Towards
a Comprehensive Model for
the Positive Electrode System of
a Lead-Acid Traction Cell

A thesis presented
in partial fulfilment of the requirements
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in Production Technology at
Massey University

Ross Richard Nilson

1989
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Abstract

This thesis develops a detailed model for the positive electrode system of an industrial lead-acid traction cell. This is referred to as the VIAM model since it relates the positive electrode voltage (V) and cell current (I) to internal distributions of current, potential, acid concentration and active mass (AM). The model can simulate both discharge and charge for a wide range of practical currents. The model takes account of microstructure, macrostructure and non-reactive structure in the positive active mass (AM). It also takes account of other cell components that affect the supply of acid to the positive electrode. The model has direct application to fundamental cell design (for example AM development) and cell systems design (for example cell charger design).

The model is based on established experimental studies, theories of electrochemical interface reactions and theories of ionic transport in electrolyte solution. From this base, three elemental models and an aggregate model are developed. The elemental models represent details of the microstructure of the positive electrode AM. The aggregate model represents the electrolyte mass (acid) and charge transport system within the positive electrode and other cell components. The combination of the elemental and aggregate models make up the VIAM model. The performance of the VIAM model (and underlying models) is assessed by comparing model results with findings from experimental studies in the literature. In addition, experiments undertaken as part of this work are used to test the model. The model and experimental results are in close agreement.
I would like to thank Dr R. I. Chaplin, Professor W. Monteith and Professor R. M. Hodgson for the encouragement and constructive criticism they have offered throughout the course of this work.

Neville Manson, Technical Manager of Chloride Batteries New Zealand Limited, kindly provided technical details for the cell used in experiments performed in this work.

My thanks also to my wife, Mary, who not only managed two very active children, Jonathan and Elissa, without help for many evenings during the preparation of this thesis, but also had enough energy left to proof-read the text.
Contents

Abstract . . . . . . . . . . . ii
Acknowledgments . . . . . . . . iii
Contents . . . . . . . . . . . . iv

List of Figures . . . . . . . . . . xi
List of Tables . . . . . . . . . . xv
Units and Symbols . . . . . . . xvi

Chapter 1: Introduction . . . . . . . . . . . . 1

Chapter 2: Lead-acid Traction Cells and Systems . . 5

2.1 Lead-acid Traction Cells . . . . . . . . . 6

2.2 Cell Service Life . . . . . . . . . . . . . . 8
  2.2.1 The Influence of the Vehicle . . . . . 9
  2.2.2 The Influence of the Charger Unit . . 10
  2.2.3 Life Reducing Processes in Perspective . 13

2.3 Cell Centred Approaches to Improving Service Life 14
  2.3.1 Fundamental Studies . . . . . . . . . 14
  2.3.2 Technological Improvements . . . . . 18
  2.3.3 Expectations from Cell Centred Work . . 19

2.4 A Complementary Approach to Improving Cell Life 19

2.5 Developing a Cell Life Prolonging Charger . . 21

2.6 A Model for the Positive Electrode System . . 23
### Chapter 3: Foundations for a Positive Electrode Model

#### 3.1 The Charge and Discharge Mechanism

- 3.1.1 A Description of the Charge and Discharge Mechanism
- 3.1.2 Conclusions Regarding the Charge and Discharge Mechanism

#### 3.2 The Mechanical Structure of the AM

- 3.2.1 A Description of the Mechanical Structure of the AM
- 3.2.2 Conclusions Regarding the AM Mechanical Structure

#### 3.3 The Electrochemical Reaction

- 3.3.1 The Equilibrium Potential
- 3.3.2 The Current/Low Overpotential Characteristic
- 3.3.3 The Current/High Overpotential Characteristic
- 3.3.4 Conclusions Regarding the Electrochemical Reaction

#### 3.4 Transport in Solution

- 3.4.1 Mass Flux Density
- 3.4.2 Electroneutrality
- 3.4.3 Charge Flux Density
- 3.4.4 An Alternative Form for Mass Flux Equations
- 3.4.5 Concentration Changes with Time
- 3.4.6 Conclusions Regarding Transport in Solution

#### 3.5 Gas Evolution in the Electrode and Cell

- 3.5.1 Current and the Gas Evolving Reaction
- 3.5.2 Electrolyte Mass Transport and Gas Evolution
- 3.5.3 Electrolyte Resistivity and Gas Evolution
- 3.5.4 Surface Masking and Gas Evolution
- 3.5.5 Conclusions Regarding Gas Evolution
3.6 Common Models for Porous Electrodes . . . 56
3.6.1 The Single Pore Model . . . 56
3.6.2 The Macrohomogeneous Model . . . 58
3.6.3 Other Models of Interest . . . 62
3.6.4 Conclusions Regarding Porous Electrode Models . . . 62

3.7 Voltage, Current and AM Experimental Data . . 63
3.7.1 Electrode Voltage and Current . . . 64
3.7.2 Electrode AM Distribution . . . 68
3.7.3 Conclusions Regarding Experimental Data . 70

3.8 Development Areas for a Positive Electrode Model 71

Chapter 4: Three Elemental Models for the Positive Electrode . . . . . . . . . . . . . . . . . 73

4.1 An Elemental Discharge Capacity Model . . . 75
4.1.1 The Microstructure: A Discharge Limiting Factor . . . . . . . . . . . . . . 75
4.1.2 Local Discharge Capacity Formulations . 76
4.1.3 Charge State Formulations . . . 78

4.2 An Elemental Discharge Surface Area Model . . 79
4.2.1 A Qualitative Description of the Discharge 79
4.2.2 Discharge Surface Area Formulations . . 81

4.3 An Elemental Charge Surface Area Model . . . 82
4.3.1 A Qualitative Description of the Charge . 84
4.3.2 A Simple Geometric Model at Full Discharge 85
4.3.3 PbSO₄ Surface Area Formulations . . . 87
4.3.4 Actual PbO₂ Surface Area Formulations . 88
4.3.5 Effective PbO₂ Surface Area Formulations . 96
4.3.6 The Elemental Charge Surface Area Model: A Practical Approach . . . . 98
Chapter 5: An Aggregate Model for the Positive Electrode

5.1 The Aggregate Model
5.1.1 Transport System Parts
5.1.2 A Minimum Representation of the Acid Transport System
5.1.3 An Aggregate Model for the Acid Transport System

5.2 Physical Dimensions of the Aggregate Model
5.2.1 The m-channel Dimensions
5.2.2 The μ-channel Dimensions
5.2.3 The h-channel Dimensions
5.2.4 The s-channel Dimensions
5.2.5 The n-channel Dimensions
5.2.6 The r-channel Dimensions

5.3 Aggregate Model Electrical Formulations
5.3.1 The h-channel Equivalent Circuit
5.3.2 The m-channel Equivalent Circuit
5.3.3 The μ-channel Equivalent Circuit
5.3.4 The Complete Equivalent Circuit
5.3.5 Supplementary Formulations for the s-channel and n-channel

5.4 Aggregate model Acid Transport Formulations
5.4.1 The General Transport, Boundary Condition and Initial Condition Equations
5.4.2 Acid Transport in the μ-channels
5.4.3 Acid Transport in the m-channel
5.4.4 Acid Transport in the h-channel
5.4.5 Acid Transport in the r-channel
5.4.6 Acid Transport in the n-channel
5.4.7 Acid Transport in the s-channel
Chapter 6: Industrial Traction Cell Experiments

6.1 The Experimental Equipment
6.1.1 Equipment Overview
6.1.2 The Data Acquisition System
6.1.3 The Purpose Built Hardware

6.2 The Cell Under Test
6.2.1 General Specification
6.2.2 Physical Construction Details
6.2.3 Grid and Paste Composition Details

6.3 Experimental Procedure and Schedule
6.3.1 Discharge/Charge Cycle Procedures
6.3.2 The Experimental Schedule

Chapter 7: Results and Discussion

7.1 The Elemental Models
7.1.1 The Discharge Capacity Model Results
7.1.2 The Discharge Capacity Model: a Discussion
7.1.3 The Discharge Surface Area Model Results
7.1.4 The Discharge Surface Model: a Discussion
7.1.5 The Charge Surface Area Model Results
7.1.6 The Charge Surface Area Model: a Discussion
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.2</td>
<td>The Aggregate Model</td>
<td>210</td>
</tr>
<tr>
<td></td>
<td>7.2.1 Aggregate Model Dimensions</td>
<td>210</td>
</tr>
<tr>
<td></td>
<td>7.2.2 Aggregate Model Dimensions: a Discussion</td>
<td>213</td>
</tr>
<tr>
<td></td>
<td>7.2.3 Functions for the Aggregate Model</td>
<td>214</td>
</tr>
<tr>
<td>7.3</td>
<td>The VIAM Model</td>
<td>215</td>
</tr>
<tr>
<td></td>
<td>7.3.1 VIAM Model Results for a Standard Case</td>
<td>216</td>
</tr>
<tr>
<td></td>
<td>7.3.2 Other VIAM Model Results</td>
<td>225</td>
</tr>
<tr>
<td></td>
<td>7.3.3 The VIAM Model μ-channel Component</td>
<td>241</td>
</tr>
<tr>
<td></td>
<td>7.3.4 The VIAM Model: a Discussion</td>
<td>241</td>
</tr>
<tr>
<td>7.4</td>
<td>Experimental Results</td>
<td>249</td>
</tr>
<tr>
<td></td>
<td>7.4.1 Calculations Performed on the Raw Data</td>
<td>249</td>
</tr>
<tr>
<td></td>
<td>7.4.2 Presentation of the Experimental Results</td>
<td>255</td>
</tr>
<tr>
<td></td>
<td>7.4.3 Experimental Results for a Standard Case</td>
<td>256</td>
</tr>
<tr>
<td></td>
<td>7.4.4 Other Experimental Results</td>
<td>259</td>
</tr>
<tr>
<td></td>
<td>7.4.5 Experimental Results: a Discussion</td>
<td>266</td>
</tr>
<tr>
<td>7.5</td>
<td>Comparing VIAM Model and Experimental Results</td>
<td>268</td>
</tr>
<tr>
<td></td>
<td>7.5.1 Acid Diffusion and Exchange Currents for the VIAM Model</td>
<td>268</td>
</tr>
<tr>
<td></td>
<td>7.5.2 The VIAM Model and Experimental Results</td>
<td>269</td>
</tr>
<tr>
<td>Chapter 8: Conclusion</td>
<td>277</td>
<td></td>
</tr>
<tr>
<td>8.1</td>
<td>The Contribution of this Work</td>
<td>277</td>
</tr>
<tr>
<td></td>
<td>8.1.1 A New Positive Electrode Model</td>
<td>277</td>
</tr>
<tr>
<td></td>
<td>8.1.2 Model Performance</td>
<td>278</td>
</tr>
<tr>
<td></td>
<td>8.1.3 Model Application</td>
<td>280</td>
</tr>
<tr>
<td>8.2</td>
<td>Extensions to the Model</td>
<td>281</td>
</tr>
<tr>
<td></td>
<td>8.2.1 Exchange Reaction Representation</td>
<td>281</td>
</tr>
<tr>
<td></td>
<td>8.2.2 Cell Gassing</td>
<td>281</td>
</tr>
<tr>
<td></td>
<td>8.2.3 A Two Dimensional Plate</td>
<td>282</td>
</tr>
<tr>
<td></td>
<td>8.2.4 Cell Temperature</td>
<td>282</td>
</tr>
</tbody>
</table>
List of Figures

Figure

1.1 The VIAM Model and Underlying Models 3

2.1 Cell Energy and Replacement Cost per Charge Cycle 8
2.2 Traction Cell Cycle Life and Discharge Depth 9
2.3 Positive Grid Corrosion and Polarisation Effects 16
2.4 Development of a Life Prolonging Charger 21

3.1 Pore Volume and Surface Area Distribution 31
3.2 Micrographs of Positive Active Mass 32
3.3 Pore Volume Distribution and Charge State 33
3.4 Experimental Surface Area and Charge State 34
3.5 Cathodic Tafel Plots for Two Lead Dioxide Electrodes 44
3.6 Dissociation of Sulphuric Acid 46
3.7 The Single Pore Model Equivalent Circuit 56
3.8 Macrohomogeneous Model Discharge Voltage Predictions 61
3.9 Experimental Cell Polarisation Components 65
3.10 Experimental Charge Voltage and Discharge Rate 66
3.11 Experimental Charge Voltage and Discharge Depth 67
3.12 Experimental AM Distribution in Positive Plate 69

4.1 A Representation of the Discharge Process 81
4.2 The Discharge Surface Area Model 82
4.3 A Representation of the Charge Process 84
4.4 A Geometric Model of the Discharged AM 85
4.5 A Rectangular System for Lead Dioxide Growth 89
4.6 An Elemental Volume for the Pb2+ Mass Balance 90
4.7 The Lead Dioxide Lobe Surface Area 92
4.8 Calculation of Charge Surface Area 105
Figure

5.1 Components of the Acid Transport System . . 109
5.2 The Aggregate Model Components . . . . 112
5.3 $\mu$-Channel Arrangement Around the $m$-Channel . . 116
5.4 The $m$-Channel Equivalent Circuit . . . . 127
5.5 The $\mu$-Channel Equivalent Circuit . . . . 129
5.6 The Aggregate Model Equivalent Circuit . . . . 133
5.7 Calculation of Electrical Parameters . . . . 154
5.8 Boundary between Channels on Same Axis . . . . 158
5.9 The $m$-Channel/$\mu$-Channel Boundary . . . . 161
5.10 Calculation of Transport Parameters . . . . 168
5.11 Over-all Calculation Procedure . . . . 169

6.1 Block Diagram of Experimental Equipment . . . . 173
6.2 Discharge/Charge Cycle Algorithm . . . . 174
6.3 Programmable Voltage Source Schematic . . . . 177
6.4 Measurement Interface Circuits . . . . 178
6.5 Temperature Controller Schematic . . . . 180
6.6 The Purpose Built Hardware . . . . 181
6.7 Cell Grid and Separator Components . . . . 183

7.1 Possible Distribution of Discharge AM in Plate 191
7.2 The Discharge Surface Area Model Approximation 195
7.3 Charge Surface Area Development . . . . 198
7.4 Lobe Profiles at Various Charge States . . . . 198
7.5 Charge Surface Area for Various Currents . . . . 201
7.6 Lobe Profiles for Various Currents . . . . 201
7.7 Charge Surface Area for Various Acid Concentrations . . . . 202
7.8 Lobe Profiles for Various Acid Concentrations . . . . 202
7.9 Charge Surface Area for Various Geometric Constants . . . . 204
7.10 Lobe Profiles for Various Geometric Constants 204
7.11 Model and Fitted Effective PbO$_2$ Charge Surface Area . . . . 206
7.12 Model and Experimental Charge Surface Area . . . . 209
7.13 Discharge Voltage for 20 Ampere Full Discharge 218
Figure

7.14 Rest and Charge Voltage for 20 Ampere Full Discharge . . . . . . 218

7.15 Discharged Plate AM for 20 Ampere Full Discharge 219
7.16 Plate Acid for 20 Ampere Full Discharge . . 219
7.17 Plate Current for 20 Ampere Full Discharge . 220
7.18 Plate Solution Potential for 20 Ampere Full Discharge . . . . . . 220
7.19 Adjacent Acid for 20 Ampere Full Discharge . 221
7.20 Reservoir Acid for 20 Ampere Full Discharge . 221
7.21 Discharge Voltage for 3.5 Ampere Full Discharge 226
7.22 Rest and Charge Voltage for 3.5 Ampere Full Discharge . . . . . . 226
7.23 Discharged Plate AM for 3.5 Ampere Full Discharge 227
7.24 Plate Acid for 3.5 Ampere Full Discharge . 227
7.25 Discharge Voltage for 140 Ampere Full Discharge 230
7.26 Rest and Charge Voltage for 140 Ampere Full Discharge . . . . . . 230
7.27 Discharged Plate AM for 140 Ampere Full Discharge 231
7.28 Plate Acid for 140 Ampere Full Discharge . 231
7.29 Discharge Voltage for More Tortuous AM . . 233
7.30 Rest and Charge Voltage for More Tortuous AM . 233
7.31 Discharged Plate AM for More Tortuous AM . 234
7.32 Plate Acid for More Tortuous AM . . . 234
7.33 Discharge Voltage with Non-participating AM . 236
7.34 Rest and Charge Voltage with Non-participating AM 236
7.35 Discharged Plate AM with Non-participating AM 237
7.36 Plate Acid with Non-participating AM . 237
7.37 Discharge Voltage for Increased Exchange Current 239
7.38 Rest and Charge Voltage for Increased Exchange Current . . . . . . 239
7.39 Discharged Plate AM for Increased Exchange Current . . . . . . 240
7.40 Plate Acid for Increased Exchange Current . 240
7.41 Experimental and Model Discharge Voltage . 244
7.42 Experimental and Model Charge Voltage . 245
7.43 Experimental and Model Discharged Plate AM . 247
Figure

7.44 Experimental and Model Cell Life . . . 248
7.45 Potential Differences in Cell Under Test . 250
7.46 AD100R20 Cell Voltage . . . . . . . 257
7.47 AD100R20 Electrode Voltage Estimate . . 257
7.48 AD100R20 Reservoir Acid Concentration . . 258
7.49 AD100R10-R20 Cell Discharge Voltage . . 260
7.50 AD100R10-R20 Electrode Discharge Voltage . 260
7.51 AD050R05-R40 Cell Discharge Voltage . . 261
7.52 AD050R05-R40 Electrode Discharge Voltage . 261
7.53 AD010R05-R40 Cell Discharge Voltage . . 262
7.54 AD010R05-R40 Electrode Discharge Voltage . 262
7.55 AD100R10-R20 Cell Rest and Charge Voltage . 263
7.56 AD100R10-R20 Electrode Rest and Charge Voltage . 263
7.57 AD050R05-R40 Cell Rest and Charge Voltage . 264
7.58 AD050R05-R40 Electrode Rest and Charge Voltage . 264
7.59 AD010R05-R40 Cell Rest and Charge Voltage . 265
7.60 AD010R05-R40 Electrode Rest and Charge Voltage . 265
7.61 VIAM model and AD100R20 Electrode Voltage . 270
7.62 VIAM model and AD100R10 Electrode Voltage . 270
7.63 VIAM model and AD050R40 Electrode Voltage . 271
7.64 VIAM model and AD050R20 Electrode Voltage . 271
7.65 VIAM model and AD050R10 Electrode Voltage . 272
7.66 VIAM model and AD050R05 Electrode Voltage . 272
7.67 VIAM model and AD010R40 Electrode Voltage . 273
7.68 VIAM model and AD010R20 Electrode Voltage . 273
7.69 VIAM model and AD010R10 Electrode Voltage . 274
7.70 VIAM model and AD010R05 Electrode Voltage . 274
**List of Tables**

**Table**

2.1 Some Features of Traction and SLI Cells . . 7  
2.2 Effect of Charge on SLI Cell Life . . . 20  

3.1 Experimental Current/Overpotential Parameters . 43  

6.1 Cell Dimensions . . . . . . . . 184  
6.2 Comparison of AM Production Processes . . 186  
6.3 Experimental Schedule . . . . . . 188  

7.1 Some Aggregate model Parameters . . . 210  
7.2 Remaining Aggregate Model Parameters . . 212  
7.3 Aggregate Model Dimensions . . . . 212
Units and Symbols

a) Units.

Normal SI units (Chiswell and Grigg (1971)) are used throughout this text with the following additions.

i) Hour(s) (abbreviated to hr(s)) is used as a measure of time (3600 s).

ii) Ampere hour(s) (abbreviated to Ahr(s)) is used as a measure of electrical charge (3600 A.s).

iii) Watt hour(s) (abbreviated to Whr(s)) is used as a measure of energy (3600 W.s).

iv) The gram (abbreviated to g) is used as a measure of mass (1x10^{-3} kg).

These additions are consistent with common practices in the battery industry.

b) Symbols for units.

Normal SI unit symbols (Chiswell and Grigg (1971)) are used throughout this text with the following additions.

i) Hr(s) for hour(s).

ii) Ahr(s) for Ampere hour(s).

iii) Whr(s) for watt hour(s).

iv) g for the gram.

v) v for the volt.
c) Symbols for variables.

Symbols for variables are fully defined in the body of text where they are first used.

d) Symbols for experiment designations.

An example designation for the experiments performed in this work is AD100R20. This should be interpreted as follows.

i) The first character (A) is the cell label that defines the cell involved (here cell A).

ii) The following four characters (D100) define the depth of discharge (here 100 Ahrs).

iii) The last three characters (R20) define the rate of discharge (here 20 A).