RENEWABLE ENERGY POTENTIAL IN NEW ZEALAND – BY THE NUMBERS

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Abstract

Renewable energy plays a very important role in New Zealand's energy supply system, with approximately 39% of energy supply from renewable sources.The idea of the thesis was inspired by the book *Sustainable Energy—Without The Hot Air* by David MacKay. The book uses basic physics and mathematics to estimate the amount of sustainable sources that are physically available before considering their economic feasibility. The goal of the thesis is to transfer some of MacKay's ideas to New Zealand, and estimate an upper limit to the energy we can get from the following renewable resources: hydro, geothermal, wind, solar and waves, then compare this to New Zealand's energy use, and hence answer the question Can New Zealand live on Renewables?

In this thesis, hydroelectricity potential was estimated using Hydroelectricity Image Processing Approach. This method is original to this thesis, and involved using some image processing to estimate an upper limit of the total hydro available using the Rainfall Map and New Zealand 100 m Digital Elevation Model. Also, some image processing has been done to estimate solar thermal and solar photovoltaic potential for every region in New Zealand using the Solar Radiation Map. Furthermore, Wind Resource Map and Rayleigh distribution were used to estimate the wind power density which is an important measure in wind industry for every region in New Zealand.

The results from this research show that it is possible for New Zealand to supply all of its energy requirements from renewable sources alone. In fact, the renewable resource available is around 9 times our current energy use. However, in reality there are many environmental, economic and social limitations that would need to be considered.

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Chapter 1

Introduction

The idea of the thesis was inspired by the book *Sustainable Energy — Without The Hot Air* by David MacKay[44], a physics professor at the University of Cambridge in the United Kingdom. The book discusses whether a country like Britain can live on its own renewable sources, and provides some strategies and plans for eliminating the gap between energy consumption and sustainable energy production. The book uses basic physics and mathematics to estimate the amount of sustainable sources that are physically available before considering their economic feasiblity.

I was interested in applying MacKay's ideas to New Zealand and to estimate an upper limit to the energy that we can get from renewable resources, and hence try to answer the question **Can New Zealand supply all of its energy requirements from renewables?** New Zealand and the United Kingdom are very different. New Zealand's population density is 15 times smaller than the United Kingdom's population density (16 people/km^2 for New Zealand, 243 people/km^2 for UK[44]). Also, New Zealand's average energy consumption is 138 kWh/d/p [53] of which 39% of energy supply is from renewables [50]. On the other hand, United Kingdom's average consumption is 125 kWh/d/p [44] of which approximately 9.4% of energy supply is from renewables [28]. In this thesis, I will consider the following renewable sources: hydro, geothermal, wind, solar and marine.

In this chapter, I will provide a brief summary of the book *Sustainable Energy — Without The Hot Air*, and an outline of the current situation of renewable energy resources in New Zealand. I will also provide an outline of the thesis.

1.1 Background

1.1.1 *Sustainable Energy—Without The Hot Air*

In the begining of *Sustainable Energy—Without The Hot Air*, MacKay points out that the sustainable energy debate is mainly about numbers. The problem is that numbers are rarely mentioned, and when they are mentioned, their meaning is often unclear and misleading. For example, we hear that Britain has a huge amount of renewable resources, but we need to know how this huge amount compares to Britain's huge consumption, and to do this comparison we need numbers(cf. [44], pp.2-4). The main aim of *Sustainable Energy—Without The Hot Air* is to provide the reader with honest, comprehensible and memorable numbers, so the reader can have a better understanding to answer questions such as "Can a country like Britain conceivably live on its own renewable energy sources?" [44, p. 3] or, "Will a switch to "advanced technologies" allow us to eliminate carbon dioxide pollution without changing our lifestyle?" [44, p. 3]

There are three motivations for discussing energy policy. First, fossil fuels are finite sources and it seems possible that cheap oil and gas will run out in our lifetime. Secondly, energy security is threatened when production cannot keep up with demand and we have to depend on other countries' fossil fuels and let them control our economy. The third motivation is that fossil fuels affect the climate. Many human activities affect the climate, but the biggest contributor is the greenhouse effect produced mainly by CO_2, which mainly comes from burning fossil fuels. (cf. [44], pp.5-16)

In his discussion, MacKay uses the units we see in our everyday energy bills.The unit for energy is kWh, and for power kWh/d/p. One reason for liking these units is that it will make it easier to move from talking about the UK to talking about other countries. It will also make the book more transportable and useful for sustainable energy discussions worldwide. (cf. [44], pp.24-26)

The first half of *Sustainable Energy—Without The Hot Air* discusses whether a country like the United Kingdom can live on its own renewables. MacKay starts by creating two 'stacks': a sustainable production stack and a consumption stack for the "typical moderately-affluent person". MacKay estimates the energy use for a typical household in the consumption stack (including transport, food, heating,. . . ,etc.) , and adds all possible sources of sustainable energy in the green stack (wind, solar,geothermal,. . . ,etc.). The two main conclusions from the first half of *Sustainable Energy—Without The Hot Air* are:

1. Renewable facilities has to be country-sized in order to make a contribution to the United Kingdom's current consumption. For example, to get a big contribution from waves, MacKay assumed covering half of the coastline with wave machines. Also, to grow energy crops, MacKay imagined covering 75% of the country with biomass plantations.
2. It is not going to be easy to find a plan that adds up using renewable resources alone. It might be possible for the average British energy consumption of 125 kWh/d/p to be provided from country sized renewable resources, but the problem is the public objections. The public needs to say yes to renewable energy proposals and stop saying no to anything other than fossil fuels. Otherwise, we can conclude that Britain's current consumption can never be met by British renewables, and Britain has to look for additional sources of energy or make huge reduction in consumption or maybe both. (cf. [44], pp.22-112)

In the second half of *Sustainable Energy—Without The Hot Air* MacKay explores six strategies for eliminating the gap between consumption and renewable production identified in the first half of the book, and then suggests five energy plans for Britain, each of which adds up.

The first three strategies for eliminating the gap involves reducing demand by population reduction, lifestyle change and changing to more efficient technology while keeping the lifestyle. The second three strategies invloves increasing supply by: clean coal, nuclear power and other countries' renewables. MacKay proposes to work with what he calls **Cartoon Britain**. Cartoon Britain consumes energy in three forms only: heating, transport and electricity, and the supply options are clean coal, nuclear and other countries' renewables. In Cartoon Britain, demand reduction involves reducing transport's and heating's energy demand and eliminating all fossil fuel use for transport and heating. This can be achieved by electrifiying transport, which will get transport off fossil fuels and makes transport more energy efficient, and by also electrifying most heating of air and water in buildings using heat pumps, which are four times more efficient than electrical heaters. What about electricity? MacKay suggests getting all the green electricity from four sources: Britain's renewables, clean coal, nuclear and from other countries' renewables. Among other countries' renewables, solar power in deserts seems to be the best option especially in countries with considerable amount of sunshine, large areas and small population density. (cf. [44], pp.114-185)

After that, MacKay suggests five energy plans for Britain that will be feasible by 2050. These plans differ in how much solar, wind, nuclear, . . . , etc they use. To allow a better understanding of each plan, MacKay deals with Cartoon Britain that consumes energy only in three forms: electricity, heating and trnasport. The common features between these plans are that the transport is largely electrified and the energy consumption of heating is reduced by improving the insulation of buildings and improving the control of temperature. MacKay concludes that it is physically possible to live on renewables and find a plan that actually adds up. But since 90% of Britain's energy is from fossil fuels, big changes will be required and needed to get off fossil fuels. (cf. [44], pp.203-213)

MacKay then transferred his ideas to the world and asked the question "Can Europe, North America and the world live on renewables?". The conclusion is Europe cannot live on renewables, and to get off fossil fuels Europe needs nuclear power or solar power in other people's deserts or both. North America's non-solar renewables are not enough; it can only live on renewables only if there is a massive expansion of solar power in its own deserts or nuclear power. The same applies to the world; the world can't live on non-solar renewables and needs to rely on one or more forms of solar power or nuclear or maybe both. (cf. [44], pp.231-239)

MacKay's ideas gained a great reputation in the world, and have been applied to many countries including USA, Australia and even New Zealand [30]. In New Zealand, *Sustainable Energy—Without All The Hot Air. A New Zealand Perspective* is a very interesting discussion about renewable energy options for New Zealand by Phil Scadden, a geoscientist at GNS Science Ltd [6]. Scadden was interested in providing a New Zealand perspective and in seeing how New Zealand's figures would look like compared to the United Kingdom's figures taking into account New Zealand's smaller population density, different level of energy use, availability of public transport, . . . , etc. Scadden focused on two questions: "Can New Zealand maintain its current per capita energy consumption without fossil fuels and live on renewables alone?", and "How can New Zealand reduce personal and national energy consumption in order to reduce the country's power requirements?" [6]

According to Scadden, New Zealand's consumer energy consumption in 2007 was 94 kWh/d/p, and 50% of this value is from oil and only 28 kWh/d/p is from renewable energy sources. The challenge is can we reduce our energy use to live on existing or expanded renewable sources alone? Scadden starts by estimating the potential for renewable resources (hydro, geothermal, wind,. . . , etc) in New Zealand. Scadden found that it is possible to maintain existing energy consumption levels and reduce dependency on fossil fuels if we are willing to accept renewable energy proposals mainly from wind, geothermal and hydro. In fact, it is possible to get nearly all the required 64 kWh/d/p from hydro, geothermal and wind alone, if we are willing to accept wind, hydro and geothermal proposals. [6]

Then, Scadden estimated the energy consumption and provided some efficiency savings that can be made. Scadden looked at the energy use for the average New Zealander instead of a typical moderately affluent person as MacKay did. Scadden found that a saving of about 25 kWh/d/p from 64 kWh/d/p (15 kWh/d/p from cars and 10 kWh/d/p from gadgets, lights and heating) is possible, but this would require huge improvements in vehicle efficiency which are at best 20 years away from now. [6]

Finally, Scadden provided a plan that could be feasible by 2025. The plan illustrates the relative cost effectiveness of various options (building top hydros, solar hot water in homes, solar photovoltaic, electric cars, . . . ,etc). This plan requires a huge number of windmills and also more hydro or geothermal generation maybe required. Scadden concluded his discussion by providing some suggestions on what to do to make a difference to our energy problem. As individuals, for example, we can reduce car usage and use bikes instead. In the work place, we can use high efficiency lightning and instead of travel we can use videoconferencing. Local goverments, for example, can build their own windmills. Central goverment can support electrification of transport and subsidise retrofitting of old houses for energy efficiency. [6]

The main goal of my thesis is to estimate an upper limit for the maximum we can get from the renewable resources: hydro, geothermal, solar, wind and marine, then compare this to New Zealand's energy use and hence try to answer the question **Can New Zealand Live on Renewables?**

In the next section, I will provide a brief outline of the current situation of renewable energy resources in New Zealand.

1.1.2 Renewable Resources in New Zealand

Renewable energy plays an important role in New Zealand's energy supply system. Approximately 39% of primary energy is from renewable energy sources in New Zealand, with around 75% of electricity generated from renewable sources mainly from hydro and geothermal. [35],[50]

Renewable resources in New Zealand include hydroelectricity, geothermal, wind, solar and marine, each of which can be explained as follows:

Hydroelectricity

Hydropower is the backbone and the main source of electricity generation in New Zealand, providing more than 50% of electricity, mainly from large dams such as Benmore, Manapouri and Clyde [35]. According to the Electricity Authority, electricity generated by hydro in 2012 was 7378.49 GWh for North Island and 14727.41 GWh for South Island. [8]

Geothermal

New Zealand has great geothermal resources and was one of the first countries to develop large-scale geothermal electricity generation in the 1950s. [33]

Currently, geothermal is the second renewable source for electricity generation in NZ, providing about 13% of electricity[10]. There are currently seven fields used for electricity generation, mainly owned and operated by Contact Energy and Mighty River Power[49]. Most of New Zealand's installed geothermal generation (about 750 MW) is situated in the Taupo Volcanic Zone, with another 25 MW installed at Ngawha in Northland[10]. As at February 2012, New Zealand has about 775MWe of geothermal electricity installed capacity and about 350MW direct energy use. [31]

Wind

New Zealand's wind resource is one of the best in the world[37]. Wind is the third renewable resource for electricity generation in New Zealand, providing about 4% of New Zealand's electricity. There are currently 17 wind farms running or under construction [45] and according to the Electricity Authority, electricity generated by wind in 2012 was 1710.96 GWh for North Island and 184.22 GWh for South Island. [8]

Solar

New Zealand has good solar radiation levels in many locations, with solar resource of about 4 kWh/m^2 per day[36]. Homes in New Zealand receive energy from the sun that is about 20 to 30 times the energy used in electricity or gas annually[36]. According to Energy Efficiency and Conservation Authority[36], around 1.6% homes in New Zealand have solar water heating systems and around 3400 systems are installed each year and this number is growing at around 30-40% annually.

Marine

New Zealand has great marine energy resources including waves and tides. The west and south-west coasts have the country's most energetic waves. Tides in New Zealand are considered moderate, except for regions such as Cook Strait and Kaipara harbor. Cook Strait has one of the strongest tidal flows in the world and Kaipara Harbour is the largest harbour in New Zealand. Although marine energy has a great potential in New Zealand, there are currently no installed marine energy facilities in the country. [34]

1.2 Thesis Outline

The main goal of the thesis is to transfer MacKay's ideas to New Zealand, and use basic physics and mathematics to estimate an upper bound for the power available from the renewable sources: hydro, geothermal, wind, solar and marine, then compare this to New Zealand's energy use, and hence answer the following question **Can New Zealand live on Renewables?**

In 2011, New Zealand consumer energy use (electricity and fuel) was 88 kWh/day/person from all sources, and the official figure is 138 kWh/day/person. However, this latter figure includes energy in coal and crude oil that is immediately exported and all the energy losses involved in converting fossil fuel to electricity. Neither figure provides a real indication of our total energy use, since they both exclude embodied energy in imports such as cars and electronics, and exports in things like aluminium. They also exclude fuel used in overseas air travel, since only fuel sourced in New Zealand is counted. [53]

In the thesis, all my calculations are in units of kWh/d/p, and I assumed that New Zealand population is 4,500,000 (Estimated Population=4,433,993 as at Thursday, 03 May 2012 at 06:21:36 pm [60]).

The thesis includes one chapter for each renewable resource (Chapters 3-7). Each chapter includes a summary of MacKay's approach for that renewable resource. In Chapter 3 (Hydroelectricity), I estimate an upper limit for hydroelectricity in New Zealand using two methods. The first method involves following MacKay's approach and dividing the country into Lowland and Highland regions. The second method is original to this thesis and involves using some image proccessing to estimate hydroelectricity available for the whole country. Then I compare the two methods and discuss the limitations associated with following MacKay's approach.

In Chapter 4 (Geothermal), I estimate the geothermal potential in New Zealand, and compare the geothermal potential of New Zealand and the United Kingdom. I will also provide a brief overview of geothermal uses in New Zealand. Chapter 5 (Wind), starts with dealing with energy in the wind, Betz's law, power coefficient and tip speed ratio. In this chapter, I estimate an upper limit of the power available in the wind in New Zealand. I also estimate the wind power density for the whole country using the wind resource map and the Rayleigh distribution, and i will discuss the limitations of MacKay's approach.

In chapter 6 (Solar), I estimate an upper limit for solar (Thermal, photovoltaic and solar farming). For solar thermal and photovoltaic, I use a different approach than MacKay's approach. This chapter also includes a brief introduction of the factors affecting solar radiation. In chapter 7 (Marine), I estimate an upper limit for Wave and Tide in New Zealand, and compare the results to the United Kingdom. I will also discuss some of the limitations associated with following MacKay's approach in Marine energy.

Finally, I provide conclusions and suggestions for future work in chapter 8. In this chapter, I sum all the renewable resources potential, and compare this to the energy use in New Zealand (138 kWh/d/p), and finally try to answer the question **Can New Zealand live on Renewables?**

Chapter 2

Data and Useful Numbers

2.1 Data Sets

The data sets used in this thesis were:

New Zealand Mean Annual Rainfall Map [1]

- Source: The National Institute of Water and Atmospheric Research (NIWA) (`http://www.niwa.co.nz/sites/default/files/images/climate_-_nz_rainfall_1971-2000.jpg`)
- Description: The map shows mean annual rainfall in millimetres as recorded by NIWA stations for the years 1971-2000.
  - Year: 2003
  - Image Resolution: 619×450

New Zealand 100 m Digital Elevation Model

- Source: Geography Department, Massey University (`https://www.massey.ac.nz`)
- Description: New Zealand 100 m digital elevation model represent elevation over the entire country with a resolution 100m/pixel. Each pixel represents an area of 100×100= 10,000 m^2.
  - Image Resolution: 15352×11645

New Zealand Solar Radiation Map ($MJ/m^2/day$) [3]

- Source: The National Institute of Water and Atmospheric Research (NIWA) (`http://www.niwa.co.nz/sites/default/files/imported/__data/assets/image/0017/50516/renewable4_large.gif`)
- Description: The map shows available solar energy per unit area per unit time measured and recorded by over 90 NIWA sites around New Zealand.
  - Year: 2005
  - Image Resolution: 506×350

New Zealand Wind Resource Map [48]

- Source: The National Institute of Water and Atmospheric Research (NIWA) (`https://www.niwa.co.nz/sites/default/files/imported/__data/assets/image/0004/50539/renewable3_large.gif`)
- Description: The map shows the average wind speed (in m/s) over the entire New Zealand.
  - Image Resolution: 500×723

Mean Daily Global Radiation (Mj/sq m) [14]

- Source: The National Institute of Water and Atmospheric Research (NIWA) (`http://www.niwa.co.nz/education-and-training/schools/resources/climate/radiation`)
- Description: The table shows mean monthly values for the years 1981-2010 for locations having atleast 5 complete years of data.

2.2 Preferred Units

m Length is measured in metres.

m^2 Area is measured in metre squared.

MWe In electricity industry, electric power is refered as megawatt electrical.

kWh Energy is measured in kilowatt-hour. The standard unit for energy is Joules (j). In electricity industry, energy is often measured in kilowatt-hour.

kWh/d/p Power is measured in kilowatt-hour per day per person. A Watt is the standard unit of power. Power is the rate of energy flow.

2.2.1 Useful Unit Conversions

1 kWh = 3,600,000 J

1 kWh/d = $(1000/24)$ W $\approx$ 40 W

1 km^2 = 10^6 m^2

1 hour = 3600 s

1 day = $24\times3600=864\times10^2$ s

1 year = 31436×10^3 s

Prefix	milli	centi	kilo	mega	giga	tera	peta
Factor	10^{-3}	10^{-2}	10^{3}	10^{6}	10^{9}	10^{12}	10^{15}

Table 2.1: Useful Conversions

2.3 Useful Numbers

New Zealand Consumer Energy Use 138 kWh/d/p (see section 1.2)

United Kingdom Consumer Energy Use [**44**] 125 kWh/d/p

New Zealand Area [**24**] 268,680 km^2

United Kingdom Area [**44**] 244,000 km^2

New Zealand Population 4,500,000 (see section 1.2)

United Kingdom Population [**44**] 60,000,000

New Zealand Population Density 16 people/km^2

United Kingdom Population Density 243 people/km^2

New Zealand Latitude 41.4395° S

United Kingdom Latitude 53.1142° N

Chapter 3

Hydroelectricity

In this chapter, I am going to calculate an upper bound for the hydroelectricity resource in New Zealand. I am also going to provide a brief summary of MacKay's approach to hydroelectricity in the UK.

3.1 Hydroelectricity—MacKay's Approach

3.1.1 Hydroelectricity in the UK

MacKay estimated an upper bound for hydroelectric power, by dividing the UK into Highland (higher altitude, higher rainfall) and Lowland (lower altitude, lower rainfall) regions, and then choosing Bedford and Kinlochewe as representatives of Lowland and Highland regions respectively. Based on the assumption that the whole Lowland (Highland) has the same altitude and rainfall rate as Bedford (Kinlochewe), MacKay concluded that an upper limit would be 8 kWh/d/p (1 kWh/d/p for Lowland,7 kWh/d/p for Highland), and assumed a practical limit would be 20% of this value, which is still around seven times the current hydroelectic power generation in the UK (0.2 kWh/d/p). (cf.[44], pp.55-56)

3.1.2 Hydroelectricity in New Zealand

Following MacKay's approach (cf.[44], pp. 55-56), I will divide New Zealand into Lowland Regions (Lower altitude and lower rainfall) and Highland Regions (Higher altitude and higher rainfall), and choose Blenheim and Rotorua as representatives of Lowland and Highland regions respectively. It is assumed that all Lowland (Highland) has the same rainfall and altitude as Blenheim (Rotorua). Now, let us calculate an upper limit for hydroelectric power for Lowland and Highland.

Definition 3.1. (MacKay [44]) Gravitational Power of Rain (GP) is the power per unit area of land on which rain falls. It is equal to the rainfall multiplied by altitude times gravity multiplied by the density of water.

$$GP=\rho ghr \tag{3.1}$$

where
GP is gravitational power in $\mathrm{W/m^2}$
r is rainfall in mm/year
g is gravity, g=9.8 $\mathrm{m/s^2}$
ρ is density of water, ρ=1000 $\mathrm{kg/m^3}$
h is altitude in m

Let us do the Lowland first. Blenheim has an average rainfall rate of 720 mm/year [15] and altitude of 33 m. The gravitational power of the rain falling on Blenheim is

$$720\ \text{mm/year} \times 33\ \text{m} \times 1000\ \mathrm{kg/m^3} \times 9.8\ \mathrm{m/s^2} = 0.01\ \mathrm{W/m^2}. \tag{3.2}$$

As the approach is already highly approximate, I will arbitrarily assume that the area of Lowland is half the area of New Zealand.The area of New Zealand is 268,680 $\mathrm{km^2}$. Multiplying the gravitational power (2.2) by the area per person (29853 $\mathrm{m^2}$/person) gives **7 kWh/d/p**.

What about Highland regions? Rotorua has an average rainfall rate of 1359 mm/year [15] and altitude of 280 m. The gravitational power of the rain falling on Rotorua is

$$1359\ \text{mm/year} \times 280\ \text{m} \times 1000\ \mathrm{kg/m^3} \times 9.8\ \mathrm{m/s^2} = 0.12\ \mathrm{W/m^2}. \tag{3.3}$$

The area of Highland is half the area of New Zealand. Multiplying the gravitational power (2.3) by the area per person (29853 $\mathrm{m^2}$/person) gives **86 kWh/d/p**.

We can conclude that an upper limit for hydroelectric power in New Zealand is **93 kWh/d/p**. That is if every single rain drop is used, every river were dammed, no evaporation occurs and assuming 100% efficient turbines. Following MacKay's approach (cf.[44], p.56), we suppose we can produce 20% of this value, which is about **19 kWh/d/p**. As at January 2012,the actual power from hydro is 13.7 kWh/d/p, so that is around a 39% increase in current hydroelectric power generated in New Zealand. [8]

One of the limitations to this approach, is that it assumes that the whole Lowland (Highland) has a rainfall rate and altitude the same as Blenheim (Rotorua). There are many regions in New Zealand which have much higher altitude and much higher rainfall rate, and I believe that this method will not provide a fair estimate for the total hydroelectricity available. In the next section, I am going to expand to MacKay's approach to cover the whole area of New Zealand.

3.2 Hydroelectricity Image Processing Approach

My goal in this section is to calculate an absolute upper bound for hydroelectricity in New Zealand, by calculating the gravitational potential power (see def. 2.1) of every drop of rain falling anywhere in New Zealand. Of course, it will never be possible to harness all of this energy; even MacKay's figure of capturing 20% of the gravitational power seems optimistic. But with this method, we can have a true upper bound. I believe that this calculation has never been carried out before.

3.2.1 What Do We Want to Calculate?

We want to calculate the energy that can be extracted from the gravitational power of all the rain falling in New Zealand.

Definition 3.2. (Giancoli [41]) Gravitational Potential Energy (GPE_{grav}) is the energy stored in an object as a result of its height.

$$GPE_{grav}=mgh \tag{3.4}$$

where
GPE_{grav} is the gravitational potential energy in joules
m is the object's mass in kg
g is gravity, g=9.8 m/ s^2
h is the height of object in metres.

Let the region Ω be New Zealand. I will divide New Zealand into rectangular regions $\Omega_{i,j}$. Each region has width Δx and height Δy and the area of each region is $\Delta x \Delta y$. For each region, I will choose a sample point (x_i,y_j). The gravitational potential energy ($GPE_{i,j}$) for region $\Omega_{i,j}$ is

$$GPE_{i,j}=\rho gh(x_i,y_j)r(x_i,y_j)\Delta x\Delta y \tag{3.5}$$

where
h(x_i,y_j) is the altitude in $\Omega_{i,j}$ in metres
r(x_i,y_j) is the rainfall in $\Omega_{i,j}$ in mm/year
ρ is the density of water, ρ=1000 kg/ m^3
g is gravity, g=9.8 m/ s^2
ΔxΔy is the area of $\Omega_{i,j}$

Since we want the energy of all the rain falling in New Zealand, we need to sum the potential energy for all the regions. The Riemann sum is
$\sum_{i=1}^{m}\sum_{j=1}^{n} \rho \mathrm{gh}(x_i,y_j)\mathrm{r}(x_i,y_j)\Delta \mathrm{x}\Delta \mathrm{y}$

Therefore, the total Gravitational Potential Energy (GPE_{NZ}) is

$$
\begin{aligned}
GPE_{NZ} &= \iint\limits_{\Omega} \rho g h(x,y) r(x,y)\, dx\, dy \\
&= \lim_{m,n\to\infty} \textstyle\sum_{i=1}^{m}\sum_{j=1}^{n}\ \rho\mathrm{gh}(x_i,y_j)\mathrm{r}(x_i,y_j)\Delta\mathrm{x}\Delta\mathrm{y}
\end{aligned}
$$

3.2.2 Image Registration

The Rainfall Map (Figure 3.1) was obtained from NIWA website [1], and New Zealand 100 m Digital Elevation Model (Figure 3.2) from Geography Department, Massey University. In this section, I am going to explain in details the image processing that has been done to calculate the total hydroelectricity available.

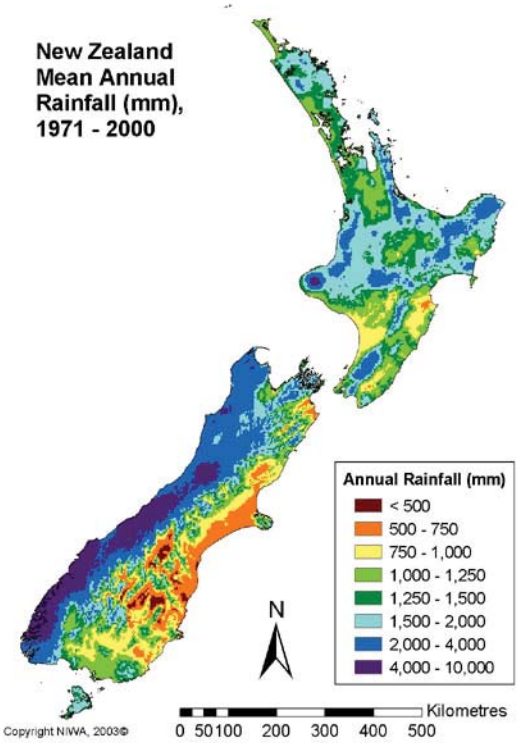

Figure 3.1: New Zealand Mean Annual Rainfall (mm). 1971-2000 [15]

Let us start by recognizing the problems that we need to overcome. These problems include:

1. No access to original rainfall data. We only have the rainfall map image, which has been processed into a JPG, causing noise, and the data has been contoured into bands as shown in the legend. Also, the coastline has been added as a black line in the rainfall map.
2. The resolution of the rainfall map is unknown, and the resolution and size of the pixels is different for the rainfall and altitude map.
3. The two maps are not well aligned together.

Figure 3.2: New Zealand 100 m Digital Elevation Model. New Zealand 100 m Digital Elevation Model was obtained from Geography Department, Massey University.

Image registration is needed to overcome the problems mentioned above. Image registration for the two images involves the following:

1. **Change the rainfall map array from RGB colour values to rainfall values**

   This step will help overcome the bleeding effect and the noise caused by processing the image into JPG. I have written a Matlab program (Rainfall.m) to carry out this step. Rainfall.m maps each colour in the rainfall to the color closest to it in the legend, and then replace the rainfall value corresponding to the colour closest. This step will also help overcome the coastline being black, by adding the RGB values for the colour black (0,0,0) and giving it a rainfall value of zero in the legend.

```
A=imread('climate_-_nz_rainfall_1971-2000.jpeg');
A=double(A);
% The RGB values for the eight colours and the colours
black and white are added
% The RGB values for the eight colours in the legend are obtained
using -impixel command in Matlab by selecting one pixel for each
colour in the legend.

R=[81 37 141 8 126 253 255 116 255 0];
G=[0 102 208 135 195 242 117 1 255 0];
B=[109 186 217 56 6 100 0 0 255 0];
legend = [7000, 3000, 1750,1375,1125,875,625,250,0,0];
```

```
%rainfall values are averaged for each colour and the colour black and white
%are assigned a rainfall value of zero
dist = zeros(1,10);
Rain = zeros(619,450);
for i=1:619
    for j=1:450
        for k=1:10
           dist(k)=sqrt((A(i,j,1)-R(k))^2+(A(i,j,2)-G(k))^2+(A(i,j,3)-B(k))^2);
        end
        [y,k]=min(dist);
        Rain(i,j) = legend(k);
    end
end
```

2. **Finding the area per pixel in the rainfall and altitude map**

   The area per pixel in the altitude map is 10000 m^2,since it is a 100 m digital elevation model. To find the area per pixel in the Rainfall map, the following method is used:

   (a) Three line segments are chosen d1,d2 and d3 (Figure 2.3)

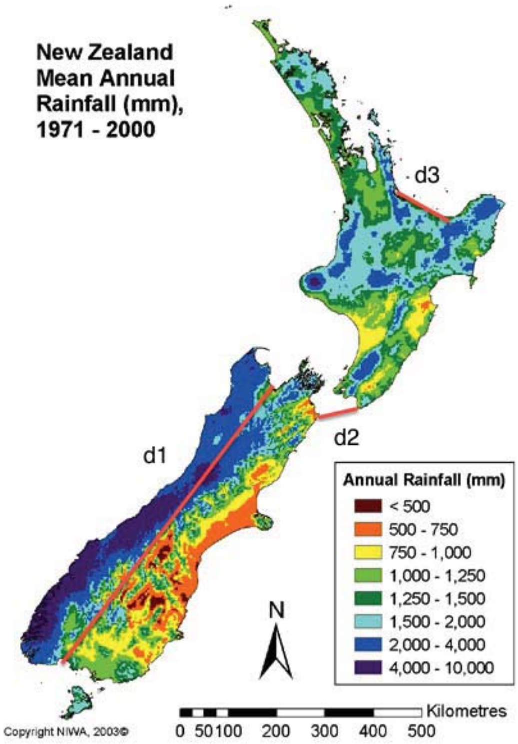

Figure 3.3: Line segments d1,d2 and d3

(b) Distance in km is calculated for d1,d2 and d3 based on latitude and longitude using the Haversine formula (see Appendix A).

Distance	Point1(lat1,long1)	Point 2(lat2,long2)	Distance (km)
d1	(-41.079,173.162)	(-46.255,167.644)	726.48
d2	(-37.633946,176.015)	(-37.996,177.251)	115.78
d3	(-41.607,175.230)	(-41.738,174.280)	80.10

Table 3.1: Distance in km for the three line segments: d1,d2 and d3, based on latitude and longitude using Haversine Formula

(c) Distance in pixels $d(x, y)$ is calculated for d1,d2 and d3 using Pythagoras , by obtaining the coordinates of pixels (x, y) (Table 3.2)

$$d(x, y) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{3.7}$$

Distance	Point1(x_1,y_1)	Point 2(x_2,y_2)	$d(x, y)$
d1	(236,332)	(73,543)	266.63
d2	(332,179)	(370,198)	42.98
d3	(301,347)	(272,352)	29.42

Table 3.2: Distance in Pixels for d1,d2 and d3

(d) The area per pixel in the rainfall map is approximately $(2.7\,\text{km})^2$=7290000 m^2 (Table 3.3)

Distance	Distance (km)	Distance in pixels	Distance(km)/$d(x, y)$
d1	726.48	266.63	2.72
d2	115.78	42.98	2.69
d3	80.10	29.42	2.72

Table 3.3: Distance per pixel (km/pixel) calculated by dividing distance obtained in Table 2.1, by distance in pixels obtained in Table 2.2

3. **Lower the resolution of the Altitude Map**

Now, we have the area per pixel for the rainfall map and the area per pixel for the altitude map. We need to lower the resolution of the altitude map, so the area per pixel approximately matches the area per pixel in the rainfall map.

The size of the altitude map is 15352×11645, and the size of the rainfall map is 619×450. An averaging method is used to lower the resolution of the altitude map. The method invloves averaging every $p \times p$ block of an $m \times n$ array. The following m-file can be used to average every $p \times p$ block for an $m \times n$ array:

```
function B=averagealt1(A,p)
[m,n]=size(A);
C=A(1:p*floor(m/p),1:p*floor(n/p));
B=zeros(floor(m/p),floor(n/p));
for i=1:p:p*floor(m/p)
    for j=1:p:p*floor(n/p)
        B((i+p-1)/p,(j+p-1)/p)=sum(sum(C(i:i+p-1,j:j+p-1))/p^2);
    end
end
```

To find p, we start by dividing the area per pixel for the rainfall by the area per pixel in altitude, and we obtain 729. The value of p is the square root of this value, which is approximately 27. So we need to average every 27×27 block in the altitude map. The size of the new altitude map is 568×431 and the size of Rainfall map is 619×450.

4. **Matching the sizes of Rainfall and Altitude**

   To perform array multiplication, the two image arrays must have the same size. I have written a Matlab programe (Altitudesize.m) to match the sizes. Altitudesize.m match the sizes of the two maps, by adding 51 rows and 19 columns of zeros to the altitude map. The elements for these rows and columns are zeros, since they represent the sea in the altitude map.

```
run Rainfall
a=Rain; %size of a is 619X450
b=imread('nz100dtm1.tif'); %size of b is 15352X11645
b(b<0)=0; % altitude map includes values that are less than zero which
represents values below sea level, this command will set all the values
that are less than zero to zero.
c=averagealt1(b,27); %resizing b by averaging every 27X27 block
% size of c is 568x431
d=zeros(51,431, 'double');
f=[c;d]; %adding 51 rows to c to match number of rows in a
g=zeros(619,19,'double');
Altitude=[f g]; % adding 19 columns to match number of columns in a
% Altitiude has the same size as Rain
```

5. **Translation**

   This step will help align the two maps together. Let's start by looking at how the two maps are aligned. First, I performed the following Matlab commands.

```
R=Rain>0
```

```
A=Altitude>0
```

R is an array of zeros and ones, which has the same size as the Rain array. It assigns a value of zero to any value in Rain equal to zero and assigns a value of 1 to any value in Rain greater than zero. Similarly, A is an array of zeros and ones, which has the same size as the Altitude array and assigns a value of zero to any value in Altitude equal to zero and assigns a value of 1 to any value in Altitude greater than zero.

`imshow(R-A)` shows the difference between the two images; the white region is the difference and each pixel in this region has a a value of 1 and the rest are zeros (where R(x,y) and A(x,y) are both ones). Figure. 2.4 shows the difference between the two maps and clearly shows that the maps are not matched and not well aligned together and this will produce errors.

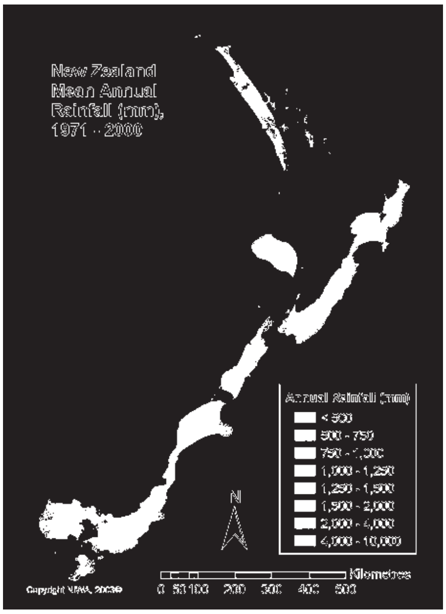

Figure 3.4: The two maps are not well aligned together, Image obtained in Matlab using the following command `imshow(R-A)`

Now, let's calcuate the norm $|R-A|$:

```
 R=Rain>0;

A=Altitude>0;

 norm(R-A)

ans =

71.0512
```

We want the value of norm $|R-A|$ to be minimum. Trying for different values of i and j, the best option is to move the rainfall map 31 pixels to the right in the x-direction.

```
run Altitudesize
a=Rain;
b=Altitude;
c=zeros(619,450);
for i=1:619;
for j=1:450;
if i<=588;
c(i,j)=a(i+31,j);
if i>588;
c(i,j)=a(i-588,j);

end
end
end
end
```

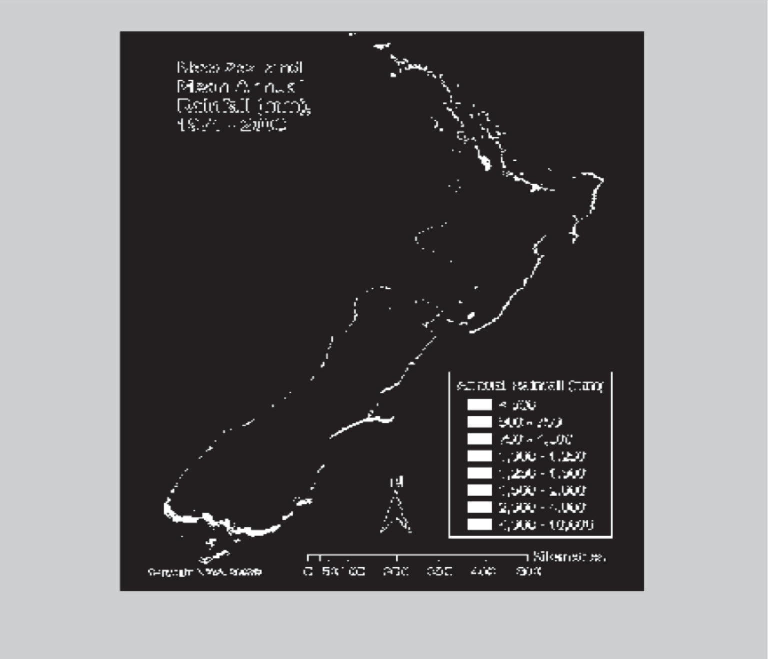

Figure 3.5: The difference between the two images after moving the rainfall map 31 pixels to the right in the x-direction. Image obtained in Matlab using the following command `imshow((c>0)-(Altitude>0))`. The alignment is still not perfect but is considered acceptable for my purposes.

The registration process is now complete, and the two maps are now ready to be used for our calculation in the next section.

Hydroelectricity calculation

The following script calculates the power for each pixel (in watts/pixel)

```
run MatchingFinal
GravitationalPowernz=1000.*9.8.*(1/31536000).*0.001.*c.*Altitude.*7290000;
```

```
%1year=31536000seconds
%area per pixel=2700X2700=7290000m2/pixel
%density of water=1000kg/m3
%gravity=9.8m/s2
sum(GravitationalPowernz(:))
```

Main Result:An Upper Bound for Hydroelectric Power in New Zealand is 546.72 kWh/d/p.

The sum is 1.0251×10^{11} W, that is about **546.72 kWh/d/p**. That is if every single rain drop is used. Let us say we can produce 20% of this value ([44], pp. 55-56), then **109.344 kWh/d/p**. As at January 2012, the actual power from hydro is 13.7 kWh/d/p [8], so that is about 9 times increase in hydroelectric power. It is surprising that such a large factor of the total gravitational power available is already in use: we are already using around 2% of the total power available!

However, there are many environmental, social and economic issues that would also need to be considered. According to East Harbour Management Services [2], hydro potential for New Zealand is around 2,500 MW and 12,000 GWh/year; that's about 9.4 kWh/d/p. According to Scadden [6], the maximum realistic potential for hydro is about 23 kWh/d/p. [6]

3.3 Comparisons

In this section, I will provide a brief comparison between MacKay's approach and my image proccessing approach. I will also compare the hydroelectricity potential between New Zealand and UK.

3.3.1 New Zealand—MacKay's Approach vs. Hydroelectricity Image Processing Approach

Approach	MacKay's	Hydroelectricity Image Processing Approach
Results (kWh/day/person)	93	546.72

Table 3.4: MacKay's Approach vs. Our Approach

From Table 3.4, the value estimated using our approach is around five times greater than the value estimated using MacKay's approach. One of the limitations to MacKay's approach is that it assumes that the whole Lowland (Highland) has rainfall rate and altitude the same as Blenheim (Rotorua). On the other hand, our approach, or what I call "Hydroelectricity Image Processing Approach", gives a better estimation of the total hydro available, since it calculates the hydro available for each region based on their rainfall rate and altitude rather than just calculating the power for two cities as a representative for the whole country.

Although our method provides a better estimation to maximum hydro available, there are many factors affecting the accuracy of our calculation including:

- Not having access to accurate and original rainfall data
- The two maps are not perfectly aligned (Figure 3.5)

Future work would be to obtain more accurate rainfall data array (monthly rainfall data), with the same resolution as the altitude map and same pixel size and then perform array multiplication to calculate an upper limit of the total hydro available.

3.3.2 New Zealand vs. United Kingdom

New Zealand has a very promising hydroelectricity potential, with an upper limit of **546.72 kWh/d/p**. Compared to UK, New Zealand's 15 times smaller population density (16 people/ km^2 for New Zealand, 243 people/ km^2 for UK [44]), approximately 4 times higher altitude (490 m for New Zealand, 166 m for UK [44]) and twice the amount of rainfall (2249 mm/year for New Zealand, 1175 mm/year for UK [9]) can be the main factors that gave the country a much higher hydroelectricity potential, with an upper limit of 546.72 kWh/d/p for New Zealand compared to 8 kWh/d/p for UK.

3.4 Hydroelectricity Conclusion

In this chapter, we estimated an upper bound for hydroelectricity available in New Zealand using two approaches: MacKay's Approach and Hydroelectricity Image Processing Approach (our approach). Our approach gives a better estimation, since it calculates the hydro available for each region based on their rainfall rate and altitude rather than just calculating the power for two cities as a representative for the whole country.

Compared to UK, New Zealand's smaller population density (16people/ km^2 for New Zealand, 243people/ km^2 for UK) , higher altitudes and higher rainfall can be the main factors that gave the country a much higher hydroelectricity potential, with an upper limit of 546.72 kWh/d/p for New Zealand compared to 8 kWh/d/p for UK.

We can conclude that New Zealand has a very promising hydroelectricity potential, with an upper limit of **546.72 kWh/d/p**, and it is surprising that we are currently using 2% of the total hydro available! However, in reality there are many social, economic and environmental issues that would need to be considered.

Chapter 4

Geothermal

Geothermal is second renewable source for electricity generation in New Zealand, providing about 13% of electricity [10]. As at February 2012, New Zealand has about 775 MWe of geothermal electricity installed capacity and about 350 MW direct energy use [35]. That is about 6 kWh/d/p (4.1 kWh/d/p electricity and 1.9 kWh/d/p direct use).

In this chapter, I will estimate the geothermal potential in New Zealand, and provide a brief summary of MacKay's approach to geothermal in the United kingdom. I will also outline geothermal uses in New Zealand, and provide a brief discussion of renewability and sustainability of geothermal resources.

4.1 Geothermal in the United Kingdom

Geothermal energy comes from the heat of the core of the earth and radioactive decay from earth's mantle. MacKay divided geothermal in the United Kngdom into two types: power available at ordinary locations, and power available in special hot spots like Iceland, and he assumed that the most resource comes from ordinary locations, since they are more numerous. (cf.[44], p. 96)

MacKay estimated the geothermal potential in the United Kingdom by first treating geothermal as a resource that would be sustainable forever, and then as a resource to be mined. According to MacKay[44], it is better to treat geothermal the same way we treat fossil fuels, as a resource to be mined rather than collected sustainably. This is because after a while of sucking the heat out of the rocks, it will take a long time to warm up again. (cf.[44], p. 96)

To estimate the geothermal resource that would be sustainable forever, MacKay assumed using geothermal energy sustainably, by sticking down straws to an appropriate depth and in order for the rocks not to get colder and colder, MacKay assumed sucking at the same natural rate at which heat is already flowing out of earth. (cf.[44], p. 97)

MacKay estimated the maximum geothermal power we can get per unit area to be 50 $\mathrm{mW/m^2}$ (The heat flow from the centre through the mantle is 10 $\mathrm{mW/m^2}$ and 40 $\mathrm{mW/m^2}$ is added from radioactive decay in the crust of the earth). However, this power is low grade heat and for electricity generation, we need higher temperature and since temperature increases with depth (heat flow decreases with depth), we need to drill deeper. According to MacKay[44], there is an optimal depth which depends on the sort of sucking method and machinery used. MacKay calculated the optimal depth to be 15 km, and estimated that an ideal heat engine would deliver 17 mW/ m^2 assuming perfect power stations, every square metre is exploited and drilling to any depth is free. If all the world land area is used and with a world population density of 43 people/ km^2 that is about 10 kWh/d/p. For the United Kingdom with a population five times greater than the world's population density an ideal heat engine would deliver **2 kWh/d/p**. (cf.[44], pp.97-98)

Second, MacKay treated **geothermal as a resource to be mined**. He explained the method of geothermal extraction from dry rocks. But the problem is that the United Kingdom has a limited hot dry rock resource, mainly located in Cornwall, and the biggest estimate of hot dry rocks resource in United Kingdom is about 1.1 kWh/d/p. (cf.[44], p.98)

MacKay also mentioned Southampton which has the only geothermal heating scheme in the United Kingdom. The scheme contributes about 15% of the total heat delivered per year by a combined system which provides hot and chilled water and sells electricity to the grid. The scheme provides geothermal power of about 0.13 kWh/d/p. (cf. [44], pp. 98-99)

Conclusion: Geothermal has a very small potential in the United Kingdom with 2 kWh/d/p sustainable forever figure, and a maximum potential of about 1 kWh/d/p from hot dry rocks.

4.2 Uses of Geothermal in New Zealand

Geothermal resources can be divided into three groups based on temperature:High (200-350°C), moderate to low and very low. In New Zealand, High temperature Resources has been used for electricity generation. The Wairakei Power Station in North Taupo is an example of high temperature geothermal field with an installed capacity of 181MW . For hundreds of years, Lower temperature geothermal energy (less than 150°C) has been directly used in New Zealand for cooking, washing, bathing and heating. Low temperature systems are used in heating greenhouses, drying timber and crop and many other agricultural and industrial processes. The Norske Skog Tasman pulp and paper mill at Kawerau is the largest geothermal direct user in the world. It uses geothermal fluids to generate clean process steam for paper drying, a source of heat in evaporators, timber drying and electricity generation. Geothermally heated green house complex developed next to Mokai geothermal plant is another example of direct use in New Zealand. [12],[49]

4.3 Geothermal Potential in New Zealand

Following MacKay's approach (cf. [44], pp.97-98), I am going to estimate the geothermal resource that would be sustainable forever ignoring any hot spots. MacKay estimated the maximum sustainable power assuming perfect power stations, drilling to any depth is free an ideal engine would deliver 17 mW/ m^2. For the United Kingdom, geothermal resource that would be sustainable forever can provide at most 2 kWh/d/p. For New Zealand with a 15 times smaller population density than that of the United Kingdom and 3 times smaller population density than the world's population density(cf.[44],p.98), geothermal power could offer at most

Country	United Kingdom	New Zealand
Geothermal Potential (kWh/d/p)	2	30

Table 4.1: Geothermal Resource that would be sustainable forever: New Zealand vs. United kingdom. These figures are sustainable forever figures ignoring any hot spots and assuming perfect power stations and drilling to any depth is free.

30 kWh/d/p. Compared to the United Kingdom, New Zealand has a better geothermal potential because of our 15 times smaller population density compared to the United Kingdom's population density.

Compared to the United Kingdom, New Zealand has more promising hot spots. In New Zealand, Geothermal has a great potential both as a source of electricity and energy use. Electricity generation is the more attractive option than direct use, as geothermal steam can not be transported for more than a few tens of kilometres.Assessment of New Zealand's High Temperature geothermal resources suggested that the total geothermal resource is estimated to be equivalent to a median value of 3600MWe of electrical generation using only existing technology[40], that is about **19.2 kWh/d/p of electricity**. As at Febrauary 2012, geothermal power was 4.1 kWh/d/p . It is very interesting that we are currently using around 20% of the available geothermal resource estimated!

Generating Capacity	10th percentile	median	90th percentile
MWe	2500	3600	5000

Table 4.2: Assessment of New Zealand's High Temperature Geothermal Resources[40]

Since we are looking for an upper bound for geothermal potential in New Zealand. Let us ignore all the limitations, and suppose we can obtain the 90th percentile value of 5000 MWe estimated by Lawless. Then, that is about **26.6 kWh/d/p** of electricity. Again, it is very surprising that we are currently using around 15% of the geothermal available resource. In reality, there are many environmental and regulatory limitations for such a potential. According to Energy Efficiency and Conservation Authority[32], taking into account all the limitations there is another geothermal potential of 1000 MWe for electricity generation, so we can say the maximum realistic potential is about **9.4 kWh/d/p of electricity**.

What about geothermal direct energy use? Geothermal energy has been directly used in New Zealand for hundreds of years (see section 4.2). As at February 2012, geothermal direct use was about 1.9 kWh/d/p. According to Kelly[7], there is a significant potential for geothermal direct energy use mainly for space heating purposes directly, and through applications such as ground source heat pumps for domestic and commercial applications. Future research work can be studying the potential for direct geothermal energy use.

Conclusion: New Zealand has a very promising geothermal potential with **30 kWh/d/p** sustainable forever figure ignoring any hot spots. An upper bound for New Zealand's high temperature geothermal resources is **26.6 kWh/d/p**. However, in reality there are many limitations for such a potential and taking into account all the limitations the maximum realistic potential is about **9.4 kWh/d/p** of electricity. Geothermal direct energy use also has a great potential in New Zealand, and future research work can be studying the direct geothermal use potential in New Zealand.

Compared to the United Kingdom, New Zealand's 15 times smaller population density and more promising hot spots locations can be the main factors that gave New Zealand a higher geothermal potential.

Sustainable Forever[1]	Maximum Realistic Potential[2]	Median and Upper Bound[3]
30	9.4	Median:19.2, Upper Bound:26.6

Table 4.3: Geothermal Potential in New Zealand (kWh/d/p):1. ignoring any hot spots and assuming perfect power stations and drilling to any depth is free. 2. As estimated by Energy Efficiency and Conservation Authority and taking into account all the limitations. 3. the median and upper bound value are based on the on the median and 90th percentile value estimated in the assessment of high temperature geothermal resources by Lawless (Table 4.2)[40].

4.4 Geothermal Renewability and Sustainability

Geothermal energy is often defined as a renewable resource by many authors (see for example [44],[4] and [7]). According to Bach[4], geothermal extraction (heat and fluid) is renewable, because the replenishment of geothermal resources (heat and fluid) will always take place at slow rates and different time scales. These time scales depend on many factors such as the type and size of the production system and the rate of extraction.[4]

What about sustainability? Geothermal is considered sustainable if the rate of withdrawal is lower than the rate of energy supply and the production levels are sustained for a long period of time. If the extraction rates exceed the supply, then the generating capacity of the geothermal field will decline as in Ohaaki in New Zealand[11],[7]. MacKay believes that it is better to treat geothermal the same way we treat fossil fuels, as a resource to be mined rather than collected sustainably. This is because after a while of sucking the heat out of the rocks, it will take a long time to warm up again. When MacKay estimated geothermal as a resource that would be sustainable forever, MacKay assumed using geothermal energy sustainably, by sticking down straws to an appropriate depth and in order for the rocks not to get colder and colder, MacKay assumed sucking at the same natural rate at which heat is already flowing out of earth[44]. On the other hand, Bach believes it is incorrect to describe geothermal extraction as a mining process, this is because when a mineral deposit is mined and the ore removed, it will be gone forever. However, this is not the case for geothermal where the replenishment will always take place at slow rates and different time scales.[4]

According to New Zealand Geothermal Association[11, para.2],"Although some geothermal resources in New Zealand have been over-exploited and had to reduce their generating capacity, as at Ohaaki, no geothermal field has ever been run to exhaustion.". On the other hand, where the field characteristics (natural recharge rate, field size,. . . ,etc) have been considered, The longevity of sustainable production can be achieved such as at Wairakei geothermal field. Wairakei has been operating for 50 years, and resource consents has been given for another 25 years at the same generation capacity. Also, another study of the Wairakei geothermal field suggested that, it will take more than 100 years of production at current rates for the field to be effectively exhausted and when this happens, it will take 400 years for the system to return to its original state. [11],[7]

We can conclude that, geothermal is a renewable and sustainable resource, under the condition that the energy extraction is limited to the rate at which heat is rising to the surface. Future research is recommended, which may include studying the renewability and sustainability of geothermal energy.

Chapter 5

Wind

In this chapter, I am going to estimate the onshore wind potential in New Zealand, and provide a brief summary of MacKay's approach to the wind in the United Kingdom. I am also going to discuss some wind energy basics including power of the wind, Betz' Law, Tip speed ratio (TSR), wind power density (WPD) and Rayleigh distribution.

Offshore wind has not been considered in this research because New Zealand doesn't have wide shallow sea areas[6]. Also, offshore power is about twice as expensive as onshore, furthermore deep offshore developments are not economically feasible at the present time[44].

5.1 Power of the Wind

Suppose a piece of air of mass m passes through an area A with constant speed v. The kinetic energy of this moving air is

$$E=\tfrac{1}{2}mv^2 \tag{5.1}$$

Derivation of Kinetic energy equation is provided in Appendix B.1. The mass of this moving air is the product of its density and volume. Suppose at time t, this piece of air has moved a distance r while passing through A, then the volume of the moving air is Ar, and hence the mass of the moving air m is

$$m=\rho Ar \tag{5.2}$$

where ρ is the density of air, ρ=1.3 kg/m^3

The kinetic energy of the moving air passing through area A per unit time is the power of the wind P_w

$$P_w=\tfrac{dE}{dt}=\tfrac{1}{2}v^2\tfrac{dm}{dt} \tag{5.3}$$

$\frac{dm}{dt}$ is the mass flow rate, and since $m=\rho Ar$, then

$$\frac{dm}{dt}=\rho A\frac{dr}{dt}=\rho Av \tag{5.4}$$

Substituting (5.4) in (5.3) gives

$$P_w=\frac{1}{2}\rho Av^3 \tag{5.5}$$

And that is the power of the wind passing through an area A with speed v

5.2 Power Coefficient (C_p) and Betz' law

Wind turbines cannot extract all the power available in the wind (P_w). The actual power that can be extracted by a wind turbine ($P_{turbine}$) is

$$P_{turbine} = P_w C_p = \frac{1}{2}\rho Av^3 C_p \tag{5.6}$$

where,
C_p is the power coefficient.

The Power Coefficient (C_p) is a measure of how efficiently a wind turbine converts wind kinetic energy into electricity, and it can be defined as the ratio between the power that can be extracted by the wind turbine (5.6) and the power available in the wind (5.5).[52]

$$C_p=\frac{P_{turbine}}{P_w} \tag{5.7}$$

The Power Coefficient can be related to Betz' law, which was developed by a German Engineer Albert Betz in 1919. Betz Law states that no wind turbine can convert more than 59.3% of the kinetic energy of the wind into mechanical energy turning a rotor, this means that the maximum achievable power coefficient is C_{pmax}= 59.3%. Betz assumes that the rotor is an ideal energy converter (the rotor does not include a hub, and has an infinite number of blades to prevent drag resistance to the wind flowing through the blades). Betz also assumes that the air flow is incompressible (constant density ρ), the flow in and out of the rotor is axial (velocity is parallel to the axis of rotation) and the flow over the whole area swept by the rotor is uniform (constant velocity).[54],[52]

Theorem 5.1. *(Betz' Law) No wind turbine can capture more than 59.3% of the kinetic energy of the wind.*

Proof. Let v be the wind velocity at the rotor, v_1 be the upstream wind velocity and v_2 be the downstream wind velocity. Let A be the area swept by the rotor, A_1 be the upwind cross sectional area before the rotor and A_2 be the downwind cross sectional area after the rotor.

Assuming incompressible flow, the continuity equation (see Appendix B.2) becomes

$$\dot{m} = \rho A_1 v_1 = \rho A v = \rho A_2 v_2 = constant \tag{5.8}$$

The force F exerted by the wind on the rotor (see Appendix B.3) is

$$F = ma = \dot{m}\frac{dv}{dt} = \rho A v (v_1 - v_2) \tag{5.9}$$

The kinetic energy can be defined as the work done (see Appendix B.1)

$$E_{kinetic} = W = Fr \tag{5.10}$$

Power P can be defined as the rate of work done, so

$$P = F\frac{dr}{dt} = \rho A v^2 (v_1 - v_2) \tag{5.11}$$

Since it is assumed that the wind velocity at the rotor v is the average of upstream v_1 and downstream v_2 wind velocities (see Appendix B.4), then the power that can be extracted by a wind turbine $P_{turbine}$ is

$$P_{turbine} = \frac{1}{4}\rho A (v_1 + v_2)({v_1}^2 - {v_2}^2) \tag{5.12}$$

The power available in the wind P_w is

$$P_w = \frac{1}{2}\rho A {v_1}^3 \tag{5.13}$$

The ratio between the power that can be extracted from the wind $P_{turbine}$ and the power available in the wind P_w is the power coefficient C_p

$$C_p = \frac{P_{turbine}}{P_w} = \frac{1}{2}(1 - (\frac{v_2}{v_1})^2)(1 + \frac{v_2}{v_1}) \tag{5.14}$$

Let b be the ratio between downstream v_2 and upstream v_1 wind velocities.

$$b = \frac{v_2}{v_1} \tag{5.15}$$

substituting 5.15 in 5.14 gives

$$C_p = \frac{1}{2}(1 - b^2)(1 + b) \tag{5.16}$$

Differentiating (5.16) with respect to b, gives

$$C_p{}' = \frac{1}{2}(3b - 1)(b+1) \tag{5.17}$$

Equating $C_p{}'$ to zero, to find the value of b that makes C_p maximum

$$C_p{}'=0 \implies b=-1 \text{ or } b=\frac{1}{3} \tag{5.18}$$

We can conclude that, the maximum value of C_p occurs at b=$\frac{1}{3}$. In other words, for optimal operation v_2 should be one third of v_1.

$$b=\frac{1}{3} \implies C_{pmax}=\frac{1}{2}(1-(\frac{1}{3})^2)(1+\frac{1}{3})=0.5926 \tag{5.19}$$

□

C_{pmax}=59.3% is often called Betz' Limit. In reality, wind turbines has efficiency values well below Betz' limit due to many factors such as friction losses, and the most efficient turbines has efficiency values between 35%–45%.[52]

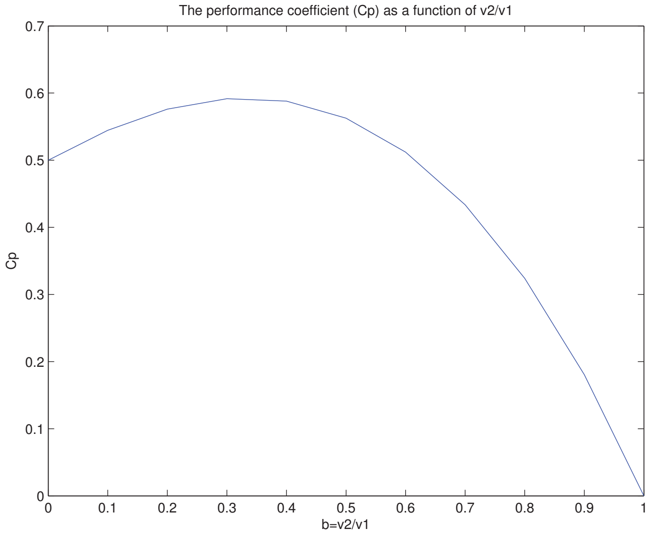

Figure 5.1: Betz' Law: The Performance coefficient C_p as a function of b = $\frac{v_2}{v_1}$. It can be seen that C_p reaches a maximum value (Betz' Limit C_p= 0.593) when the ratio between downstream and upstream wind velocities is b=$\frac{1}{3}$

5.3 Tip Speed Ratio (TSR)

Tip Speed Ratio (TSR) is a very important factor in wind turbine design, which help maximize the efficiency and the power output of a wind turbine. Tip Speed Ratio λ can be defined as the ratio between the blade's tip speed v_b and the wind speed v_w. [52]

$$\lambda = \frac{v_b}{v_w} \tag{5.20}$$

where,
v_b is the velocity of rotor tip in m/s
v_w is the wind velocity in m/s

The blade's tip speed v_b is the product of the rotor rotational speed ω (see Appendix B.5.1) and the blade's length r

$$v_b = \omega r \tag{5.21}$$

where,
ω=2πf, f is the frequency of rotation in Hz (rotations/sec)
r is the blade's length in metres

If the rotor of the wind turbine rotates too slowly, most of the wind will pass between the blades. On the other hand, if the rotor rotates too fast, the rotating blades will act like a solid wall to the wind. Therefore, wind turbines need to operate at their optimal tip speed ratio in order achieve higher efficiency (higher C_p value), and extract as much power as possible from the wind.[54]

Example 5.2. [52] If the wind is blowing at 15 m/s and the turbine is rotating at 1 revolutions per second. What is the tip speed ratio λ, if the blade's length is 10 m?

$f = 1$ rotations/sec
$\omega = 2\pi f = 2\pi\times1 = 2\pi$ rad/sec
$v_r = \omega r = 2\pi\times10 = 20\pi$ m/s
$\lambda = \frac{v_r}{v_w} = \frac{20\pi}{15} = 4$

Optimal TSR depends on the number of blades. The fewer the number of blades, the faster the wind turbine has to rotate, and the larger the number of blades the slower the wind turbine has to rotate to extract the maximum power from the wind. It has been found that for n bladed rotor the optimal tip speed ratio is

$$\lambda_{optimal}=\frac{4\pi}{n} \tag{5.22}$$

The derivation of Equation 5.22 is provided in Appendix B.5.2. The optimal TSR for a three bladed rotor is approximately equal to 4.19, and for a two bladed rotor is 6.28. In reality, if we want to design a highly efficient aerofoil, we can increase these optimal values by as much as 25%–30%, this can be achieved by increasing the rotor rotational speed ω and therefore generating more power. For example, for a two bladed rotor values will range between 7.85–8.16.[52]

Optimal TSR also depends on many other factors including airfoil profile used and wind turbine type. It has been found that a three bladed rotor has optimal TSR values ranging between 6–8 (most observed TSR value for a three bladed rotor is around 7). It is also desirable to have a higher TSR values, as this will result in higher rotor rotational speed ω, and hence generating more power. [52],[54]

For example, three bladed 45 m long in diameter wind turbine in Tararua Wind Farm has a tip speed ratio of approximately 7, as shown in Example 5.3. Tararua Wind farm is the largest wind farm in New Zealand in terms of number of turbines and the power output. Tararua has 134 turbines with an annual output of 620,000 MWh, that is approximately about 0.38 kWh/d/p.[46]

Example 5.3. [46] The average wind speed in Tararua Wind farm is 35 km/hr. What is the tip speed ratio for a turbine with three 45 m blades rotating at 14 revolutions/minute?

$f = 14$ rotations/minute $= 0.23$ rotations/sec
$v_w = 35$ km/hr $= 9.72$ m/s $\omega = 2\pi f = 2\pi\times0.23 = 0.46\pi$ rad/sec
$v_r = \omega r = 0.46\pi\times45 = 20.7\pi$ m/s
$\lambda = \frac{v_r}{v_w} = \frac{20.7\pi}{9.72} \approx 7$

5.4 Wind Power Density (WPD) and Wind Speed Distribution (Weibull and Rayleigh Distribution)

In wind industry, wind power density is used to estimate the wind resource available for any location and is also used to determine the best locations for wind energy generation (wind farms).

Definition 5.4. [56] Wind Power Density (WPD) is is a measure of the wind resource available at a specific location, and is measured in W/m^2.

$$P_{wind}=\frac{1}{2}\rho v^3 \tag{5.23}$$

where,
P_{wind} is Wind Power Density (WPD) in W/m^2
ρ is air density, ρ= 1.3 kg/m^3
v is wind velocity in m/s

WPD values are divided into classes, these classes range between 1-7 (1 indicating poorest resource, and 7 indicating excellent resource) as shown in Table 5.1. Locations with wind power density classes of 4 or higher are normally considered to be potential locations for windfarms. Wind power density is often measured at two standard heights 10 meters and 50 meters. 50 meters is the approximate hubheight for wind turbines.[56]

	10 m height		50 m height	
Wind power class	Mean wind speed (m/s)	Wind power density (W/m^2)	Mean wind speed (m/s)	Wind power density (W/m^2)
1	< 4.4	< 100	<5.6	<200
2	4.4-5.1	100-150	5.6-6.4	200-300
3	5.1-5.6	150-200	6.4-7.0	300-400
4	5.6-6.0	200-250	7.0-7.5	400-500
5	6.0-6.4	250-300	7.5-8.0	500-600
6	6.4-7.0	300-400	8.0-8.8	600-800
7	>7.0	>400	>8.8	>800

Table 5.1: Wind Power Density (WPD) classes at two standard heights 10 m and 50 m.[56]

Equation 5.23 can be used to estimate the wind power density at a specific location using the average wind speed. However, in reality wind changes its speed and direction continuously throughout the year and even throughout the day. Since power is proprtional to the cube of the wind speed and the sum of the cubes is not the same as the cube of the sum, the average wind speed will not provide a good estimate of the wind power potential at a specific location, and it is very important to know the wind speed distribution (frequency of occurence of certain wind speeds) in order to estimate the power output at a specific location.[44],[59]

Also, wind turbines do not deliver power proporional to the average wind speed cubed, but rather wind turbines have a range of wind speeds within which they deliver power. The cut-in speed is the speed at which the turbine starts generating power (cut in speed normally ranges between 3-5 m/s). The cut-out speed is the wind speed at which the turbine stops generating power to prevent any damage that may be caused to the turbine (the cut-out speed is usually around 25 m/s). At lower speeds than the cut-in speed, and higher speeds than the cut-out speed the wind turbine generates little or no power.[44]

It has been found that Weibull distribution and Rayleigh distribution provide a good fit for the wind speed variation at many locations around the world, and hence help in providing a good approximation of the power output at any potential location.[56]

Definition 5.5. Weibull Distribution is a continuous probability distribution. The probability density function (PDF) for Weibull is

$$F(x) = \frac{k}{c}(\frac{x}{c})^{k-1}e^{-(x/c)^2} \tag{5.24}$$

where,
$F(x)$ is the frequency of occurence of wind speed x
k is the shape parameter
c is the scale parameter

Rayleigh Distribution is a special case of Weibull Distribution with shape parameter k=2. It has been found that Rayleigh Distribution provides a good wind speed distribution for many locations around the world, since many locations have the shape parameter approximately close to 2.

Definition 5.6. Rayleigh Distribution is a special case of Weibull Distribution with the shape parameter k=2.

For Weibull Distribution, the average wind speed v, scale parameter c and the shape parameter k can be related by

$$v{=}c\Gamma(1+\frac{1}{k}) \tag{5.25}$$

where,
$\Gamma(1+\frac{1}{k})$ is the Gamma function, $\Gamma(1+\frac{1}{k}) = \frac{1}{k}\Gamma\frac{1}{k}$

The shape parameter k specifies the shape of the distribution, and it ranges between 1 and 3. If wind speeds tend to a certain value then the distribution will have a higher k value, on the other hand, lower k value means broader distribution with a variable wind speeds.[58],[56]

The scale parameter c relates to the average wind speed v, and is measured in m/s. It tells how windy the location is. Figure shows Rayleigh distribution (shape parameter k=2) for different mean wind speeds k =2). From Figure 5.2, as the scale parameter gets larger, the distribution gets stretched to the right and height decreases while maintaining shape and location, on the other hand, as c gets smaller the distribution is pushed towards the left and height increases.[58],[56]

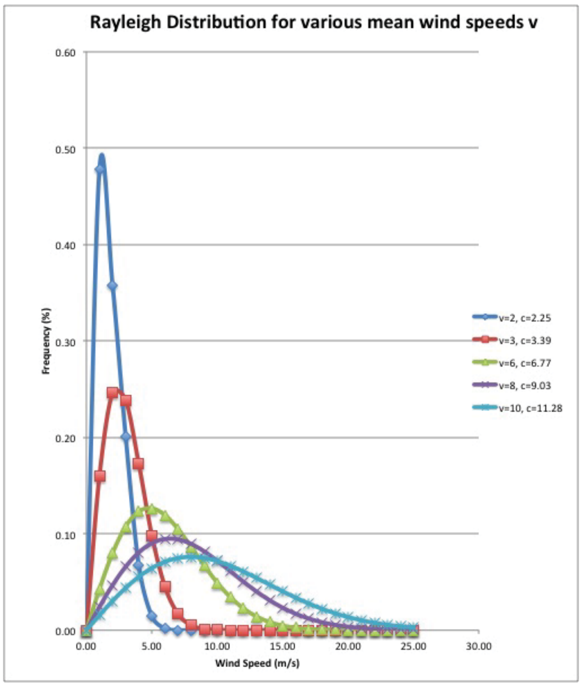

Figure 5.2: Rayleigh Distribution for various mean wind speeds v. Figure obtained from tables B.1 and B.2 in Appendix B.7.1

5.5 Wind - MacKay's Approach

5.5.1 Wind in the United Kingdom

MacKay estimated the wind potential in the United Kingdom by multiplying the average power per unit land area of a wind farm by the area per person in the United Kingdom. MacKay assumed the average wind speed in the United Kingdom to be 6 m/s, and estimated the power per unit area of a wind farm with a wind speed of 6 m/s to be 2.2 W/m^2. Then MacKay estimated that covering the United kingdom with 50% efficient wind turbines could deliver 200 kWh/d/p, and covering the windiest 10% of the United Kingdom could deliver 20 kWh/d/p. However, the windmills that would be required to provide the United Kingdom with 20 kWh/d/p would be really huge (50 times wind hardware in Denmark, 7 times the wind farms in Germany and twice all the wind farms in the world). MacKay concludes that the wind energy resource in the United Kingdom is huge, however it is much less than the United Kingdom huge consumption, and the maximum plausible production from onshore wind in the United Kingdom is 20 kWh/d/p.(cf.[44], pp.32-33,263-265)

5.5.2 Wind in New Zealand

In this section, I am going to follow MacKay's approach to estimate the onshore wind potential in New Zealand.

Suppose the average wind speed in New Zealand is 6 m/s (that is the mean wind speed in Wellington [13]). In reality that is an overestimate for some locations in New Zealand and an underestimate for others (see Appendix B.6, and Figure 5.3). According to Energy Efficiency and Conservation Authority[38], there are many regions in New Zealand with an average wind speed above 10 m/s. Following MacKay's approach, I will estimate the onshore wind potential in New Zealand by multiplying wind power per unit area by the area per person in New Zealand.

$$\text{Power per person}=\text{Wind power per unit area} \times \text{area per person} \tag{5.26}$$

Let us first cestimate the wind power density (Equation 5.24). The wind power density assuming an average wind speed of 6 m/s is

$$0.5\times 1.3\times(6\ \text{m/s})^3 = 140\ \text{W/m}^2 \tag{5.27}$$

In reality, not all this energy can be extracted from the wind and according to Betz' Law the maximum energy that can be extracted by a wind turbine is around 59% from the wind kinetic energy.

To esttimate the power of a single windmill, let us assume a windmill with a diameter of $d = 25$ m and a hub height of $h = 32$ m.[44]

$$\text{Power of a single windmill} = 50\% \times\text{power per unit area}\times\text{area} = 50\% \times 0.5\rho \text{v}^3\times\tfrac{\pi}{4}(25^2) = 34\ \text{kW} \tag{5.28}$$

To estimate the power per unit land area of a wind farm $P_{windfarm}$ we need to decide how close should we pack our windmills. According to MacKay[44], windmills can not be spaced closer than five times their diameter without losing power.

$$\text{P}_{windfarm}=\frac{\text{Power of a single windmill}}{\text{Land area per windmill}}= \frac{34}{(5d)^2}=2.2\ \text{W/m}^2 \tag{5.29}$$

Assuming that the power per unit area of a windfarm is 2.2 W/m^2, suppose we cover the whole country of New Zealand with 50% efficient wind turbines. Then that is about **3152 kWh/d/p**. Let us be realistic and suppose we only cover the windiest 10% of the country with windmills then this would deliver about **315 kWh/d/p**

Compared to the United Kingdom, New Zealand has a more promising wind potential, around nine times greater than the United kingdom's wind potential. Our 15 times smaller population density can be one of the main factors that gave New Zealand a higher wind potential.

One of the limitations to this approach is that it assumes that the wind speed for the whole country is 6 m/s. In reality wind changes its speed and direction continuously, and the average wind speed will not provide a good approximation of the potential power at a specific location. That is why it is very important to know the wind speed distribution to estimate the wind potential at a specific location. In the next section, I am going to estimate wind potential for the whole country using the wind resource map and Rayleigh distribution.

5.6 Onshore Wind Potential in New Zealand

5.6.1 Wind in New Zealand-Rayleigh Distribution and Image Processing Approach

My goal in this section is to use the Wind Resource Map (Figure 5.3) obtained from NIWA website [48],and Rayleigh distribution to estimate the wind power density (WPD) for each location in New Zealand.

I will assume that the wind speed in New Zealand follow a Rayleigh Distribution. Wind speeds are obtained from the Wind Resource Map (Wind speed measurements in the Wind Resource map are taken at 10 m height[38]), then Rayleigh distribution is used to obtain the wind power density in W/m^2 for each wind speed (see Appendix B.7.1), then some image processing has been done to obtain Figure 5.3 (see Appendix B.7.2).

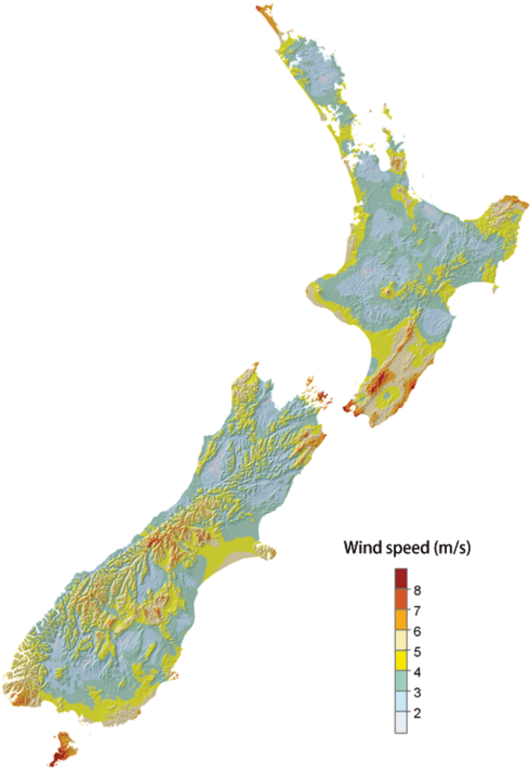

Figure 5.3: Wind Resource Map. [48]

From Figure 5.4, New Zealand has an excellent wind resource in many locations, and there are many potential wind farm locations with wind power density of class 4 or higher. Our model is based on mean wind speeds, and to provide more accurate results we need to include the variations in wind speed e.g., maximum, minimum and standard deviation. Also, the standard deviation will be needed to produce a full model and see when turbines will work.

There are currently 17 wind farms running or under construction generating about 4% of New Zealand's electricity. There are also many potential wind projects (proposed wind farms) which are under investigation, waiting for a consent or received a consent and waiting to be built. Although not all potential wind projects will be built, wind will play a significant role in achieving New Zealand's future renewable energy generation targets, according to New Zealand Wind Energy Association, Wind will play a very important role in achieving New Zealands government goal of generating 90% of electricity from renewables, and wind is also expected to generate around 20% of New Zealand's electricity by 2030. [8],[47]

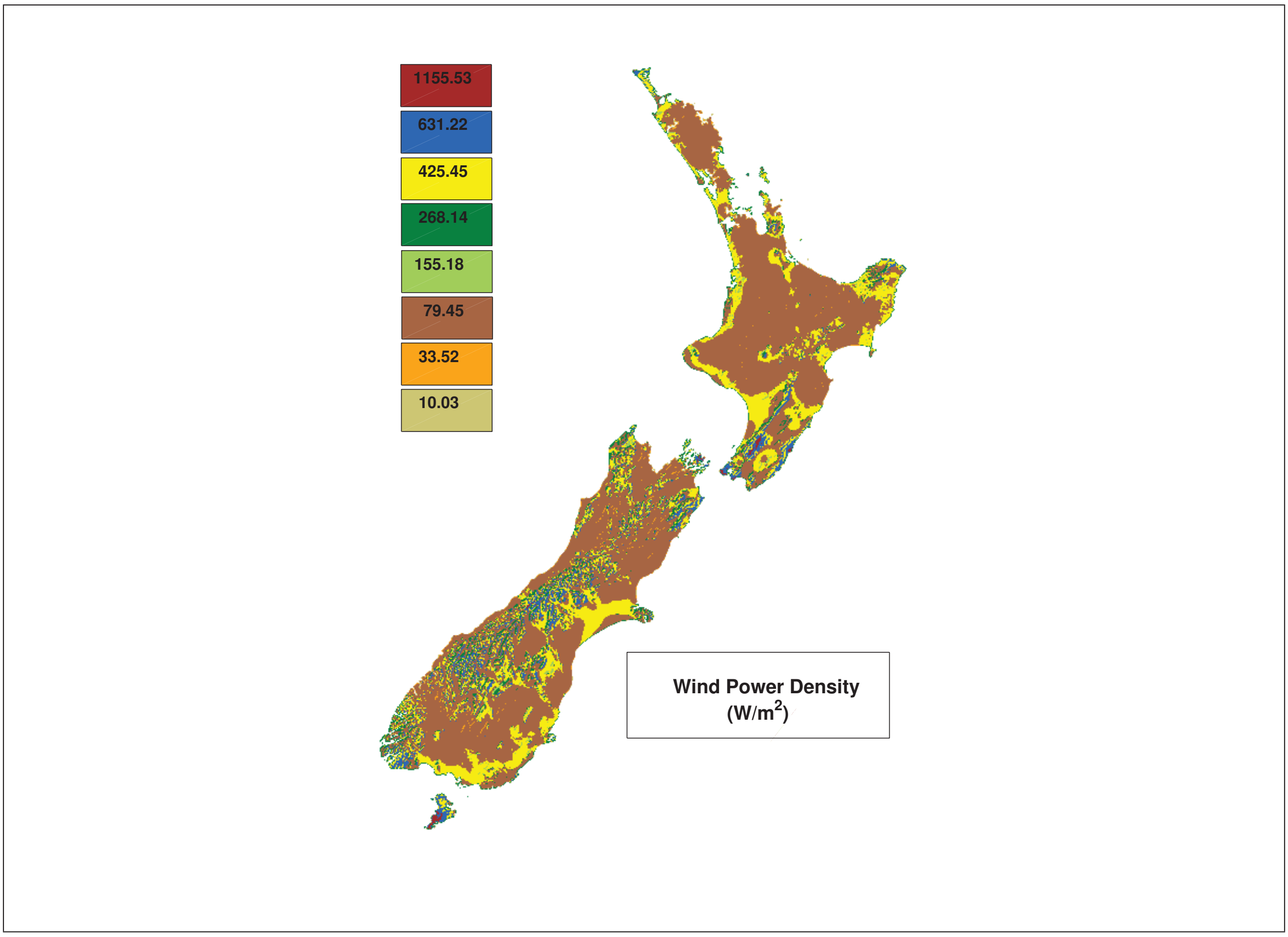

Figure 5.4: Wind Power Density (WPD) at 10 m estimated using Rayleigh Distribution (see Appendix B.7).

5.6.2 Onshore Wind Potential–An Upper Limit

In 2008, the Electricity Commission conducted a study to identify the potential wind resource available. The potential wind resource in this study has been divided into three Economic Tranches, such that Tranche 1 represents areas that will be economic in the near future, while Tranches 2 and 3 represent areas that will become economic with a larger rise in the value of electricity. These Tranches are also divided based on wind speeds, capacity factor (capacity factor is the average power production divided by the rated power [57]), the expected cost ($/MWh) and the available resource in GWh. The study excludes any land use areas (forestry, tourism,...,etc.), areas of altitude over 1500 m, areas within 500 km of structures such as buildings and urban areas and waterways. The study also assumes 50% willingness by landowners.[57]

The study estimated an upper limit of the available wind resource to be approximately around 127,370 GWh/year, that is about 77.5 kWh/d/p. However, in reality there are many environmental, economic and social constraints that would need to be considered [57]. According to Scadden [6], covering 0.6% of the area of New Zealand (around the area of Stewart Island) with wind turbines could deliver around 33 kWh/d/p at competitive pricing.

We can conclude that New Zealand has a significant wind resource, and there many potential locations for wind power development which can play a very integral part in achieving New Zealand's future energy goals. An upper bound for the onshore wind in New Zealand is about 77.5 kWh/d/p, however in reality there are many environmental, economic and social limitations that would need to be considered.

Chapter 6

Solar

In this chapter, I am going to estimate the solar potential in New Zealand, and provide a brief summary of MacKay's approach to solar in the United Kingdom. I am also going to discuss the factors that affect the intensity of solar radiation.

6.1 Factors Affecting Solar Radiation

Solar intensity is the power of sunshine per unit land area, and is measured in Watts per square metre. At midday on the equator and in the absence of cloud cover, earth surface receives approximately a raw power of 1000 W for every square metre facing the sun[44]. There are many factors that affect the solar intensity. The first factor is **Latitude**. As close as we get to the equator, solar intensity increases and vice versa. The maximum value occurs at the equator, where sunlight rays are perpendicular to the surface and the least at the poles. To find the solar intensity at a specific location, the solar intensity value at the equator (1000 $\mathrm{W/m^2}$) is multiplied by $\cos\theta$, where θ is latitude (Equation 6.1) [44]. Solar intensity for three cities in New Zealand in the absence of cloud cover is calculated in Table 6.1

$$I_\theta = I_{equator} \times \cos\theta \tag{6.1}$$

where,
$I_{equator}$ is solar intensity at midday in the absence of cloud cover on the equator, $I_{equator}$=1000 $\mathrm{W/m^2}$
θ is latitude
I_θ is solar intensity in $\mathrm{W/m^2}$ at a specific location with latitude θ

City	Latitude	Power per $\mathrm{m^2}$ of land ($\mathrm{W/m^2}$)
Auckland	36.8500° S	800.21
Wellington	41.2889° S	751.49
Invercargill	46.4250° S	689.30

Table 6.1: Solar Intensity ($\mathrm{W/m^2}$) at noon calculated based on latitude in the absence of cloud cover for three cities in New Zealand: Auckland[22], Wellington[29] and Invercargill[26]

Time of the day is another factor. Variation of solar intensity throughout the day approximately follows a sin function, and is highest at noon where sunrays are perpendicular to the surface, and lowest at sunrise and sunset where sunrays are approximately parallel to surface. Figure 6.2, shows variation in solar intensity throughout the day in the absence of cloud cover.

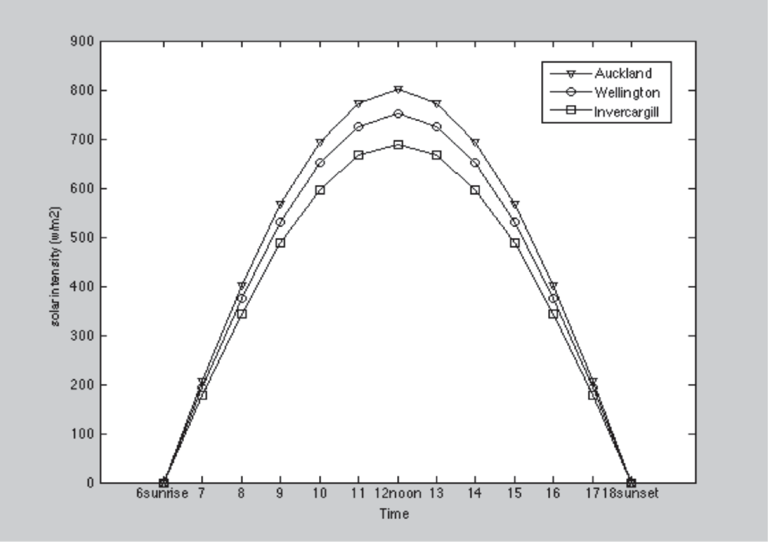

Figure 6.1: Average Solar Intensity vs.Time of the day on a flat surface in the absence of cloud cover for Auckland, Wellington and Invercargill (see Appendix C.1). The variation of solar intensity throughout the day approximately follows a sin function.

Seasonal variations also affect the intensity of solar radiation (Figure 6.2). Summer is warmer than winter, because the sun rays hit the earth at more direct angle and days are longer in summer than in winter.

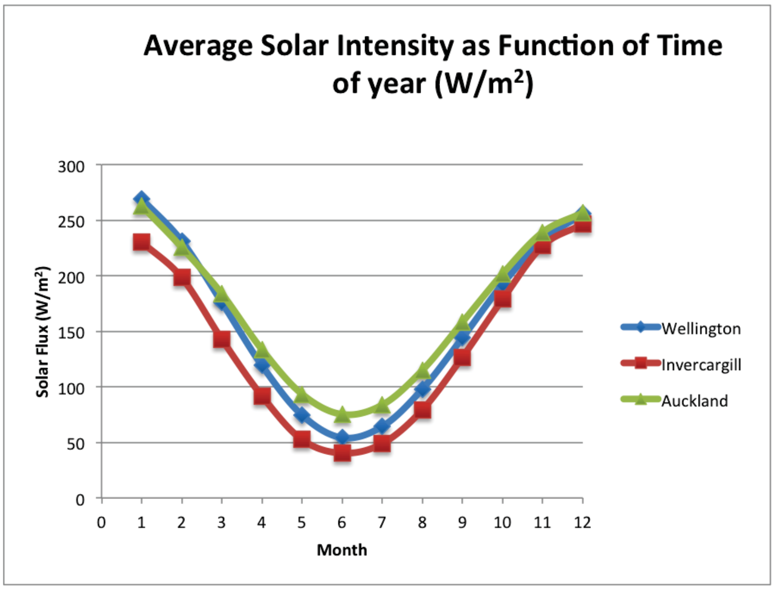

Figure 6.2: Average solar intensity in Auckland, Wellington and Invercargill as a function of time of year (see Appendix C.4)[14]

Note that the solar intensity values in Figure 6.2 are smaller than the solar intensity values obtained in Table 6.1, this is because solar intensity values in Table 6.1 are based on latitude and they are solar intensity values at noon and in the absence of cloud cover. We lose out because it is not midday all the time and solar intensity varies throughout the day and is highest at noon where sun rays are perpendicular to the surface as shown in Figure 6.1. We also lose power because of the cloud cover. According to NIWA[16], on average cloud reflects about 20% and absorbs 4% of solar radiation.

6.2 Solar—MacKay's Approach

6.2.1 Solar in the United Kingdom

South facing roofs in Britain recieve about 110 $\mathrm{W/m^2}$, and flat ground recieves about 100 $\mathrm{W/m^2}$. MacKay estimated that covering south facing roofs with 50% efficient solar thermal panels ($10\,\mathrm{m^2}$ of panels per person) and 20% efficient photovoltaic panels (10 $\mathrm{m^2}$ of panels per person) can provide about **13 kWh/d/p** and **5 kWh/d/p** respectively. According to MacKay[44], there will not be enough roof space for both solar thermal and solar photovoltaic panels, and he believes that going thermal is the better option, since the cost for installing photovoltaic is four times more than thermal and provides half as much energy. Although covering south facing roofs with photovoltaic may provide a big amount of electricity, but again there is not enough roof space to make a large contribution to energy consumption. (cf.[44], pp.38-40)

MacKay also discussed the idea of solar farming and estimated that covering 5% of the United Kingdom with 10% efficient photovoltaic panels can provide **50 kWh/d/p**. But this requires more than 100 times all photovoltaic in the world, and the electricity from solar farms is four times as expensive as the market rate. MacKay then discussed the idea of biomass. He estimated the maximum power available to be 36 kWh/d/p, but there will be losses along the processing chain, and he assumed that these losses will be as small as 33%, so solar biomass including biofuel, wood and waste incineration could deliver at most **24 kWh/d/p**. (cf.[44], pp.41-44)

MacKay concludes that:

- Covering south facing roofs with photovoltaic may provide a big amount of electricity, however there is not enough roof space for photovoltaic to make a large contribution to energy consumption.
- Solar farming is not an attractive option for a country like the United Kingdom, and it will be better to put panels in a sunnier country and use power lines to use the energy in the United Kingdom.
- Biofuels can deliver so little power, they do not add up and they can not replace all transport fuels especially in the United Kingdom.

Solar Thermal [1]	Solar Photovoltaic [2]	Solar Farming [3]	Biomass [4]
13	5	50	24

Table 6.2: Solar Potential in the United Kingdom: 1. Power (in kWh/d/p) delivered by covering south facing roofs with 50% efficient thermal panels (10 m^2 array per person). 2. Power (in kWh/d/p) delivered by covering south facing roofs with 20% efficient solar photovolatic panels (10 m^2 array per person). 3. Solar Farming involves covering 5% of the United Kingdom with 10% efficient photovoltaic panels. 4. Solar biomass including biofuel, wood and waste incineration

6.2.2 Solar in New Zealand

I believe that New Zealand has a better solar potential than the United Kingdom, because of our lower latitude than United Kingdom's latitude. According to Energy Efficiency and Conservation Authority [36], New Zealand solar energy resource is about 4 kWh/m^2. In this section, I will follow MacKay's approach (cf. [44], pp.38-49) and make rough estimates of the power that solar thermal, solar photovoltaic and solar farming could deliver in kWh/d/p.

Solar Thermal involves using sunshine for heating of buildings and making hot water. Suppose we cover all north facing roofs with 50% efficient solar thermal panels (10 m^2 of panels per person). In reality, 10 m^2 of panels per person is large, and according to Smarter Homes [18] , solar panels should be around 1 m^2 of collectors per person in the house, and the panels should face north (north west and north east can also be satisfactory). We find that 50% efficient solar thermal panels could deliver

$50\% \times 10\ m^2 \times 4\ kWh/m^2 = 20\ kWh/d/p$

Solar Photovoltaic panels turn energy from sunlight into electricity. Suppose we cover all north facing roofs with 20% efficient photovoltaic panels ($10\,m^2$ per person). The power that could be delivered by 20% efficient photovoltaic panels is

$20\% \times 10\ m^2 \times 4\ kWh/m^2 = 8\ kWh/d/p$

I believe that New Zealand's lower latitude can be the main factor that gave New Zealand a higher solar thermal and solar photovoltaic potential than the United Kingdom. According to MacKay [44], solar heating and solar photovoltaic could deliver about 13 kWh/d/p and 5 kWh/d/p in the United Kingdom respectively.

How about solar farming? Following MacKay's approach, suppose we cover 5% of New Zealand with 10% efficient solar photovoltaic panels, we would have

$10\% \times 4\ kWh/m^2 \times 2985.6\ m^2/person = 1194\ kWh/d/p$

That is about 23 times the United Kingdom figure of 50 kWh/d/p. Compared to the United Kingdom, New Zealand's lower latitude and New Zealand's 15 times smaller population density can be the main factors that gave New Zealand a higher solar potential than the United Kingdom.

Let us be realistic, 1194.2 kWh/d/p is a huge number and the cost will be definitely huge! According to Scadden [6], The solar costs currently 2-4 times hydro, geothermal and wind. Suppose we have these solar farms to cover the area of Invercargill (491 km^2) and New Plymouth (2209 km^2). Invercargill has radiation levels as high as in Germany, where solar panels are commonly used [36]. Invercargill receives about 3.31 kWh/ m^2/day (see Appendix B.3) of solar radiation, so suppose we cover 50% of Invercargill with 10% efficient solar panels.

$50\% \times 10\% \times 3.31$ kWh/ $\text{m}^2 \times 109.1$ m^2/person=18 kWh/d/p

How about New Plymouth? New Plymouth receives about 4.11 kWh/m^2/day (see Appendix B.3) of solar radiation, so suppose we cover 50% of New Plymouth (2209 km^2) with 10% efficient solar panels.

$50\% \times 10\% \times 4.11$ kWh/ $\text{m}^2 \times 490.9$ m^2/person=101 kWh/d/p

So covering 50% of Invercargill and New Plymouth (that is about 0.5% the area of New Zealand) with 10% efficient photovoltaic panels can deliver about **119 kWh/d/p**! Although this number seems promising, there are many factors to be considered such as environmental factors and the economics. According to Scadden[6], the solar cost is currently two to four times hydro, geothermal and wind, but it is likely to come down in the near future. Furthermore, the price for photovoltaic is currently 1/3 of their price four years ago. It is also important to note that these solar farms would have a lower effect on New Zealand's landscape, as it can deliver 5-8 times as much power per square meter as wind.[6]

Conclusion: New Zealand has a more promising solar potential than the United Kingdom, because of our lower latitude and 15 times smaller population density. In New Zealand, solar thermal and solar photovoltaic could deliver 20 kWh/d/p and 8 kWh/d/p respectively. Covering 5% of New Zealand with 10% efficient photovoltaic panels could deliver 1194 kWh/d/p. In reality, There are many limitations for such a potential including the economics. The solar cost is currently around two to four times hydro, geothermal and wind, but it is likely to come down in the near future.

Country	**Solar Thermal**	**Solar Photovoltaic**	**Solar Farming**
United Kingdom	13	5	50
New Zealand	20	8	1194

Table 6.3: New Zealand vs. United Kingdom: Power (in kWh/d/p) delivered by covering roofs with 50% efficient thermal panels (10 m^2 array per person) and 20% efficient solar photovolatic panels (10 m^2 array per person). Solar Farming involves covering 5% of the country with 10% efficient photovoltaic panels

6.3 Solar in New Zealand—The Numbers

My goal in this section is to estimate solar thermal and solar photovoltaic potential for every region in New Zealand.

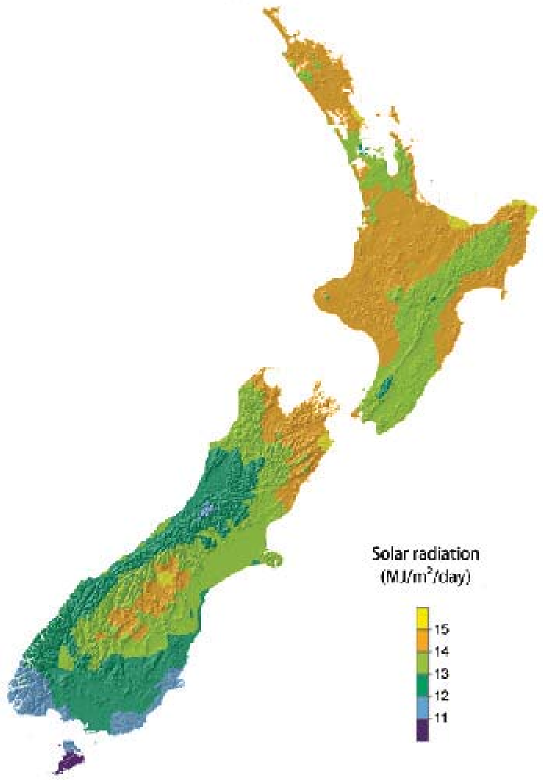

Figure 6.3: Solar Radiation in New Zealand (MJ/ m^2/day) [3]

6.3.1 Solar Thermal

Solar thermal involves using sunshine for heating of buildings and making hot water. Suppose we cover all north facing roofs with 50% efficient solar thermal panels (10 m^2 of panels per person).

How much could solar thermal deliver in kWh/d/p for each region in New Zealand if we install 50% efficient solar thermal panels (10 m^2 of panels per person)?

The Solar Radiation Map (Figure 6.3) was obtained from NIWA website, and some image processing has been done to obtain Figure 6.4 (see Appendix C.2).

From Figure 6.4, 10 m^2 of north facing solar thermal panels could deliver between **17-20 kWh/d/p**. Most values in the North Island lie between 19-20 kWh/d/p. In the South Island most values lie between 17-19 kWh/d/p. Smaller values (14-16 kWh/d/p) also exist in the lower South Island. According to MacKay, United Kingdom receives about 13 kWh/d/p. Compared to the United Kingdom, New Zealand has higher values, as New Zealand has lower latitude compared to the United Kingdom.

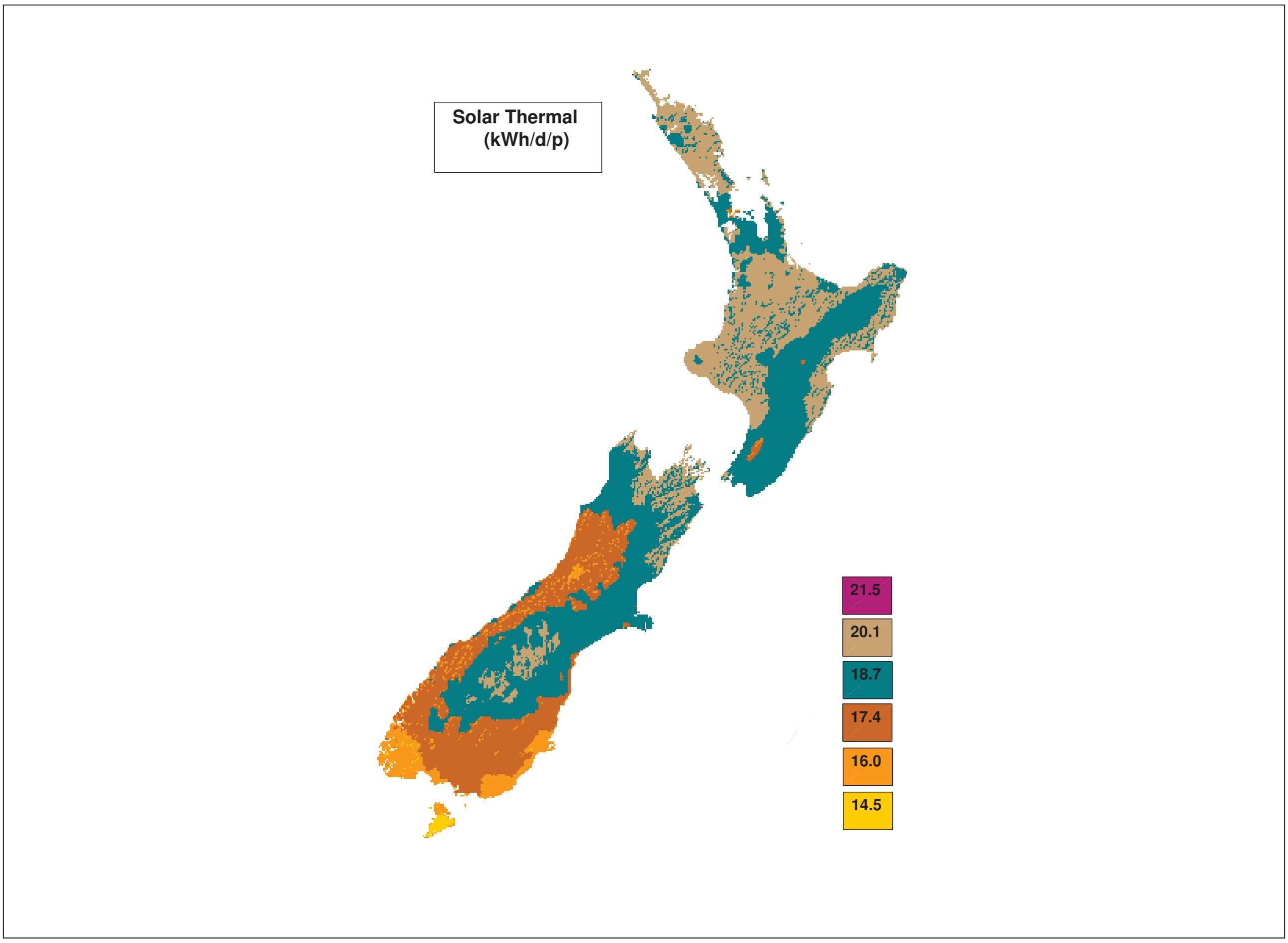

Figure 6.4: Solar power generated by installing 50% efficient solar thermal panels (10 m^2 of panels per person). See Appendix C.2 for the image processing that has been done to obtain this figure.

Although solar thermal potential seems promising. In reality, we will not be able to get this amount in winter or cloudy days. In summer, it may be possible to heat all the water that the household needs with solar energy, but in winter solar water heating will only meet part of the household's hot water needs. [17]

In New Zealand, Water heating accounts for about 30%-35% of the average Household's energy bill. On average, solar water heating system would provide around 50%-70% of annual household's hot water needs, that is about 20% of the total New Zealand household's power bill. Also we need to consider the costs, the cost for installing solar hot water system ranges between $4,000-$8,000. [17],[20]

6.3.2 Solar Photovoltaic

Solar photovoltaic panels turn energy from sunlight into electricity. Suppose we cover all north facing roofs with 20% efficient photovoltaic panels ($10\,\mathrm{m}^2$ of panels per person).

How much could solar photovoltaic deliver in kWh/d/p for each region in New Zealand if we install 20% efficient photovoltaic panels (10 m^2 of panels per person)?

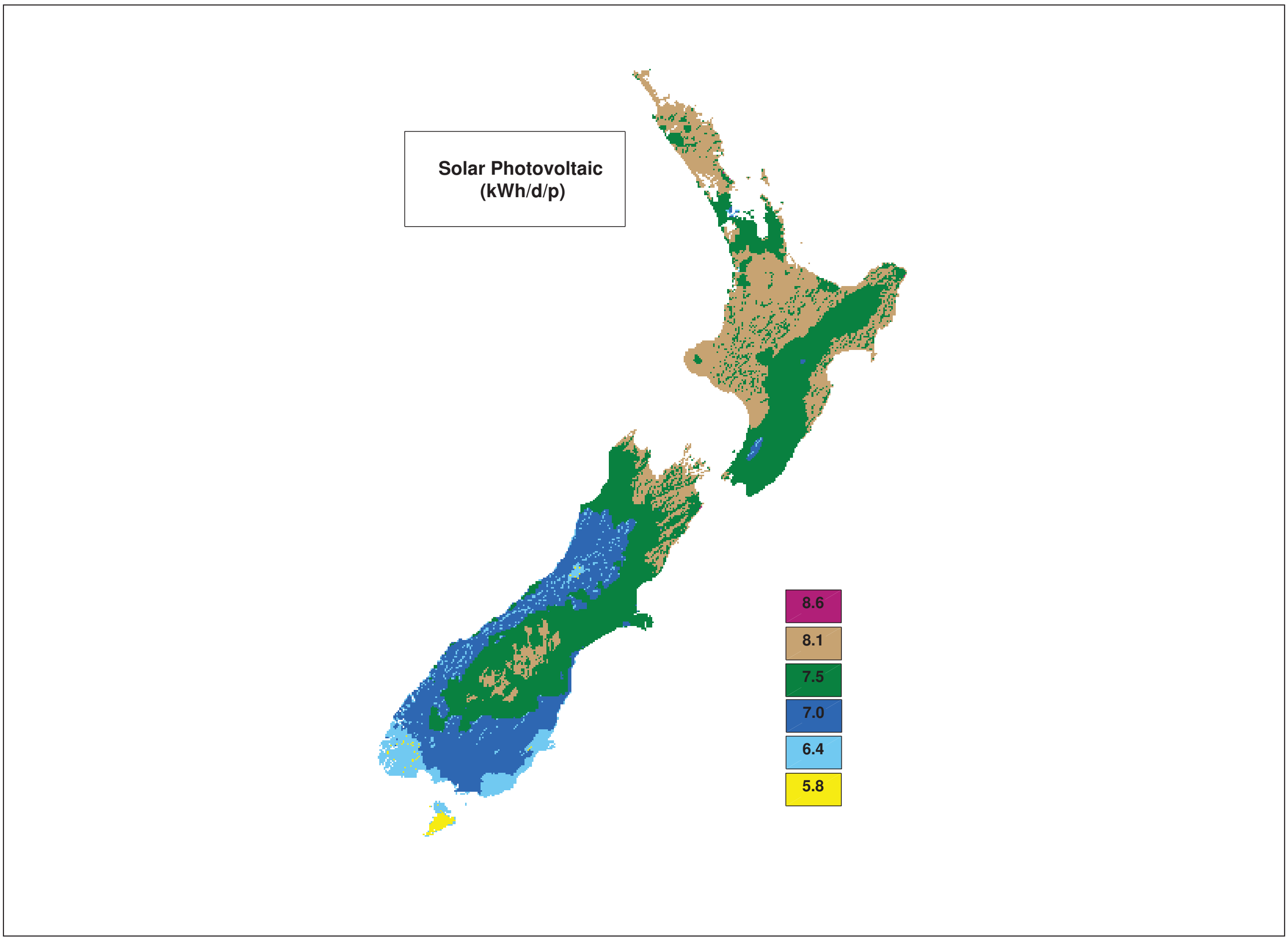

Figure 6.5: Solar power generated by installing 20% efficient solar photovoltaic panels (10 m^2 of panels per person). See Appendix C.3 for the image processing that has been done to obtain this figure.

The Solar Radiation Map (Figure 6.3) was obtained from NIWA website, and some image processing (see Appendix C.3) has been done to obtain Figure 6.5.

From Figure 6.5, Most values lie between **7-8 kWh/d/p**. According to MacKay, United Kingdom receives about 5 kWh/d/p. Again compared to the United Kingdom, New Zealand has higher values as New Zealand has lower latitude than United Kingdom.

In reality, we need to consider the economic cost.The cost for installing 1.5-kW system (the smallest system with eight 190 W panels) starts from $7500. This system size would eliminate roughly as much of household's power as would a solar hot-water system (around 20%

of household's total power). The Average New Zealand home would need between 5-kW and 6-kW system to eliminate its power bill and the cost will range between \$20,000 to \$28,000. [19]

6.4 Solar Thermal vs. Solar Photovoltaic

We need to choose between either solar thermal or solar photovoltaic, since there may not be enough roof space for installing both. MacKay believes that going thermal is the better option,since the cost for installing photovoltaic is four times more than thermal and provides half as much energy. Another smart solution would be to use a combined system generating both electricity and hot water. (cf.[44], p.40)

For New Zealand, although it was believed that solar thermal is the better option, because of their lower price compared to solar photovoltaic. Currently, solar thermal and photovoltaic systems have become comparable, as solar photovolatic prices have decreased and is now about 1/3 of their price four years ago. On average, solar hot water system would roughly eliminate around 20% of the total power bill. The smallest photovoltaic system would eliminate as much as would a solar hot water system. [20],[19]

Some of the differences between solar thermal and solar photovoltaic system include:

- The goverment provides subsidies for installing solar heating system, on the other hand, no subsidies for installing solar photovoltaic systems. [19]
- Solar thermal system costs between \$4,000-\$,8000, while solar photovoltaic system cost ranges between \$7000-\$28000. [20]
- Solar thermal system would roughly eliminate around 20% of the total household's power bill. Solar photovoltaic systems comes in different sizes, and the smallest solar photovoltaic system(1.5-kW) would eliminate as much as would a solar thermal system. A 5-kW to 6-kW system would roughly eliminate all of the household's power bill. [21]
- There is an energy loss in solar water heating, hot water needs to be used as it is generated. On the other hand, photovoltaic system involves no power loss as excess power is exported to the grid for credits or cash. [20]

Chapter 7

Marine

In this chapter, I am going to estimate the marine potential in New Zealand. I am also going to provide a brief summary of MacKay's approach to marine in the United Kingdom.

7.1 Marine in the United Kingdom

MacKay found an upper bound for the maximum wave power, by estimating the incoming power per unit length of coastline, and multiply it by the length of the coastline. Atlantic waves power is about 40 kW/m of coastline, and Britannia rules about 1000 km of the Atlantic coastline, so the total raw power is 16 kWh/d/p. However this power cannot all be extracted, and MacKay assumed covering half of the coastline with 50% efficient wave machines, then the power that can be extracted is about 4 kWh/d/p (around 25% of the total theoretical power estimated). MacKay then compared his calculations with the current wave technology. There are currently three Pelamis energy collectors, and according to Pelamis makers, a 2 km long wave farm could deliver about 6 kW/m. So the power delivered by 500 km wave farm would be 1.2kWh/d/p. (cf.[44], pp.73-75)

How about tides? MacKay believes that tide is a very attractive option for Britain for many reasons including that tides are predictable (work day and night), last for millions of years. Furthermore, when compared to photovoltaic power, tide does not require high cost hardware. Tides can also make constant contribution to electrical grid, this is because high and low tides take about 12 hours to progress around British Isles, so strongest currents occur at different times from each other. (cf.[44], p.86)

According to MacKay[44], the power that can be extracted from tides can never be more than the total power of tidal waves from the Atlantic. The total power is about 100 kWh/d/p. Assuming 10% of the total tidal power available can be extracted, and the conversion and processes are 50% efficient, then that is about 5 kWh/d/p. Then MacKay estimated the power that could be delivered by tidal farms, barrages and offshore tidal lagoons which all sum up to about 11 kWh/d/p. According to MacKay[44],tidal power has never been used in an industrial scale in Britain, and its hard to know the economic and technical challenges associated with it. (cf.[44], pp.82-87)

MacKay concludes that wave power can not play a significant role in sustainable energy in Britain, and building tide turbines may have many economic and technical challenges. The maximum contribution of Wave and Tide to sustainable energy in Britain is about 4 kWh/d/p and 11 kWh/d/p respectively.

7.2 Marine in New Zealand

My goal in this section is to estimate an upper bound for Marine potential in New Zealand including waves and tides.

Let us first estimate the wave potential for New Zealand following MacKay's approach. According to the Ministry for the Enivronment [39], New Zealand can be divided into four regions based on the range of the long-term mean for significant wave height (H_{av}), the average wave period between wave crests (T_{av}) and wind direction (see Appendix D). Table 7.1, shows the power per unit wave front $P_{wavefront}$ for each of the four regions in New Zealand.

Definition 7.1. [44] Power per unit length of wave front $P_{wavefront}$ is

$$P_{wavefront}=\frac{1}{16}\rho g h^2 v \tag{7.1}$$

where
P$_{wavefront}$ is per unit length of wave front in W/m
ρ is density of water, ρ=1000 kg/m^3
g is gravity, g=9.8 m/s^2
h is wave height in metres
v is speed of waves in m/s, v = $\frac{gT}{2\pi}$, T is time period in seconds

Region	Wave Height	Time	Wave speed (m/s)	$P_{wavefront}$ (kW/m)
South Facing Coasts (Fiordland to Catlins)- South Island	3.5	11	17.16	128.73
Western New Zealand Coasts	2.5	7	10.92	41.80
Eastern New Zealand up to East Cape	2.25	7.5	11.70	36.27
North Eastern North Island	1.5	6	9.36	12.90

Table 7.1: Power per unit length of wave front P$_{wavefront}$. Wave Height and Time are the average values of H_{av} and T_{av} in Table D.1 (see Appendix D) respectively.

To estimate the total incoming raw power $P_{incoming}$ in kWh/d/p, we need to multiply the power per unit length of wave front $P_{wavefront}$, by the length of coastline $I_{coastline}$ in km.

$$P_{incoming} = P_{wavefront} \times I_{coastline} \tag{7.2}$$

I estimated the coastline length for each of the four regions in New Zealand as shown in Figure 7.1. Table 7.2, shows the raw incoming power in kWh/d/p. **The total raw incoming power is 867 kWh/d/p**. In reality, wave machines can not extract all this power, and some power will be lost during conversion from mechanical energy to electricity[44]. Suppose we cover 50% of our coastline with 50% efficient wave machines, then **the total power that can**

Figure 7.1: Coastline Length in km. I estimated the coastline length using the scale (ruler) in the map. [42]

be extracted is 217 kWh/d/p.

Due to Southern ocean winds, New Zealand's West and South-West coasts have the most energetic waves[27]. Suppose we only cover 50% of our Southern and Western coastline with 50% efficient wave machines, then that is about **147 kWh/d/p**. Although New Zealand's wave potential looks promising, in reality there are many factors that would affect where wave generators could be built, and most wave machines are built for survivability more than efficiency[44],[6]. A study of the development potential of marine resources in New Zealand undertaken by Power Projects in 2008[5], concluded that New Zealand's wave power potential is more than 7000 MW, that is about 37 kWh/d/p. However, this estimate should be treated with caution as it is not based on a detailed economic evaluation [5]. According to Scadden[6], studies suggest that there are sites in New Zealand that can offer about 2 kWh/d/p and a maximum potential of 27 kWh/d/p based on existing technology however,there are many environmental and economic barriers.

Region	Power(kW/m)	Coastline Length(km)	$P_{incoming}$ (kWh/d/p)
South Facing Coasts (Fiordland to Catlins-South Island)	128.73	400	274.624
Western New Zealand Coasts	41.80	1400	312.11
Eastern New Zealand up to East Cape	36.27	1200	232.13
North Eastern North Island	12.90	700	48.16

Table 7.2: Raw incoming wave power $P_{incoming}$ in kWh/d/p

How about Tides? Tides in New Zealand are considered moderate, except for regions such as Cook Strait and Kaipara Harbour. Cook Strait has one of the strongest tidal flows in the world, and Kaipara Harbour is the largest harbour in New Zealand. [27],[51]

According to Power Projects Limited[5], the tidal flows into and out of Kaipara Harbour are equivalent to 11,000 MW, and tidal/ocean currents in Cook Strait are equivalent to 13,000 MW of potential electricity generation. That is about 128 kWh/d/p of electricity. However in reality the amount that can be extracted from available tidal power is very small[5]. Studies also indicate that only small part of the Cook Strait is attractive for electricity generation, where tidal currents reaches sufficient speed, but even under these conditions there are many limitations against electricity generation tidal schemes.[5]

Resource consents have been given to many tidal energy projects. In 2011, Crest Energy received a resource consent to install 200 under water tidal turbines each with a maximum capacity of 1.2 MW in Kaipara Harbour. A resource consent was also granted to Neptune power in 2008 for a tidal turbine off Sinclair Head in Cook Strait. Neptune believes that tides in cook Strait can generate 12 GW (that is about more than $1\frac{1}{2}$ times New Zealand's current requirements). Energy Pacifica has also applied for a resource consent to install ten turbines in Tory Channel In Cook Strait. Each turbine is able to produce up to 1.2 MW. [5],[27]

Suppose All the resource consents mentioned above become a reality. Then tides in New Zealand could deliver about **65.34 kWh/d/p**. A study of the development potential of marine resources in New Zealand undertaken by Power Projects in 2008[5], concluded that New Zealand's tidal power potential is more than 500 MW, that is about 3 kWh/d/p. However, this estimate should be treated with caution as it is not based on a detailed economic evaluation. According to Scadden[6], 0.4 kWh/d/p can be obtained from sites in Cook Strait, Stewart island, Cape Reinga and Kaipara Harbour, and 11 kWh/d/p can be extracted from Cook strait.

According to Power Projects[5], wave and tidal energy reserves can exceed New Zealand's requirements for electricity provided that marine power costs are reduced to be competitive with other renewable energy sources such hydro, geothermal and the wind.

Conclusion: New Zealand has a very promising wave and tide future potential, with an incoming raw wave power of **867 kWh/d/p**, and an upper limit of **128 kWh/d/p** for tides from the tidal flows in Cook Strait and Kaipara Harbour. In the future, it is possible that New Zealand can provide most of its energy requirements from marine resources, if marine technology costs become competitive with other renewable sources. However, in reality there are many limitations, and economic evaluation of marine technologies is needed.

Compared to the United Kingdom, New Zealand has a huge promising wave and tide potential. Our 15 times smaller population density and more energetic waves can be the main factors that gave New Zealand a much higher marine potential compared to the United Kingdom.

Chapter 8

Conclusions and Future Work Recommendations

In this chapter, I will provide the main conclusions that can be derived from this research. I will also provide a brief outline of the research process in terms of its aim,methods used to estimate the potential for each renewable resource and some of the limitations associated with following MacKay's approach. Finally, I will conclude with some recommendations for future research work.

8.1 Research Process

The main goal of this research has been to estimate an upper bound for the power available from the renewable sources: hydro, geothermal, wind, solar and marine, then compare this to New Zealand's energy use, and hence answer the following question Can New Zealand supply all of its energy requirements from renewables?

In chapter 3, I estimated the hydroelectricity potential using two methods: MacKay's Approach and Hydroelectricity Image Processing Approach. The first method involved following MacKay's approach, and dividing the country into Lowland and Highland regions (Blenheim and Rotorua were chosen as representatives for Lowland and Highland regions respectively). The goal of Hydroelectricity Image Processing Approach was to calculate an absolute upper bound for hydroelectricity in New Zealand, by calculating the gravitational potential power (see def. 2.1) of every drop of rain falling anywhere in New Zealand. To achieve this goal, the Rainfall Map and New Zealand 100 m Digital Elevation Model were obtained and some image processing has been done to estimate the total hydroelectricity available.

One of the limitations to MacKay's approach is that it assumes that the whole Lowland (Highland) has a rainfall rate and altitude the same as Blenheim (Rotorua). On the other hand, Hydroelectricity Image Processing Approach gives a better estimation of the total hydro available, since it calculates the hydro available for each region based on their rainfall rate and altitude rather than just calculating the power for two cities as a representative for the whole country.

In chapter 4, I estimated the geothermal potential in New Zealand. Geothermal potential was estimated following MacKay's approach, by treating geothermal as a resource that would be sustainable forever ignoring any hot spots, assuming perfect power stations and drilling to any depth is free. I also estimated the potential of high temperature geothermal resources in New Zealand based on an assessment of New Zealand's high temperature geothermal resources by Lawless[40].

In chapter 5, I estimated the onshore wind potential in New Zealand following MacKay's approach, and Wellington with its average wind speed of 6 m/s was chosen as a representative for the whole country. One of the limitations to this approach is that it assumes that the wind speed for the whole country is 6 m/s. In reality, wind changes its speed and direction continuously, and the average wind speed will not provide a good approximation of the potential power at a specific location. I also estimated the onshore wind potential in New Zealand based on a study conducted by the Electricity Commission in 2008[57]. The aim of the study was to identify the potential wind resource available[57]. Furthermore, I estimated the wind power density which is a very important measure in wind industry for the whole country using the Wind Resource Map and Rayleigh distribution.

In chapter 6, I estimated the solar potential (solar thermal, solar photovoltaic and solar farming) in New Zealand following MacKay's approach. To estimate solar thermal and photovoltaic potential, I assumed covering north facing roofs with 50% efficient solar thermal and 20% efficient solar photovoltaic panels respectively (10 m^2 of panels per person). I also estimated the solar thermal and solar photovoltaic potential for every region in New Zealand using the Solar Radiation Map.

Finally, I estimated marine potential in chapter 7. The wave potential in New Zealand was estimated following MacKay's approach. An upper bound for the total raw incoming wave power was estimated by calculating the power per unit length of wave front (kW/m), and multiplying it by the coastline length. I also estimated the power that can be extracted from waves, if we cover 50% of New Zealand's coastline with 50% efficient wave machines. Furthermore, I estimated the power that can be extracted from southern and western coastline (have the most energetic waves) if we cover 50% of New Zealand's southern and western coastline with 50% efficient wave machines. In reality there many factors that would affect where wave machines could be built and wave machine are built for survivability more than efficiency[44],[6]. An upper bound for tide potential from the tidal flows in Cook Strait and Kaipara Harbour in New Zealand was estimated based on a study of the development potential of marine resources in New Zealand conducted by Power Projects in 2008[34].

8.2 Conclusions

This research has achieved very interesting results to do with the likelihood, of obtaining all of New Zealand's energy requirements from renewable resources only. The total renewable resource available from the renewable sources: hydro, goethermal, onshore wind, solar and marine is approximately about 891 kWh/d/p, that is about 9 times our energy use of 138 kWh/d/p. This clearly indicates that, it is possible for New Zealand to supply all of its energy requirements from renewable sources alone. However, in reality there are many economical, environmental and social considerations that would need to be considered, for this upper limit.

Compared to the United Kingdom, New Zealand's 15 times smaller population density and the availablity of renewable resources (higher altitude and rainfall for hydroelectricity, more hot spots locations for geothermal,...,etc.) can be the main factors that gave New Zealand a much higher potential.

Renewable Resource	**United Kingdom**	**New Zealand**
Hydroelectricity	1.5	MacKay's Approach: 93 Hydroelectricity Image Processing Approach: 546.7
Geothermal	Sustainable forever: 2 Hot dry rocks: 1	Sustainable forever: 30 High temperature geothermal resources: 26.6
Wind	20	MacKay's Approach: 315 Onshore wind potential-upper limit: 77.5
Solar Thermal	13	17-20
Solar Photovoltaic	5	7-8
Wave	4	147
Tide	11	65.3
Total (kWh/d/p)	**55.5**	**891**

Table 8.1: Renewable potential (kWh/d/p): New Zealand vs. United Kingdom. Hydro, geothermal and wind values included in the total renewable potential for New Zealand (Total (kWh/d/p)) in this table are hydroelectricity value estimated using Hydroelectricity Image Processing Approach, high temperature geothermal resources for geothermal and onshore wind potential-upper limit for onshore wind.

Some of the main conclusions that can be derived from this research are:

- **Hydroelectricity**
  New Zealand has a very promising hydroelectricity potential with an upper limit of 546.72 kWh/d/p, that is if every single rain drop is used. However, in reality it is impossible to harness all this energy from the rain and there are many economic, environmental and social limitations that would need to be considered. It is also very surprising that we are currently using around 2% of the total hydro available!

  Compared to the United Kingdom, New Zealand's 15 times smaller population density, higher altitudes and higher rainfall can be the main factors that gave New Zealand a much higher hydroelectricity potential.

- **Geothermal**
  An upper bound for New Zealand's high temperature geothermal resources is 26.6 kWh/d/p. However, in reality there are many limitations for such a potential, and taking into account all the limitations the maximum realistic potential is about 9.4 kWh/d/p of electricity.

  Compared to the United Kingdom, New Zealand's 15 times smaller population density and more promising hot spots locations can be the main factors that gave New Zealand a higher geothermal potential.

- **Onshore Wind**
  New Zealand has a significant wind resource, and there many potential locations for wind power development which can play a very integral part in achieving New Zealand's future

energy goals. An upper limit of the onshore wind potential in New Zealand is approximately about 77.5 kWh/d/p, however there are many limitations for such a potential.

- **Solar**
  New Zealand has a promising solar potential. 10 m^2 of north facing, solar thermal panels that are 50% efficient could deliver between 17-20 kWh/d/p of hot water, however in reality we will not be able to get this amount in winter or cloudy days. Also, 10 m^2 of north facing, solar photovoltaic panels that are 20% efficient could deliver between 7-8 kWh/d/p. Furthermore, solar farming has a huge potential in New Zealand and covering 5% of New Zealand with 10% efficient photovoltaic panels could deliver about 1194 kWh/d/p.

  Although these figures seem promising, in reality there are many limitations for such a potential including the economics. Solar costs are currently two to four times hydro and wind, but it is expected to come down in the near future.

  Compared to the United Kingdom, New Zealand's 15 times smaller population density and New Zealand's lower latitude can be the main factors that gave New Zealand a higher solar potential.

- **Marine**
  New Zealand has a very promising wave and tide future potential with total incoming raw wave power of 867 kWh/d/p, and an upper limit of 128 kWh/d/p for tides from the tidal flows in Cook Strait (has the strongest tidal flows in the world) and Kaipara Harbour (the largest harbour in New Zealand).

  Covering 50% of New Zealand's coastline with 50% efficient wave machines could deliver about 217 kWh/d/p, and covering 50% of New Zealand's Southern and Western coastline with 50% efficient wave machines could deliver about 147 kWh/d/p. However, in reality there many factors that would affect where wave machines could be built and wave machine are built for survivability more than efficiency.

  Although the potential for tidal flows in Cook Strait and Kaipara Harbour looks promising, in reality there are many limitations and the amount that can be extracted from the available tidal power is much smaller. Resource consents have been given to many tidal projects in Cook Strait and Kaipara Harbour which if they become a reality could deliver about 65 kWh/d/p. Again, there are many limitations that would need to be considered. It is also estimated that waves and tides in New Zealand could provide about 37 kWh/d/p and 3 kWh/d/p. However, these results are not based on a detailed economic evaluation.

8.3 Future Work

In order to verify and confirm the results obtained from this reseach, future work is recommended. For example, the method used to estimate an upper limit for hydroelectricity potential in New Zealand (Hydroelectricity Image Processing Approach) was based on public data, and the result can be made more accurate. Although Hydroelectricity Image Processing Approach provides a good estimation to the maximum hydro available, there are many factors affecting the accuracy of our calculation including not having access to accurate and original rainfall data, and the accuracy of this calculation could further be developed by obtaining more accurate rainfall data e.g., monthly rainfall data. Also, the wind power density was estimated using the mean wind speeds and Rayleigh distribution. The wind results could be developed, if we include the variations in wind speed e.g., maximum, minimum and standard deviation. Furthermore, the full model will need the standard deviation to see when wind turbines will work.

This research also point to several other directions for future work. One direction would be to investigate the other side of energy balance (energy consumption), and see where potential savings can be made (smarter heating, better transport, efficient electricity use,. . . , etc.). Another direction would be to explore some energy plans for New Zealand that can be economically feasible in the near future. For simplicity, we can follow MacKay's approach and deal with **Cartoon New Zealand**, and assume that Cartoon New Zealand consumes energy only in three forms: heating, electricity and transport. The goal will be to reduce demand (high efficiency lightning, electrifiying transport and heating,. . . ,etc), and increase renewable energy production (building more wind turbines, solar thermal panels, photovoltaic panels,. . . ,etc.). The simplification of New Zealand into Cartoon New Zealand will help provide a better understanding of our energy issues, and can be the starting point for more detailed plans in the future.

Finally, in terms of applications to real world, studying the potential for geothermal direct energy use in New Zealand is another future research direction. Geothermal direct use potential has not been studied in this research. Geothermal energy has been directly used in New Zealand for hundreds of years (see section 4.2). According to Kelly[7], there is a signicant potential for geothermal direct energy use mainly for space heating purposes directly, and through applications such as ground source heat pumps for domestic and commercial applications.

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Appendix A

Haversine Formula

Haversine Formula calculates the great circle distance between two points on a sphere based on longitude and Latitude.[25]

$$d{=}Rc \tag{A.1}$$

where,
R is earth's radius, R=6371 km[23]

$$a{=}\sin^2(\frac{diff\ lat}{2}){+}\cos(lat1)\cos(lat2)\sin^2(\frac{diff\ long}{2}) \tag{A.2}$$

$$c{=}2\arctan\sqrt{\frac{a}{1-a}} \tag{A.3}$$

How accurate is Haversine? The earth has an oblate spheroid shape and haversine assumes that the earth is spherical, but its accuracy is accepted for most purposes.[23],[43]

The longitude and latitude values are obtained using google maps. The following M-file can be used to calculate the distance in km using the Haversine Formula between any two points based on their latitude and longitude.

```
function d=haversine(lat1,long1,lat2,long2)

 % haversine(lat1,long1,lat2,long2) calculates the distance in km between
two points on a sphere (earth of radius 6371 km)

%convert degrees to radians
lat1=lat1*pi/180;
long1=long1*pi/180;
lat2=lat2*pi/180;
long2=long2*pi/180;
difflat=abs(lat1-lat2); %difference in latitude
```

```
difflong=abs(long1-long2); %difference in longitude
R=6371; % average radius of earth in km
a = (sin(difflat/2))^2 + cos(lat1) * cos(lat2) * (sin(difflong/2))^2;
c=2*atan2(sqrt(a),sqrt(1-a));
d=R*c;
double(d)

end
```

Appendix B

Wind

B.1 Derivation of Kinetic Energy Equation, E=$\frac{1}{2}$mv^2 [41]

Definition B.1. [41] The Kinetic Energy $E_{kinetic}$ of an object with mass m and velocity v is the work done W in displacing that object from rest to a distance r under force F

$$E_{kinetic}=W=Fr \tag{B.1}$$

According to Newton's Second Law:

$$F=ma \tag{B.2}$$

Substitute B.2 in B.1, gives

$$E_{kinetic}=mar \tag{B.3}$$

According to the Third Law of Motion:

$$v^2=u^2+2ar \tag{B.4}$$

Where u is the initial velocity, letting $u = 0$, and solving for a gives

$$a = \frac{v^2}{2r} \tag{B.5}$$

Substituting B.5 in B.3 gives

$$E_{kinetic}=\frac{1}{2}mv^2 \tag{B.6}$$

B.2 Continuity Equation (Conservation of Mass)[41]

Suppose we have a steady flow in a pipe of varying parameter. The mass of the fluid m_1 with density ρ_1 passing through area A_1 is

$$m_1 = \rho_1 V_1 \tag{B.7}$$

The volume of the fluid V_1 passing through area A_1 at time $\triangle$t is

$$V_1 = A_1 v_1 \triangle t \tag{B.8}$$

where v_1 is the fluid velocity passing through area A_1.

Substituting B.8 in B.7 gives

$$m_1 = \rho_1 A_1 v_1 \triangle t \tag{B.9}$$

m_1 is the mass of the fluid passing through area A_1 in time $\triangle$t. Similarly, the mass of the fluid m_2 with density ρ_2 passing through area A_2 is

$$m_2 = \rho_2 V_2 \tag{B.10}$$

The volume of the fluid V_2 passing through area A_2 at time $\triangle$t is

$$V_2 = A_2 v_2 \triangle t \tag{B.11}$$

where v_2 is the fluid velocity passing through area A_2.

Substituting B.11 in B.10 gives

$$m_2 = \rho_2 A_2 v_2 \triangle t \tag{B.12}$$

m_2 is the mass of the fluid passing through area A_2 in time $\triangle$t.

The mass flow rates are constant through the pipe ($\dot{\mathrm{m}}_1 = \dot{\mathrm{m}}_2$), since no fluid flow in or out the sides, so

$$\dot{m} = \rho_1 A_1 v_1 = \rho_2 A_2 v_2 \tag{B.13}$$

In the case of incompressible flow (constant density ρ), the continuity equation becomes

$$\dot{m}{=}A_1v_1{=}A_2v_2 \tag{B.14}$$

B.3 Newton's Second Law for Fluids[55]

Suppose we have a steady flow in a streamtube. The momentum of the fluid entering the tube p_1 is the product of the mass of the fluid entering the tube m_1 and its velocity v_1

$$p_1 = m_1v_1 \tag{B.15}$$

The mass of the fluid entering the tube m_1 is the product of its density ρ_1 and volume V_1. The volume of the fluid V_1 entering the tube at time $\triangle$t is the product of the area A_1, velocity v_1 and $\triangle$t

$$m_1 = \rho_1V_1 {=}\rho_1A_1v_1\triangle \mathrm{t} \tag{B.16}$$

Substiting B.16 in B.15 gives

$$p_1 = \rho_1A_1v_1\triangle \mathrm{t}v_1 \tag{B.17}$$

Similarly, the momentum of the fluid leaving the tube p_2 is the product of the mass of the fluid leaving the tube m_2 and its velocity v_2

$$p_2 = m_2v_2 \tag{B.18}$$

The mass of the fluid leaving the tube m_2 is the product of its density ρ_2 and volume V_2. The volume of the fluid V_2 leaving the tube at time $\triangle$t is the product of the area A_2, velocity v_2 and $\triangle$t

$$m_2 = \rho_2V_2 {=}\rho_2A_2v_2\triangle \mathrm{t} \tag{B.19}$$

Substiting B.19 in B.18 gives

$$p_2 = \rho_2A_2v_2\triangle \mathrm{t}v_2 \tag{B.20}$$

According to Newton's second law for fluids,the rate of change of momentum of a body is equal to the resultant force acting on the body, and takes place in the direction of the force.[55]

$$F=\frac{p_2-p_1}{\triangle t} = \rho_2 A_2 {v_2}^2 - \rho_1 A_1 {v_1}^2 \tag{B.21}$$

We know from the continuity equation that $\dot{m}=\rho_1 A_1 v_1 = \rho_2 A_2 v_2$, then B.21 becomes

$$F = \dot{m}(v_2 - v_1) \tag{B.22}$$

In the case of a wind turbine, Equation B.22 is the force exerted by the wind on the rotor.

B.4 Wind Velocity at the Rotor v[52]

Let v_1 and v_2 be the upstream and downstream wind velocities respectively, and let v be the wind velocity at the rotor. Assuming incompressible flow, then the continuity equation (see appendix B.2) is

$$\dot{m} = \rho A_1 v_1 = \rho A_2 v_2 \tag{B.23}$$

The force exerted by the wind on the rotor F (see Appendix B.3) is

$$F = ma = \dot{m}a = \rho A v (v_2 - v_1) \tag{B.24}$$

The work done by this force can be written incrementally [52] as

$$dE = F dr \tag{B.25}$$

Power P can be defined as the rate of work done

$$P = \frac{dE}{dt} = F\frac{dr}{dt} \tag{B.26}$$

Substituting B.24 in B.26 gives

$$P = \frac{dE}{dt} = \rho A v^2 (v_1 - v_2) \tag{B.27}$$

The power P can also be defined as the rate of change of kinetic energy from upstream to downstream

$$P = \frac{\triangle E}{\triangle t} = \frac{1}{2}\dot{m}({v_1}^2 - {v_2}^2) \tag{B.28}$$

Equating B.27 and B.28 gives

$$\rho A v^2 (v_1 - v_2) = \frac{1}{2}\rho A v({v_1}^2 - {v_2}^2) \tag{B.29}$$

Simplifying B.29 gives

$$v(v_1 - v_2) = \frac{1}{2}({v_1}^2 - {v_2}^2) \tag{B.30}$$

From B.30, we can conclude that

$$v = \frac{v_1 + v_2}{2} \tag{B.31}$$

Therefore, the velocity at the rotor v is the average of upstream v_1 and downstream v_2 wind velocities.

B.5 Optimal Tip Speed Ratio $\lambda_{optimal}$

B.5.1 Some Useful Definitions [41]

Frequency f is the number of rotations per unit time, and is measured in Hertz (Hz). 1 Hz= 1 rotation/second.

Time Period T is the time taken for one complete cycle. and is measured in seconds.

$$\mathrm{T} = \frac{1}{f} \tag{B.32}$$

Angular Frequency (or angular speed) ω is a measure of rotation rate, and is measured in radians/second.

$$\omega = 2\pi \mathrm{f} \tag{B.33}$$

B.5.2 Optimal Tip Speed Ratio — The Proof [52]

Optimal Tip Speed Ratio $\lambda_{optimal}$ can be found by relating the time taken for the distrubed wind to reestablish itself T_w to the time taken for the next blade to move to the position of the preceding blade T_b. Let v be the wind speed, ω the rotational frequency of the rotor, n the number of blades and s is the length of the distrubed wind stream.

For n bladed rotor, the time taken for the next blade to move to the position of the preceding blade is

$$T_b=\frac{1}{f} = \frac{2\pi}{n\omega} \tag{B.34}$$

The time taken for the distrubed wind to reestablish itself T_w is

$$T_w=\frac{s}{v} \tag{B.35}$$

If $T_w>T_b$ (the rotor is rotating too slowly), then some wind will be unaffected. On the other hand, if $T_w<T_b$ (the rotor rotating too fast), then the blades will act as a solid wall against the wind, and some wind will not pass through the rotor. The maximum power extraction occur when T_b approximately equals T_w

$$T_b\approx T_w \tag{B.36}$$

$$\frac{2\pi}{n\omega}\approx\frac{s}{v} \tag{B.37}$$

Rearranging Equation B.37 gives the optimal rotational frequency $\omega_{optimal}$

$$\omega_{optimal} \approx \frac{2\pi v}{ns} \tag{B.38}$$

The optimal tip speed ratio $\lambda_{optimal}$ can be written as

$$\lambda_{optimal} = \frac{r\omega_{optimal}}{v} = \frac{2\pi r}{ns} \tag{B.39}$$

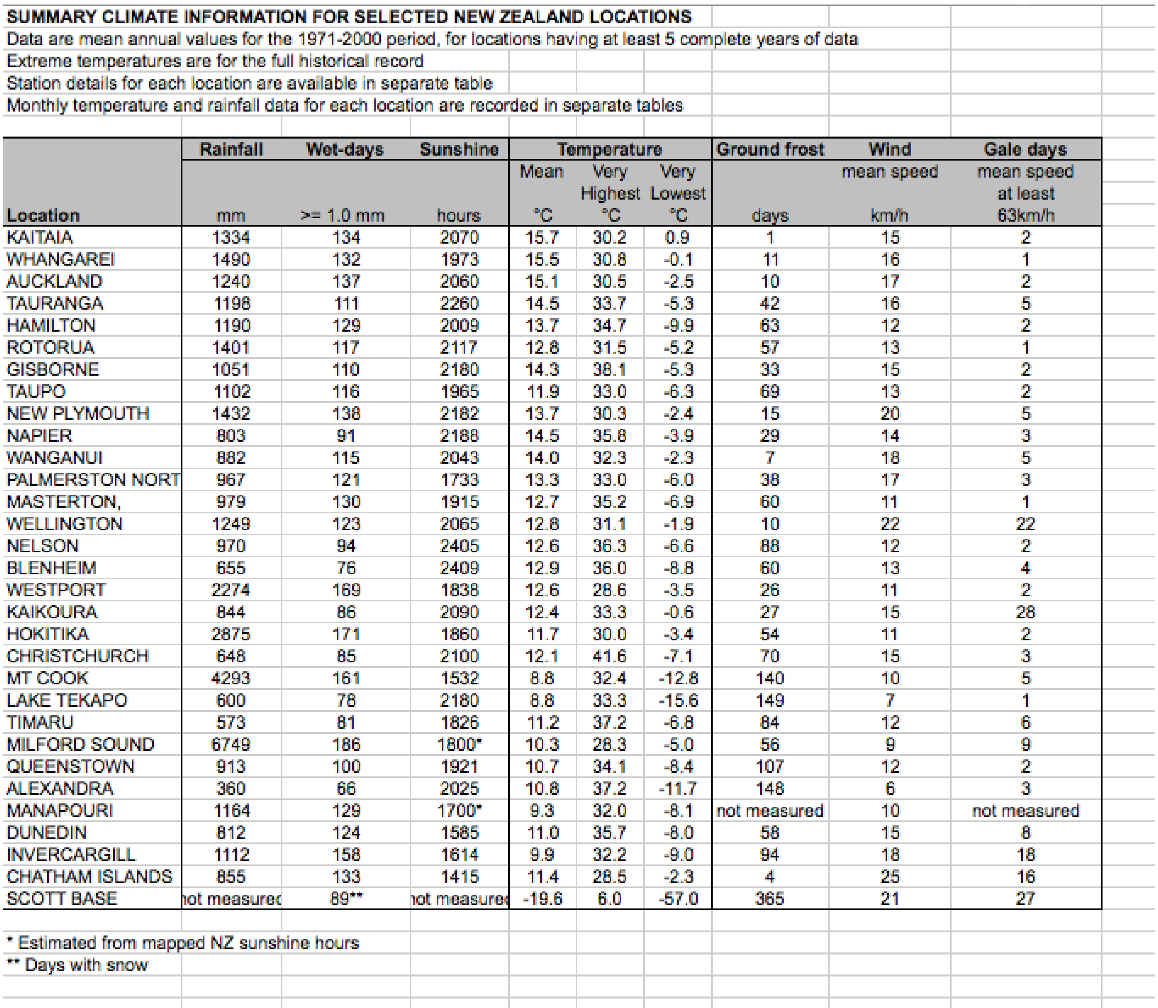
It has been observed that s is approximately equal to the half the rotor blade's length r

$$\frac{s}{r} = \frac{1}{2} \tag{B.40}$$

Substituting B.40 in B.39 gives the optimal tip speed ratio $\lambda_{optimal}$

$$\lambda_{optimal} = \frac{r\omega_{optimal}}{v} = \frac{4\pi}{n} \tag{B.41}$$

B.6 Average Wind Speed in New Zealand



SUMMARY CLIMATE INFORMATION FOR SELECTED NEW ZEALAND LOCATIONS

Data are mean annual values for the 1971-2000 period, for locations having at least 5 complete years of data

Extreme temperatures are for the full historical record

Station details for each location are available in separate table

Monthly temperature and rainfall data for each location are recorded in separate tables

	Rainfall	Wet-days	Sunshine	Temperature			Ground frost	Wind	Gale days
				Mean	Very Highest	Very Lowest		mean speed	mean speed at least
Location	mm	>= 1.0 mm	hours	°C	°C	°C	days	km/h	63km/h
KAITAIA	1334	134	2070	15.7	30.2	0.9	1	15	2
WHANGAREI	1490	132	1973	15.5	30.8	-0.1	11	16	1
AUCKLAND	1240	137	2060	15.1	30.5	-2.5	10	17	2
TAURANGA	1198	111	2260	14.5	33.7	-5.3	42	16	5
HAMILTON	1190	129	2009	13.7	34.7	-9.9	63	12	2
ROTORUA	1401	117	2117	12.8	31.5	-5.2	57	13	1
GISBORNE	1051	110	2180	14.3	38.1	-5.3	33	15	2
TAUPO	1102	116	1965	11.9	33.0	-6.3	69	13	2
NEW PLYMOUTH	1432	138	2182	13.7	30.3	-2.4	15	20	5
NAPIER	803	91	2188	14.5	35.8	-3.9	29	14	3
WANGANUI	882	115	2043	14.0	32.3	-2.3	7	18	5
PALMERSTON NORT	967	121	1733	13.3	33.0	-6.0	38	17	3
MASTERTON,	979	130	1915	12.7	35.2	-6.9	60	11	1
WELLINGTON	1249	123	2065	12.8	31.1	-1.9	10	22	22
NELSON	970	94	2405	12.6	36.3	-6.6	88	12	2
BLENHEIM	655	76	2409	12.9	36.0	-8.8	60	13	4
WESTPORT	2274	169	1838	12.6	28.6	-3.5	26	11	2
KAIKOURA	844	86	2090	12.4	33.3	-0.6	27	15	28
HOKITIKA	2875	171	1860	11.7	30.0	-3.4	54	11	2
CHRISTCHURCH	648	85	2100	12.1	41.6	-7.1	70	15	3
MT COOK	4293	161	1532	8.8	32.4	-12.8	140	10	5
LAKE TEKAPO	600	78	2180	8.8	33.3	-15.6	149	7	1
TIMARU	573	81	1826	11.2	37.2	-6.8	84	12	6
MILFORD SOUND	6749	186	1800*	10.3	28.3	-5.0	56	9	9
QUEENSTOWN	913	100	1921	10.7	34.1	-8.4	107	12	2
ALEXANDRA	360	66	2025	10.8	37.2	-11.7	148	6	3
MANAPOURI	1164	129	1700*	9.3	32.0	-8.1	not measured	10	not measured
DUNEDIN	812	124	1585	11.0	35.7	-8.0	58	15	8
INVERCARGILL	1112	158	1614	9.9	32.2	-9.0	94	18	18
CHATHAM ISLANDS	855	133	1415	11.4	28.5	-2.3	4	25	16
SCOTT BASE	not measured	89**	not measured	-19.6	6.0	-57.0	365	21	27

* Estimated from mapped NZ sunshine hours

** Days with snow

Figure B.1: Summary Climate Information for Selected New Zealand Locations [13]

B.7 Rayleigh Distribution and Image Processing Approach

My goal in this section is to use the Rayleigh distribution, and the Wind Resource Map to estimate the wind power density for each region in New Zealand.

Wind speeds are obtained from the Wind Resource Map (Figure 5.3)[48], these wind speeds are: 2,3,4,5,6,7,8 and 10 (I assumed the maximum wind speed in New Zealand is 10 m/s. According to Energy Efficiency and Conservation Authority[38], there are many regions in New Zealand with an average wind speed above 10 m/s.). Then, Rayleigh distribution is used to estimate the wind power density for each wind speed as shown in Tables B.1 and B.2.

B.7.1 Wind speeds Rayleigh Distribution Tables

Average Wind Speeds v = 2,3,4,5,6,7,8,10

Wind speeds x_i = 0,1,2,3,...,25 , i=1,2,...,26

The scale parameter c

$$\mathrm{c}=\frac{2v}{\sqrt{\pi}} \tag{B.42}$$

The Frequency of Occurence F(x_i) of wind speed x_i at a location with an average wind speed v

$$\mathrm{F}(x_i) = 2\frac{x_i}{c^2}\mathrm{e}^{-(x_i/c)^2} \tag{B.43}$$

Total WPD at a specific location with an average wind speed v

$$\text{Total WPD} = 0.5\rho\sum_{i=1}^{26} {x_i}^3\mathrm{F}(x_i) \tag{B.44}$$

ρ is the density of air, $\rho = 1.3$ kg/m^3

Average Wind Speed (m/s)	2.00		3.00		4.00		5.00	
Scale Parameter c (m/s)	2.26		3.39		4.51		5.64	
Wind Speed x (m/s)	Frequency(%)	WPD (W/m^2)	Frequency(%)	WPD (W/m^2)	Frequency(%)	WPD (W/m^2)	Frequency(%)	WPD (W/m^2)
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.00	0.48	0.31	0.16	0.10	0.09	0.06	0.06	0.04
2.00	0.36	1.86	0.25	1.28	0.16	0.84	0.11	0.58
3.00	0.20	3.53	0.24	4.19	0.19	3.32	0.14	2.49
4.00	0.07	2.82	0.17	7.19	0.18	7.45	0.15	6.32
5.00	0.01	1.18	0.10	8.00	0.14	11.69	0.14	11.64
6.00	0.00	0.28	0.05	6.35	0.10	14.13	0.12	17.08
7.00	0.00	0.04	0.02	3.79	0.06	13.83	0.09	21.03
8.00	0.00	0.00	0.01	1.74	0.03	11.30	0.07	22.40
9.00	0.00	0.00	0.00	0.63	0.02	7.85	0.04	21.03
10.00	0.00	0.00	0.00	0.18	0.01	4.71	0.03	17.65
11.00	0.00	0.00	0.00	0.04	0.00	2.46	0.02	13.36
12.00	0.00	0.00	0.00	0.01	0.00	1.13	0.01	9.19
13.00	0.00	0.00	0.00	0.00	0.00	0.45	0.00	5.77
14.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00	3.32
15.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	1.76
16.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.86
17.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.39
18.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16
19.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06
20.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02
21.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
22.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
23.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
24.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Total WPD (W/m^2)		10.03		33.52		79.45		155.18

Table B.1: Wind Power Density (WPD) in W/m^2 estimated using Rayleigh distribution for average wind speeds v = 2,3,4 and 5.

Average Wind Speed v	6.00		7.00		8.00		10.00	
Scale Parameter c (m/s)	6.77		7.90		9.03		11.28	
Wind Speed x (m/s)	Frequency(%)	WPD (W/m^2)	Frequency(%)	WPD (W/m^2)	Frequency(%)	WPD (W/m^2)	Frequency(%)	WPD (W/m^2)
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.00	0.04	0.03	0.03	0.02	0.02	0.02	0.02	0.01
2.00	0.08	0.42	0.06	0.31	0.05	0.24	0.03	0.16
3.00	0.11	1.89	0.08	1.46	0.07	1.16	0.04	0.77
4.00	0.12	5.12	0.10	4.13	0.08	3.36	0.06	2.31
5.00	0.13	10.27	0.11	8.72	0.09	7.34	0.06	5.24
6.00	0.12	16.76	0.11	15.16	0.09	13.29	0.07	9.97
7.00	0.10	23.38	0.10	22.81	0.09	20.99	0.07	16.68
8.00	0.09	28.75	0.09	30.60	0.09	29.79	0.08	25.30
9.00	0.07	31.79	0.08	37.32	0.08	38.74	0.07	35.46
10.00	0.05	32.01	0.06	41.95	0.07	46.76	0.07	46.55
11.00	0.03	29.64	0.05	43.87	0.06	52.91	0.07	57.79
12.00	0.02	25.41	0.04	42.97	0.05	56.51	0.06	68.33
13.00	0.01	20.29	0.03	39.65	0.04	57.27	0.05	77.33
14.00	0.01	15.14	0.02	34.59	0.03	55.30	0.05	84.14
15.00	0.00	10.60	0.01	28.64	0.02	51.06	0.04	88.30
16.00	0.00	6.98	0.01	22.56	0.02	45.18	0.03	89.60
17.00	0.00	4.33	0.01	16.94	0.01	38.41	0.03	88.12
18.00	0.00	2.54	0.00	12.15	0.01	31.42	0.02	84.14
19.00	0.00	1.40	0.00	8.34	0.01	24.77	0.02	78.11
20.00	0.00	0.74	0.00	5.48	0.00	18.84	0.01	70.60
21.00	0.00	0.37	0.00	3.45	0.00	13.85	0.01	62.19
22.00	0.00	0.17	0.00	2.09	0.00	9.84	0.01	53.44
23.00	0.00	0.08	0.00	1.21	0.00	6.77	0.01	44.83
24.00	0.00	0.03	0.00	0.68	0.00	4.51	0.00	36.74
25.00	0.00	0.01	0.00	0.36	0.00	2.91	0.00	29.44
Total WPD (W/m^2)		268.14		425.45		631.22		1155.53

Table B.2: Wind Power Density (WPD) in W/m^2 estimated using Rayleigh distribution for average wind speeds v = 6,7,8,9 and 10.

B.7.2 Matlab Code

Some of the problems we need to overcome is that we do not have access to original wind speed data, we only have the Wind Resource Map, and the data has been contoured into bands as shown in the legend (Figure 5.3). The following Matlab program maps each colour in the Wind Resource Map to the color closest to it in the legend, and then replace the Wind Power Density value (Table B.3) corresponding to the colour closest. After that each value is given a certain colour.

Wind speed (m/s)	2	3	4	5	6	7	8	10
WPD (W/m^2)	10.03	33.52	79.45	155.18	268.14	425.45	631.22	1155.53

Table B.3: Wind Power Density (WPD) in W/m^2 estimated using Rayleigh Distribution (see Appendix B.7.1). Wind speed values are obtained from the Wind Resource Map.

```
[A, map]=imread('WIND.gif');
b=ind2rgb(A,map); %convert indexed image to RGB

% the RGB values of  the colour white (1,1,1) are  added
and are given a a value of zero in the legend.
% the RGB values are obtained from image b using impixel in Matlab by selecting
one pixel for each colour in the legend.
R=[0.6157     0.8353    0.9686    0.9765    0.9725  0.6118  0.7922  0.9098 1 ];
G=[0.1216    0.3373    0.6667    0.9373   0.9059     0.8118    0.9098   0.9373 1 ];
B= [0.1176 0.1373 0.0549 0.6941 0  0.7294   0.9569   0.9608   1 ];

% The legend values are Wind Power Density values in Table B.3
legend = [ 1155.53 , 631.22 , 425.45, 268.14, 155.18, 79.45 , 33.52, 10.03,0];
dist = zeros(1,9);
Wind = zeros(723,500);
for i=1:723
    for j=1:500
        for k=1:9
           dist(k)=sqrt((b(i,j,1)-R(k))^2+(b(i,j,2)-G(k))^2+(b(i,j,3)-B(k))^2);
        end
        [y,k]=min(dist);
        Wind(i,j) = legend(k);
    end
end

% RGB values are obtained from pixelr (http://pixlr.com/editor/)
Red=[165 0 255 0 162   167   255 205   255];
Green=[42 102 255 128 205 100 165 198  255];
Blue=[42 204 0 0 90  67 0 115  255];
C = zeros(723,500,3, 'uint8');

for i=1:723
```

```
    for j=1:500
        for k=1:9
           if Wind(i,j)== legend(k)
            C(i,j,1) = Red(k);
            C(i,j,2) = Green(k);
            C(i,j,3) = Blue(k);
           end
        end
    end
end
b= im2uint8(b);
imtool(C);
```

Appendix C

Solar

C.1 Solar Intensity vs. Time of day

The variation of solar intensity throughout the day approximately follows a sin function. Highest intensity is at noon, where sun rays are perpendicular to the surface, and lowest at sunrise and sunset where sun rays are approximately parallel to the surface.

$$I_{\phi}=I_{\theta}\times\sin\phi \tag{C.1}$$

Where,
ϕ is the angle at which sunrays strike the ground, $0\leq\phi\leq\pi$
I_{θ} is solar intensity at a specific location with latitude θ

Equation C.1 is used to obtain Figure 6.1, as shown in the following Matlab programe. I_{θ} values for Auckland, Wellington and Invercargill are from Table 6.1.

```
x=0:pi/12:pi; % x is the angle at which sunrays strike the surface.

Auckland=800.21*sin(x);
Wellington=751.49*sin(x);
Invercargill=689.30*sin(x);
plot(x,Auckland,x,Wellington,x,Invercargill);
set(gca,'XTick',0:pi/12:pi);
set(gca,'xTickLabel',{'6sunrise','7','8','9','10','11',
'12noon','13','14','15','16','17','18sunset'});
xlabel('Time');
ylabel('solar intensity (w/m^2)');
 h=legend('Auckland','Wellington','Invercargill',3)
```

C.2 Solar Thermal

Suppose we cover all north facing roofs with 50% efficient solar thermal panels (10 m^2 of panels per person). Figure 6.3 (section 6.3) is used to produce Figure 6.4 as shown below.

The following steps are taken to estimate solar thermal potential (in kWh/d/p) for each region in New Zealand:

1. **Convert the solar radiation values from MJ/m^2/day to kWh/m^2/day.**

   The values of solar radiation in the Solar Radiation Map are in units of MJ/ m^2/day. To convert to kWh/m^2/day we multiply by $\frac{1,000,000}{3,600,000}$

   ```
   Solar =[10.50 11.50 12.50 13.50 14.50 15.50]
   % solar radiation values in MJ/m^2/day
   % Solar are the average values of solar radiation for each colour
   in the legend in the Solar Radiation Map (Figure 6.3)

   Solar =

   10.5000 11.5000 12.5000 13.5000 14.5000 15.5000

   Solar1=1000000*(1/3600000)*Solar  %Solar1 is solar radiation values
                                                           in kWh/m^2/day

   Solar1 =

   2.9167 3.1944 3.4722 3.7500 4.0278 4.3056
   ```

2. **Estimate how much could solar thermal deliver in kWh/d/p, if we install 50% efficient solar thermal panels (10 m^2 of panels per person)**

   $$\text{Solar Thermal (kWh/d/p)}=50\% \times 10 \text{ m}^2 \times \text{Power of sunshine (kWh/m}^2\text{/day)} \tag{C.2}$$

   Equation C.2 is used to estimate how much could solar thermal deliver if we install 50% efficient solar panels (10 m^2 of panels per person)

   ```
   Solar Thermal=0.50*10*Solar1

   Solar Thermal =

   14.5833 15.9722 17.3611 18.7500 20.1389 21.5278

   % these are the values of the legend in the following Matlab program.
   ```

3. **Change the solar map array from RGB colour values to Solar Thermal values, then change the Solar Thermal values to RGB colour values.**

Solar Radiation (MJ/m^2/day)	10.5	11.5	12.5	13.5	14.5	15.5
Solar Thermal (kWh/d/p)	14.5	16.0	17.4	18.7	20.1	21.5

Table C.1: Solar Thermal (kWh/d/p)

We do not have access to original solar radiation data, we only have the Solar Radiation Map and the data has been contoured into bands as shown in the legend (Figure 6.3). The following Matlab program maps each colour in the solar map to the color closest to it in the legend, and then replace the Solar Thermal value (Table C.1) corresponding to the colour closest. After that each value is given a certain colour.

```
[A, map]=imread('solarradiation1.gif');
b=ind2rgb(A,map); %convert indexed image to RGB

% the RGB values of  the colour white (1,1,1) are  added
and are given a a value of zero in the legend.
% the RGB values are obtained from b using impixel in Matlab by selecting
one pixel for each colour in the legend.

R=[0.3098 0.3922 0.0078 0.6039 0.9647 0.9686 1.000 ];
G=[0.1569 0.6353 0.6078 0.7608 0.6627 0.9059 1.000 ];
B=[0.4157 0.8235 0.3843 0.0863 0.0784 0.000 1.000 ];
% The legend values are Solar Thermal values in Table C.1
legend = [ 14.5 , 16.0 , 17.4 , 18.7 , 20.1 , 21.5, 0];
dist = zeros(1,7);
Solar = zeros(506,350);
for i=1:506
    for j=1:350
        for k=1:7
           dist(k)=sqrt((b(i,j,1)-R(k))^2+(b(i,j,2)-G(k))^2+(b(i,j,3)-B(k))^2);
         end
         [y,k]=min(dist);
         Solar(i,j) = legend(k);
   end
end

% RGB values are obtained from pixelr (http://pixlr.com/editor/)
Red=[255 255 204 6 199 179 255];
Green=[204 153 102 123 163 0 255];
Blue=[0 0 0 131 114 118 255];
C = zeros(506,350,3, 'uint8');
for i=1:506
     for j=1:350
          for k=1:7
               if Solar(i,j)== legend(k)
                C(i,j,1) = Red(k);
```

```
                C(i,j,2) = Green(k);
                C(i,j,3) = Blue(k);
               end
          end
    end
end
b= im2uint8(b);
imtool(C);
```

C.3 Solar Photovoltaic

Suppose we cover all north facing roofs with 20% efficient solar photovoltaic panels (10 m^2 of panels per person). Again, Figure 6.3 (section 6.3) is used to produce Figure 6.5 as shown below.

The following steps are taken to estimate solar photovoltaic potential (in kWh/d/p) for each region in New Zealand:

1. **Convert solar radiation values from MJ/m^2/day to kWh/m^2/day**

```
Solar =[10.50 11.50 12.50 13.50 14.50 15.50]
 % solar radiation values in MJ/m^2/day
% Solar are the average values of solar radiation for each colour
in the legend in the Solar Radiation Map (Figure 6.3)

Solar =

10.5000 11.5000 12.5000 13.5000 14.5000 15.5000

%Solar1 is solar radiation values in kWh/m^2/day

Solar1=1000000*(1/3600000)*Solar

Solar1=

2.9167 3.1944 3.4722 3.7500 4.0278 4.3056
```

2. **Estimate how much could solar photovoltaic deliver in kWh/d/p, if we install 20% efficient solar photovoltaic panels (10 m^2 of panels per person)**

$$\text{Solar Photovoltaic (kWh/d/p)}=20\% \times 10\ \text{m}^2 \times \text{Solar1 (kWh/m}^2\text{/day)} \tag{C.3}$$

Equation C.3 is used to estimate how much could solar photovoltaic deliver if we install 50% efficient solar panels (10 m^2 of panels per person)

```
Solar Photovoltaic=0.20*10*Solar1

Solar Photovoltaic =

5.8333 6.3889 6.9444 7.5000 8.0556 8.6111
```

Solar Radiation (MJ/m^2/day)	10.5	11.5	12.5	13.5	14.5	15.5
Solar Photovoltaic (kWh/d/p)	5.8	6.4	7.0	7.5	8.1	8.6

Table C.2: Photovoltaic (kWh/d/p)

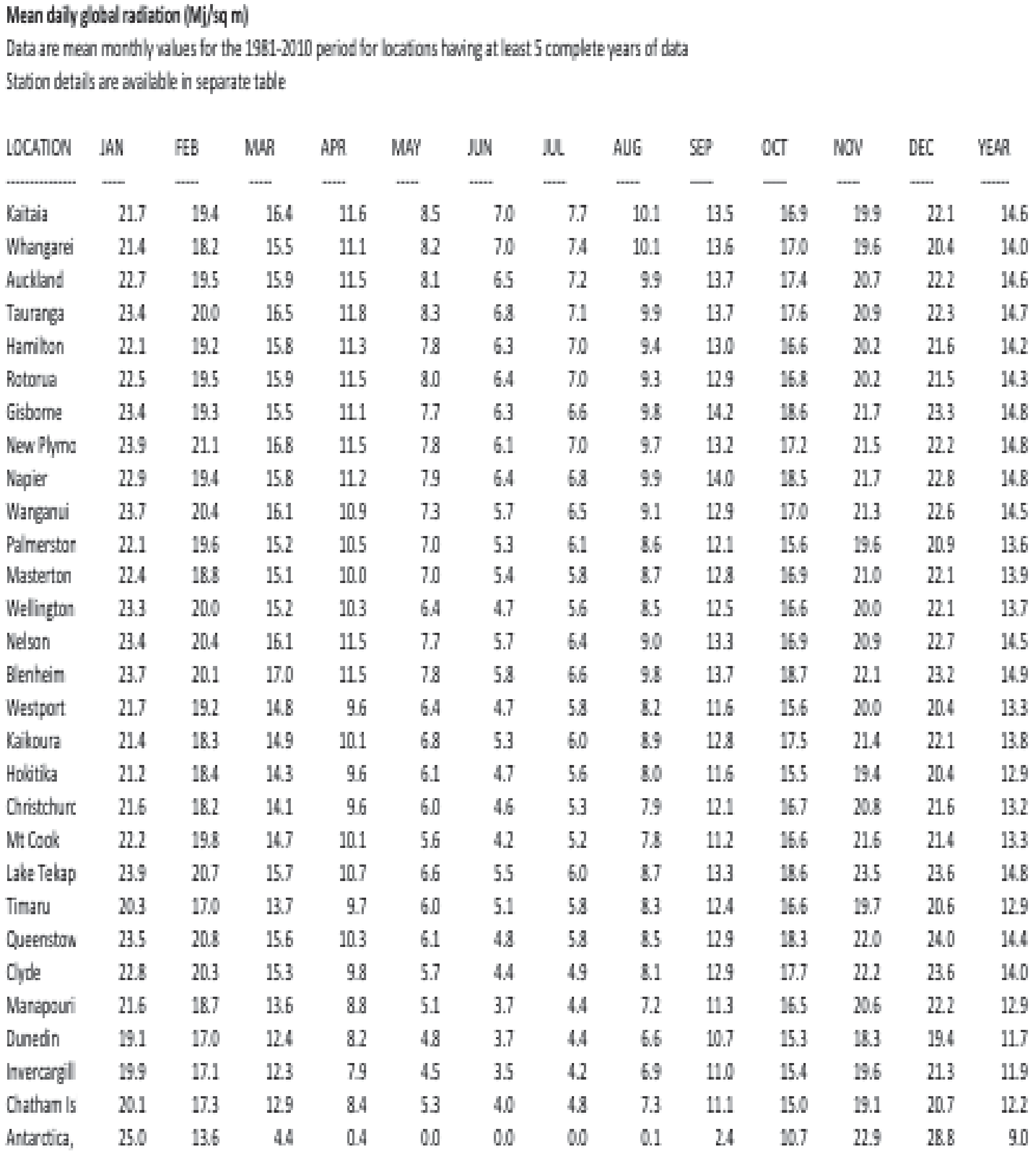
3. **Change the solar map array from RGB colour values to Solar Photovoltaic values, then change the solar values to RGB colour values.**

   The following Matlab program maps each colour in the Solar Radiation Map to the color closest to it in the legend and then replace the Solar Photovoltaic value (Table C.2) corresponding to the colour closest. After that each value is given a certain colour.

```
[A, map]=imread('solarradiation1.gif');
b=ind2rgb(A,map);

R=[0.3098    0.3922    0.0078    0.6039    0.9647    0.9686 1.000 ];
G=[0.1569    0.6353   0.6078   0.7608   0.6627    0.9059  1.000 ];
B=[0.4157 0.8235  0.3843  0.0863 0.0784 0.000  1.000  ];

 % Legend values are solar photovoltaic values (in kWh/d/p) in Table C.2
legend = [ 5.8   ,  6.4 ,   7.0 ,  7.5 ,  8.1 ,  8.6, 0];
dist = zeros(1,7);
Solar = zeros(506,350);
for i=1:506
    for j=1:350
        for k=1:7
           dist(k)=sqrt((b(i,j,1)-R(k))^2+(b(i,j,2)-G(k))^2+(b(i,j,3)-B(k))^2);
        end
        [y,k]=min(dist);
        Solar(i,j) = legend(k);
    end
end
Red=[255 102 0 0 199 179 255];
Green=[255 204 102 128 163 0 255];
Blue=[0 255 204 0 114 118 255];
C = zeros(350,506,3, 'uint8');

for i=1:506
    for j=1:350
        for k=1:7
```

```
                if Solar(i,j)== legend(k)
                 C(i,j,1) = Red(k);
                 C(i,j,2) = Green(k);
                 C(i,j,3) = Blue(k);
                end
             end
         end
     end
     b= im2uint8(b);
     imtool(C);
```

C.4 Mean Daily Global Radiation (MJ/m^2)



Mean daily global radiation (Mj/sq m)

Data are mean monthly values for the 1981-2010 period for locations having at least 5 complete years of data

Station details are available in separate table

LOCATION	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	YEAR
Kaitaia	21.7	19.4	16.4	11.6	8.5	7.0	7.7	10.1	13.5	16.9	19.9	22.1	14.6
Whangarei	21.4	18.2	15.5	11.1	8.2	7.0	7.4	10.1	13.6	17.0	19.6	20.4	14.0
Auckland	22.7	19.5	15.9	11.5	8.1	6.5	7.2	9.9	13.7	17.4	20.7	22.2	14.6
Tauranga	23.4	20.0	16.5	11.8	8.3	6.8	7.1	9.9	13.7	17.6	20.9	22.3	14.7
Hamilton	22.1	19.2	15.8	11.3	7.8	6.3	7.0	9.4	13.0	16.6	20.2	21.6	14.2
Rotorua	22.5	19.5	15.9	11.5	8.0	6.4	7.0	9.3	12.9	16.8	20.2	21.5	14.3
Gisborne	23.4	19.3	15.5	11.1	7.7	6.3	6.6	9.8	14.2	18.6	21.7	23.3	14.8
New Plymo	23.9	21.1	16.8	11.5	7.8	6.1	7.0	9.7	13.2	17.2	21.5	22.2	14.8
Napier	22.9	19.4	15.8	11.2	7.9	6.4	6.8	9.9	14.0	18.5	21.7	22.8	14.8
Wanganui	23.7	20.4	16.1	10.9	7.3	5.7	6.5	9.1	12.9	17.0	21.3	22.6	14.5
Palmerston	22.1	19.6	15.2	10.5	7.0	5.3	6.1	8.6	12.1	15.6	19.6	20.9	13.6
Masterton	22.4	18.8	15.1	10.0	7.0	5.4	5.8	8.7	12.8	16.9	21.0	22.1	13.9
Wellington	23.3	20.0	15.2	10.3	6.4	4.7	5.6	8.5	12.5	16.6	20.0	22.1	13.7
Nelson	23.4	20.4	16.1	11.5	7.7	5.7	6.4	9.0	13.3	16.9	20.9	22.7	14.5
Blenheim	23.7	20.1	17.0	11.5	7.8	5.8	6.6	9.8	13.7	18.7	22.1	23.2	14.9
Westport	21.7	19.2	14.8	9.6	6.4	4.7	5.8	8.2	11.6	15.6	20.0	20.4	13.3
Kaikoura	21.4	18.3	14.9	10.1	6.8	5.3	6.0	8.9	12.8	17.5	21.4	22.1	13.8
Hokitika	21.2	18.4	14.3	9.6	6.1	4.7	5.6	8.0	11.6	15.5	19.4	20.4	12.9
Christchurc	21.6	18.2	14.1	9.6	6.0	4.6	5.3	7.9	12.1	16.7	20.8	21.6	13.2
Mt Cook	22.2	19.8	14.7	10.1	5.6	4.2	5.2	7.8	11.2	16.6	21.6	21.4	13.3
Lake Tekap	23.9	20.7	15.7	10.7	6.6	5.5	6.0	8.7	13.3	18.6	23.5	23.6	14.8
Timaru	20.3	17.0	13.7	9.7	6.0	5.1	5.8	8.3	12.4	16.6	19.7	20.6	12.9
Queenstow	23.5	20.8	15.6	10.3	6.1	4.8	5.8	8.5	12.9	18.3	22.0	24.0	14.4
Clyde	22.8	20.3	15.3	9.8	5.7	4.4	4.9	8.1	12.9	17.7	22.2	23.6	14.0
Manapouri	21.6	18.7	13.6	8.8	5.1	3.7	4.4	7.2	11.3	16.5	20.6	22.2	12.9
Dunedin	19.1	17.0	12.4	8.2	4.8	3.7	4.4	6.6	10.7	15.3	18.3	19.4	11.7
Invercargill	19.9	17.1	12.3	7.9	4.5	3.5	4.2	6.9	11.0	15.4	19.6	21.3	11.9
Chatham Is	20.1	17.3	12.9	8.4	5.3	4.0	4.8	7.3	11.1	15.0	19.1	20.7	12.2
Antarctica,	25.0	13.6	4.4	0.4	0.0	0.0	0.0	0.1	2.4	10.7	22.9	28.8	9.0

Figure C.1: Mean Daily Global Radiation (MJ/m^2)[14]

Appendix D

Waves

New Zealand can be divided into four regions based on the range of the long-term mean for significant wave height (H_{av}), the average wave period between wave crests (T_{av}) and wind direction as shown in the table below. [39]

Region	H_{av}	T_{av}	Wind Direction
South Facing Coasts (Fiordland to Catlins)- South Island	3-4	10-12	SW-W
Western New Zealand Coasts	2-3	6-8	SW-W
Eastern New Zealand up to East Cape	1.5-3	6-9	S
North Eastern North Island	1-2	5-7	N-E

Table D.1: New Zealand's four regions based on the range of the long-term mean for significant wave height (H_{av}) , the average wave period between wave crests (T_{av}) and wind direction.[39]

Significant wave height H_{av} can be defined as the average wave height (trough to crest) of the one-third the largest waves.[39]