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RENEWABLE ENERGY POTENTIAL IN NEW ZEALAND
– BY THE NUMBERS

A thesis presented in partial fulfilment of the requirements for the Degree of
Master of Science
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Abstract

Renewable energy plays a very important role in New Zealand’s energy supply system, with approximately 39% of energy supply from renewable sources. The idea of the thesis was inspired by the book *Sustainable Energy—Without The Hot Air* by David MacKay. The book uses basic physics and mathematics to estimate the amount of sustainable sources that are physically available before considering their economic feasibility. The goal of the thesis is to transfer some of MacKay’s ideas to New Zealand, and estimate an upper limit to the energy we can get from the following renewable resources: hydro, geothermal, wind, solar and waves, then compare this to New Zealand’s energy use, and hence answer the question Can New Zealand live on Renewables?

In this thesis, hydroelectricity potential was estimated using Hydroelectricity Image Processing Approach. This method is original to this thesis, and involved using some image processing to estimate an upper limit of the total hydro available using the Rainfall Map and New Zealand 100 m Digital Elevation Model. Also, some image processing has been done to estimate solar thermal and solar photovoltaic potential for every region in New Zealand using the Solar Radiation Map. Furthermore, Wind Resource Map and Rayleigh distribution were used to estimate the wind power density which is an important measure in wind industry for every region in New Zealand.

The results from this research show that it is possible for New Zealand to supply all of its energy requirements from renewable sources alone. In fact, the renewable resource available is around 9 times our current energy use. However, in reality there are many environmental, economic and social limitations that would need to be considered.
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