A DESCRIPTOR APPROACH
TO
SINGULAR LQG CONTROL PROBLEMS
USING
WIENER-HOPF METHODS

A thesis presented in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Production Technology at Massey University.

Ian Harvey Noell

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Errata

Page 53 Line 6:

... the function f(t) must be absolutely integrable over $(-\infty,\infty)$.

Page 116 Section 5.3.1, 2nd line:

... the state-space representations (4.22) and (4.23).

Page 135 Equation (5.76):

$$J = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \Big(Tr\Big\{ \Delta T\Gamma\Gamma^{*}T^{*}\Delta^{*} - M_{\oplus}\Gamma^{*}T^{*}\Delta^{*} - \Delta T\Gamma M_{\oplus}^{*} - TM_{\vartheta}^{*} + M_{\vartheta}T^{*} \Big\} + Tr\Big\{QP_{d}\Phi_{d}P_{d}^{*}\Big\} \Big)\, ds \qquad (5.76)$$

Page 157 Example 6.1 the state-space representation of P_d is:

$$P_d = (sI-A)^{-1}E = \left[\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right]$$

Abstract

Wiener-Hopf methods are used in this thesis to solve the output feedback Linear Quadratic Gaussian (LQG) control problem for continuous, linear time-invariant systems where the weighting on the control inputs and the measurement noise intensity may be singular. Some outstanding issues regarding the closed loop stability of Wiener-Hopf solutions and its connection with partial fraction expansion are resolved.

The main tools in this study are state-space representations and Linear Matrix Inequalities. The relationship between Linear Matrix Inequalities and Wiener-Hopf solutions is studied; the role of the Linear Matrix Inequality in determining spectral factors, the partial fraction expansion step, the form of the controller, and the value of the performance index is demonstrated.

One of the main contributions of this thesis is the derivation of some new descriptor forms for singular LQG controllers which depend on the solution to the Linear Matrix Inequalities. These forms are used to establish the separation theorem for singular LQG control problems and to investigate the order of singular LQG controllers.

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He would have been at a loss to explain what was so arresting about this notion; he simply felt stricken to the heart and stood there in a terror that was almost mystical. A moment passed and everything before him seemed to expand; instead of horror - light and gladness, ecstasy; he began to struggle for breath and . . . but the moment passed. Thank God, it wasn't that! He took a deep breath and looked about him.

Fyodor Dostoevsky
The Idiot.

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