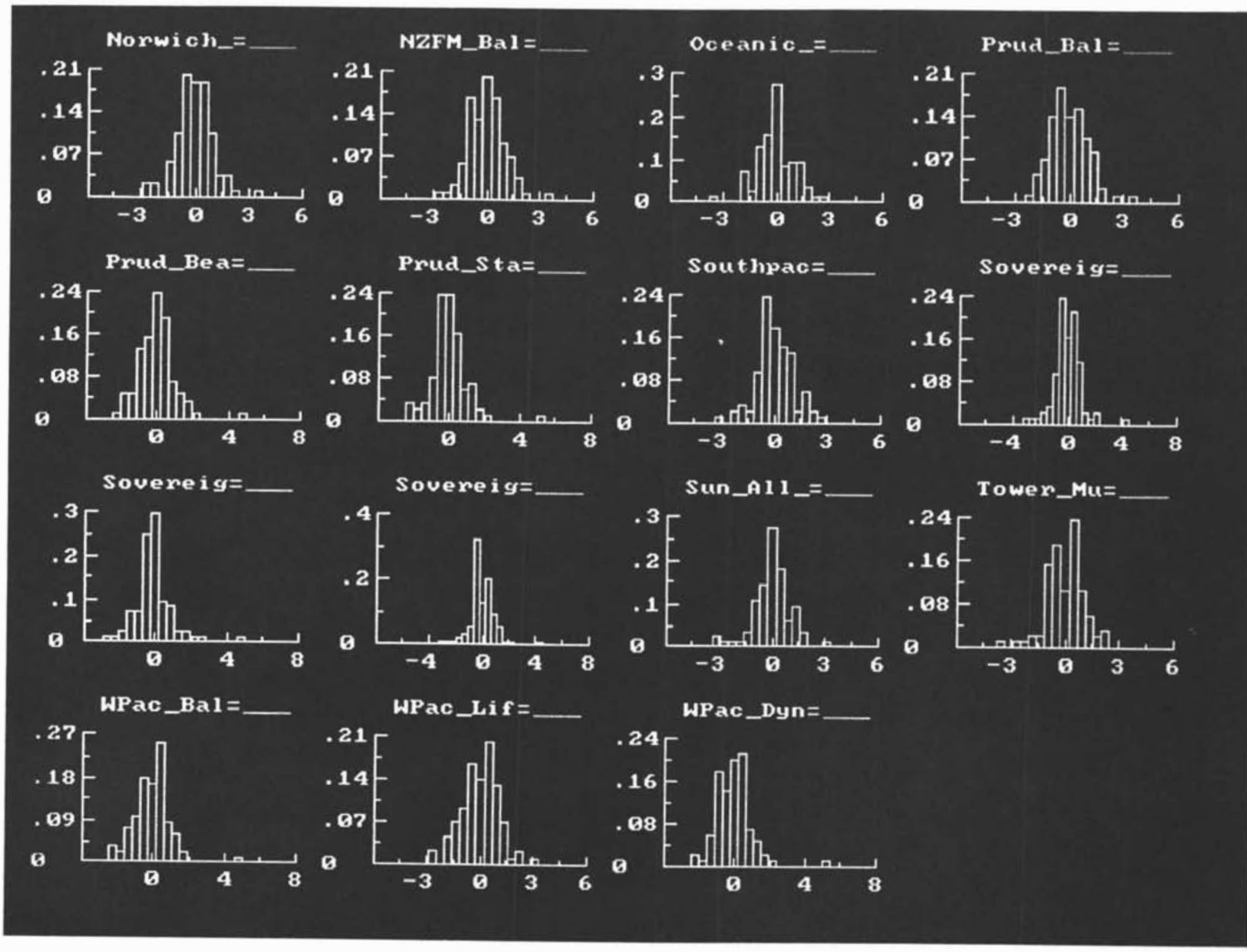
Distribution of Returns - Managed Funds: New Zealand Balanced

Nominal Data



Distribution of Returns - Managed Funds: New Zealand Balanced

Nominal Data



Normality Test Results

Managed Funds: New Zealand Balanced

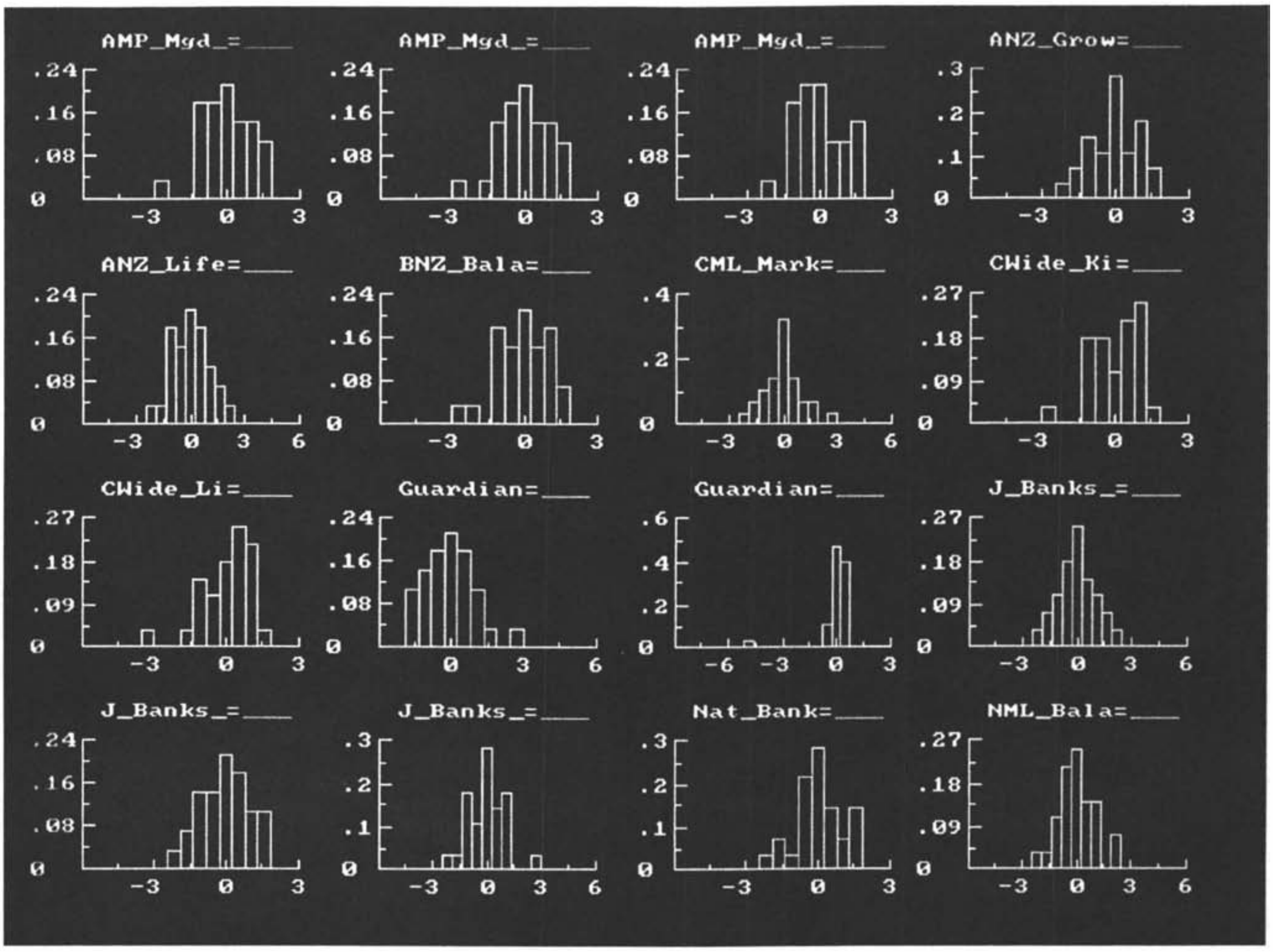
Real Data

		Mean	Standard Deviation	Skewness	Kurtosis	Normality χ^2
1	AMP Managed Fund Balanced (B)	0.007172	0.028421	-0.245456	-0.337765	0.43728
2	AMP Managed Fund Balanced (C)	0.006472	0.027389	-0.346016	-0.224235	0.81711
3	AMP Managed Fund Balanced (M)	0.007666	0.028531	-0.168397	-0.434085	0.20497
4	ANZ Growth Trust	0.012893	0.045995	-0.160555	-0.845666	0.6728
5	ANZ Life Managed Fund	0.011324	0.031477	-0.059857	-0.073421	0.16648
6	BNZ Balanced	0.007341	0.027009	-0.539809	-0.236968	1.8763
7	Colonial Mutual Life Market Linked	0.00778	0.032994	0.495134	0.901922	4.2292
8	Countrywide Bank Kiwi Trust	0.006341	0.038629	-0.774853	0.717196	3.6102
9	Countrywide Bank Life Multi Fund	0.00388	0.026174	-1.015147	0.986432	5.5398
10	Guardian Balanced	0.008586	0.031717	0.405928	0.189865	1.7049
11	Guardian Assurance Balanced	0.000583	0.105573	-4.154042	17.792401	156.02 **
12	Joseph Banks Asset Growth	0.009154	0.03239	0.161838	-0.144101	0.62788
13	Joseph Banks Capital	0.007539	0.021447	-0.102889	-0.61663	0.092403
14	Joseph Banks Growth	0.009206	0.040335	0.257565	0.166356	1.5587
15	National Bank Fund of Funds Balanced	0.008892	0.025571	-0.351848	-0.250463	0.8154
16	National Mutual Life Balanced	0.011366	0.041333	0.198282	0.029632	1.1106
17	Norwich Life Global	0.012655	0.027049	-0.074486	-0.841993	0.42946
18	New Zealand Funds Management Balanced	0.007434	0.03193	-0.332745	-0.556825	0.84115
19	Oceanic Managed	0.013256	0.024512	-0.408465	-0.206519	1.0586
20	Prudential Balanced Growth	0.009024	0.032822	-0.01581	-0.573103	0.00256
21	Prudential Beaver	0.010003	0.03499	0.706426	1.337708	5.4575
22	Prudential Stag	0.01389	0.037593	0.393224	1.906203	10.333 **
23	Southpac Balanced	0.008372	0.027285	0.002559	-0.264437	0.30141
24	Sovereign Balanced Growth	0.011127	0.043052	-0.017911	0.441128	2.7281
25	Sovereign Conservative	0.009316	0.029822	-0.39897	-0.483261	1.1458
26	Sovereign High Growth	0.011378	0.048478	-0.155212	-0.010517	0.97542
27	Sun Alliance Bond Managed	0.002838	0.036266	-1.027681	0.640082	6.6411
28	Tower Multi Sector	0.008372	0.029716	-0.55372	-0.152356	1.8988
29	Westpac Balanced	0.008587	0.019851	0.139771	-0.412464	0.17383
30	Westpac Life Investment	0.0087	0.023399	-0.188362	-1.186661	2.9625
31	Westpac Retirement Balanced	0.013531	0.027809	0.32963	-0.105904	0.93111

** Statistically different from normal 1% level.

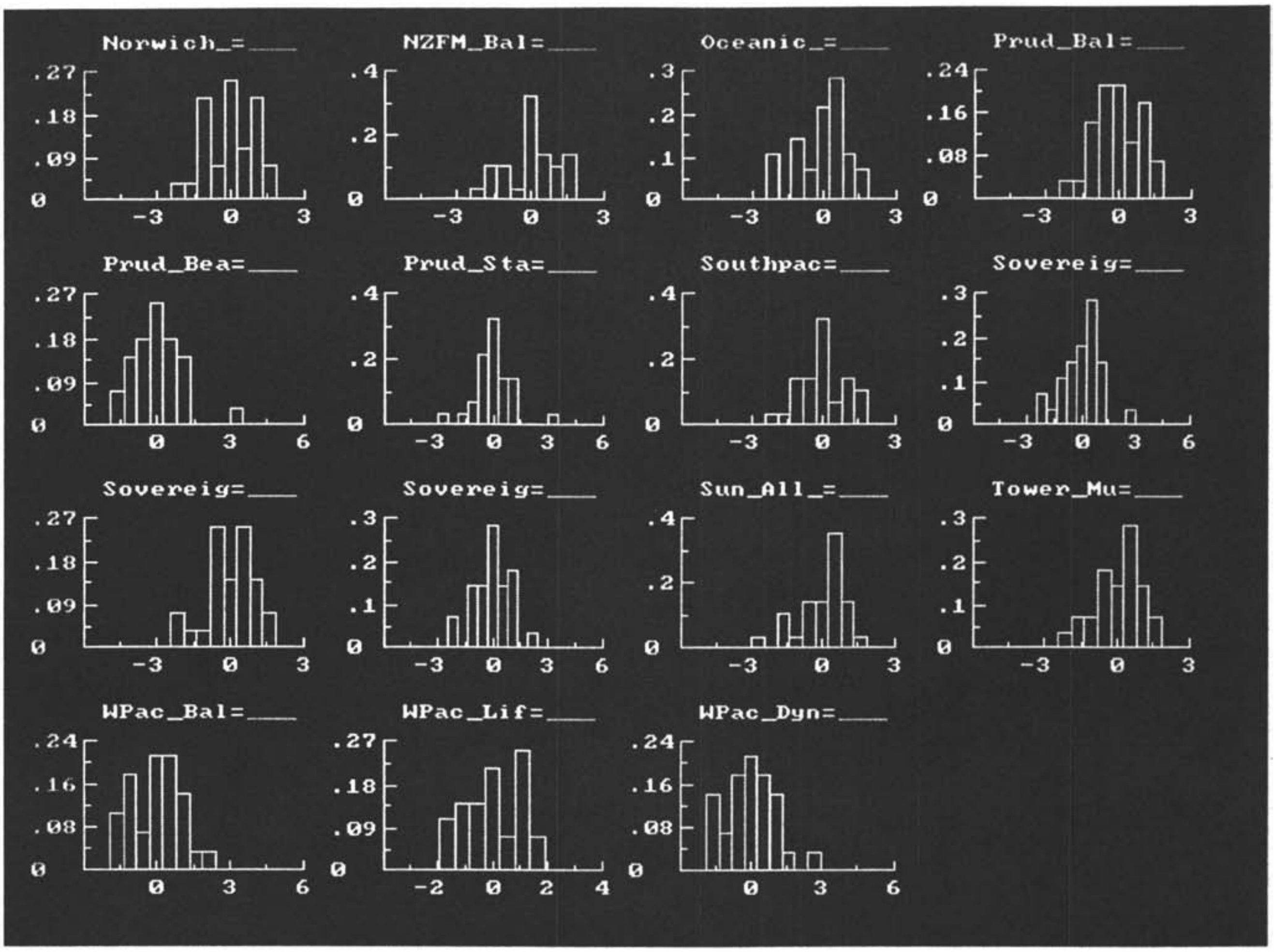
Distribution of Returns - Managed Funds: New Zealand Balanced

Real Data



Distribution of Returns - Managed Funds: New Zealand Balanced

Real Data



Normality Test Results

Fixed Interest Term Deposit

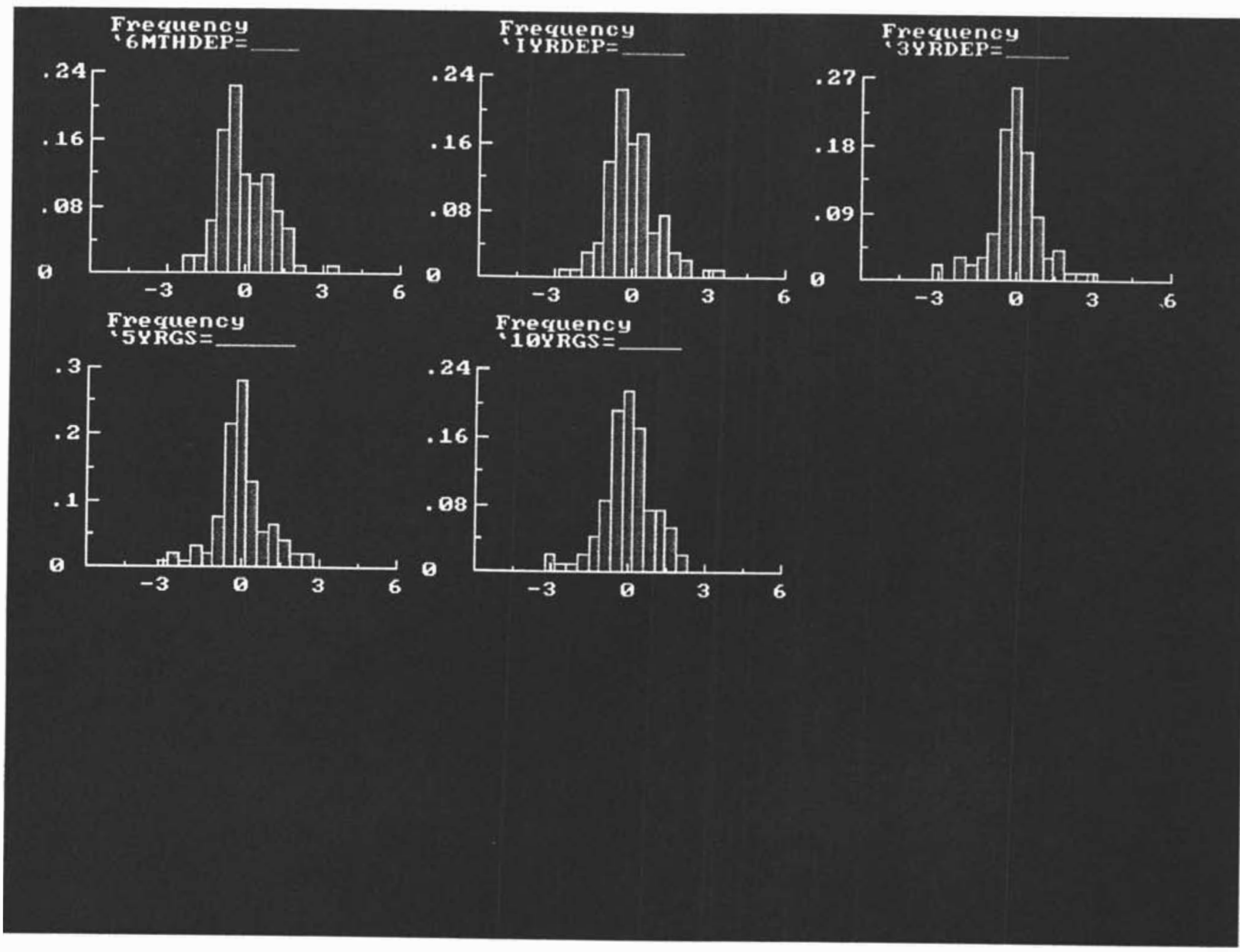
Nominal Data

		Mean	Standard Deviation	Skewness	Kurtosis	Normality χ^2
1	6 Month Deposit	0.006076	0.00265	0.386359	-0.084733	3.119
2	1 Year Deposit	0.006249	0.004163	0.750802	1.023003	9.2834 **
3	3 Year Deposit	0.0066	0.009545	-0.587046	3.369185	28.291 **
4	5 Year Government Stock	0.008315	0.009605	-0.132415	0.898312	5.7704
5	10 Year Government Stock	0.008657	0.013443	-0.396865	0.789482	4.5724

** Statistically different from normal 1% level.

Distribution of Returns - Fixed Interest Time Deposit

Nominal Data



Normality Test Results

Fixed Interest Term Deposit

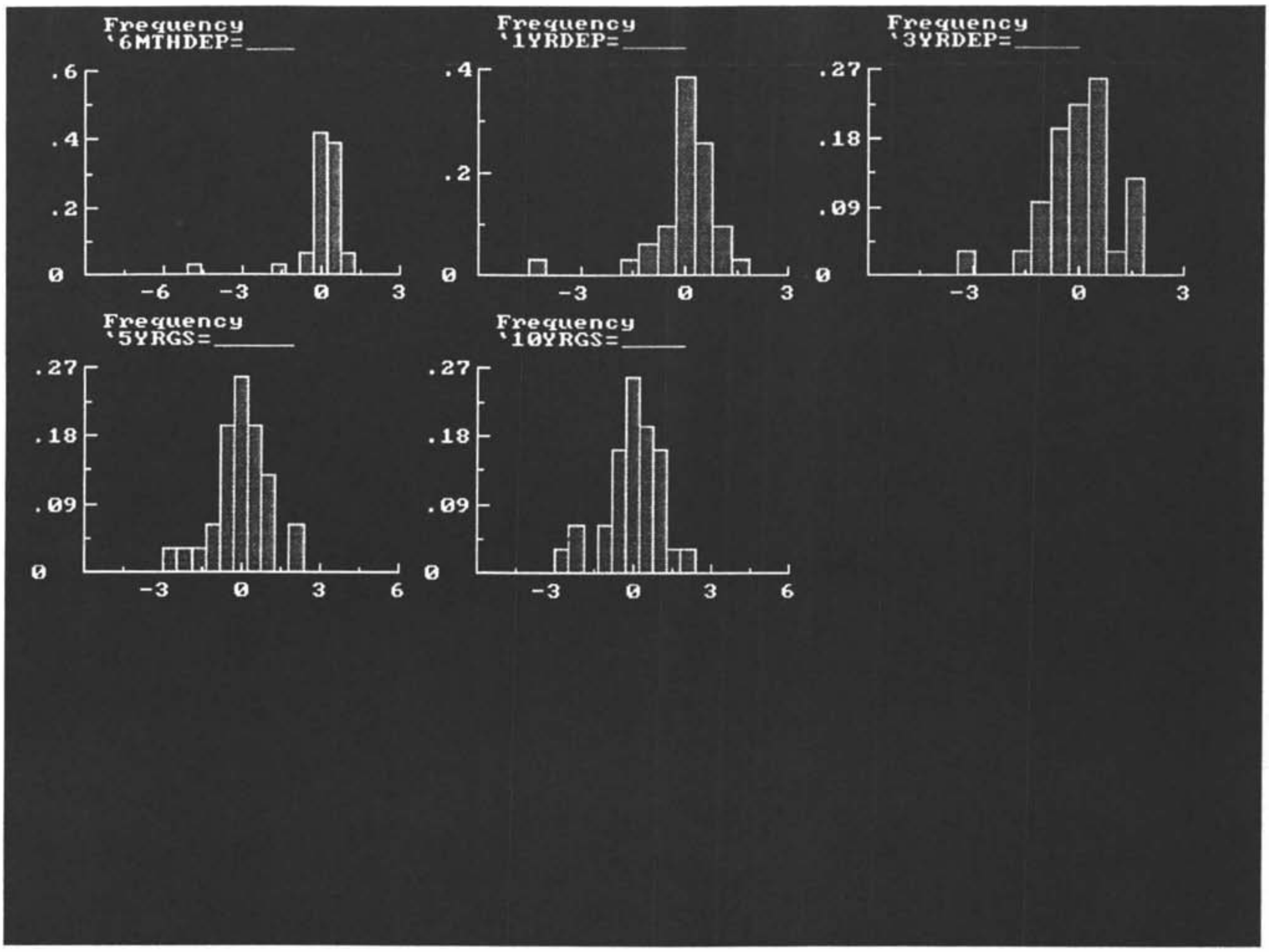
Real Data

		Mean	Standard Deviation	Skewness	Kurtosis	Normality χ^2
1	6 Month Deposit	0.006345	0.014045	-3.168709	12.152569	62.638 **
2	1 Year Deposit	0.00682	0.015391	-2.091355	6.527333	19.235 **
3	3 Year Deposit	0.007967	0.020882	-0.568169	0.56797	2.9379
4	5 Year Government Stock	0.012793	0.026588	-0.31568	0.208125	1.5875
5	10 Year Government Stock	0.014851	0.032216	-0.606455	0.331889	2.5051

** Statistically different from normal 1% level.

Distribution of Returns - Fixed Interest Time Deposit

Real Data



APPENDIX SIX

Unit Root Test Results

Selected Companies

		Nominal Data		Real Data	
		T-adf*	α	T-adf**	α
1	Air New Zealand	-9.708	0.057782		
2	Bank of New Zealand	-12.403	0.085328		
3	BNZ Finance	-16.819	0.070891	-7.2071	0.17934
4	Brierley Investments	-12.632	0.064705	-6.7704	0.19169
5	Carter Holt Harvey	-13.109	0.069892	-6.0781	0.17492
6	Cavalier Corporation	-14.187	0.078956	-5.4164	0.26441
7	Ceramco Corporation	-11.327	0.072201	-4.0597	0.23753
8	Colonial Motors	-15.751	0.058694	-5.1915	0.14826
9	Corporate Investments	-15.491	0.11117	-5.1842	0.25205
10	DB Group	-13.237	0.067851	-4.794	0.18105
11	Donaghys	-13.889	0.05061	-7.2686	0.14957
12	Ernest Adams	-18.135	0.071097	-6.0194	0.17524
13	Elders Resources NZFP	-11.517	0.065515		
14	Enerco New Zealand	-9.5025	0.047163		
15	Fay Richwhite	-14.46	0.1231	-6.2718	0.25344
16	Fernz Corporation	-17.532	0.077994	-6.0225	0.2091
17	Fisher & Paykel	-15.487	0.062499	-5.7213	0.16824
18	Fletcher Challenge	-12.628	0.060769	-8.5185	0.15918
19	Goodman Fielder	-14.335	0.06138	-6.409	0.15695
20	Guinness Peat Group	-9.1038	0.061804		
21	Hallenstein Glasson	-16.865	0.097654	-4.243	0.25645
22	Independent Newspapers	-13.716	0.057502	-6.4375	0.15468
23	Jarden Corporation	-10.476	0.10704		
24	Lion Nathan	-14.685	0.061892	-5.7328	0.15234
25	Macraes Mining	-10.407	0.081959		
26	Mair Astley	-15.148	0.10429	-4.6025	0.32199
27	Milburn New Zealand	-14.929	0.059052	-6.0147	0.18689
28	Natural Gas Corporation	-7.6477	0.070308		

Unit Root Test Results, Selected Companies Cont....

		Nominal Data		Real Data	
		T-adf*	α	T-adf**	α
29	New Zealand Oil & Gas	-15.775	0.10876	-6.4698	0.22313
30	New Zealand Refining	-17.365	0.074629	-4.918	0.19133
31	Owens Group	-14.445	0.085135	-4.7639	0.20179
32	PDL Holdings	-12.829	0.084239	-3.7876	0.31699
33	Progressive Enterprises	-9.5142	0.046305		
34	Salmond Smith Biolab	-14.754	0.091316	-4.3519	0.26303
35	Sanford	-15.442	0.065407	-5.3753	0.17496
36	Southern Petroleum	-14.574	0.14743	-6.9125	0.32536
37	Steel & Tube	-15.956	0.088472	-3.8422	0.23111
38	Telecom Corporation	-9.221	0.043255		
39	Trans Tasman Properties	-18.928	0.15985	-6.4733	0.3577
40	Whitcoulls Group	-10.938	0.094502		
41	Wilson Neill	-18.742	0.16136	-4.6576	0.39516
42	Wilson & Horton	-12.561	0.053539	-6.5163	0.16671

*	Critical Values	5% = 2.874, 1% = 3.461 except for the following:
	Air New Zealand	5% = -2.88, 1% = -3.474
	Bank of New Zealand	5% = -2.881, 1% = -3.476
	Elders Resources NZFP	5% = -2.889, 1% = -3.494
	Enerco New Zealand	5% = -2.9, 1% = -3.517
	Guinness Peat Group	5% = -2.89, 1% = -3.496
	Jarden Corporation	5% = -2.885, 1% = -3.485
	Macraes Mining	5% = -2.881, 1% = -3.476
	Natural Gas Corporation	5% = -2.901, 1% = -3.52
	Progressive Enterprises	5% = -2.894, 1% = -3.505
	Telecom Corporation	5% = -2.888, 1% = -3.492
	Whitcoulls Group	5% = -2.89, 1% = -3.495
**	Critical Values	5% = -2.95, 1% = -3.635

All lags = 0

Unit Root Test Results

Stock Market Sectors

		Nominal Data		Real Data	
		T-adf*	α	T-adf**	α
1	Agriculture	-7.8548	0.084317	-4.1051	0.16991
2	Automotive	-9.7354	0.085221	-4.3379	0.14942
3	Building	-7.8075	0.083856	-5.107	0.18203
4	Chemicals	-7.8466	0.078851	-5.2372	0.19308
5	Electrical	-9.1482	0.091365	-4.6453	0.17262
6	Energy & Fuel	-8.9093	0.077286	-5.3613	0.14526
7	Engineering	-8.2792	0.084212	-3.8549	0.16596
8	Finance & Banks	-8.9045	0.097404	-6.5666	0.16944
9	Food	-7.3973	0.071406	-5.1909	0.15877
10	Forestry	-9.8077	0.084607	-9.115	0.12957
11	Investment	-7.652	0.091829	-6.3631	0.18441
12	Liquor & Tobacco	-10.495	0.066555	-6.5717	0.096827
13	Meat & By-products	-10.439	0.10608	-5.2135	0.19683
14	Media & Communication	-8.6074	0.070403	-5.2237	0.14137
15	Medical Supplies	-8.6409	0.099568	-3.1469	0.1833
16	Miscellaneous	-9.0269	0.085722	-4.0638	0.16546
17	Property	-9.4053	0.10828	-6.8698	0.18016
18	Retailers	-9.6555	0.091308	-3.4328	0.15753
19	Textile & Apparel	-10.192	0.090812	-4.9236	0.16751
20	Transport & Tourism	-8.6879	0.08231	-6.1897	0.1757

* Critical Values 5% = -2.889, 1% = -3.494

** Critical Values 5% = -2.95, 1% = -3.635

All lags = 0

Unit Root Test Results

Managed Funds: New Zealand Fixed Interest

		Nominal Data		Real Data	
		T-adf*	α	T-adf**	α
1	AMP Fixed Interest Security	-7.6332	0.008487	-3.2211	0.015286
2	ANZ New Zealand Fixed Interest	-6.9492	0.007883	-4.0722	0.016338
3	BNZ New Zealand Strategic Bonds	-7.6202	0.008598	-3.1018	0.015981
4	Joseph Banks New Zealand Bonds	-10.449	0.013061	-3.3671	0.018421
5	National Bank Income	-7.9975	0.009976	-3.396	0.018408
6	National Mutual Life Fixed Interest	-10.667	0.012123	-4.3299	0.020207
7	New Zealand Funds Management Fixed Interest	-10.083	0.011944	-5.4742	0.018499
8	Prudential Income Trust	-9.2387	0.010442	-3.1757	0.015586
9	Sovereign New Zealand Fixed Interest	-11.039	0.015517	-3.2218	0.016512

* Critical Values 5% = -2.896, 1% = -3.51

** Critical Values 5% = -2.975, 1% = -3.696

All lags = 0

Unit Root Test Results

Managed Funds: New Zealand Equities

		Nominal Data		Real Data	
		T-adf*	α	T-adf**	α
1	AMP Share Fund	-9.0146	0.041929	-4.97	0.07374
2	BNZ New Zealand Blue Chip	-9.2596	0.052025	-4.7145	0.087008
3	Guardian Assurance Equity	-8.1422	0.033005	-3.5477	0.060774
4	Guardian New Zealand Equity	-9.4277	0.05587	-4.6648	0.085221
5	Joseph Banks New Zealand Equity	-8.5178	0.049815	-5.1495	0.091793
6	Joseph Banks New Zealand Equity Imputation	-7.8548	0.05329	-4.308	0.10517
7	National Mutual Life Share	-9.5174	0.045847	-4.6255	0.084778
8	New Zealand Funds Management Equity	-8.6379	0.046848	-4.9393	0.085342
9	Prudential New Zealand Equity	-9.2106	0.048994	-4.9441	0.081674
10	Sovereign New Zealand Equity	-8.2974	0.043978	-4.9797	0.077772
11	Tower New Zealand Equity	-8.2199	0.047194	-4.5019	0.088094

* Critical Values 5% = -2.896, 1% = -3.51

** Critical Values 5% = -2.975, 1% = -3.696

All lags = 0

Unit Root Test Results

Managed Funds: New Zealand Balanced

		Nominal Data		Real Data	
		T-adf*	α	T-adf**	α
1	AMP Managed Fund Balanced(B)	-9.8887	0.016646	-4.8984	0.030075
2	AMP Managed Fund Balanced (C)	-10.227	0.017464	-4.9435	0.028985
3	AMP Managed Fund Balanced (M)	-10.174	0.017244	-4.6901	0.030129
4	ANZ Growth Trust	-8.5454	0.025859	-4.9835	0.048618
5	ANZ Life Managed Fund	-8.3435	0.016621	-4.0356	0.032322
6	BNZ Balanced	-9.3298	0.015556	-4.2872	0.028211
7	Colonial Mutual Life Market Linked	-9.6389	0.020266	-5.0883	0.034598
8	Countrywide Bank Kiwi Trust	-10.004	0.024213	-5.1118	0.040316
9	Countrywide Bank Life Multi Fund	-8.929	0.014259	-5.3454	0.027594
10	Guardian Balanced	-9.0427	0.019502	-4.3274	0.032932
11	Guardian Assurance Balanced	-8.9774	0.018868	-1.4954	0.11157
12	Joseph Banks Asset Growth	-8.9133	0.018106	-4.6632	0.033858
13	Joseph Banks Capital	-10.934	0.015585	-4.4843	0.022409
14	Joseph Banks Growth	-9.056	0.022823	-4.7892	0.042071
15	National Bank Fund of Funds Balanced	-9.2586	0.015357	-4.126	0.026561
16	National Mutual Life Balanced	-11.334	0.02349	-4.3781	0.043366
17	Norwich Life Global	-9.2421	0.014924	-4.1566	0.028165
18	New Zealand Funds Management Balanced	-9.1364	0.018757	-4.5601	0.03361
19	Oceanic Managed	-9.1096	0.014193	-2.6389	0.021411
20	Prudential Balanced Growth	-9.379	0.021111	-4.549	0.034546
21	Prudential Beaver	-8.9434	0.019855	-6.2584	0.036055
22	Prudential Stag	-9.314	0.022769	-5.9259	0.039191
23	Southpac Balanced	-9.0084	0.014944	-4.2037	0.028413
24	Sovereign Balanced Growth	-7.726	0.021396	-4.7104	0.045235
25	Sovereign Conservative	-7.5649	0.015545	-4.2607	0.030856
26	Sovereign High Growth	-7.6175	0.024671	-4.9809	0.051088

Unit Root Test Results,

Managed Funds: New Zealand Balanced Cont....

		Nominal Data		Real Data	
		T-adf*	α	T-adf**	α
27	Sun Alliance Bond Managed	-7.6202	0.017541	-4.34	0.037838
28	Tower Multi Sector	-9.4194	0.016702	-5.8141	0.030558
29	Westpac Balanced	-8.216	0.010878	-3.8839	0.019646
30	Westpac Life Investment	-8.8221	0.012572	-3.841	0.023923
31	Westpac Retirement Balanced	-8.5856	0.015939	-4.038	0.028765

* Critical Values 5% = -2.896, 1% = -3.51

** Critical Values 5% = -2.975, 1% = -3.696

All lags = 0

Unit Root Test Results

Fixed Interest Term Deposit

		Nominal Data		Real Data	
		T-adf*	α	T-adf**	α
1	6 Month Deposit	-4.4249	0.001942	-6.3597	0.014448
2	1 Year Deposit	-8.2074	0.00413	-5.9346	0.015485
3	3 Year Deposit	-11.67	0.009583	-5.9325	0.019382
4	5 Year Government Stock	-6.3154	0.008751	-5.5658	0.024826
5	10 Year Government Stock	-6.2898	0.012502	-4.3843	0.032654

* Critical Values 5% = -2.889, 1% = -3.494 except for the following:

10 year Government Stock 5% = -2.893, 1% = -3.502

** Critical Values 5% = -2.963, 1% = -3.666 except for the following:

10 year Government Stock 5% = -2.971, 1% = -3.684

All lags = 0

APPENDIX SEVEN

Optimal Portfolio Results

STOCK PORTFOLIOS PERIOD JANUARY 1987 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	1.33803	1.902268	5.642377	27, 30	8, 92
0.2	0.868702	1.734538	4.329185	27, 30	39, 61
0.3	0.476792	1.502623	3.419434	3, 11, 16, 27, 30, 32, 36, 37	4, 7, 2, 35, 39, 7, 1, 5
0.4	0.184883	1.196728	2.529613	3, 8, 11, 16, 25, 27, 30, 32, 35, 36, 37	4, 6, 11, 3, 14, 23, 25, 7, 3, 1, 3

STOCK PORTFOLIOS PERIOD JANUARY 1988 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	1.470249	1.989876	7.008707	30	100
0.2	1.073534	1.754199	3.403326	6, 21, 27, 30, 36, 37	2, 1, 33, 51, 3, 10
0.3	0.748286	1.69572	3.15811	6, 27, 30, 36, 37	6, 38, 40, 4, 12
0.4	0.446906	1.646525	2.999046	6, 27, 30, 32, 36, 37	7, 36, 34, 6, 4, 13
0.5	0.207176	1.367396	2.320439	6, 8, 11, 12, 27, 30, 32, 35, 36, 37	3, 10, 14, 4, 27, 24, 5, 3, 2, 8

STOCK PORTFOLIOS PERIOD JANUARY 1989 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	1.353367	1.733732	3.803645	21, 27, 30	21, 27, 52
0.2	1.016937	1.650677	3.1687	21, 27, 30, 32, 37	8, 34, 39, 10, 9
0.3	0.709552	1.555386	2.819444	11, 21, 27, 30, 31, 32, 36, 37	6, 3, 33, 32, 2, 12, 2, 10
0.4	0.453974	1.389929	2.339886	8, 11, 27, 30, 31, 32, 36, 37	5, 17, 28, 23, 5, 9, 4, 9
0.5	0.231829	1.299326	2.134991	8, 11, 27, 30, 31, 32, 36, 37	11, 20, 24, 19, 5, 8, 5, 8

STOCK PORTFOLIOS PERIOD JANUARY 1990 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	1.294511	1.801993	5.074819	21, 27, 30, 37	27, 19, 52, 2
0.2	0.854685	1.631897	3.886057	21, 27, 30, 31, 32, 37	11, 37, 35, 7, 6, 4
0.3	0.494518	1.519602	3.416947	21, 27, 30, 31, 32, 36, 37	6, 38, 26, 16, 7, 3, 4
0.4	0.169804	1.390064	3.050649	11, 25, 27, 30, 31, 32, 36, 37	9, 2, 35, 22, 18, 6, 4, 4

STOCK PORTFOLIOS PERIOD JANUARY 1991 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	1.966234	2.610285	6.440506	21, 37	47, 53
0.2	1.419309	2.320529	4.506101	21, 27, 32, 37	26, 32, 10, 32
0.3	0.998655	2.162467	3.879371	21, 27, 31, 32, 37	18, 41, 7, 10, 24
0.4	0.635397	2.017983	3.456467	21, 27, 31, 32, 36, 37	11, 42, 15, 8, 4, 20
0.5	0.301927	1.937009	3.270163	21, 27, 30, 31, 32, 36, 37	8, 41, 3, 18, 8, 5, 17

STOCK PORTFOLIOS PERIOD JANUARY 1992 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	1.807105	2.409156	6.020492	3, 27, 37	7, 27, 66
0.2	1.329953	2.194576	4.323116	3, 27, 37	5, 60, 35
0.3	0.925241	2.109701	3.948199	3, 27, 32, 37	4, 62, 9, 25
0.4	0.535519	2.079458	3.859848	3, 27, 32, 37	4, 63, 11, 22
0.5	0.175067	1.962674	3.575215	3, 18, 27, 32, 37	3, 8, 59, 11, 19

STOCK PORTFOLIOS PERIOD JANUARY 1987 - DECEMBER 1991					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	1.697204	2.49109	7.956989	30	100
0.2	0.959555	2.286324	6.633843	16, 30	22, 78
0.3	0.378489	1.794032	4.718479	11, 12, 16, 27, 30, 35	9, 8, 19, 11, 48, 5

STOCK PORTFOLIOS PERIOD JANUARY 1993 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	1.31354	1.93188	6.183395	36, 14	31, 69
0.2	0.805092	1.664193	4.295508	14, 18, 25, 33, 36	32, 19, 9, 24, 16
0.3	0.395912	1.585833	3.966403	14, 18, 25, 33, 36	22, 21, 18, 28, 11

STOCK PORTFOLIOS PERIOD JANUARY 1988 - DECEMBER 1992					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	2.02304	2.660743	6.377023	30, 36	97, 3
0.2	1.47681	2.471061	4.971257	6, 21, 30, 32, 36	17, 3, 65, 12, 3
0.3	1.013487	2.258796	4.151029	6, 16, 21, 27, 30, 31, 32, 35, 36	15, 8, 2, 4, 48, 2, 14, 5, 2
0.4	0.627083	2.089593	3.656282	6, 16, 27, 30, 31, 32, 35, 36	12, 10, 12, 38, 3, 12, 11, 2
0.5	0.308907	1.841425	3.065029	6, 12, 16, 27, 30, 32, 35, 36	8, 18, 10, 11, 31, 11, 10, 1
0.6	0.040813	1.714026	2.788688	6, 11, 12, 16, 27, 30, 32, 35, 36	4, 13, 20, 9, 11, 25, 9, 8, 1

STOCK PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1993					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	2.087742	2.568673	4.809305	21, 30, 32	24, 60, 16
0.2	1.629228	2.472439	4.216056	21, 27, 30, 32, 34	14, 7, 52, 24, 3
0.3	1.239579	2.317972	3.594644	21, 27, 30, 32, 34, 35	7, 18, 41, 21, 7, 6
0.4	0.904502	2.154198	3.124237	11, 21, 27, 30, 32, 34, 35, 36, 37	4, 2, 20, 32, 18, 7, 12, 3, 2
0.5	0.60739	2.043068	2.871355	11, 27, 30, 32, 34, 35, 36, 37	12, 21, 27, 15, 6, 12, 4, 3
0.6	0.33324	1.960145	2.711506	11, 27, 30, 31, 32, 34, 35, 36, 37	17, 20, 23, 5, 13, 4, 11, 4, 3
0.7	0.076255	1.823388	2.495904	8, 11, 27, 30, 31, 32, 34, 35, 36	8, 18, 18, 20, 4, 12, 4, 11, 5

STOCK PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1994					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	1.485277	2.083047	5.977705	3, 21, 27, 30	4, 20, 2, 74
0.2	1.007328	1.859758	4.262151	3, 21, 27, 30, 32, 37	4, 8, 35, 44, 5, 4
0.3	0.617128	1.706723	3.63198	3, 21, 27, 30, 31, 32, 36, 37	4,3, 37, 32, 9, 6, 4, 5
0.4	0.270074	1.58551	3.288572	3, 11, 25, 27, 30, 31, 32, 36, 37	3, 5, 2, 36, 26, 13, 6, 5, 4

STOCK PORTFOLIOS PERIOD JANUARY 1987 - DECEMBER 1989					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	1.448831	1.974151	5.253196	10, 16, 30	29, 30, 41
0.2	0.95261	1.866679	4.570348	10, 12, 16, 29, 30	33, 12, 24, 1, 30
0.3	0.513796	1.796155	4.274528	10, 12, 16, 29, 30	32, 20, 21, 3, 24
0.4	0.09771	1.734384	4.091684	6, 10, 12, 16, 29, 30	4, 31, 22, 19, 3, 21

STOCK PORTFOLIOS PERIOD JANUARY 1988 - DECEMBER 1990					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	1.742146	2.417105	6.749598	30, 36	90, 10
0.2	0.430393	1.826682	6.397027	6, 29, 30, 36	47, 18, 14, 21
0.3	0.547172	1.872745	4.418578	6, 12, 16, 29, 30, 36	10, 17, 8, 7, 55, 3
0.4	0.156556	1.604373	3.619543	6, 12, 16, 27, 29, 30, 31	8, 26, 9, 9, 6, 40, 2

STOCK PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1991					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	1.75841	2.403073	6.446636	6, 30	19, 81
0.2	1.147459	2.281098	5.668195	6, 30, 35	28, 63, 9
0.3	0.661441	1.931346	4.233014	6, 16, 25, 30, 35	15, 10, 14, 44, 17
0.4	0.249693	1.741391	3.729247	6, 11, 27, 30, 31, 35, 36	9, 24, 9, 31, 10, 13, 4

STOCK PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1992					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	2.377308	2.982318	6.050101	21, 30, 32	23, 50, 27
0.2	1.784846	2.957609	5.863812	21, 30, 32	15, 56, 29
0.3	1.248488	2.680388	4.772996	21, 30, 31, 32, 34, 35	8, 45, 5, 24, 6, 12
0.4	0.818152	2.376566	3.896035	16, 21, 27, 30, 31, 32, 34, 35, 36	3, 4, 6, 34, 11, 18, 5, 16, 3
0.5	0.453787	2.177726	3.447879	16, 21, 25, 27, 30, 31, 32, 34, 35, 36	4, 2, 4, 10, 29, 12, 16, 4, 15, 4
0.6	0.11196	2.077696	3.276226	25, 27, 30, 31, 32, 34, 35, 36	6,13, 26, 14, 14, 5, 17, 5

STOCK PORTFOLIOS PERIOD JANUARY 1991 - DECEMBER 1993					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	3.512724	4.172229	6.595041	21, 32, 37	44, 26, 30
0.2	2.866851	4.106197	6.196733	21, 32, 34, 37	34, 29, 7, 30
0.3	2.32148	3.744624	4.743811	21, 27, 32, 34, 37	21, 22, 24, 12, 21
0.4	1.883146	3.501539	4.045982	21, 25, 27, 30, 32, 34, 36, 37	13, 3, 29, 3, 20, 12, 3, 17
0.5	1.507883	3.277575	3.539384	6, 21, 25, 27, 30, 31, 32, 34, 36, 37	3, 8, 5, 30, 5, 6, 16, 9, 5, 13
0.6	1.165806	3.162499	3.327821	6, 21, 25, 27, 30, 31, 32, 34, 36, 37	5, 5, 6, 30, 6, 9, 14, 8, 6, 11
0.7	0.853122	2.959758	3.00948	11, 21, 25, 27, 30, 31, 32, 34, 36, 37	10, 2, 6, 30, 7, 9, 12, 6, 7, 11
0.8	0.558519	2.865381	2.883576	11, 25, 27, 30 31, 32, 34, 36, 37	13, 30, 6, 8, 10, 11, 5, 7, 10
0.9	0.272725	2.82471	2.835541	11, 25, 27, 30, 31, 32, 34, 36, 37	15, 6, 30, 7, 11, 10, 4, 7, 10
1	0.000121	2.765452	2.764239	11, 25, 27, 30, 31, 32, 34, 35, 36, 37	15, 6, 29, 7, 10, 9, 4, 4, 7, 9

STOCK PORTFOLIOS PERIOD JANUARY 1992 - DECEMBER 1994					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	2.081159	2.858189	7.770294	32, 37, 40	9, 79, 12
0.2	1.485279	2.40781	4.612657	27, 32, 37, 40	42, 18, 33, 7
0.3	1.052858	2.298204	4.151151	27, 32, 37, 40	54, 17, 24, 5
0.4	0.647155	2.245813	3.996646	21, 27, 32, 37, 40	3, 58, 16, 19, 4
0.5	0.254429	2.171634	3.834408	18, 21, 27, 32, 37, 40	4, 4, 58, 15, 17, 2

STOCK PORTFOLIOS (REAL VALUES) PERIOD JANUARY 1987 - SEPTEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	9.867044	11.73234	18.65299	30	100
0.2	8.001746	11.73234	18.65299	30	100
0.3	6.145579	11.55446	18.02962	27, 30	6, 94
0.4	4.411052	11.15148	16.85108	27, 30, 32	18, 80, 2
0.5	2.753065	10.93761	16.36909	27, 30, 32	24, 73, 3
0.6	1.164859	10.20681	15.06992	27, 30, 32, 35	26, 64, 1, 9

STOCK PORTFOLIOS (REAL VALUES) PERIOD JANUARY 1988 - SEPTEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Companies	Proportion %
0.1	10.53304	12.25619	17.2316	30	100
0.2	8.809874	12.25619	17.2316	30	100
0.3	7.142645	11.8756	15.776501	27, 30, 32	9, 87, 4
0.4	5.64257	11.37828	14.339274	6, 27, 30, 32, 36	2, 20, 70, 4, 4
0.5	4.250213	11.06548	13.630524	6, 27, 30, 32, 36, 37	3, 25, 60, 3, 6, 3
0.6	2.905529	10.88402	13.29748	6, 27, 30, 32, 36, 37	5, 28, 54, 2, 7, 4
0.7	1.70574	9.590714	11.264248	27, 30, 35, 36, 37	26, 42, 20, 8, 4
0.8	0.558951	9.272322	10.891717	27, 30, 31, 35, 36	28, 39, 5, 20, 8

SECTOR PORTFOLIOS PERIOD JANUARY 1987 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	2.521675	3.301095	7.794203	6	100
0.2	1.742255	3.301095	7.794203	6	100
0.3	0.992407	3.122134	7.09909	3, 4, 6	1, 14, 85
0.4	0.312399	2.935159	6.5569	3, 4, 6, 7	7, 20, 70, 3

SECTOR PORTFOLIOS PERIOD JANUARY 1988 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	3.219115	3.976656	7.575418	6	100
0.2	2.461573	3.976656	7.575418	6	100
0.3	1.710672	3.879043	7.227904	3, 6	7, 93
0.4	1.033803	3.573522	6.349295	3, 4, 6, 7	16, 7, 73, 4
0.5	0.379009	3.39438	6.200526	4, 6, 7	7, 67, 26

SECTOR PORTFOLIOS PERIOD JANUARY 1989 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	3.123895	3.876691	7.527973	6	100
0.2	2.371214	3.86615	7.474674	3, 6	1, 99
0.3	1.659524	3.728572	6.896823	3, 6	82, 18
0.4	0.981471	3.621117	6.599116	3, 4, 6, 7	24, 1, 73, 2
0.5	0.342984	3.454599	6.223228	3, 4, 6, 7	24, 7, 63, 6

SECTOR PORTFOLIOS PERIOD JANUARY 1990 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	3.33462	4.069217	7.345974	6	100
0.2	2.600031	4.066204	7.330865	3, 6	1, 99
0.3	1.909764	3.902948	6.643947	3, 6	19, 81
0.4	1.257381	3.814438	6.392894	3, 6, 7	26, 73, 1
0.5	0.639882	3.603921	5.928076	3, 5, 6, 7	28, 5, 63, 4
0.6	0.06513	3.435894	5.617939	3, 5, 6, 7, 16	28, 10, 56, 3, 3

SECTOR PORTFOLIOS PERIOD JANUARY 1991 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	3.212698	3.808495	5.957975	3, 6, 7	24, 46, 30
0.2	2.631309	3.763241	5.659657	3, 6, 7, 18	31, 35, 25, 9
0.3	2.071469	3.738958	5.558294	3, 6, 7, 18	33, 31, 22, 14
0.4	1.5176	3.727234	5.524083	3, 6, 7, 18	34, 29, 21, 16
0.5	0.966061	3.720292	5.508461	3, 6, 7, 18	35, 27, 20, 18
0.6	0.415675	3.715694	5.50003	3, 6, 7, 18	35, 27, 19, 19

SECTOR PORTFOLIOS PERIOD JANUARY 1992 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	3.272467	3.932418	6.599525	3, 7	70, 30
0.2	2.614264	3.928926	6.573311	3, 7	66, 34
0.3	1.958557	3.888534	6.433255	3, 5, 7	64, 3, 33
0.4	1.337348	3.759728	6.05595	3, 5, 7	61, 13, 26
0.5	0.74038	3.691589	5.902419	3, 5, 7	59, 19, 22
0.6	0.198772	3.492239	5.489111	3, 5, 7, 20	52, 15, 14, 19

SECTOR PORTFOLIOS PERIOD JANUARY 1993 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	2.56382	3.295242	7.314224	3	100
0.2	1.839856	3.217938	6.890147	3, 14	87, 13
0.3	1.179786	3.106261	6.421581	3, 14	69, 31
0.4	0.548478	2.995247	6.11692	3, 14, 20	58, 36, 6

SECTOR PORTFOLIOS PERIOD JANUARY 1987 - DECEMBER 1991					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	2.867356	3.683967	8.166105	6	100
0.2	2.050746	3.683967	8.166105	6	100
0.3	1.285459	3.420672	7.117375	4, 6	20, 80
0.4	0.597008	3.284862	6.719636	4, 6	30, 70

SECTOR PORTFOLIOS PERIOD JANUARY 1988 - DECEMBER 1992					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	4.141052	4.9347	7.93648	6	100
0.2	3.347404	4.9347	7.93648	6	100
0.3	2.560432	4.781733	7.404333	4, 6	8, 92
0.4	1.879215	4.447521	6.420766	4, 6, 19	24, 74, 2
0.5	1.268219	4.186137	5.835834	4, 6, 9,	28, 63, 9
0.6	0.705795	3.972178	5.443971	3, 4, 6, 9	4, 28, 55, 13
0.7	0.173018	3.835452	5.23205	3, 4, 6, 9	7, 28, 50, 15

SECTOR PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1993					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	4.370572	5.158584	7.880076	6	100
0.2	3.582568	5.158584	7.880076	6	100
0.3	2.823336	4.913198	6.966205	3, 4, 6	4, 12, 84
0.4	2.163463	4.7025	6.34759	3, 4, 6	10, 21, 69
0.5	1.544074	4.574942	6.061735	3, 4, 6, 19	13, 23, 61, 3
0.6	0.953928	4.401613	5.746142	3, 4, 6, 9, 19	14, 22, 54, 6, 4
0.7	0.38966	4.279504	5.55692	3, 4, 6, 9, 19	15, 21, 49, 11, 4

SECTOR PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1994					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	3.565429	4.340767	7.753379	6	100
0.2	2.790091	4.340767	7.753379	6	100
0.3	2.030062	4.207928	7.259553	3, 6	11, 89
0.4	1.329364	4.011676	6.705778	3, 6, 16	21, 74, 5
0.5	0.684649	3.803695	6.238091	3, 6, 7, 15, 16	25, 60, 2, 2, 11
0.6	0.079165	3.605984	5.878031	3, 5, 6, 16	27, 7, 53, 13

SECTOR PORTFOLIOS PERIOD JANUARY 1987 - DECEMBER 1989					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	2.01569	2.80225	7.865605	4	100
0.2	1.229129	2.80225	7.865605	4	100
0.3	0.443462	2.77361	7.767161	4, 6	97, 3

SECTOR PORTFOLIOS PERIOD JANUARY 1988 - DECEMBER 1990					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	3.388475	4.137222	7.487483	6	100
0.2	2.639726	4.137222	7.487483	6	100
0.3	1.890977	4.137222	7.487483	6	100
0.4	1.155076	3.870574	6.788746	4, 6, 9,	4, 90, 6
0.5	0.530564	3.465126	5.869121	4, 6, 9,	11, 74, 15

SECTOR PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1991					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	4.468764	5.234306	7.65542	6	100
0.2	3.703221	5.234306	7.65542	6	100
0.3	2.937718	5.222323	7.615349	6, 9	99, 1
0.4	2.241796	4.856525	6.536821	6, 9	82, 18
0.5	1.611991	4.664285	6.104586	6, 9 ,19	73, 23, 4
0.6	1.0133	4.548652	5.892251	6, 9, 19	68, 26, 6
0.7	0.430477	4.472594	5.774451	6, 9, 19	64, 28, 8

SECTOR PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1992					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	4.985761	5.750806	7.650453	6	100
0.2	4.220714	5.750806	7.650453	6	100
0.3	3.45567	5.750806	7.650453	6	100
0.4	2.697108	5.628795	7.329219	6, 16,	93, 7
0.5	2.017964	5.116042	5.824388	3, 4, 6, 16	5, 11, 71, 13
0.6	1.449312	4.65264	5.338879	3, 4, 6, 9, 16	10, 14, 58, 6, 12
0.7	0.935664	4.418046	4.974831	3, 4, 6, 9, 16	13, 15, 51, 10, 11
0.8	0.387396	4.245865	4.823086	3, 5, 6, 9, 15	20, 8, 49, 16, 7
0.9	0.087552	4.422667	4.816795	3, 4, 6, 9, 18	15, 19, 49, 14, 3

SECTOR PORTFOLIOS PERIOD JANUARY 1991 - DECEMBER 1993					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	5.953372	6.792056	8.386816	15	100
0.2	5.298841	6.408019	5.545894	3, 15	31, 69
0.3	4.774732	6.272355	4.992078	3, 7, 15, 16	36, 4, 56, 4
0.4	4.289377	6.19122	4.754607	3, 7, 15, 16	36, 5, 49, 10
0.5	3.815116	6.148629	4.658362	3, 7, 15, 16	37, 5, 45, 13
0.6	3.356718	6.119317	4.604331	3, 7, 15, 16	37, 5, 42, 16
0.7	2.882235	6.105123	4.604126	3, 15, 16	38, 41, 21
0.8	2.42285	6.090764	4.584893	3, 15, 16	38, 40, 22
0.9	1.965052	6.079713	4.571845	3, 15, 16	38, 39, 23
1	1.508356	6.070936	4.56258	3, 15, 16	38, 38, 24
1.1	1.052455	6.063792	4.555762	3. 15, 16	38, 38, 24
1.2	0.597148	6.057863	4.550595	3, 15, 16	38, 37, 25
1.3	0.142298	6.052861	4.546587	3, 15, 16	38, 37, 25

SECTOR PORTFOLIOS PERIOD JANUARY 1992 - DECEMBER 1994					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	3.516577	4.202615	6.860386	3, 7	59, 41
0.2	2.831682	4.20033	6.843242	3, 7	62, 38
0.3	2.149588	4.152404	6.676053	3, 5, 7	63, 4, 34
0.4	1.502925	4.030124	6.317998	3, 5, 7	61, 13, 26
0.5	0.879395	3.964736	6.170681	3, 5, 7	60, 18, 22
0.6	0.285293	3.799291	5.856662	3, 5, 7, 20	55, 16, 16, 12

SECTOR PORTFOLIOS (REAL VALUES) PERIOD JANUARY 1987 - SEPTEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	7.466706	8.925342	14.58634	6	100
0.2	6.008076	8.925342	14.58634	6	100
0.3	4.549442	8.925342	14.58634	6	100
0.4	3.090808	8.925342	14.58634	6	100
0.5	1.640698	8.768135	14.254872	3, 4, 6	3, 2, 95
0.6	0.238911	8.534692	13.826301	3, 4, 6	7, 6, 87

SECTOR PORTFOLIOS (REAL VALUES) PERIOD JANUARY 1988 - SEPTEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Sectors	Proportion %
0.1	9.734721	11.01403	12.7931	6	100
0.2	8.455411	11.01403	12.7931	6	100
0.3	7.176104	11.01403	12.7931	6	100
0.4	5.896793	11.01403	12.7931	6	100
0.5	4.648542	10.73578	12.174481	3, 6,	8, 92
0.6	3.450316	10.5463	11.826635	3, 6,	13, 87
0.7	2.285679	10.31167	11.465705	3, 6, 7	15, 81, 3
0.8	1.161785	9.894659	10.916093	3, 6, 7, 9	15,.76, 5, 4
0.9	0.096564	9.481872	10.42812	3, 6, 7, 9	15, 71, 5, 9

FIXED INTEREST FUNDS PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Funds	Proportion %
0.1	0.622279	0.779012	1.567332	9	100
0.2	0.49378	0.709754	1.079865	7, 8, 9	13, 40, 47
0.3	0.392632	0.679792	0.957199	3, 7, 8, 9	22, 16, 33, 29
0.4	0.313061	0.645756	0.831738	1, 2, 8, 9	24, 37, 22, 17
0.5	0.230959	0.637282	0.812647	1, 2, 8, 9	25, 42, 21, 12
0.6	0.150233	0.631921	0.802813	1, 2, 8, 9	26, 45, 19, 10
0.7	0.070664	0.627952	0.796126	1, 2, 6, 8, 9	26, 46, 3, 17, 8

FIXED INTEREST FUNDS PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Funds	Proportion %
0.1	0.593749	0.748445	1.546951	9	100
0.2	0.458866	0.680014	1.105735	6, 8, 9	13, 35, 52
0.3	0.355014	0.654435	0.998069	6, 8, 9	13, 53, 34
0.4	0.265067	0.617635	0.881421	2, 6, 8	43, 8, 30, 19
0.5	0.178496	0.605261	0.853531	2, 6, 8, 9	55, 6, 25, 14
0.6	0.094708	0.591516	0.828012	1, 2, 6, 9	25, 59, 5, 11
0.7	0.012299	0.586827	0.820754	1, 2, 6, 9	25, 63, 4, 8

FIXED INTEREST FUNDS PORTFOLIOS PERIOD JANUARY 1991 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Funds	Proportion %
0.1	0.628747	0.732567	1.0382	9	100
0.2	0.524927	0.732567	1.0382	9	100
0.3	0.421107	0.732567	1.0382	9	100
0.4	0.317287	0.732567	1.0382	9	100
0.5	0.213467	0.732567	1.0382	9	100
0.6	0.109752	0.725122	1.025612	2, 9	4, 96
0.7	0.010116	0.690894	0.972537	2, 9	23, 77

FIXED INTEREST FUNDS PORTFOLIOS PERIOD JANUARY 1992 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Funds	Proportion %
0.1	0.511606	0.6135	1.018945	9	100
0.2	0.409711	0.6135	1.018945	9	100
0.3	0.307817	0.6135	1.018945	9	100
0.4	0.205922	0.6135	1.018945	9	100
0.5	0.104028	0.6135	1.018945	9	100
0.6	0.002133	0.6135	1.018945	9	100

FIXED INTEREST FUNDS PORTFOLIOS PERIOD JANUARY 1993 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Funds	Proportion %
0.1	0.433343	0.536472	1.031294	9	100
0.2	0.330214	0.536472	1.031294	9	100
0.3	0.227084	0.536472	1.031294	9	100
0.4	0.123955	0.536472	1.031294	9	100
0.5	0.020825	0.536472	1.031294	9	100

FIXED INTEREST FUNDS PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1993					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Funds	Proportion %
0.1	0.799659	0.9703	1.706414	9	100
0.2	0.682406	0.849279	0.834364	1, 7, 8, 9	32, 8, 29, 31
0.3	0.606539	0.821162	0.715411	1, 7, 8, 9	49, 11, 22, 18
0.4	0.537257	0.807431	0.675434	1, 5, 7, 8, 9	53, 5, 11, 19, 12
0.5	0.472785	0.794557	0.643545	1, 2, 5, 7, 8, 9	46, 16, 6, 7, 17, 8
0.6	0.408999	0.788912	0.633188	1, 2, 5, 7, 8, 9	45, 19, 7, 7, 16, 6
0.7	0.346005	0.785031	0.62718	1, 2, 5, 7, 8, 9	45, 21, 8, 6, 15, 5
0.8	0.283491	0.782188	0.623371	1, 2, 5, 7, 8,. 9	44, 23, 9, 6, 14, 4
0.9	0.22129	0.780009	0.620799	1, 2, 5, 7, 8, 9	44, 24, 9, 6, 14, 3
1	0.159306	0.778285	0.618978	1, 2, 5, 7, 8, 9	44, 25, 10, 5, 14, 2
1.1	0.097479	0.776885	0.617642	1, 2, 5, 7, 8, 9	44, 26, 10, 5, 13, 2
1.2	0.035767	0.775725	0.616631	1, 2, 5, 7, 8, 9	44, 26, 11, 5, 13, 1

FIXED INTEREST FUNDS PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1994					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Funds	Proportion %
0.1	0.585492	0.750167	1.646749	9	100
0.2	0.443915	0.672212	1.141481	6, 8, 9	10, 40, 50
0.3	0.33862	0.629764	0.970479	2, 6, 8, 9	24, 8, 40, 28
0.4	0.245642	0.606212	0.901421	2, 6, 8, 9	45, 6, 31, 18
0.5	0.157039	0.594083	0.874082	2, 6, 8, 9	56, 5, 26, 13
0.6	0.071564	0.578388	0.844701	1, 2, 6, 9	26, 58, 5, 11

FIXED INTEREST FUNDS PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1991					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Funds	Proportion %
0.1	0.843881	0.912918	0.690354	1, 7, 8, 9	53, 6, 23, 18
0.2	0.784604	0.895333	0.553642	1, 7, 8, 9	75, 6, 12, 7
0.3	0.7303	0.891162	0.536199	1, 7, 8, 9	81, 6, 9, 4
0.4	0.677065	0.888374	0.52827	1, 6, 7, 8, 9	83, 1, 6, 8, 2
0.5	0.624526	0.886086	0.523115	1, 6, 7, 8, 9	84, 3, 5, 6, 2
0.6	0.572363	0.884594	0.520383	1, 6, 7, 8, 9	84, 4, 5, 6, 1
0.7	0.520413	0.883541	0.518754	1, 6, 7, 8,. 9	85, 4, 5, 5, 1
0.8	0.468594	0.882758	0.517705	1, 6, 7, 8, 9	85, 5, 4, 5, 1
0.9	0.416861	0.882152	0.516989	1, 6, 7, 8, 9	85, 5, 4, 5, 1
1	0.365189	0.881668	0.516479	1, 6, 7, 8, 9	85, 5, 4, 5, 1
1.1	0.313616	0.880189	0.515067	1, 5, 6, 7, 8, 9	85, 1, 5, 4, 4, 1
1.2	0.262164	0.87898	0.514013	1, 5, 6, 7, 8, 9	84, 2, 6, 3, 4, 1
1.3	0.210806	0.877962	0.513198	1, 5, 6, 7, 8	84, 3, 6, 3, 4
1.4	0.15951	0.877315	0.512717	1, 5, 6, 7, 8	83, 4, 6, 3, 4
1.5	0.108259	0.876742	0.512322	1, 5, 6, 7, 8	83, 4, 6, 3, 4
1.6	0.057043	0.876241	0.511999	1, 5, 6, 7, 8	83, 4, 6, 3, 4
1.7	0.005857	0.875801	0.511731	1, 5, 6, 7, 8	82, 5, 6, 3, 4

FIXED INTEREST FUNDS PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1992					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Funds	Proportion %
0.1	0.77019	0.921916	1.517257	6, 8, 9	8, 17, 75
0.2	0.668668	0.834407	0.828691	1, 6, 8, 9	41, 6, 31, 22
0.3	0.58729	0.798687	0.704648	1, 3, 9	50, 41, 9
0.4	0.520614	0.792311	0.679242	1, 2, 3, 8, 9	40, 27, 18, 10, 5
0.5	0.453862	0.78063	0.653528	1, 2, 3, 5, 9	38, 18, 39, 4, 1
0.6	0.388954	0.776963	0.646676	1, 2, 3, 5	36, 18, 39, 7
0.7	0.324348	0.77622	0.64553	1, 2, 3, 5	35, 19, 38, 8
0.8	0.259835	0.775665	0.644786	1, 2, 3, 5	34, 20, 37, 9
0.9	0.195383	0.775235	0.644274	1, 2, 3, 5	34, 19, 37, 10
1	0.130974	0.774891	0.643909	1, 2, 3, 5	33, 20, 36, 11
1.1	0.066596	0.77461	0.643645	1, 2, 3, 5	33, 20, 36, 11
1.2	0.002242	0.774376	0.643591	1, 2, 3, 5	33, 20, 36, 11

FIXED INTEREST FUNDS PORTFOLIOS PERIOD JANUARY 1991 - DECEMBER 1993					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Funds	Proportion %
0.1	0.925206	1.020417	0.95211	9	100
0.2	0.829995	1.020417	0.95211	9	100
0.3	0.734784	1.020417	0.95211	9	100
0.4	0.639573	1.020417	0.95211	9	100
0.5	0.544362	1.020417	0.95211	9	100
0.6	0.449786	1.011187	0.935665	6, 9	4, 96
0.7	0.359187	0.970442	0.873218	1, 6, 8, 9	8, 4, 11, 77
0.8	0.274852	0.928857	0.817502	1, 2, 6, 8, 9	11, 9, 2, 20, 58
0.9	0.194911	0.900494	0.783977	1, 2, 6, 8, 9	13, 15, 1, 26, 45
1	0.117664	0.880108	0.762436	1, 2, 8, 9	14, 20, 30, 36
1.1	0.042148	0.864881	0.747903	1, 2, 8, 9	14, 23, 33, 30

FIXED INTEREST FUNDS PORTFOLIOS PERIOD JANUARY 1992 - DECEMBER 1994					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Funds	Proportion %
0.1	0.466329	0.571389	1.050599	9	100
0.2	0.361629	0.571389	1.050599	9	100
0.3	0.256209	0.571389	1.050599	9	100
0.4	0.15115	0.571389	1.050599	9	100
0.5	0.04609	0.571389	1.050599	9	100

FIXED INTEREST FUNDS PORTFOLIOS (REAL VALUES) PERIOD JANUARY 1989 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Funds	Proportion %
0.1	1.363268	1.547929	1.846604	9	100
0.2	1.178608	1.547929	1.846604	9	100
0.3	0.993947	1.547929	1.846604	9	100
0.4	0.809287	1.547929	1.846604	9	100
0.5	0.624879	1.537152	1.824543	7, 9	3, 97
0.6	0.446066	1.501393	1.758877	7, 9	11, 89
0.7	0.272153	1.47794	1.72255	7, 9	16, 84
0.8	0.101057	1.460704	1.699505	7, 9	20, 80

EQUITY FUNDS PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Equity Funds	Proportion %
0.1	0.806303	1.1375	3.31197	3	100
0.2	0.475106	1.1375	3.31197	3	100
0.3	0.143909	1.1375	3.31197	3	100

EQUITY FUNDS PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Equity Funds	Proportion %
0.1	0.672412	1.010097	3.376851	3	100
0.2	0.334727	1.010097	3.376851	3	100

EQUITY FUNDS PORTFOLIOS PERIOD JANUARY 1991 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Equity Funds	Proportion %
0.1	1.250922	1.569917	3.189952	3	100
0.2	0.931926	1.569917	3.189952	3	100
0.3	0.612931	1.569917	3.189952	3	100
0.4	0.293936	1.569917	3.189952	3	100

EQUITY FUNDS PORTFOLIOS PERIOD JANUARY 1992 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Equity Funds	Proportion %
0.1	0.912929	1.233021	3.200914	3	100
0.2	0.592838	1.233021	3.200914	3	100
0.3	0.272747	1.233021	3.200914	3	100

EQUITY FUNDS PORTFOLIOS PERIOD JANUARY 1993 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Equity Funds	Proportion %
0.1	0.731248	1.103389	3.721407	3	100
0.2	0.359108	1.103389	3.721407	3	100

EQUITY FUNDS PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1993					
Risk Factor	Lambda	Mean	Standard Deviation	Equity Funds	Proportion %
0.1	1.14656	1.500808	3.542482	3, 6	91, 9
0.2	0.802084	1.488267	3.430915	3	100
0.3	0.458992	1.488267	3.430915	3	100
0.4	0.115901	1.488267	3.430915	3	100

EQUITY FUNDS PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1994					
Risk Factor	Lambda	Mean	Standard Deviation	Equity Funds	Proportion %
0.1	0.659924	1.020133	3.602092	3	100
0.2	0.299715	1.020133	3.602092	3	100

EQUITY FUNDS PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1991					
Risk	Lambda	Mean	Standard Deviation	Equity Funds	Proportion %
0.1	0.660502	1.010139	3.496374	3	100
0.2	0.310864	1.010139	3.496374	3	100

EQUITY FUNDS PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1992					
Risk Factor	Lambda	Mean	Standard Deviation	Equity Funds	Proportion %
0.1	0.715094	1.065028	3.499341	3	100
0.2	0.36516	1.065028	3.499341	3	100
0.3	0.015226	1.065028	3.499341	3	100

EQUITY FUNDS PORTFOLIOS PERIOD JANUARY 1991 - DECEMBER 1993					
Risk Factor	Lambda	Mean	Standard Deviation	Equity Funds	Proportion %
0.1	2.156626	2.504663	3.480369	3, 6	49, 51
0.2	1.827753	2.467876	3.200614	3, 6	79, 21
0.3	1.510393	2.457146	3.155843	3, 6	88, 12
0.4	1.195682	2.451926	3,140612	3, 6	92, 8
0.5	0.882009	2.448826	3.133636	3, 6	95, 5
0.6	0.56885	2.446769	3.129865	3, 6	97, 3
0.7	0.255985	2.445305	3.127601	3. 6	98, 2

EQUITY FUNDS PORTFOLIOS PERIOD JANUARY 1992 - DECEMBER 1994					
Risk Factor	Lambda	Mean	Standard Deviation	Equity Funds	Proportion %
0.1	0.970985	1.324056	3.530702	3	100
0.2	0.617915	1.324056	3.530702	3	100
0.3	0.264845	1.324056	3.530702	3	100

EQUITY FUNDS PORTFOLIOS (REAL VALUES) PERIOD JANUARY 1989 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Equity Funds	Proportion %
0.1	2.068672	2.67725	6.085784	3	100
0.2	1.460093	2.67725	6.085784	3	100
0.3	0.851515	2.67725	6.085784	3	100
0.4	0.242937	2.67725	6.085784	3	100

BALANCED FUNDS PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Balanced Funds	Proportion %
0.1	0.579193	0.757631	1.784379	11	100
0.2	0.400755	0.757631	1.784379	11	100
0.3	0.222317	0.757631	1.784379	11	100
0.4	0.089413	0.511551	1.055344	11, 19, 29	24, 9, 67

BALANCED FUNDS PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Balanced Funds	Proportion %
0.1	0.714211	0.897778	1.835668	11	100
0.2	0.530644	0.897778	1.835668	11	100
0.3	0.347078	0.897778	1.835668	11	100
0.4	0.173235	0.77671	1.508687	11, 29, 31	65, 11, 24
0.5	0.057625	0.554833	0.994416	11, 28, 29	20, 9, 71

BALANCED FUNDS PORTFOLIOS PERIOD JANUARY 1991 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Balanced Funds	Proportion %
0.1	0.55828	0.746633	1.883532	11	100
0.2	0.369927	0.746633	1.883532	11	100
0.3	0.181574	0.746633	1.883532	11	100
0.4	0.054443	0.439813	0.963425	11, 28, 29	17, 11, 72

BALANCED FUNDS PORTFOLIOS PERIOD JANUARY 1992 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Balanced Funds	Proportion %
0.1	0.40775	0.593677	1.859272	4, 11	5, 95
0.2	0.241929	0.487139	1.226046	1, 11, 28	13, 23, 64
0.3	0.131032	0.455403	1.081235	1, 28	20, 80
0.4	0.032395	0.39185	0.898637	28, 29	59, 41

BALANCED FUNDS PORTFOLIOS PERIOD JANUARY 1993 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Balanced Funds	Proportion %
0.1	0.116384	0.204583	0.881997	28	100
0.2	0.031436	0.197297	0.829308	1, 28	15, 85

BALANCED FUNDS PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1993					
Risk Factor	Lambda	Mean	Standard Deviation	Balanced Funds	Proportion %
0.1	0.696601	0.897867	2.012656	11	100
0.2	0.495336	0.897867	2.012656	11	100
0.3	0.29407	0.897867	2.012656	11	100
0.4	0.112318	0.665293	1.382439	11, 19, 29	42, 13, 45

BALANCED FUNDS PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1994					
Risk Factor	Lambda	Mean	Standard Deviation	Balanced Funds	Proportion %
0.1	0.880907	1.077333	1.964259	11	100
0.2	0.684482	1.077333	1.964259	11	100
0.3	0.488056	1.077333	1.964259	11	100
0.4	0.293636	1.021613	1.819945	11, 31	85, 15
0.5	0.141838	0.74369	1.203703	5, 11, 28, 29	1, 35, 2, 62
0.6	0.033394	0.634435	1.001735	11, 28, 29	14, 11, 75

BALANCED FUNDS PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1991					
Risk Factor	Lambda	Mean	Standard Deviation	Balanced Funds	Proportion %
0.1	0.808304	0.97514	1.668367	11, 19	99, 1
0.2	0.67869	0.911902	1.16606	11, 19	25, 75
0.3	0.565237	0.899068	1.112771	11, 19, 22	9, 90, 1
0.4	0.456973	0.88363	1.066643	19, 22	93, 7
0.5	0.350574	0.881512	1.061877	19, 22	91, 9
0.6	0.245301	0.869356	1.040092	19, 22, 25	87, 8, 5
0.7	0.142229	0.858198	1.022812	19, 22, 25	83, 8, 9
0.8	0.042194	0.824768	0.978217	19, 22, 25, 29	74, 6, 9, 11

BALANCED FUNDS PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1992					
Risk Factor	Lambda	Mean	Standard Deviation	Balanced Funds	Proportion %
0.1	1.538708	1.73075	1.920424	11	100
0.2	1.346665	1.73075	1.920424	11	100
0.3	1.154623	1.73075	1.920424	11	100
0.4	0.962581	1.73075	1.920424	11	100
0.5	0.770538	1.73075	1.920424	11	100
0.6	0.578733	1.704862	1.876883	11, 31	96, 4
0.7	0.407801	1.496151	1.554786	11, 25, 31	67, 13, 20
0.8	0.281156	1.176292	1.11892	11, 25, 27, 29	37, 15, 2, 46
0.9	0.177228	1.055144	0.975462	11, 19, 25, 27, 29	21, 7, 15, 2, 55
1	0.083806	0.983778	0.899972	11, 19, 25, 27, 29	11, 11, 16, 2, 60

BALANCED FUNDS PORTFOLIOS PERIOD JANUARY 1991 - DECEMBER 1993					
Risk Factor	Lambda	Mean	Standard Deviation	Balanced Funds	Proportion %
0.1	0.744021	0.973028	2.290068		
0.2	0.515014	0.973028	2.290068		
0.3	0.286008	0.973028	2.290068		
0.4	0.656173	0.744463	1.697113	11, 29	60, 40

BALANCED FUNDS PORTFOLIOS PERIOD JANUARY 1992 - DECEMBER 1994					
Risk Factor	Lambda	Mean	Standard Deviation	Balanced Funds	Proportion %
0.1	0.579891	0.791664	2.117725	4, 11	3, 97
0.2	0.376296	0.702743	1.632234	1, 11, 28	1, 54, 45
0.3	0.242896	0.607943	1.216819	1, 28	21, 79
0.4	0.123132	0.570226	1.117734	1, 28, 29	9, 72, 19
0.5	0.018493	0.524405	1.011825	28, 29	58, 42

BALANCED FUNDS PORTFOLIOS (REAL VALUES) PERIOD MARCH 1989 - DECEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Balanced Funds	Proportion %
0.1	1.115711	1.344849	2.291383	19, 22, 31	57, 21, 22
0.2	0.888228	1.341554	2.266629	19, 22, 31	61, 14, 25
0.3	0.661835	1.340475	2.262133	19, 22, 31	62, 13, 25
0.4	0.435711	1.339938	2.260566	19, 22, 31	63, 12, 25
0.5	0.209695	1.339616	2.259842	19, 22, 31	63, 11, 26

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1987 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.638446	0.831486	0.965202	4	100
0.4	0.513457	0.648857	0.338526	1, 3, 4	81, 1, 18
0.6	0.452277	0.624744	0.287402	1, 3, 4	91, 2, 7
0.8	0.396313	0.615805	0.274408	1, 3, 4	95, 2, 3
1	0.342054	0.610941	0.268886	1, 3, 4	97, 2, 1
1.2	0.288577	0.608402	0.266521	1, 3	98, 2
1.4	0.235282	0.6083	0.266441	1, 3	99, 1
1.6	0.181999	0.608223	0.266389	1, 3	99, 1
1.8	0.128725	0.608163	0.266355	1, 3	99, 1
2	0.095666	0.61864	0.266329	1, 3	79, 21
2.2	0.022193	0.608076	0.266311	1, 3	99, 1

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1988 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.59526	0.865591	1.351656	5	100
0.4	0.475077	0.593267	0.295466	1, 3, 5	87, 4, 9
0.6	0.420544	0.576134	0.259229	1, 3, 5	93, 4, 3
0.8	0.369824	0.569406	0.249399	1, 3, 5	96, 3, 1
1	0.318144	0.567912	0.248193	1, 3	97, 3
1.2	0.271026	0.566624	0.246374	1, 3	97, 3
1.4	0.221783	0.566345	0.246171	1, 3	98, 2
1.6	0.172575	0.566136	0.245975	1, 3	98, 2
1.8	0.12339	0.565974	0.245879	1, 3	98, 2
2	0.074221	0.565844	0.245811	1, 3	99, 1
2.2	0.025064	0.565738	0.245761	1, 3	99, 1

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1989 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.576674	0.850988	1.371567	5	100
0.4	0.442357	0.545332	0.257488	1, 3, 5	87, 6, 7
0.6	0.395875	0.526834	0.218174	1, 3, 5	93, 6, 1
0.8	0.353286	0.521877	0.210713	1, 3	94, 6
1	0.31119	0.521463	0.210238	1, 3	95, 5
1.2	0.269163	0.521189	0.210022	1, 3	96, 4
14	0.227175	0.520994	0.209871	1, 3	96, 4
1.6	0.185211	0.520848	0.209773	1, 3	96, 4
1.8	0.143264	0.520734	0.209706	1, 3	96, 4
2	0.101328	0.520644	0.209658	1, 3	96, 4
2.2	0.0594	0.520569	0.209622	1, 3	96, 4

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1990 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.526333	0.813217	1.43442	5	100
0.4	0.41009	0.507398	0.243311	1, 3, 5	89, 6, 5
0.6	0.365349	0.492742	0.212368	1, 3	93, 7
0.8	0.323066	0.492017	0.211187	1, 3	94, 6
1	0.280853	0.491819	0.210966	1, 3	94, 6
1.2	0.238673	0.491688	0.210846	1, 3	95, 5
1.4	0.196512	0.491594	0.210768	1, 3	95, 5
1.6	0.150963	0.491512	0.210733	1, 3	95, 5
1.8	0.11222	0.491469	0.210694	1, 3	95, 5
2	0.070084	0.491426	0.210671	1, 3	95, 5

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1991 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.528661	0.828877	1.501083	5	100
0.4	0.367446	0.462097	0.236643	1, 3, 5	81, 15, 4
0.6	0.327311	0.439629	0.187083	1, 3	92, 8
0.8	0.290499	0.435918	0.181659	1, 3	96, 4
1	0.254407	0.433826	0.179444	1, 3	98, 2
1.2	0.218659	0.432472	0.178045	1, 3	99, 1
1.4	0.183087	0.431947	0.177757	1	100
1.6	0.147536	0.431947	0.177757	1	100
1.8	0.111984	0.431947	0.177757	1	100
2	0.076433	0.431947	0.177757	1	100
2.2	0.040881	0.431947	0.177757	1	100
2.4	0.00533	0.431947	0.177757	1	100

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1992 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.359153	0.399736	0.202978	1, 5	93, 7
0.4	0.329353	0.382956	0.134007	1	100
0.6	0.302551	0.382956	0.134007	1	100
0.8	0.27575	0.382956	0.134007	1	100
1	0.248948	0.382956	0.134007	1	100
1.2	0.222147	0.382956	0.134007	1	100
1.4	0.195345	0.382956	0.134007	1	100
1.6	0.168544	0.382956	0.134007	1	100
1.8	0.141742	0.382956	0.134007	1	100
2	0.114941	0.382956	0.134007	1	100
2.2	0.08814	0.382956	0.134007	1	100
2.4	0.061338	0.382956	0.134007	1	100
2.6	0.034537	0.382956	0.134007	1	100
2.8	0.007735	0.382956	0.134007	1	100

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1993 - OCTOBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.358886	0.38603	0.13572	1	100
0.4	0.331742	0.38603	0.13572	1	100
0.6	0.304598	0.38603	0.13572	1	100
0.8	0.277454	0.38603	0.13572	1	100
1	0.25031	0.38603	0.13572	1	100
1.2	0.223166	0.38603	0.13572	1	100
1.4	0.196022	0.38603	0.13572	1	100
1.6	0.168878	0.38603	0.13572	1	100
1.8	0.141734	0.38603	0.13572	1	100
2	0.11459	0.38603	0.13572	1	100
2.2	0.087446	0.38603	0.13572	1	100
2.4	0.060302	0.38603	0.13572	1	100
2.6	0.033158	0.38603	0.13572	1	100
2.8	0.006014	0.38603	0.13572	1	100

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1987 - December 1991					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.908151	1.08575	0.887992	4	100
0.4	0.738309	0.9598	0.553715	1, 2, 4	24, 18, 58
0.6	0.672411	0.830253	0.263059	1, 3, 4	82, 1, 17
0.8	0.623174	0.811079	0.234947	1, 3, 4	88, 1, 11
1	0.577365	0.801942	0.224499	1, 3, 4	91, 1, 8
1.2	0.533008	0.796429	0.219545	1, 3, 4	93, 1, 6
1.4	0.489417	0.792697	0.216564	1, 3, 4	94, 1, 5
1.6	0.446286	0.789986	0.214709	1, 3, 4	95, 1, 4
1.8	0.403453	0.787922	0.213542	1, 3, 4	95, 1, 4
2	0.360825	0.786294	0.212838	1, 3, 4	96, 1, 3
2.2	0.318344	0.784976	0.212132	1, 3, 4	96, 1, 3
2.4	0.275973	0.783886	0.21166	1, 3, 4	97, 1, 2
2.6	0.233685	0.78297	0.211187	1, 3, 4	97, 1, 2
2.8	0.191462	0.782188	0.21095	1, 3, 4	97, 1, 2
3	0.149292	0.781513	0.210713	1, 3, 4	97, 2, 1
3.2	0.107163	0.780923	0.210549	1, 3, 4	97, 2, 1
3.4	0.065069	0.780405	0.210393	1, 3, 4	98, 1, 1
3.6	0.023004	0.779945	0.210238	1, 3, 4	98, 1, 1

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1988 - December 1992					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.829191	1.063417	1.171128	5	100
0.4	0.61643	0.869828	0.633482	2, 3, 4, 5	41, 7, 28, 24
0.6	0.542721	0.716258	0.289309	1, 3, 5	83, 5, 12
0.8	0.48838	0.69629	0.259808	1, 3, 5	88, 5, 7
1	0.437633	0.686657	0.248998	1, 3, 5	91, 5, 4
1.2	0.388418	0.680818	0.243721	1, 3, 5	92, 5, 3
1.4	0.340044	0.677004	0.240624	1, 3, 4, 5	93, 5, 1, 1
1.6	0.292117	0.674069	0.238747	1, 3, 4, 5	93, 5, 1, 1
1.8	0.244514	0.671832	0.237487	1, 3, 4	94, 5, 1
2	0.197131	0.670097	0.236432	1, 3	95, 5
2.2	0.149863	0.66968	0.236281	1, 3	95, 5
2.4	0.102616	0.669472	0.23619	1, 3	95, 5
2.6	0.055385	0.669296	0.236119	1, 3	95, 5
2.8	0.008167	0.669146	0.236064	1, 3	96, 4

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1989 - December 1993					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.903005	1.1234	1.101977	5	100
0.4	0.68261	1.1234	1.101977	5	100
0.6	0.497949	0.743324	0.408901	1, 5	66, 34
0.8	0.43525	0.647657	0.265518	1, 3, 5	80, 6, 14
1	0.385633	0.621343	0.235797	1, 3, 5	84, 7, 9
1.2	0.339887	0.607746	0.223159	1, 3, 5	87, 7, 6
1.4	0.295969	0.599178	0.216564	1, 3, 5	88, 7, 5
1.6	0.253084	0.593203	0.212603	1, 3, 5	89, 7, 4
1.8	0.210849	0.588766	0.209954	1, 3, 5	90, 7, 3
2	0.16905	0.585326	0.208086	1, 3, 5	91, 7, 2
2.2	0.12756	0.582574	0.206882	1, 3, 5	91, 7, 2
2.4	0.086298	0.580319	0.205913	1, 3, 5	92, 7, 1
2.6	0.045208	0.578434	0.205183	1, 3, 5	92, 7, 1
2.8	0.004252	0.576835	0.20445	1, 3, 5	92, 7, 1

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1994					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.479324	0.7697	1.451882	5	100
0.4	0.397197	0.495421	0.245561	1, 3, 5	93, 3, 4
0.6	0.350976	0.485698	0.224537	1, 3	95, 5
0.8	0.311644	0.485642	0.224472	1, 3	95, 5
1	0.261194	0.485598	0.224403	1, 3	96, 4
1.2	0.216316	0.485573	0.224381	1, 3	96, 4
1.4	0.171442	0.485555	0.224367	1, 3	96, 4
1.6	0.126569	0.485542	0.224357	1, 3	96, 4
1.8	0.081698	0.485531	0.224352	1, 3	96, 4
2	0.036828	0.485523	0.224348	1, 3	96, 4

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1987 - DECEMBER 1989					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.945384	1.131139	0.928775	4	100
0.4	0.786293	0.932466	0.365377	1, 4	67, 33
0.6	0.729166	0.878767	0.249399	1, 4	85, 15
0.8	0.681725	0.864804	0.22891	1, 4	90, 10
1	0.636849	0.857782	0.220907	1, 4	92, 8
1.2	0.593099	0.853455	0.217025	1, 4	93, 7
1.4	0.549955	0.850493	0.214709	1, 4	94, 6
1.6	0.499772	0.848667	0.213307	1, 4	95, 5
1.8	0.464635	0.846674	0.212132	1, 4	96, 4
2	0.42226	0.845365	0.21166	1, 4	96, 4
2.2	0.380002	0.844304	0.21095	1, 4	96, 4
2.4	0.337833	0.843424	0.210713	1, 4	97, 3
2.6	0.295731	0.842684	0.210476	1, 4	97, 3
2.8	0.253682	0.842052	0.210238	1, 4	97, 3
3	0.211675	0.841506	0.21	1, 4	97, 3
3.2	0.169702	0.841029	0.209762	1, 4	98, 2
3.4	0.127758	0.840609	0.209662	1, 4	98, 2
3.6	0.060778	0.840294	0.209598	1, 4	98, 2
3.8	0.009883	0.839942	0.209523	1, 4	98, 2
4	0.002049	0.839604	0.209285	1, 4	98, 2

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1988 - DECEMBER 1990					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.759635	0.861442	0.509019	2, 4	51, 49
0.4	0.704839	0.785944	0.202731	1, 4	92, 8
0.6	0.665917	0.779595	0.189473	1, 3, 4	96, 1, 3
0.8	0.628499	0.776782	0.185472	1, 3, 4	98, 1, 1
1	0.591628	0.775292	0.183664	1, 3	99, 1
1.2	0.554898	0.775263	0.183637	1, 3	99, 1
1.4	0.518173	0.775242	0.183621	1, 3	99, 1
1.6	0.48145	0.775226	0.183609	1, 3	99, 1
1.8	0.444728	0.775213	0.183603	1, 3	99, 1
2	0.408008	0.775204	0.183598	1, 3	99, 1
2.2	0.371289	0.775196	0.183594	1, 3	99, 1
2.4	0.334571	0.775189	0.183591	1, 3	99, 1
2.6	0.297853	0.775183	0.183588	1, 3	99, 1
2.8	0.261135	0.775178	0.183587	1, 3	99, 1
3	0.224418	0.775174	0.183585	1, 3	99, 1
3.2	0.187701	0.77517	0.183584	1, 3	99, 1
3.4	0.150985	0.775167	0.183583	1, 3	99, 1
3.6	0.114268	0.775164	0.183582	1, 3	99, 1
3.8	0.077552	0.775162	0.183581	1, 3	99, 1
4	0.040835	0.775159	0.183581	1, 3	99, 1
4.2	0.004119	0.775157	0.18358	1, 3	99, 1

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1989 - DECEMBER 1991					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.9145	1.153639	1.195694	5	100
0.4	0.691796	0.949182	0.643459	2, 3, 4	21, 2, 77
0.6	0.623282	0.733578	0.183848	1, 3, 4	62, 6, 32
0.8	0.581574	0.717194	0.169412	1, 3, 4	86, 7, 7
1	0.548882	0.707796	0.159059	1, 3, 4	89, 7, 4
1.2	0.517652	0.702361	0.153948	1, 3, 4	91, 7, 2
1.4	0.48717	0.698758	0.150997	1, 3, 4	92, 7, 1
1.6	0.457129	0.696172	0.149332	1, 3	93, 7
1.8	0.427326	0.695362	0.148896	1, 3	93, 7
2	0.39755	0.695249	0.148828	1, 3	93, 7
2.2	0.367785	0.695156	0.148795	1, 3	93, 7
2.4	0.338028	0.695078	0.148761	1, 3	93, 7
2.6	0.308276	0.695013	0.148728	1, 3	93, 7
2.8	0.273142	0.694964	0.148716	1, 3	93, 7
3	0.248786	0.694908	0.148707	1, 3	94, 6
3.2	0.219046	0.694865	0.148693	1, 3	94, 6
3.4	0.189309	0.694828	0.148682	1, 3	94, 6
3.6	0.159573	0.694794	0.148672	1, 3	94, 6
3.8	0.12984	0.694764	0.148664	1, 3	94, 6
4	0.100108	0.694738	0.148657	1, 3	94, 6
4.2	0.070377	0.694713	0.148651	1, 3	94, 6
4.4	0.040647	0.694691	0.148646	1, 3	94, 6
4.6	0.010918	0.694671	0.148642	1, 3	94, 6

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1990 - DECEMBER 1992					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.847883	1.094917	1.235169	5	100
0.4	0.600849	1.094917	1.235169	5	100
0.6	0.482596	0.657265	0.291033	1, 3, 5	76, 13, 11
0.8	0.430834	0.621975	0.238956	1, 3, 5	82, 13, 5
1	0.384908	0.607604	0.222711	1, 3, 5	85, 13, 2
1.2	0.341204	0.599394	0.215174	1, 3	87, 13
1.4	0.298289	0.598407	0.214476	1, 3	87, 13
1.6	0.255443	0.598008	0.214009	1, 3	87, 13
1.8	0.212642	0.597698	0.213916	1, 3	88, 12
2	0.169872	0.597451	0.213776	1, 3	88, 12
2.2	0.127124	0.597249	0.213682	1, 3	88, 12
2.4	0.084393	0.597081	0.213542	1, 3	88, 12

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1991 - DECEMBER 1993					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	1.041983	1.27	1.140088	5	100
0.4	0.813965	1.27	1.140088	5	100
0.6	0.585947	1.27	1.140088	5	100
0.8	0.397038	0.97385	0.721041	3, 5	50, 50
1	0.310534	0.595712	0.285132	1, 3, 5	66, 23, 11
1.2	0.260236	0.534981	0.22891	1, 3, 5	78, 17, 5
1.4	0.216828	0.507706	0.207749	1, 3, 5	83, 15, 2
1.6	0.176473	0.491359	0.196723	1, 3, 5	85, 14, 1
1.8	0.137656	0.484645	0.192873	1, 3	88, 12
2	0.09925	0.481991	0.191311	1, 3	89, 11
2.2	0.061082	0.47986	0.190263	1, 3	90, 10
2.4	0.023092	0.478109	0.189473	1, 3	91, 9

FIXED INTEREST PORTFOLIOS PERIOD JANUARY 1992 - DECEMBER 1994					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.2	0.323462	0.34883	0.126807	1, 5	99, 1
0.4	0.298675	0.348195	0.123798	1	100
0.6	0.273916	0.348195	0.123798	1	100
0.8	0.249156	0.348195	0.123798	1	100
1	0.224396	0.348195	0.123798	1	100
1.2	0.199637	0.348195	0.123798	1	100
1.4	0.174877	0.348195	0.123798	1	100
1.6	0.150117	0.348195	0.123798	1	100
1.8	0.125358	0.348195	0.123798	1	100
2	0.100598	0.348195	0.123798	1	100
2.2	0.075838	0.348195	0.123798	1	100
2.4	0.051079	0.348195	0.123798	1	100
2.6	0.026319	0.348195	0.123798	1	100

FIXED INTEREST PORTFOLIOS (REAL VALUES) PERIOD JANUARY 1987- SEPTEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.1	1.009448	1.2792	2.697523	4	100
0.2	0.739696	1.2792	2.697523	4	100
0.3	0.469943	1.2792	2.697523	4	100
0.4	0.204553	1.146672	2.355292	1, 3, 4	20, 1, 79

FIXED INTEREST PORTFOLIOS (REAL VALUES) PERIOD JANUARY 1988 - SEPTEMBER 1995					
Risk Factor	Lambda	Mean	Standard Deviation	Fixed Interest Securities	Proportion %
0.1	1.157615	1.485097	3.274816	5	100
0.2	0.830134	1.485097	3.274816	5	100
0.3	0.502652	1.485097	3.274816	5	100
0.4	0.224826	1.116022	2.227988	2, 3, 4, 5	34, 11, 20, 35
0.5	0.025316	0.946409	1.842184	2, 3, 5	61, 14, 25

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