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**A PRE-FILTER
FOR RECURSIVE
CONTINUOUS-TIME-MODEL
PARAMETER ESTIMATION**

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the degree of

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ABSTRACT

This thesis reports the development of a prefiltering technique for estimating the parameters of continuous-time models given by differential equations. This technique is based on some special integrals, named the Fixed Interval Integrals (FII), which result from multiple integrations over intervals of fixed length. An estimation method using this prefiltering technique has several significant features:

- it is capable of estimating both system delay and the parameters of a second order dynamical model.
- it is able to be implemented on discrete-time digital devices.
- it is independent of initial conditions, hence these do not need to be known.
- it uses well-established discrete-time estimation algorithms.

This development starts with the definition of a new operator notation system. It then studies in detail the properties of the fixed interval integration and its relationship to traditional calculus operations. Several possible methods to realize the FII operation using analog and digital filters are also given.

Using these results and some simulation examples, the use of FII in parameter and delay estimation of continuous-time models is demonstrated.

Since the FII is likely to be useful in engineering and mathematics beyond parameter estimation, some other possible applications of the FII are outlined.

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LIST OF CONTRIBUTIONS

The major contributions in this thesis are :

1. the development of an operator notation system for some calculus operations which is convenient for both theoretical and implementational work.
2. the determination of the relationships between Fixed Interval Integration and some other calculus operations.
3. the development of several realization methods for Fixed Interval Integration.
4. the development of a prefiltering technique based on the Fixed Interval Integration, for estimating parameters and pure delay of continuous-time models described by differential equations.

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