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Developing Early Algebraic Understanding in an Inquiry Classroom

A thesis presented in partial fulfilment of the requirements for the degree of Master of Education at Massey University, Palmerston North, New Zealand

Jodie Margaret Roberta Hunter
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ABSTRACT

This study explores Year 5 and 6 students’ construction of early algebraic concepts within an inquiry classroom context. Also under consideration are the tools—the instructional tasks and models, the forms of notation and symbolisation, the discourse and interaction, and the teacher’s pedagogical actions—which mediate student development of early algebraic reasoning.

An emergent theoretical perspective which brings together social and constructivist theories of learning underpins the focus of the study. Relevant literature is drawn on to illustrate the need for student focus to shift from a procedural perspective of number operations and relations to understanding their structural aspects. Comprehensive evidence in the literature is provided of the significant role of the teacher in developing the students’ early algebraic reasoning through facilitating their participation in making conjectures, generalising, justifying and formalising.

A classroom-based qualitative research approach—teaching experiment—matched the emergent theoretical frame taken in the study. The teaching experiment approach supported a collaborative teacher-researcher partnership. Student interviews, participant and video recorded observations, and classroom artefacts formed the data collection. On-going and retrospective data analysis was used to develop the findings as one classroom case study.

Important changes in student reasoning were revealed in the findings as the teacher guided development of productive discourse and facilitated extended time and space for student discussion and exploration within an inquiry context. Students were provided with many rich opportunities to engage with tasks and models which explicitly focused on developing relational thinking, understanding of algebraic notation, the exploration of the properties and relationships of numbers, and functional patterns. Evidence is provided that through engaging with the tasks and models, the students learnt to make conjectures, represent, justify, generalise and formalise their observations. Of significance in deepening student understanding of early algebraic concepts were the repeated challenges to their partial understandings.

The research findings provide insights into ways teachers can assist students to use their implicit understanding of number relations and properties as a foundation for the construction of early algebraic reasoning. The results of this study suggest that student participation in mathematical activity which included explanation, argumentation and justification supported their development of rich algebraic reasoning.
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# TABLE OF CONTENTS

| ABSTRACT | ii |
| ACKNOWLEDGEMENTS | iii |
| TABLE OF CONTENTS | iv |

## CHAPTER 1: INTRODUCTION

1.1 Background to the study 1
1.2 Research Objectives 3
1.3 Overview 3

## CHAPTER 2: LITERATURE REVIEW

2.1 Introduction 5
2.2 Constructing mathematical knowledge in the classroom 6
  2.2.1 Socio-constructivist learning theory 6
2.3 Inquiry classrooms 8
  2.3.1 Developing students’ mathematical dispositions in an inquiry classroom 8
  2.3.2 Role of the teacher in an inquiry classroom 9
2.4 Collaborative interaction and classroom mathematical discourse 11
2.5 The construction of early algebraic understanding in the classroom 13
2.6 Early algebraic understanding as generalised arithmetic 14
  2.6.1 Understanding of variables and algebraic notation 15
  2.6.2 Understanding equivalence 17
  2.6.3 Understanding arithmetic operations and operational laws 19
2.7 Early algebraic understanding as functional thinking 20
  2.7.1 Generalisation strategies to support functional thinking 20
  2.7.2 Tools and representations to support functional thinking 21
2.8 Developing the processes of early algebraic reasoning 22
  2.8.1 Making conjectures 22
5.2.1 Students’ initial understanding of the equal sign 53
5.2.2 Classroom activity to develop student understanding of the equal sign 55
5.2.3 Classroom activity to develop relational thinking 56
5.2.4 Classroom activity to enrich understanding of relational equivalence 59
5.2.5 Students’ progress in understanding the equal sign 60

5.3 Algebraic notation 62
5.3.1 Students’ initial understanding of algebraic notation 62
5.3.2 Classroom activity to develop student understanding of algebraic notation 63
5.3.3 Confronting a misconception about algebraic notation 66
5.3.4 Students’ progress in understanding algebraic notation 67

5.4 Understanding properties of numbers 69
5.4.1 Students’ initial understanding of the properties of numbers 69
5.4.2 Classroom activity to develop student understanding of the properties of numbers 70
5.4.3 Developing understanding of the commutative principle 70
5.4.4 Developing understanding of the properties of odd and even numbers 72
5.4.5 Developing understanding of the properties of zero 75
5.4.6 Developing understanding of the relationship between addition and subtraction and multiplication and division 76
5.4.7 Students’ progress in understanding the properties of numbers 78

5.5 Summary 80

CHAPTER 6: EXPLORING EARLY ALGEBRAIC UNDERSTANDING THROUGH PATTERNING 82
6.1 Introduction 82
6.2 Students’ initial understanding of functional relationships 82
6.3 Classroom activities to develop student understanding of functional relationships 84
6.3.1 Using t-charts to support functional thinking 84
6.3.2 Classroom activity to develop students’ generalisation strategies for algebraic patterns 86
6.3.3 Classroom activities to develop student justification of functional rules 90
6.3.4 Classroom activities to develop students formalisation of functional rules 94
6.4 Students’ progress in understanding functional relationships 96
6.5 Summary 97

CHAPTER 7: CONCLUSION 99
7.1 Introduction 99
7.2 The complex nature of teaching and learning 99
  7.2.1 Instructional tasks and models 100
  7.2.2 Notation and symbolisation 102
  7.2.3 Inquiry classroom practice: Social and socio-mathematical norms 104
7.3 Opportunities for further research 105
7.4 Concluding thoughts 106

REFERENCES 107

APPENDICES:
Appendix A: Interview questions (pre-unit) 118
Appendix B: Interview questions (post-unit) 120
Appendix C: Tasks and problems 122
Appendix D: Teacher information sheet and consent form 126
Appendix E: Student and parent information sheet and consent form 128
Appendix F: Board of Trustees information sheet and consent form 130

LIST OF TABLES
3.1 Summary time-line 35
5.1 Percentage of all students’ errors when solving equivalence problems 54
5.2 Percentage of students using relational or computational strategies 54
5.3 Percentage of students using relational or computational strategies 61
5.4 Percentage of students using algebraic notation for an unknown quantity 62
5.5 Percentage of students using algebraic notation for an unknown quantity 67
6.1 Percentage of students correctly using the functional relationship 83
6.2 Percentage of students correctly using the functional relationship 96

LIST OF FIGURES
5.1 Recording relational strategies using arrows 58
5.2 Solution strategy for CD player problem 65
5.3 Patterns of odd and even numbers 73
5.4 The link between multiplication and division 77
5.5 Hamish’s reflection 78
5.6 Josie’s array 79
6.1 T-chart 85
6.2 T-chart for the mouse-cage problem 86
6.3 T-chart 88
6.4 Using a geometric representation to show a functional rule 91