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Legendre Foliations on Contact Metric Manifolds

**A thesis presented in partial fulfilment of the requirements for
the degree of**

Doctor of Philosophy

in

Mathematics

at

Massey University

Nicola Jayne

1992

ABSTRACT

This thesis develops the theory of Legendre foliations on contact manifolds by associating a contact metric structure with a contact manifold and investigating Legendre foliations on the resultant contact metric manifold. The contact metric structure introduces a metric for the Legendre foliation which enables us to study the curvature properties of a Legendre foliation, furthermore when this metric is bundle-like we have a semi-Riemannian foliation hence we can define a semi-Riemannian Legendre foliation and study its properties.

We use the invariant Π as defined by Pang to define a family of contact metric structures for a non-degenerate Legendre foliation and from this family we pick out a unique contact metric structure the canonical contact metric structure. Furthermore a canonical contact metric structure is identified for a flat Legendre foliation and shown to be a Sasakian structure.

Under some circumstances a Legendre foliation on a contact metric manifold has a second Legendre foliation, the conjugate Legendre foliation, associated with it. We investigate the conditions for the existence and the properties of the conjugate Legendre foliation.

By using a definition similar to that of a Legendre foliation on a contact metric manifold we conclude this thesis by defining a complex Legendre foliation on a complex contact metric manifold and beginning an investigation of its properties.

I would like to thank Dr Gillian Thornley for her guidance, encouragement and many helpful suggestions throughout my work. Thanks also to Biff Gaia for her constant support and love.

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