

Copyright is owned by the Author of the thesis. Permission is given for a copy to be downloaded by an individual for the purpose of research and private study only. The thesis may not be reproduced elsewhere without the permission of the Author.

# **Developing Early Algebraic Understanding in an Inquiry Classroom**

A thesis presented in partial fulfilment of the  
requirements for the degree of  
Master of Education  
at Massey University, Palmerston North, New Zealand

**Jodie Margaret Roberta Hunter**

**2007**

## ABSTRACT

This study explores Year 5 and 6 students' construction of early algebraic concepts within an inquiry classroom context. Also under consideration are the tools—the instructional tasks and models, the forms of notation and symbolisation, the discourse and interaction, and the teacher's pedagogical actions—which mediate student development of early algebraic reasoning.

An emergent theoretical perspective which brings together social and constructivist theories of learning underpins the focus of the study. Relevant literature is drawn on to illustrate the need for student focus to shift from a procedural perspective of number operations and relations to understanding their structural aspects. Comprehensive evidence in the literature is provided of the significant role of the teacher in developing the students' early algebraic reasoning through facilitating their participation in making conjectures, generalising, justifying and formalising.

A classroom-based qualitative research approach—teaching experiment—matched the emergent theoretical frame taken in the study. The teaching experiment approach supported a collaborative teacher-researcher partnership. Student interviews, participant and video recorded observations, and classroom artefacts formed the data collection. On-going and retrospective data analysis was used to develop the findings as one classroom case study.

Important changes in student reasoning were revealed in the findings as the teacher guided development of productive discourse and facilitated extended time and space for student discussion and exploration within an inquiry context. Students were provided with many rich opportunities to engage with tasks and models which explicitly focused on developing relational thinking, understanding of algebraic notation, the exploration of the properties and relationships of numbers, and functional patterns. Evidence is provided that through engaging with the tasks and models, the students learnt to make conjectures, represent, justify, generalise and formalise their observations. Of significance in deepening student understanding of early algebraic concepts were the repeated challenges to their partial understandings.

The research findings provide insights into ways teachers can assist students to use their implicit understanding of number relations and properties as a foundation for the construction of early algebraic reasoning. The results of this study suggest that student participation in mathematical activity which included explanation, argumentation and justification supported their development of rich algebraic reasoning.

## ACKNOWLEDGEMENTS

I would like to acknowledge and thank the many people who made this study possible. Most importantly I wish to thank the teacher who so willingly and enthusiastically gave of herself and her time. Her personal belief in this study and collaborative spirit contributed greatly to the project and was a source of strength for me. I would also like to thank the students in her classroom for their enthusiasm and keen participation in the mathematical activities which were undertaken.

I wish to acknowledge and thank my main supervisor, Dr Glenda Anthony who showed unstinting interest and offered invaluable support in the writing of this study. Glenda so willingly gave of her time and provided important professional suggestions and support. My thanks extend to Ngaire Davies, my second supervisor who aided the writing process through her supportive professional comments.

Finally, I must acknowledge members of my family, my mother Bobbie for her unwavering belief in me and perpetual support, my father Les for his unfailing confidence and my supportive, caring husband, Gavin who remained tolerant and positive throughout this research study. Their support enabled me to believe in my ability to complete this project.

# TABLE OF CONTENTS

|  |     |
|--|-----|
| <b>ABSTRACT</b>  | ii  |
| <b>ACKNOWLEDGEMENTS</b>  | iii |
| <b>TABLE OF CONTENTS</b>   | iv  |
| <b>CHAPTER 1: INTRODUCTION</b>   | 1   |
| 1.1 Background to the study  | 1   |
| 1.2 Research Objectives  | 3   |
| 1.3 Overview   | 3   |
| <b>CHAPTER 2: LITERATURE REVIEW</b>  | 5   |
| 2.1 Introduction   | 5   |
| 2.2 Constructing mathematical knowledge in the classroom                     | 6   |
| 2.2.1 Socio-constructivist learning theory                                   | 6   |
| 2.3 Inquiry classrooms   | 8   |
| 2.3.1 Developing students' mathematical dispositions in an inquiry classroom | 8   |
| 2.3.2 Role of the teacher in an inquiry classroom                            | 9   |
| 2.4 Collaborative interaction and classroom mathematical discourse           | 11  |
| 2.5 The construction of early algebraic understanding in the classroom       | 13  |
| 2.6 Early algebraic understanding as generalised arithmetic                  | 14  |
| 2.6.1 Understanding of variables and algebraic notation                      | 15  |
| 2.6.2 Understanding equivalence  | 17  |
| 2.6.3 Understanding arithmetic operations and operational laws               | 19  |
| 2.7 Early algebraic understanding as functional thinking                     | 20  |
| 2.7.1 Generalisation strategies to support functional thinking               | 20  |
| 2.7.2 Tools and representations to support functional thinking               | 21  |
| 2.8 Developing the processes of early algebraic reasoning                    | 22  |
| 2.8.1 Making conjectures   | 22  |

|                   |   |           |
|-------------------|---|-----------|
| 2.8.2             | Making generalisations  | 23        |
| 2.8.3             | Justifying conjectures and generalisations                              | 24        |
| 2.8.4             | Formalising conjectures and generalisations                             | 25        |
| 2.9               | Summary   | 25        |
| <b>CHAPTER 3:</b> | <b>RESEARCH DESIGN</b>  | <b>27</b> |
| 3.1               | Introduction  | 27        |
| 3.2               | Justification for methodology   | 27        |
| 3.3               | Researcher role   | 29        |
| 3.4               | Data collection   | 30        |
| 3.4.1             | Observation   | 30        |
| 3.4.2             | Interviews  | 31        |
| 3.4.3             | Classroom artefacts   | 31        |
| 3.5               | The research study: Settings, sample and schedule                       | 32        |
| 3.5.1             | The setting and the sample  | 32        |
| 3.5.2             | The research study schedule   | 32        |
| 3.6               | Data analysis   | 35        |
| 3.7               | Validity and reliability  | 36        |
| 3.8               | Ethical considerations  | 37        |
| 3.9               | Summary   | 39        |
| <b>CHAPTER 4:</b> | <b>DEVELOPING EARLY ALGEBRAIC UNDERSTANDING IN AN INQUIRY CLASSROOM</b> | <b>40</b> |
| 4.1               | Introduction  | 40        |
| 4.2               | The classroom context   | 40        |
| 4.2.1             | Structure of the learning sessions                                      | 40        |
| 4.3               | Developing collaborative interaction                                    | 41        |
| 4.4               | The emergence of inquiry and justification                              | 47        |
| 4.5               | Shifting focus to algebraic reasoning                                   | 50        |
| 4.5               | Summary   | 52        |
| <b>CHAPTER 5:</b> | <b>EXPLORING EARLY ALGEBRAIC UNDERSTANDING THROUGH NUMBER</b>           | <b>53</b> |
| 5.1               | Introduction  | 53        |
| 5.2               | Understanding the equal sign  | 53        |

|       |   |    |
|-------|---|----|
| 5.2.1 | Students' initial understanding of the equal sign   | 53 |
| 5.2.2 | Classroom activity to develop student understanding of the equal sign   | 55 |
| 5.2.3 | Classroom activity to develop relational thinking   | 56 |
| 5.2.4 | Classroom activity to enrich understanding of relational equivalence  | 59 |
| 5.2.5 | Students' progress in understanding the equal sign  | 60 |
| 5.3   | Algebraic notation  | 62 |
| 5.3.1 | Students' initial understanding of algebraic notation   | 62 |
| 5.3.2 | Classroom activity to develop student understanding of algebraic notation                                     | 63 |
| 5.3.3 | Confronting a misconception about algebraic notation  | 66 |
| 5.3.4 | Students' progress in understanding algebraic notation  | 67 |
| 5.4   | Understanding properties of numbers   | 69 |
| 5.4.1 | Students' initial understanding of the properties of numbers  | 69 |
| 5.4.2 | Classroom activity to develop student understanding of the properties of numbers                              | 70 |
| 5.4.3 | Developing understanding of the commutative principle   | 70 |
| 5.4.4 | Developing understanding of the properties of odd and even numbers  | 72 |
| 5.4.5 | Developing understanding of the properties of zero  | 75 |
| 5.4.6 | Developing understanding of the relationship between addition and subtraction and multiplication and division | 76 |
| 5.4.7 | Students' progress in understanding the properties of numbers   | 78 |
| 5.5   | Summary   | 80 |

**CHAPTER 6:            EXPLORING EARLY ALGEBRAIC UNDERSTANDING THROUGH PATTERNING**            82

|       |  |    |
|-------|--|----|
| 6.1   | Introduction   | 82 |
| 6.2   | Students' initial understanding of functional relationships                              | 82 |
| 6.3   | Classroom activities to develop student understanding of functional relationships        | 84 |
| 6.3.1 | Using t-charts to support functional thinking  | 84 |
| 6.3.2 | Classroom activity to develop students' generalisation strategies for algebraic patterns | 86 |

|                              |  |     |
|------------------------------|--|-----|
| 6.3.3                        | Classroom activities to develop student justification of functional rules  | 90  |
| 6.3.4                        | Classroom activities to develop students formalisation of functional rules | 94  |
| 6.4                          | Students' progress in understanding functional relationships               | 96  |
| 6.5                          | Summary  | 97  |
| <b>CHAPTER 7: CONCLUSION</b> |  | 99  |
| 7.1                          | Introduction   | 99  |
| 7.2                          | The complex nature of teaching and learning                                | 99  |
| 7.2.1                        | Instructional tasks and models   | 100 |
| 7.2.2                        | Notation and symbolisation   | 102 |
| 7.2.3                        | Inquiry classroom practice: Social and socio-mathematical norms            | 104 |
| 7.3                          | Opportunities for further research   | 105 |
| 7.4                          | Concluding thoughts  | 106 |
| <b>REFERENCES</b>            |  | 107 |
| <b>APPENDICES:</b>           |  |     |
| Appendix A:                  | Interview questions (pre-unit)   | 118 |
| Appendix B:                  | Interview questions (post-unit)  | 120 |
| Appendix C:                  | Tasks and problems   | 122 |
| Appendix D:                  | Teacher information sheet and consent form                                 | 126 |
| Appendix E:                  | Student and parent information sheet and consent form                      | 128 |
| Appendix F:                  | Board of Trustees information sheet and consent form                       | 130 |
| <b>LIST OF TABLES</b>        |  |     |
| 3.1                          | Summary time-line  | 35  |
| 5.1                          | Percentage of all students' errors when solving equivalence problems       | 54  |
| 5.2                          | Percentage of students using relational or computational strategies        | 54  |
| 5.3                          | Percentage of students using relational or computational strategies        | 61  |

|     |   |    |
|-----|---|----|
| 5.4 | Percentage of students using algebraic notation for an unknown quantity | 62 |
| 5.5 | Percentage of students using algebraic notation for an unknown quantity | 67 |
| 6.1 | Percentage of students correctly using the functional relationship      | 83 |
| 6.2 | Percentage of students correctly using the functional relationship      | 96 |

## **LIST OF FIGURES**

|     |  |    |
|-----|--|----|
| 5.1 | Recording relational strategies using arrows               | 58 |
| 5.2 | Solution strategy for CD player problem                    | 65 |
| 5.3 | Patterns of odd and even numbers                           | 73 |
| 5.4 | The link between multiplication and division               | 77 |
| 5.5 | Hamish's reflection  | 78 |
| 5.6 | Josie's array  | 79 |
| 6.1 | T-chart  | 85 |
| 6.2 | T-chart for the mouse-cage problem                         | 86 |
| 6.3 | T-chart  | 88 |
| 6.4 | Using a geometric representation to show a functional rule | 91 |