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Optimal harvesting strategies  
for fisheries:  
A differential equations approach

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## **Abstract**

The purpose of fisheries management is to achieve a sustainable development of the activity, so that future generations can also benefit from the resource. However, the optimal harvesting strategy usually maximizes an economically important objective function formed by the harvester which can lead to the extinction of the resource population. Therefore, sustainability has been far more difficult to achieve than is commonly thought; fish populations are becoming increasingly limited and catches are declining due to overexploitation.

The aim of this research is to determine an optimal harvesting strategy which fulfills the economic objective of the harvester while maintaining the population density over a pre-specified minimum viable level throughout the harvest. We develop and investigate the harvesting model in both deterministic and stochastic settings. We first employ the Expected Net Present Value approach and determine the optimal harvesting policy using various optimization techniques including optimal control theory and dynamic programming. Next we use real options theory, model fish harvesting as a real option, and compute the value of the harvesting opportunity which also yields the optimal harvesting strategy. We further extend the stochastic problem to include price elasticity of demand and present results for different values of the coefficient of elasticity.

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