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# **Developing Early Algebraic Understanding in an Inquiry Classroom**

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## ABSTRACT

This study explores Year 5 and 6 students' construction of early algebraic concepts within an inquiry classroom context. Also under consideration are the tools—the instructional tasks and models, the forms of notation and symbolisation, the discourse and interaction, and the teacher's pedagogical actions—which mediate student development of early algebraic reasoning.

An emergent theoretical perspective which brings together social and constructivist theories of learning underpins the focus of the study. Relevant literature is drawn on to illustrate the need for student focus to shift from a procedural perspective of number operations and relations to understanding their structural aspects. Comprehensive evidence in the literature is provided of the significant role of the teacher in developing the students' early algebraic reasoning through facilitating their participation in making conjectures, generalising, justifying and formalising.

A classroom-based qualitative research approach—teaching experiment—matched the emergent theoretical frame taken in the study. The teaching experiment approach supported a collaborative teacher-researcher partnership. Student interviews, participant and video recorded observations, and classroom artefacts formed the data collection. On-going and retrospective data analysis was used to develop the findings as one classroom case study.

Important changes in student reasoning were revealed in the findings as the teacher guided development of productive discourse and facilitated extended time and space for student discussion and exploration within an inquiry context. Students were provided with many rich opportunities to engage with tasks and models which explicitly focused on developing relational thinking, understanding of algebraic notation, the exploration of the properties and relationships of numbers, and functional patterns. Evidence is provided that through engaging with the tasks and models, the students learnt to make conjectures, represent, justify, generalise and formalise their observations. Of significance in deepening student understanding of early algebraic concepts were the repeated challenges to their partial understandings.

The research findings provide insights into ways teachers can assist students to use their implicit understanding of number relations and properties as a foundation for the construction of early algebraic reasoning. The results of this study suggest that student participation in mathematical activity which included explanation, argumentation and justification supported their development of rich algebraic reasoning.

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