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MODELLING THE INITIATION OF A
HYDROTHERMAL ERUPTION – THE SHOCK TUBE
MODEL

A THESIS PRESENTED IN PARTIAL FULFILMENT OF THE
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Abstract

Modelling of hydrothermal eruption phenomenon has been a growing topic of research for the past 20 years. To date, models have focussed on the underground quasi-steady flow and on the above ground eruption jet including particle deposition. Very little has been able to be said about the first few seconds of an eruption, nor has much modelling work been done on eruption causes. In this thesis we develop a shock tube model for the initiation of a hydrothermal eruption which aims to answer some of the remaining questions about causality of a hydrothermal eruption.

The new shock tube model reported in this thesis is the first model able to simulate the initiation of a hydrothermal eruption. We take into account the three phases present during the eruption; in the geothermal reservoir below ground, liquid water and water vapour are present, in the above ground flow we also include air. The fact that the flow below ground is moving through a porous medium is accounted for, and new numerical methods using the finite volume framework are developed to solve the arising system of equations. Numerical simulations are described which simulate various eruptions, including ones with steam caps and rapidly developing cracks in the geothermal reservoir. Results from numerical simulations will be able to guide the design of future lab experiments of hydrothermal eruptions. The work of this thesis results in the first model able to simulate the initiation of a hydrothermal eruption.

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