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HEAT TRANSFER DURING FREEZING OF FOODS AND PREDICTION OF FREEZING TIMES

A thesis presented in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Biotechnology at Massey University.

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HEAT TRANSFER DURING FREEZING OF FOODS AND
PREDICTION OF FREEZING TIMES

ABSTRACT

A study of methods for predicting the freezing time of foods was made. Four shapes — finite slabs, cylinders, spheres and rectangular bricks were considered.

For each shape experimental measurements of the freezing time were made over a wide range of conditions using Karlsruhe test substance, a defined analogue material. Experiments with slabs of minced lean beef and mashed potato were also conducted.

Practical food freezing problems, where the material is initially superheated above its freezing point and the third kind of boundary condition (convective cooling) is applied, have not been solved analytically because of the non-linear boundary conditions. Food materials when freezing release latent heat over a range of temperature which further complicates any attempt at solution.

The accuracy of the various solutions to the freezing problem proposed in the literature was evaluated by comparison of the various calculated freezing times with the experimentally determined values over a total of 187 freezing experiments.

For those solutions requiring numerical evaluation, the best was found to be a three-level finite difference scheme which generally gave a prediction of the freezing time to within ±9% of the experimental values with 95% confidence. With the regular geometric shapes investigated finite elements have no advantage over finite differences and were not considered.

For the existing exact and approximate analytical solutions, it is shown that these do not give accurate prediction of the freezing time for any of the four shapes,
mainly because all but two of these solutions do not take account of initial superheat in the material to be frozen, and these two solutions are for initial superheat in a semi-infinite slab, not a finite slab.

For the existing empirically modified solutions and empirical relationships, it is shown that they do not give accurate prediction of freezing time; at best the 95% confidence limits of the percentage difference between the calculated and experimental freezing times are 0% to +20% for slabs, cylinders and spheres.

All solutions for freezing of rectangular bricks with the third kind of boundary condition use the geometric factors derived by Plank. These factors are shown to be subject to error, the error increasing as the ratios of the two larger dimensions to the smallest increase.

A group of formulae is proposed which are simple to use and give accurate prediction of freezing time. They modify the geometric factors in Plank's equation, taking initial superheat into account, and in the case of rectangular bricks correct the errors inherent in these geometric factors. This group of simple formulae are shown to predict the freezing time with 95% confidence to within 5% of the experimental values for slabs, to within 7% for cylinders and spheres, and to within 10% for rectangular bricks.

The prediction accuracy of the simple formulae and the three level finite difference scheme are similar but the simple formulae can be calculated quickly without the use of a computer which is a big advantage. In addition to simplicity and accuracy the simple formulae are also versatile, and by use of suitable approximations can handle some practical problems in which conditions change with time.
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