The disjunctivities of $\omega$-languages

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Abstract
An $\omega$-language over a finite alphabet $X$ is a set of infinite sequences of letters of $X$. Consider congruences $I$, $P_{\omega}$, and $O_{\omega}$ on $X^*$ and a congruence $O_{\omega}$ on $X^\omega$ introduced by an $\omega$-language $L$. $I$, $P_{\omega}$, and $O_{\omega}$ are called the infinitary syntactic-congruence, the principal congruence and the $\omega$-syntactic congruence of $L$, respectively. If $I$ ($P_{\omega}$, $O_{\omega}$) is the equality then $L$ is called an $I$-disjunctive ($P$-disjunctive, $O$-disjunctive, respectively) $\omega$-language. Properties concerning such $\omega$-languages are explored and relations between these $\omega$-languages are also studied.

The disjunctivity concerning the infinitary syntactic-congruence $I$
Given an $\omega$-language $L$, by the infinitary syntactic-congruence $I$ of $L$ we mean the relation $I$ on $X^*$ given by $u \equiv v (I) \iff \forall x, y \in X^*, x(uy)^\omega \in L \iff x(vy)^\omega \in L$. If $I$ is the equality then $L$ is called $I$-disjunctive. Every $I$-discrete $I$-dense $\omega$-language is $I$-disjunctive. An $\omega$-language is $I$-dense iff it contains an $I$-disjunctive language. A periodically generated $\omega$-language $L$ is $I$-dense iff $L$ can be expressed as a disjoint union of infinitely many $I$-disjunctive $\omega$-languages.

The disjunctivity concerning the principal congruence $P_{\omega}$
Given an $\omega$-language $L$, by the principal congruence $P_{\omega}$ of $L$ we mean the relation $P_{\omega}$ on $X^*$ given by $u \equiv v (P_{\omega}) \iff \forall x \in X^*$ and $\alpha \in X^\omega$, $xu\alpha \in L \iff xv\alpha \in L$. If $P_{\omega}$ is the equality then $L$ is called $P$-disjunctive. Every $P$-discrete $P$-dense $\omega$-language is $P$-disjunctive. A $P$-discrete $\omega$-language is $P$-disjunctive iff the set of all its finite subwords is $X^*$.

The disjunctivity concerning the $\omega$-syntactic congruence $O_{\omega}$
Given an $\omega$-language $L$, by the $\omega$-syntactic congruence $O_{\omega}$ of $L$ we mean the relation $O_{\omega}$ on $X^\omega$ given by $\alpha \equiv \beta (O_{\omega}) \iff \forall x \in X^*$, $x\alpha \in L \iff x\beta \in L$. If $O_{\omega}$ is the equality then $L$ is called $O$-disjunctive. If $S$ is a left singular language then $SL$ is $O$-disjunctive for any $O$-disjunctive $\omega$-language $L$. If $P$ is a finite prefix code then $L$ is an $O$-disjunctive $\omega$-language iff $PL$ is $O$-disjunctive.

Families of Disjunctive $\omega$-languages
Every $I$-closed $I$-disjunctive $\omega$-language is $P$-disjunctive while not every $P$-disjunctive $\omega$-language is $I$-disjunctive. Every $O$-disjunctive $\omega$-language is $P$-disjunctive while not every $P$-disjunctive $\omega$-language is $O$-disjunctive. Every $O$-disjunctive $\omega$-language is $I$-disjunctive while not every $I$-disjunctive $\omega$-language is $O$-disjunctive. Every $P$-disjunctive $\omega$-language is $P$-dense

References


