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A MODEL FOR DEEP GEOTHERMAL BRINES:
STATE SPACE DESCRIPTION
AND THERMODYNAMIC PROPERTIES

A THESIS PRESENTED IN PARTIAL FULFILMENT
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Abstract

To facilitate the simulation of heat and fluid transport in deep geophysical environments, this thesis proposes correlations for calculation of density, enthalpy and viscosity values for brine ($\text{H}_2\text{O} + \text{NaCl}$) over a wide range of temperature T , pressure p and mass fraction of sodium chloride X . Although geothermal fluids are not pure H_2O - NaCl systems, they are mainly composed of H_2O and NaCl and in this thesis we model geothermal fluids as brines. Firstly T - p - X state space delineations of such a model fluid are described. Then using experimental and calculated data, approximate correlations for the three properties are given in terms of the primary variables T , p and X . These correlations cover the entire T - p - X state space and can be used in subroutines suitable for use in numerical simulation programs. The case of one-dimensional, steady vertical flows is described and our correlations for the state space delineations and the thermodynamic properties are tested on such flows.

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Nomenclature

The unit volume and area referred to here are the unit volume and area of the fluid-saturated porous medium, i.e. the fluid-filled geothermal system.

A_e	Internal energy per unit volume, J/m ³
A_m	Total mass per unit volume, kg/m ³
A_m^γ	Mass per unit volume of the sodium chloride ($\gamma = c$), rock ($\gamma = r$) or water ($\gamma = w$), kg/m ³
C	Chemical composition
c	Specific heat at constant volume, kJ/kg K = J/g K
\mathbf{g}, g	Acceleration due to gravity, m/s ²
h	Specific enthalpy, kJ/kg = J/g
k	Permeability, m ²
k_{ri}	Relative permeability of the liquid ($i = \ell$) or the gas ($i = g$), dimensionless
K	Thermal conductivity, W/m K
m_γ	Mass flux per unit area of the sodium chloride ($\gamma = c$) or the water ($\gamma = w$), kg/m ² s
NCG	Non-condensable gas(es)
p	Pressure, bars absolute
q_e	Rate of production or injection of energy per unit volume, J/m ³ s
q_m^γ	Rate of production or injection of mass per unit volume of the sodium chloride ($\gamma = c$) or the water ($\gamma = w$), kg/m ³ s
Q_e, q	Energy flux per unit area, J/m ² s
Q_m	Total mass flux per unit area, kg/m ² s
Q_m^γ	Mass flux per unit area of the sodium chloride ($\gamma = c$) or the water ($\gamma = w$), kg/m ² s

Q_{mi}	Mass flux of liquid ($i = \ell$) or gas ($i = g$) per unit area, kg/m ² s
Q_{mi}^{γ}	Mass flux per unit area of γ in the i phase, $\gamma = w$ (water) or c (sodium chloride), $i = \ell$ (liquid) or g (gas), kg/m ² s
t	Time, s
T	Temperature, °C
TDS	Total dissolved solids
S	Saturation, dimensionless
u	Specific internal energy, kJ/kg = J/g
V	Volume, m ³
VPL	Vapour pressure lowering
\mathbf{x}	Position vector, m
X	Mass fraction of sodium chloride, dimensionless
z	Vertical coordinate, m

Subscripts

b	Brine
bg	Brine gas or vapour
bl	Brine liquid
c	Sodium chloride
cap	Capillary
cg	Sodium chloride gas
cl	Sodium chloride liquid
$c\ sat$	Sodium chloride saturation
$cg\ sat$	Saturated sodium chloride gas
$cl\ sat$	Saturated sodium chloride liquid
$CRIT$	Critical
e	Energy
f	Fluid
g	Gas or vapour
gs	Gas phase of the two-phase fluid
$g\ SAT (= bg\ sat)$	Halite-saturated gas on the three-phase surface
$g\ sol$	Maximum solubility in water gas (vapour) or halite-saturated gas

ℓ	Liquid
ℓ_s	Liquid phase of the two-phase fluid
$\ell SAT (= bl sat)$	Halite-saturated liquid on the three-phase surface
ℓsol	Maximum solubility in liquid water or halite-saturated liquid
m	Mass
mg	Mass of gas
$m\ell$	Mass of liquid
r	Rock
SAT	Saturation
w	Water
$w sat$	Water saturation
$wg sat$	Saturated water gas
$w\ell sat$	Saturated liquid water
1000	1000 bars absolute
2b	Region 2b

Superscripts

c	Sodium chloride
w	Water

Greek letters

ε	Volume fraction of pore space occupied by the fluid or voidage, dimensionless
μ	Dynamic viscosity, kg/m s
$\nu = \mu/\rho$	Kinematic viscosity, m ² /s
ρ	Density, kg/m ³
ϕ	Porosity, dimensionless

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