# Travelling Wave Solutions of Multisymplectic Discretisations of Wave Equations

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## Abstract

Symplectic integrators for Hamiltonian ODEs have been well studied over the years and a lot is known about these integrators. They preserve the symplecticity of the system which automatically ensures the preservation of other geometric properties of the system, such as a nearby Hamiltonian and periodic and quasiperiodic orbits.

It is then natural to ask how this situation generalises to Hamiltonian PDEs, which leads us to the concept of multisymplectic integration. In this thesis we study the question of how well multisymplectic integrators capture the long-time dynamics of multi-Hamiltonian PDEs. We approach this question in two ways—numerically and through backward error analysis (BEA). As multi-Hamiltonian PDEs possess travelling wave solutions, we wish to see how well multisymplectic integrators preserve these types of solutions.

We mainly use the leapfrog method applied to the nonlinear wave equation as our test problem and look for the preservation of periodic travelling waves. We call the resulting equation the *discrete travelling wave equation*. It cannot be solved exactly. Therefore, our analysis begins with numerically solving the discrete travelling wave equation for simplified nonlinearities.

Next, we mov on to analysing periodic solution for a smooth nonlinearity. This results in the presence of resonances in the solutions for certain combinations of the parameters. Finally, we use backward error analysis to compare and back up our results from numerical analysis.

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