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**Constructing Decimal Concepts in an
Inquiry Classroom**

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Abstract

This study examines the construction of decimal concepts of primary aged students in the classroom. It builds on previous work which has promoted the use of percentages as a means for rational number thinking and for the enhancement of such thinking through multiple modes of representation. In this study percentages provide a foundation for rational number understanding as represented through the decimal system.

The study is set within an inquiry classroom. In this classroom the pedagogical approach maps out an alternative to customary practice by shifting the traditional teacher-student relationship to one of partnership in knowledge construction. In this classroom both student engagement with well-designed learning activities, and mathematical discussion and debate are all deemed highly important to the production of decimal understandings.

The investigation revealed that students had a wealth of informal rational number knowledge. This informal knowledge created a useful context and springboard for the development of new conceptual understandings of decimal fractions. That development was not immediate—it traced out a lengthy, unpredictable and recursive path and required students to reflect on their thinking and allowed for subtle teacher and peer reconstruction of students' misconceptions. From those findings recommendations are made for a productive approach to the teaching of decimals in primary school classrooms.

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