Simplified Modelling of Pollutant Transport in Naturally-layered Aquifers

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Abstract

Chemical species such as tracers or dissolved pollutants flow along with the slow-moving water as it makes its way through the complex porous structure of the aquifer; during this process they are dispersed in different directions. The rate of dispersion depends on the geometric characteristics of the porous structure and speed of the fluid.

Generally, groundwater systems have layered structures determined by different events in the geological processes that formed them. The layers in a system have different physical properties, and their thicknesses are not uniform. This naturally layered structure is used here to advantage by discretizing them into almost horizontal layers, where each may have different geometrical characteristics such as thickness permeability, dispersivitiy, porosity, etc. The system of advection-dispersion equations that model the fluid and species transport then have coefficients that depend mainly on depth, but with a layer composition that may change with horizontal distance.

The mean dynamic pressure (or mean hydraulic head) may be assumed constant vertically at each horizontal point if it is not in the vicinity of a well or where there is very small vertical flow. In the vicinity of recharge or pumping wells, the mean dynamic pressures or hydraulic heads for each sub-layer of the aquifer may be allowed to have different values for each different sub-layer. Steady-state fluid flow is considered in this thesis in both confined and phreatic (unconfined) aquifers.

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Contents

Nomenclature

$\bar{c}$	average concentration of pollutant in the fluid averaged over sub-layer thickness [kg m^{-3}]
$\mathbf{D}$	three-dimensional tensor coefficient of mechanical dispersion of a dissolved pollutant while it flows in the porous media [m^2 s^{-1}]
D	scalar coefficient of mechanical dispersion of a dissolved pollutant while it flows in the porous media [m^2 s^{-1}]
F	steady rate of fluid recharge in or pumping out of a sub-layer of the aquifer (F has dimensions of [m s^{-1}] when thickness varies in one horizontal direction and of [m^2 s^{-1}] when thickness varies in both horizontal directions)
f	mass flux function in a sub-layer of the aquifer or fluid recharge flux function in or pumping flux function out of a sub-layer of the aquifer (if f is used for mass flux of a chemical, it has dimensions of [kg m^{-3} s^{-1}] and if it is used for fluid flux it has dimensions of [s^{-1}])
g	gravitational acceleration [m s^{-2}]
H	hydraulic head [m]
$\bar{H}$	mean hydraulic head averaged over the sub-layer thickness [m]
h	thickness of the sub-layer or the aquifer [m]
$\mathbf{K}$	permeability tensor of the porous media [m^2]
K	isotropic permeability of the porous media [m^2]

$\bar{K}$ mean (weighted) isotropic permeability of the aquifer averaged over the aquifer thickness, i.e., $\bar{K} = \left(\sum h_i K_i\right)/h$ [m^2]

M steady mass flux of fluid in or out of a sub-layer of the aquifer [kg m^{-3} s^{-1}]

P dynamic pressure of the fluid [kg m^{-1} s^{-2}]

$\bar{P}$ mean dynamic pressure of the fluid averaged over the sub-layer thickness [kg m^{-1} s^{-2}]

p absolute pressure of the fluid [kg m^{-1} s^{-2}]

Q instantaneous flow of mass in a sub-layer of the aquifer (Q has dimensions of [kg m^{-2}] when thickness varies in one horizontal direction and of [kg m^{-1}] when thickness varies in both horizontal directions)

$\mathbf{q}$ total horizontal volume flux $\mathbf{q} = (q_x, q_y)$ through the whole aquifer [m^2 s^{-1}]

q steady mass or volume flux of fluid or a chemical in a sub-layer of the aquifer (if q is used for mass flux of a chemical, it has dimensions of [kg m^{-2} s^{-1}] when thickness varies in one horizontal direction and [kg m^{-1} s^{-1}] when thickness varies in both horizontal directions and if q is used for the fluid flux, it has dimensions of [m s^{-1}] when thickness varies in one horizontal direction and [m^2 s^{-1}] when thickness varies in both horizontal directions)

r the fluid flux normal to the layer interface from the lower sub-layer to the upper sub-layer [m s^{-1}]

$\mathbf{u}$ Darcy velocity vector of fluid $\mathbf{u} = (u, v, w)$ in the porous media [m s^{-1}]

$\bar{\mathbf{u}}$ average two-dimensional horizontal Darcy velocity vector of fluid $\bar{\mathbf{u}} = (\bar{u}, \bar{v})$ in the porous media averaged over sub-layer thickness [m s^{-1}]

z height above datum [m]

Greek Symbols

α dispersivity of the porous medium [m]

δ	Dirac delta function
κ	hydraulic conductivity [m s^{-1}]
μ	dynamic viscosity of the fluid [kg m^{-1} s^{-1}]
ρ	density of the fluid [kg m^{-3}]
τ	interlayer dispersive transfer coefficient [m s^{-1}]
Φ	the function Φ_i for the ith interface level between the sub-layers (starting from bottom) is defined by $\Phi_i(x, y, z) = z - z_i(x, y)$, where z_i is the interface level between the $i - 1$th and the ith sub-layers [m]
ϕ	porosity of the porous media [-]
ψ	the two-dimensional stream function [m^3 s^{-1}]

Subscripts

atm	atmospheric
d	downstream
I	injection
L	longitudinal
P	pollutant
R	remediating agent
ret	retention
s	curve along phreatic surface
st	stream
T	transverse
t	top
tot	total
u	upstream
V	volume
W	withdrawal
x, y, z	principal directions of Cartesian coordinate system