ON A FLEXIBLE MODEL FOR NEW ZEALAND’S HYDRO- THERMAL ELECTRICITY GENERATION SYSTEM

A thesis presented in partial fulfilment of the requirements for the degree of Ph.D. in Operations Research at Massey University

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1994
Abstract

This thesis investigates the modelling of the New Zealand hydro-thermal electricity generation system in order to determine an optimal strategy for generation, in terms of minimizing fuel costs. The model currently used by ECNZ (Electricity Corporation of New Zealand) uses an SDP (Stochastic Dynamic Programming) method for solution; this allows little detail of the physical system, and models two explicit hydro reservoirs. The model developed in this thesis is flexible, in order to allow the balance between ensuring stochastically stable solutions and the detail of the physical system, to be altered, whilst ensuring computational tractibility. The areas of the system which are important to be modelled accurately are isolated, as are those which may lead to computational intractibility if they are modelled in too much detail. The flexibility in the model also allows the effects of the approximations used on solutions to be explored in a wider framework.

The time horizon of the model is one to two years, with time steps of the order of a week. The time horizon describes the level to which many aspects of the system are to be modelled. Transmission is modelled explicitly so as to include information on the geographical locations of power stations and power users; this takes the form of a network structure underlying the model. The load at each geographic location is represented by a Load Duration Curve (which is more robust, in terms of forecasting, than a direct representation of load with respect to time). Hydro river chains are modelled as single power stations with a single reservoir and connect the model temporally; we model six explicit hydro river chains. Thermal stations are modelled individually, and the generation from run-of-river and geothermal stations is removed from load before solution begins.

The initial approach considers a model which, upon further investigation, is unacceptable. However, examination of the issues highlighted by this approach provide insight into the system. The resultant re-modelling of the problem leads to a linear model which does not explicitly model the uncertainty in the generating
capacity of stations due to forced outages. This accentuates the reason why the usual approach to explicitly modelling the uncertainty of supply (within a week) cannot be used in the case where the geographic distribution of generation has been explicitly modelled. The deterministic model may then be formulated as a Generalized Network with side constraints.

The deterministic model developed can be extended stochastically in many ways. The stochastic extension investigated uses Rockafellar and Wets’ Progressive Hedging Algorithm. This takes a scenario approach, in which the stochastic variables are approximated by a number of scenarios of observed values. A policy is required which minimizes the expected cost of generation over these scenarios, ensuring that information on the observed values of the stochastic variables is not used before it would be available in practice.

Results and implementation issues are discussed for both the deterministic and stochastic models. Consideration is given to the implementation of a finished product, as well as implementation for the purposes of investigating the feasibility and examining the computational effectiveness of approximations made in the model.
I would like to thank ECNZ for their funding of this project and for sending me to the 15th Symposium on Mathematical Programming which gelled many of the ideas in this thesis. My chief supervisor Dr. John Giffin deserves a lot of praise for his help, support, extensive knowledge of the English Language, and, "in some ways," for telling me when to stop. I would also like to thank my second supervisor Dr. Jonathan Lermit for his help, ideas (it’s a shame about the cumulants), valiantly trying to print the colour figures within this thesis, for coming up every month or so to ensure I was actually doing some work, and for his nice comments about markets (which I have studiously tried to avoid). I would also like to thank Philippa, my wife, for her love and support, for her proof reading (even though she complained about not understanding most of it), and especially for her interest and enthusiasm in my thesis.

There are many other people I would like to thank but don’t have the room for, although a special note of thanks goes to Richard “Vancouver Coast Guard” Rayner for getting Saruman and Matlab back up again (well, for calling the repairman) when I had only two weeks to finish this think, and for being away the six days before I sent it in to be bound, while I was trying to get the colour pages printed.

Special thanks goes to Philippa (my wife) and Mark Johnston for helping me to “colour in” the coloured pictures, by-hand, the night before binding. Thankyou.
Addendum

Page 25, line 11: “optimality” should read “optimality”.

Page 109, §6.2, sentences two and three should become:

“The general stochastic program can be written as a multi-stage stochastic program with recourse. The two-stage stochastic program with recourse can be written as follows:

$$\text{Min } f_1(x) + E \left[ f_2(x, \xi) \right]$$

subject to:

$$Ax = b$$

where $x$ is the decision variable representing a decision that must be implemented prior to the realization of the random variable $\xi$, $f_1(x)$ represents the cost of decision $x$, and $f_2(x, \xi)$ (where $\xi$ is a single observation of $\xi$) is defined as:

$$f_2(x, \xi) = \text{Min } g(y)$$

subject to:

$$Wy = \xi - Tx,$$

$$y \geq 0$$

where this involves the determination of the optimal recourse variable $y$ given the initial decision $x$. Extension to the multi-stage case involves defining (6.0b) in a similar manner to (6.0a). In our case the recourse variables are the releases of the subsequent weeks.”

Page 113, paragraph 3; should be appended with the sentence:

“While there are many other stochastic techniques which could be considered, since the focus here is to show that it is feasible to extend the deterministic model developed to a full stochastic model and we cannot cover every method here, the following are a selection of approaches which have been used in the past to model such a system.”

Page 115, §6.5, line 2; “...as it offers the greatest flexibility in the extent ...” should read “...as it appears to offer the greatest flexibility, of any of the many possible stochastic approaches which could be used, in the extent ...”
Contents

Acknowledgements ____________________________________________ iv
Table of Contents ___________________________________________ v
List of Figures ______________________________________________ x
List of Tables _______________________________________________ xi

1 Problem Definition _________________________________________ 1
  1.1 Description of the System .................................................. 2
  1.2 Objectives of the Model ...................................................... 4
  1.3 Uncertain Supply ............................................................... 5
  1.4 Hydro Detail ................................................................. 6
  1.5 Transmission Detail ......................................................... 8
  1.6 Putting it in Perspective .................................................. 8
  1.7 An Implementable Model .................................................. 9

2 Past Solutions _____________________________________________ 11
  2.1 Maximizing Generation .................................................... 12
  2.2 Purely Deterministic ....................................................... 13
  2.3 Stochastic Aspects ........................................................... 15
  2.4 Load Duration Curves ...................................................... 15
  2.5 Filling an LDC using Thermal Stations ................................ 17
  2.6 Hydro Stations Filling LDC ............................................... 19
  2.7 Geographical Distribution of Power ................................... 21
  2.8 How Many Hydro Reservoirs? ............................................ 23
3 Desirable Features of the Model

3.1 A Flexible Model
3.2 Internal Consistency
3.3 Geographic Distribution
3.4 Time
3.5 Transmission
3.6 Load
3.7 Stations
  3.7.1 Thermal Stations
  3.7.2 Hydro Stations
  3.7.3 Auxiliary Stations
3.8 Non-Supply
3.9 Stochastic Elements
3.10 The Electricity Curve Approximation
3.11 Discussion

4 Inappropriate Approximations

4.1 Filling Contract Curves
4.2 Hydro Stations Filling Contract Curves
4.3 Transmission Capacity
4.4 The Objective Function and Convexity
4.5 A Cumulant Approximation
  4.5.1 Contract Curve Filling Functions
  4.5.2 Transmission Revisited
  4.5.3 Dispense with Piecewise Quadratics?
4.6 Why a Normal Approximation?
  4.6.1 Contract Curve Corners
  4.6.2 Convexity of the New $E$
  4.6.3 Hydro Creases
4.7 The really bad news!
4.8 Discussion

5 The Model

5.1 Removing the Hydro Crease
5.2 Thermal Contract Curves
5.2.1 Approximations to Handle Breakdowns
5.3 Transmission
5.4 A Better Basis
5.5 Exploiting Flexibility
5.6 End Effects and Discount Factors
5.7 The Deterministic Model
5.8 Generalized Network with Side Constraints
5.9 Discussion

6 Modelling Stochastic Inflows
6.1 Optimality in a Stochastic Setting
6.2 The General Problem
6.3 A New Method
6.4 Stochastic Approaches
   6.4.1 Stochastic Dynamic Programming
   6.4.2 Scenario-Aggregation
6.5 Applying Progressive Hedging
   6.5.1 A Brief Description of Progressive Hedging
6.6 Choosing Scenarios
6.7 Reducing Effort when Progressive Hedging
6.8 Non-anticipativity
6.9 Advantages of a Scenario Method
6.10 Discussion

7 House Rules
7.1 Huntly and Stratford
7.2 Gas Deliverability
7.3 Security of Supply
7.4 Implementation of these Constraints

8 Function Formulation
8.1 The Generalization
8.2 Unsettling Results
8.3 Discussion
List of Figures

1.1 Amalgamated hydro system and representative inflows ........ 8

2.1 A Load Curve and corresponding Load Duration Curve ........ 18
2.2 An LDC and its inverse, a probability distribution function .... 18
2.3 Totally reliable stations filling an LDC .................. 20
2.4 Unreliable stations filling an LDC .......................... 20
2.5 Hydro station splitting an LDC ............................... 22
2.6 Hydro station splitting a thermal station’s generation .......... 22

3.1 The representative geographic network .......................... 28
3.2 The North Island load for a whole year and a single day ......... 30
3.3 The effect of allowing negative power transmission ............ 32
3.4 Interaction between two Load Duration Curves .................. 35
3.5 An example of a waterflow network ............................. 40
3.6 The best least squares 3-piecewise quadratic approximation ..... 46
3.7 Intended generation of some station ............................ 46
3.8 Best fit 12-nomial approximating a non-smooth curve ........... 47
3.9 Various piecewise approximations to an LDC ................... 47
3.10 A 4-piecewise quadratic approximating an LDC ............... 49

4.1 The two cases for a hydro station filling a C.C. ................. 55
4.2 Two hydro stations filling a C.C. with no overlap .............. 55
4.3 Two hydro stations filling a C.C. with overlap .................. 57
4.4 Values of [a b c] for which \(at^2 + bt + c \in [0, 1] \forall t \in [0, 1]\) .... 59
4.5 A truncated Gram-Charlier Type A expansion of an LDC ......... 70
4.6 A normal approximation of an LDC ............................ 70
4.7 Area above zero for the normal approximation of \(b(t - \frac{1}{2}) + c\) .... 75
4.8 Crease in hydro station non-supply ............................ 75
4.9 A C.C. and its corresponding normal approximation ........................................ 77
4.10 Percentage of actual generation wrongly non-supplied ........................................ 77
4.11 The fraction of actual transmission wrongly penalized ........................................ 80

5.1 A simple hydro river chain ......................................................................................... 83
5.2 A maximum level release constraint applied in both forms ........................................ 85
5.3 Non-supply in a hydro station's C.C., and its “approximation” ................................... 91
5.4 Capacity constraints solely through line loss ............................................................ 91
5.5 Values of $a$ and $b$ for which $at^2 + bt + A \in [A, B] \forall t \in [0,1]$ ................................ 93
5.6 Effective smoothing of piecewise linear line losses .................................................... 93
5.7 Structure of the new basis elements ............................................................................ 96
5.8 Part of the waterflow network with the addition of $V$ ................................................ 105

6.1 Ten years of inflows into lake Te Anau ................................................................. 107
6.2 Te Anau inflows against those into Pukaki and Hawea ............................................ 107
6.3 Rockafellar and Wets' Progressive Hedging Algorithm .............................................. 118
6.4 A scenario tree ........................................................................................................ 120
6.5 Various scenarios from the scenario tree .................................................................... 120
6.6 Split sub-trees ........................................................................................................... 123
6.7 Joined sub-trees ........................................................................................................ 123
6.8 Suggested break-up of a particular scenario tree ....................................................... 125

8.1 The projection/restriction of two sets onto two subspaces ........................................ 140
9.1 Convergence using sub-optimal subproblem solutions ............................................. 154
9.2 Convergence of $W$ with and without a weighted inner product ............................... 154
9.3 Various measures of convergence for an eight scenario problem .............................. 156
9.4 Convergence measures from an exhaustive run ....................................................... 158
9.5 Convergence for differing $r$ values .......................................................................... 160
List of Tables

5.1 Connections between the release and total generation .............. 86
5.2 The Full Deterministic Working Model .............................. 101
5.3 Notation used in Table 5.2 ........................................... 102

8.1 Function formulation ............................................. 135
8.2 Variables used in Table 8.1 ........................................ 136
8.3 Approximation of the formulation of Table 8.1 ....................... 137
8.4 \( F^{(1)} \) ............................................................ 138
8.5 \( F^{(1)}_n \) ............................................................ 138
8.6 \( F^{(2)} \) ............................................................... 139

9.1 Solution times for various \( r \) values ............................... 159
9.2 Solution times under differing numbers of scenarios ............... 161