Interactions Destroy Dynamical Localization with Strong & Weak Chaos

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in collaboration with

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Condensed Matter Division

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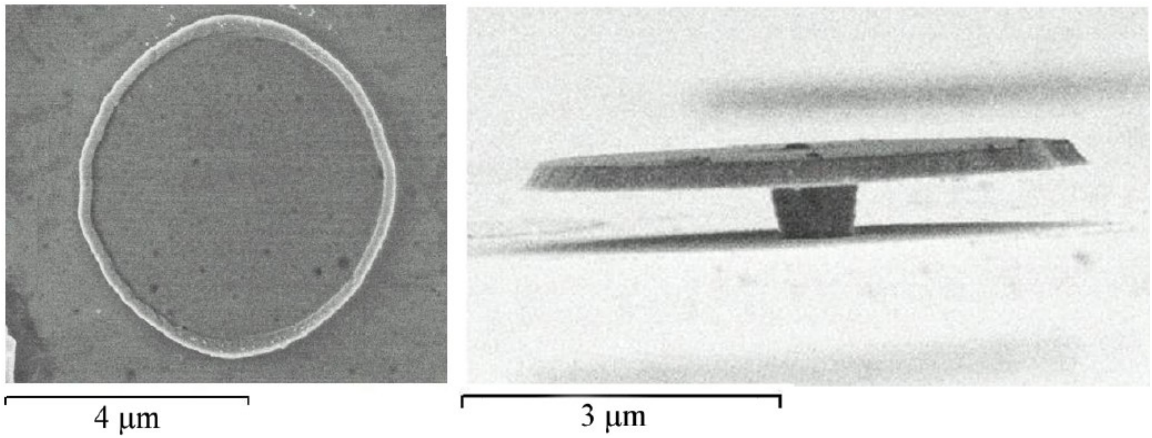
9:45-10:15, Dienstag, 20.09.2011

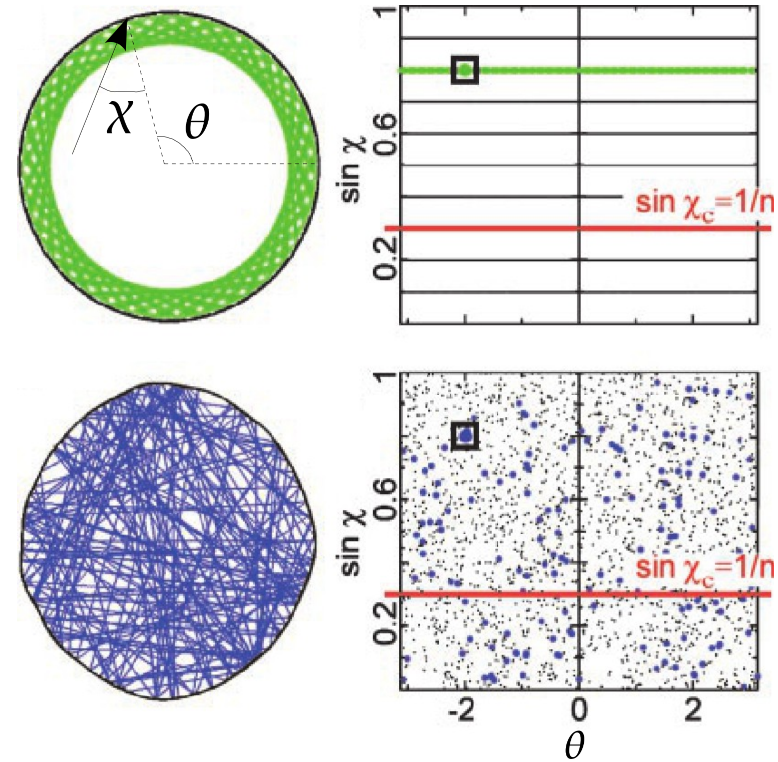
ArXiv: 1108.2217, submitted *Europhys. Lett.*

Experimental Motivation

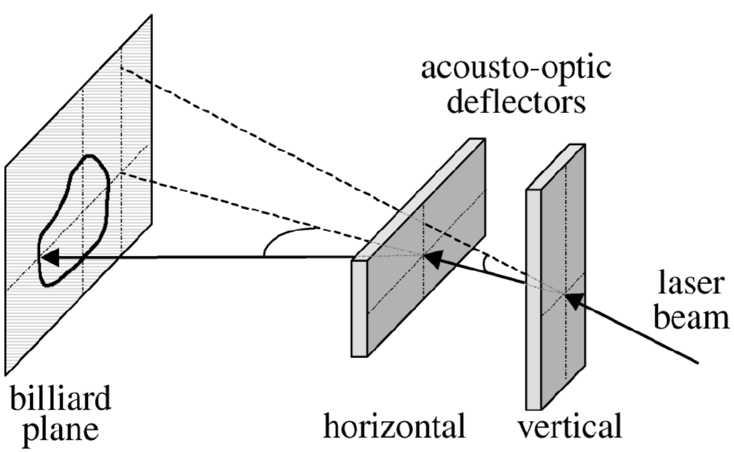
- Microcavity Lasing

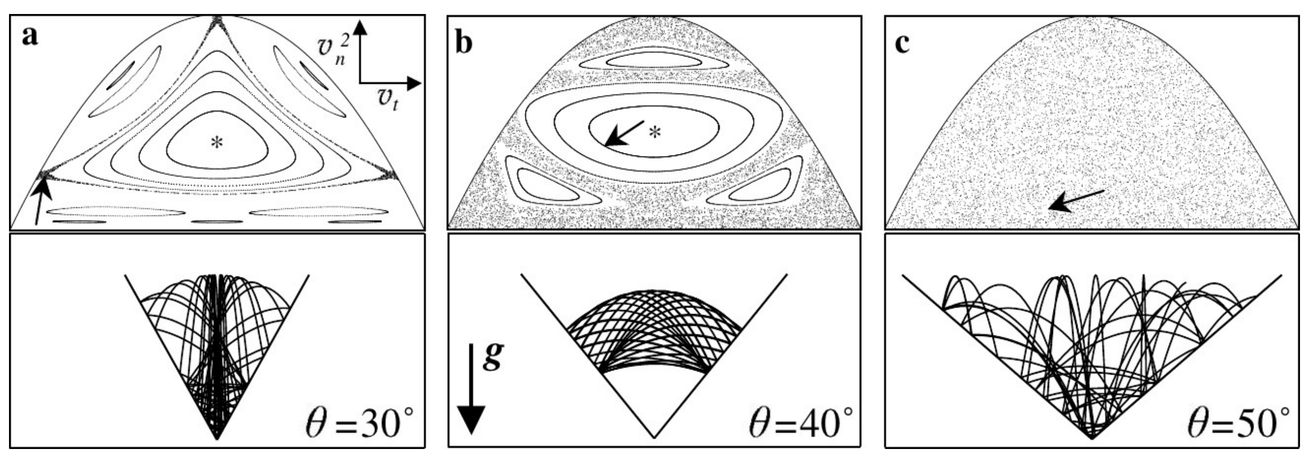
Podolskiy et al., Proc.Nat.Acad.Sci. ***101****, 10498 (2004)*





- Ultracold Atomic Gases (BEC)



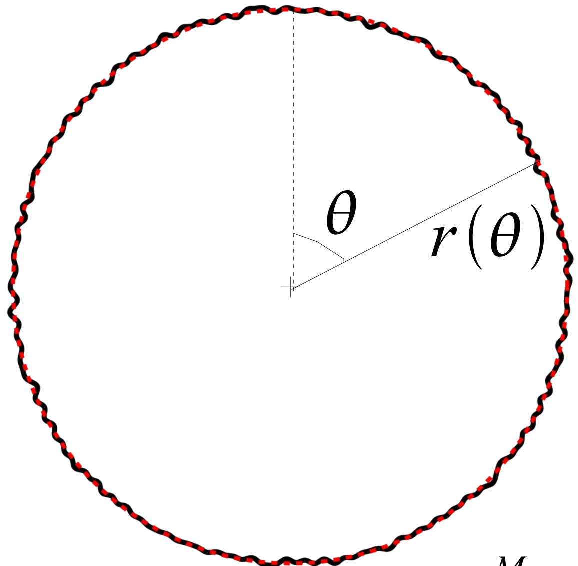


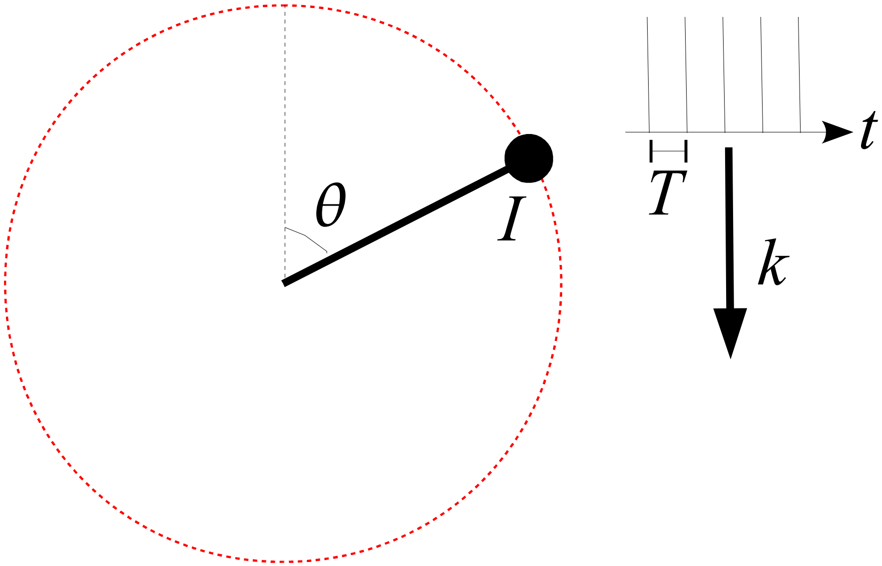
Raizen et al., Phys.Rev.Lett. ***86****, 1514 (2001)*

A Brief Synopsis

- An Introduction
  - ➢ Billiards & Kicked Rotors
  - ➢ The Quantum Case: Dynamical Localization
- The Nonlinear Quantum Kicked Rotor
  - ➢ Spreading: Measures of Interest
  - ➢ The Incoherent Heating Conjecture
  - ➢ Defining a Parameter Space
  - ➢ Numerical Results
- Outlook

Billiards & Kicked Rotors





$$r(\theta)=r_0+\sigma_r,\ \sigma_r=r_0\Re\sum_{m=2}^{M}\gamma_m e^{im\theta}$$

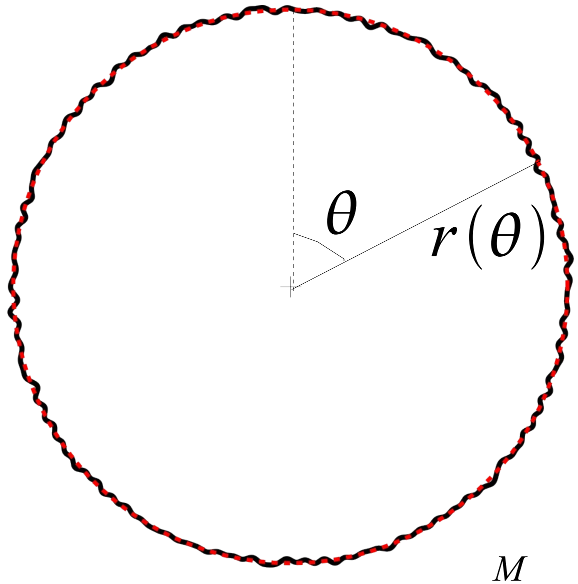
$$\kappa(\theta)=\frac{1}{r_0}\frac{dr}{d\theta},\ \tilde{\kappa}=\langle\kappa(\theta)\rangle_\theta$$

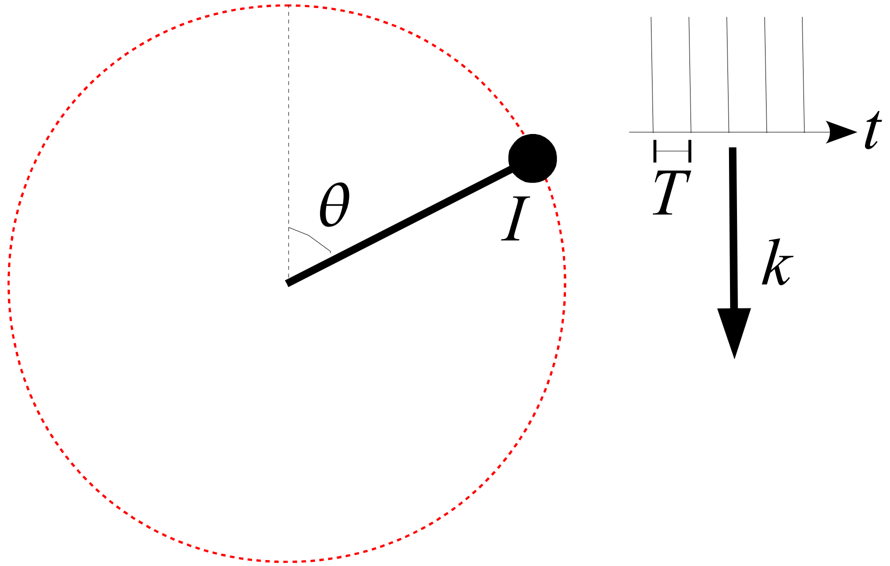
$$V(t)=k\cos(\theta)\sum\delta(t-mT)$$

Frahm & Shepelyansky, Phys.Rev.Lett. ***78****, 1440 (1997);*
Sirko et al, Phys.Lett.A ***266****, 331 (2000)*

Casati et al., Lect. Notes Phys. ***93****, 334 (1979)*

Billiards & Kicked Rotors





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$$\kappa(\theta)=\frac{1}{r_0}\frac{dr}{d\theta},\ \tilde{\kappa}=\langle\kappa(\theta)\rangle_\theta$$

$$l_{n+1}=l_n+2\sqrt{l_{\max}^2-l_r^2}\cdot\kappa(\theta_n)$$

$$\theta_{n+1}=\theta_n+\pi+\operatorname{asin}(l_{n+1}/l_{\max})$$

$$V(t)=k\cos(\theta)\sum\delta(t-mT)$$

$$p_{n+1}=p_n+k\sin(\theta_n)$$

$$\theta_{n+1}=\theta_n+p_{n+1}$$

Frahm & Shepelyansky, Phys.Rev.Lett. ***78****, 1440 (1997);*
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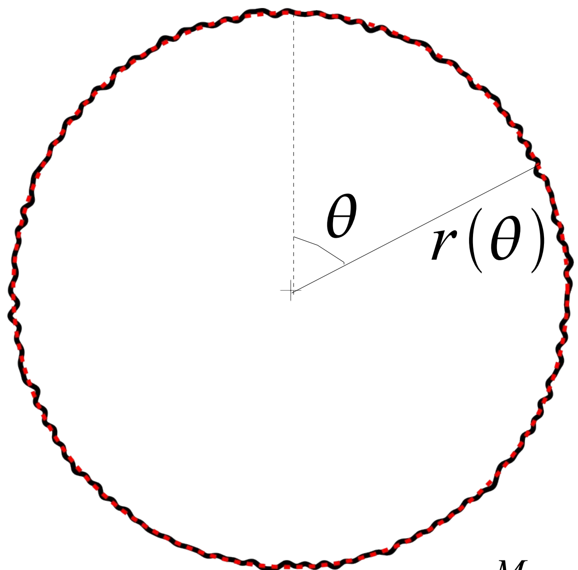
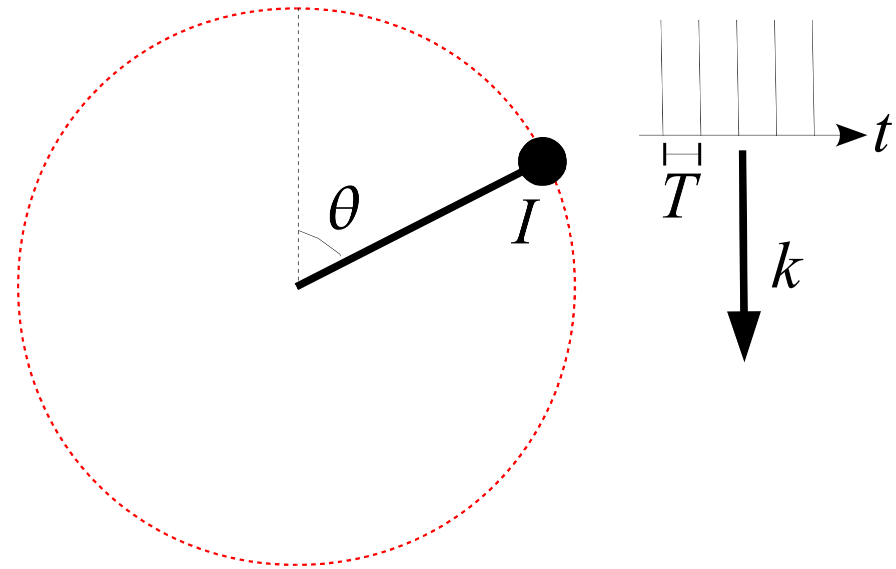
Billiards & Kicked Rotors





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$$\sigma_r(\theta)\propto\cos(\theta)$$

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Casati et al., Lect. Notes Phys. ***93****, 334 (1979)*

The Quantum Case: Dynamical Localization

$$i\,\partial_t \psi = -\frac{2\pi}{\tilde{T}}\partial_\theta^2 \psi + k\cos(\theta)\,\psi \sum_m \delta(t - mT)$$

$$\psi(\theta, t+1) = \hat{U}\cdot\psi(\theta, t) = \hat{B}\cdot\hat{G}\cdot\psi(\theta, t)$$

Free Rotation:

$$\hat{G}\left(\frac{\tau}{2}, \theta\right) = \exp\left(-i\frac{\tau}{2}\partial_\theta^2\right), \quad \tau = 4\pi T/\tilde{T}$$

Single Kick:

$$\hat{B}(k, \theta) = \exp(-i\,k\cos\theta)$$

$$\psi(\theta, t) = \sum_n A_n(t)\, e^{i n \theta}$$

$$\downarrow$$

$$A_n(t+1) = \sum_m (-i)^{n-m} J_{n-m}(k)\, A_m(t) \exp[-i\tau m^2/2]$$

Floquet theory: $\psi(\theta, t) = e^{-i\chi t}\phi_\chi(\theta, t)$; $\phi_\chi(\theta, t+1) = \phi_\chi(\theta, t)$

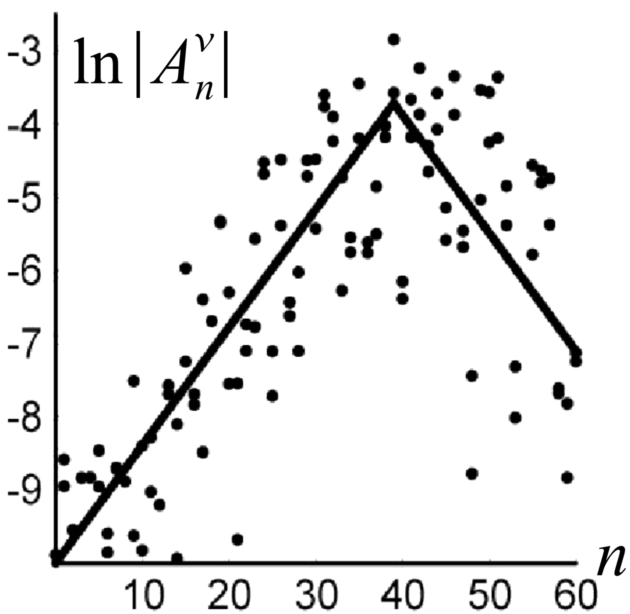
Eigenproblem! $\rightarrow$ $\lambda_\nu A_n^\nu = \sum_m (-i)^{n-m} J_{n-m}(k) \exp[-i\tau m^2/2]\cdot A_m^\nu$

Fishman et al., Phys.Rev.Lett. **49**, 509 (1982)

The Quantum Case: Dynamical Localization

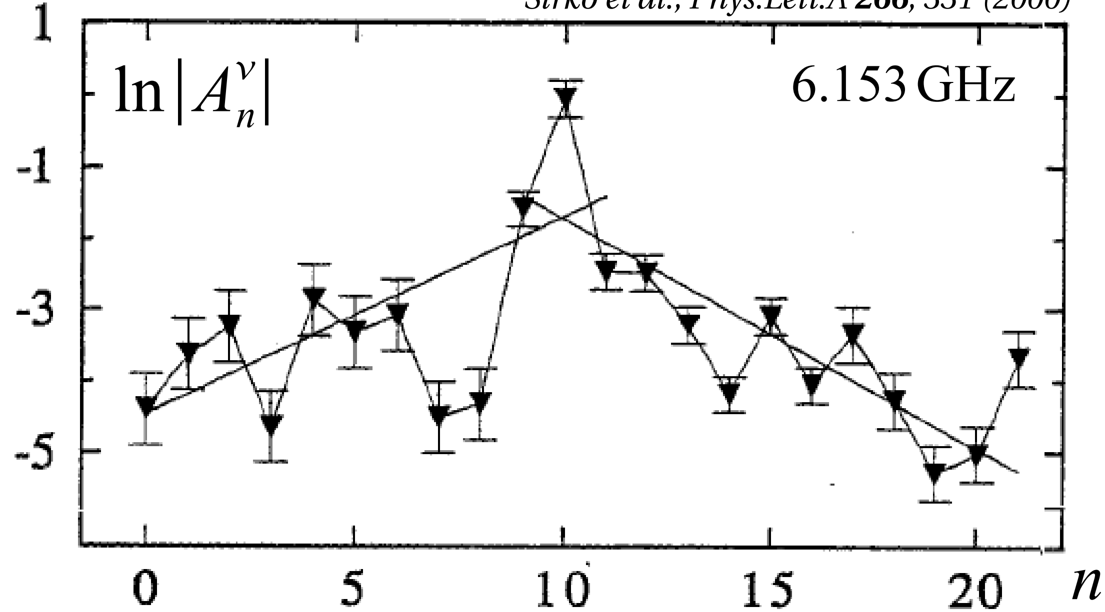
$$\lambda_\nu A_n^\nu = \sum_m (-i)^{n-m} J_{n-m}(k) \exp[-i\tau m^2/2] \cdot A_m^\nu$$

$$\ln|A_n^\nu| \sim \frac{|n-n_0|}{\xi_\nu}$$



Feng et al., Opt. Express ***13****, 5641 (2005)*

Sirko et al., Phys.Lett.A ***266****, 331 (2000)*



$$\lambda_\nu = \exp(i\chi_\nu) \;\rightarrow\; \Delta = 2\pi$$

$$\xi \sim V^{-1} = \langle \sum_n |A_n^\nu|^4 \rangle_\nu \;\rightarrow\; d = \Delta / V$$

The Nonlinear Kicked Rotor

$$i\,\partial_t \psi = -\frac{2\pi}{\tilde{T}}\partial_\theta^2 \psi + k\cos(\theta)\,\psi \sum_m \delta(t - mT) + \tilde{\beta}|\psi|^2\psi$$

$$\Big\downarrow \; \beta = \tilde{\beta}\,T/2\pi\hbar^2$$

$$A_n(t+1) = \sum_m (-i)^{n-m} J_{n-m}(k)\,A_m(t)\exp\left[\frac{-i\tau m^2}{2} + i\beta|A_m|^2\right]$$

Shepelyansky, Phys.Rev.Lett. ***70****, 1787 (1993)*

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$$A_n(t) = \sum_\nu \phi_\nu(t) A_n^\nu \qquad \exp(i\beta|A_m|^2) \approx i\beta|A_m|^2$$

$$\boxed{\phi_\nu(t+1) = \lambda_\nu \phi_\nu(t) + \beta \sum_{\mu_1,\mu_2,\mu_3} I_{\nu,\mu_1,\mu_2,\mu_3} \phi_{\mu_1} \phi_{\mu_2}^* \phi_{\mu_3}}$$

$$I_{\nu,\mu_1,\mu_2,\mu_3} \propto \sum_{n,m} A_n^{\nu *} A_m^{\mu_1} A_m^{\mu_2 *} A_m^{\mu_3} \qquad \boxed{\beta\langle|A_m|^2\rangle_m \sim \beta\langle|\phi_\nu|^2\rangle_\nu = \beta\rho}$$

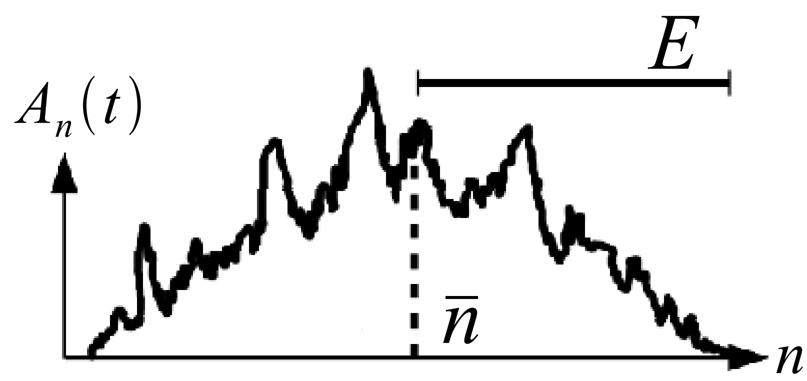
Shepelyansky, Phys.Rev.Lett. ***70****, 1787 (1993)*

Spreading: Measures of Interest

Moments:

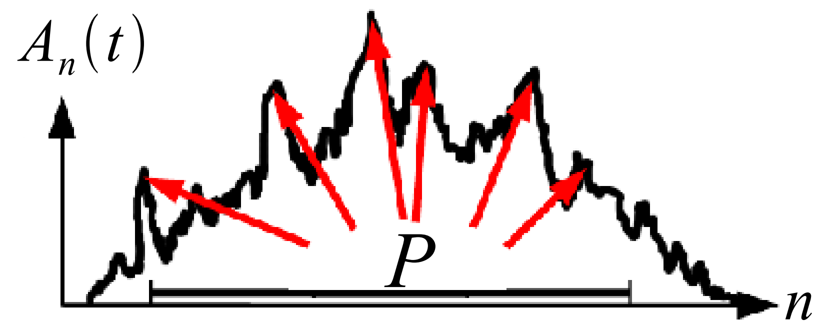
$$\bar{n}=\sum_n n|A_n(t)|^2$$

$$E=\sum_n \frac{1}{2}(n-\bar{n})^2|A_n(t)|^2$$



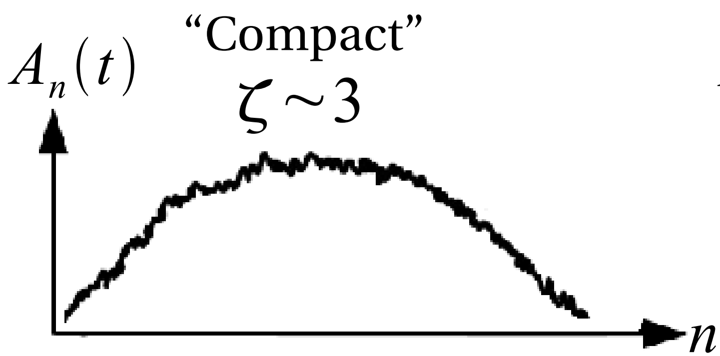
Participation:

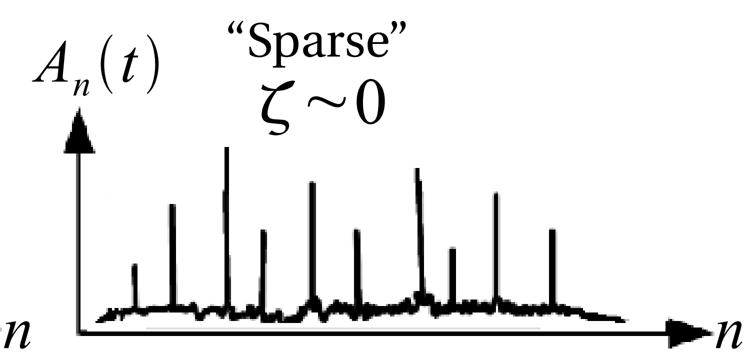
$$P=\left[\sum_n |A_n(t)|^4\right]^{-1}$$



Compactness Index:

$$\zeta=P^2/2E$$





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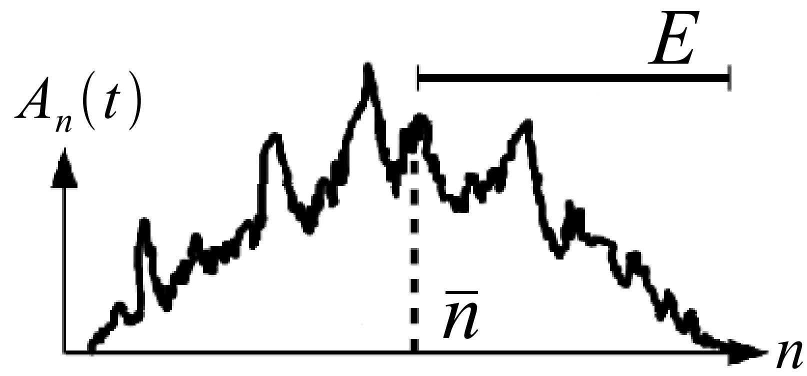
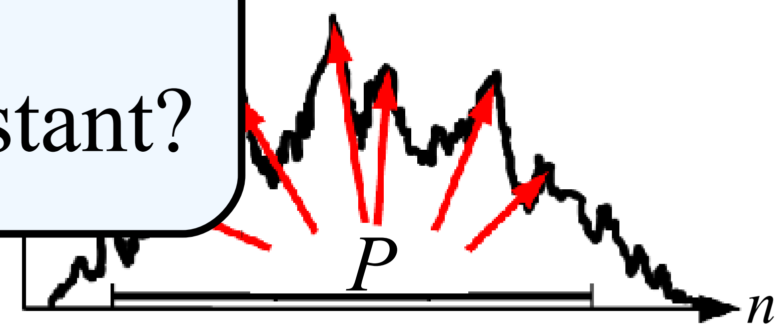


$$\lim_{t\to\infty} E\sim t^{\alpha}, \quad \alpha=?$$

$$\lim_{t\to\infty} \zeta = \text{constant?}$$

Participation:

$$P=\Big[\sum_n |A$$



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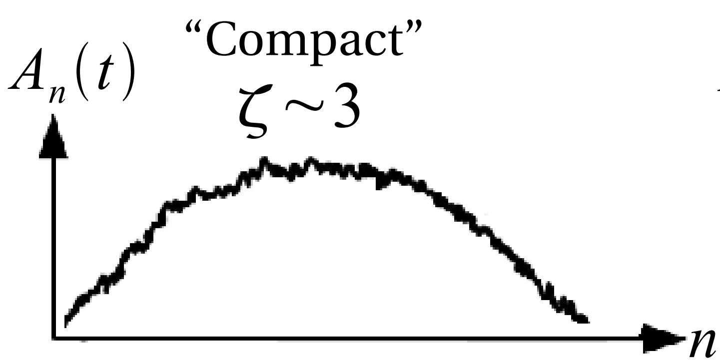
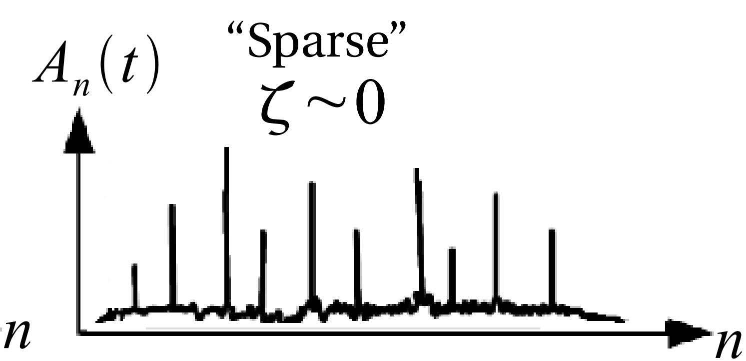
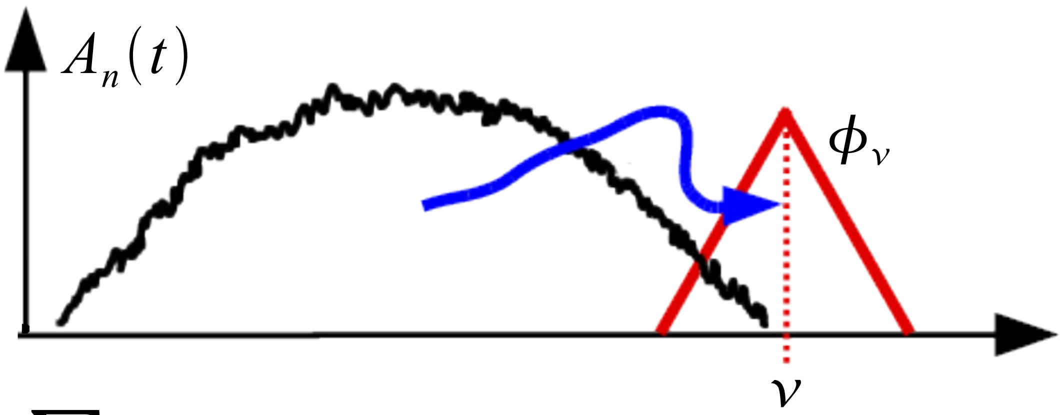




The Incoherent Heating Conjecture

Chaos = Nonintegrability + Resonance

"Resonance" = Incoherent Heating of an Exterior Eigenmode



$$A_n(t) = \sum_{\mu} \phi_{\mu}(t)\, A_n^{\nu}$$

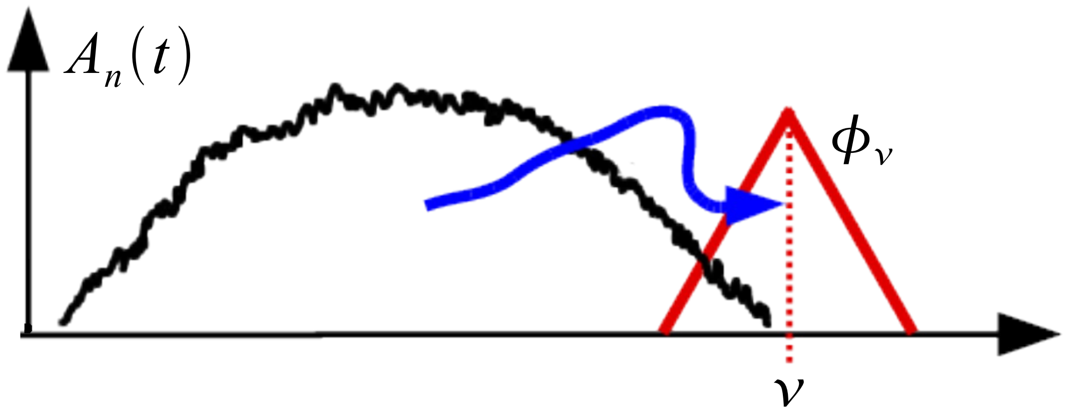
$$\partial_t \phi_\nu = \lambda_\nu \phi_\nu + \beta \sum_{\mu_1,\mu_2,\mu_3} I_{\nu,\mu_1,\mu_2,\mu_3} \phi_{\mu_1} \phi_{\mu_2}^{*} \phi_{\mu_3}, \quad I_{\nu,\mu_1,\mu_2,\mu_3} \propto \sum_{n,m} A_n^{\nu *} A_m^{\mu_1} A_m^{\mu_2 *} A_m^{\mu_3}$$

Minimum Resonance Condition: $R_\nu = \min_{\{\mu_j\}} \left| \frac{\lambda_\nu - \lambda_{\mu_1} + \lambda_{\mu_2} - \lambda_{\mu_3}}{I_{\nu,\mu_1,\mu_2,\mu_3}} \right|$

Probability of a Resonance: $P(\beta\rho) = \int_0^{\beta\rho} W(R)\, dR \sim 1 - \exp(\beta\rho / d)$

*Krimer & Flach, Phys. Rev. E **82,** 046221 (2010)*

The Incoherent Heating Conjecture



$$\partial_t \phi_\nu = \lambda_\nu \phi_\nu + \beta \sum_{\mu_1, \mu_2, \mu_3} I_{\nu, \mu_1, \mu_2, \mu_3} \phi_{\mu_1} \phi^*_{\mu_2} \phi_{\mu_3}, \quad P(\beta\rho) \sim 1 - \exp(\beta\rho/d)$$

Conjecture

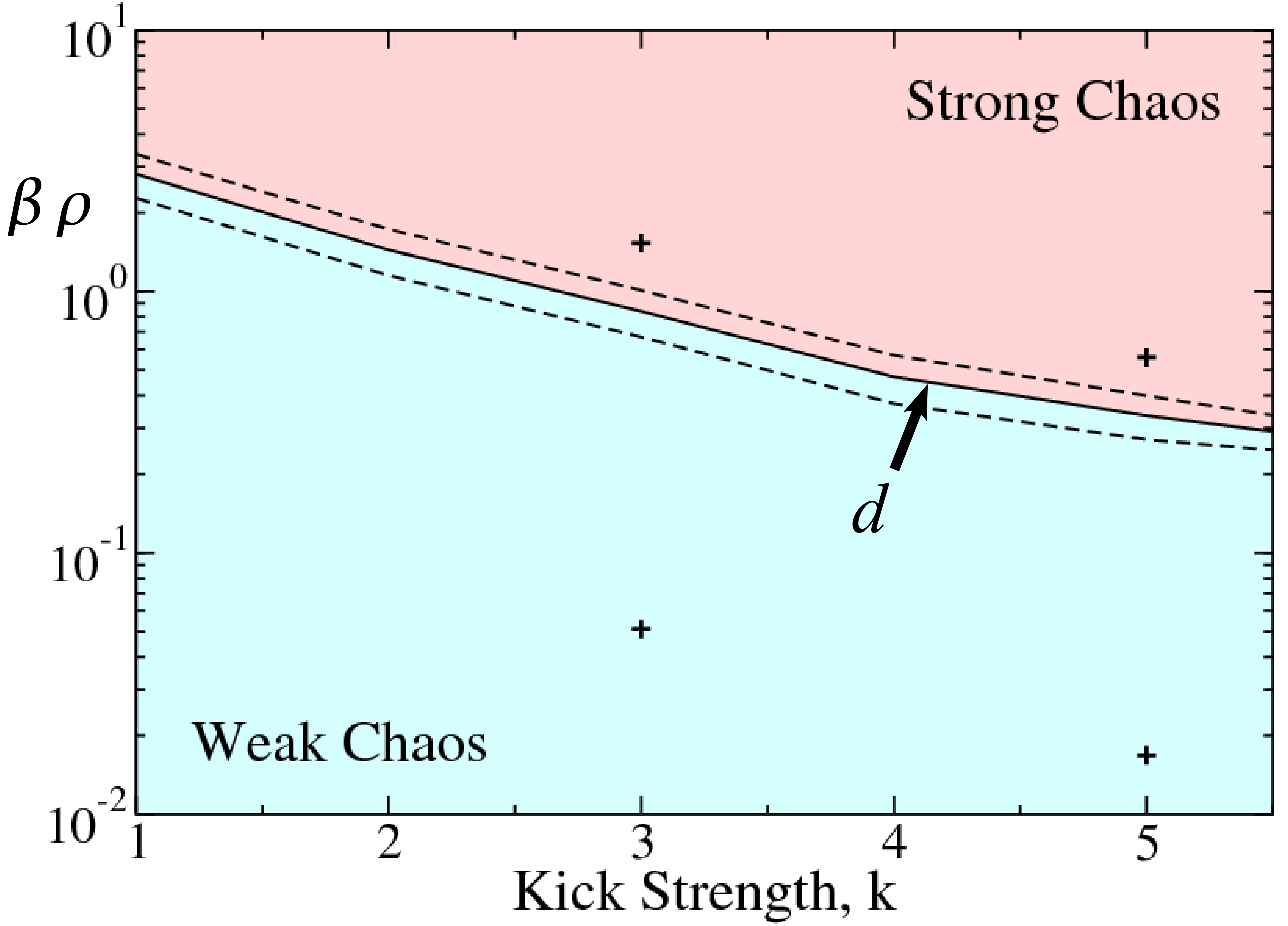
$$\partial_t \phi_\nu = \lambda_\nu \phi_\nu + \beta \rho^{3/2} P(\beta\rho) \cdot f(t), \quad \langle f(t) f(t') \rangle = \delta(t - t')$$

$$\partial_t \rho \sim (\beta \rho P)^2 \rho t \rightarrow D = (\beta \rho P)^2, E = Dt$$

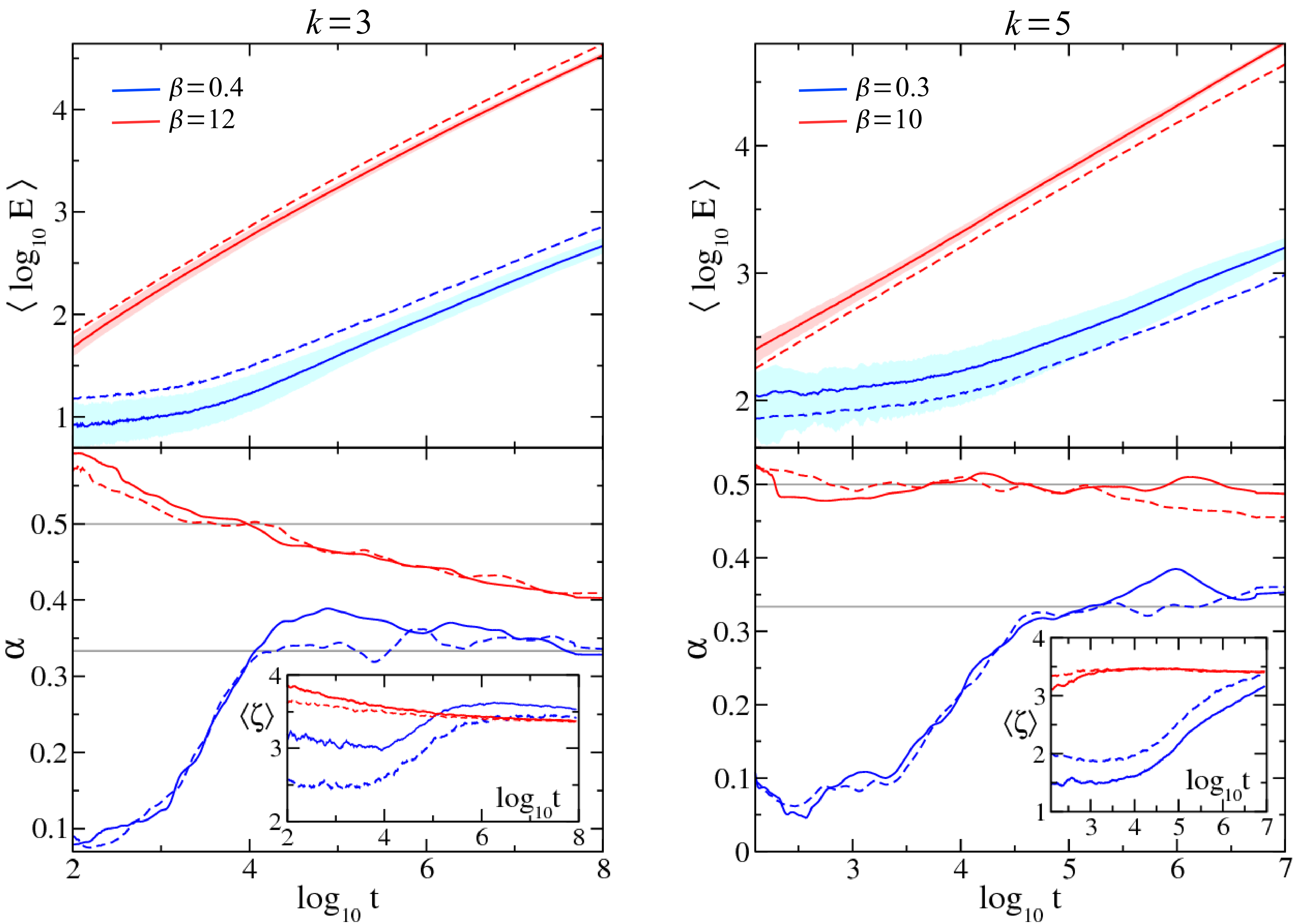
"Strong Chaos":	$P \sim 1$	$\rightarrow$	$E \sim \beta t^{1/2}$	$\rightarrow$	$\alpha = 1/2$
"Weak Chaos":	$P \sim \beta\rho/d$	$\rightarrow$	$E \sim \beta^{4/3} t^{1/3}$	$\rightarrow$	$\alpha = 1/3$

Flach, Chem. Phys. ***375****, 548 (2010)*

Defining a Parameter Space



Numerical Results



Outlook & Foods for Thought

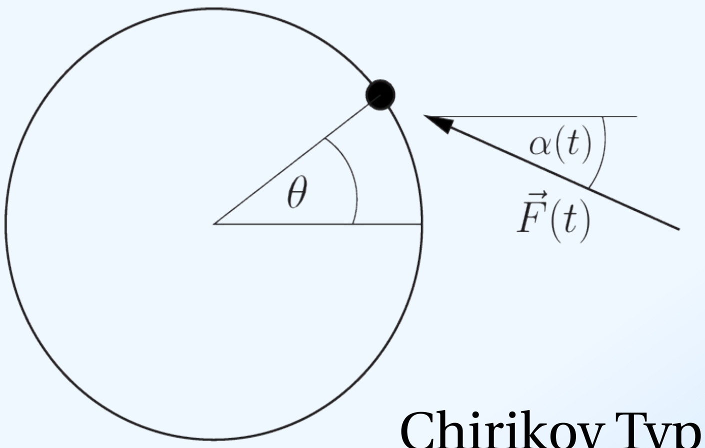
Regimes hold for kicked rotor, albeit NO self-trapping.

- Incommensurate Multiple Kicking → '2d' and '3d' rotors
- Nonlinear Powers → $|\psi|^2 \psi \rightarrow |\psi|^\sigma \psi$

Generalized Conjecture true also?

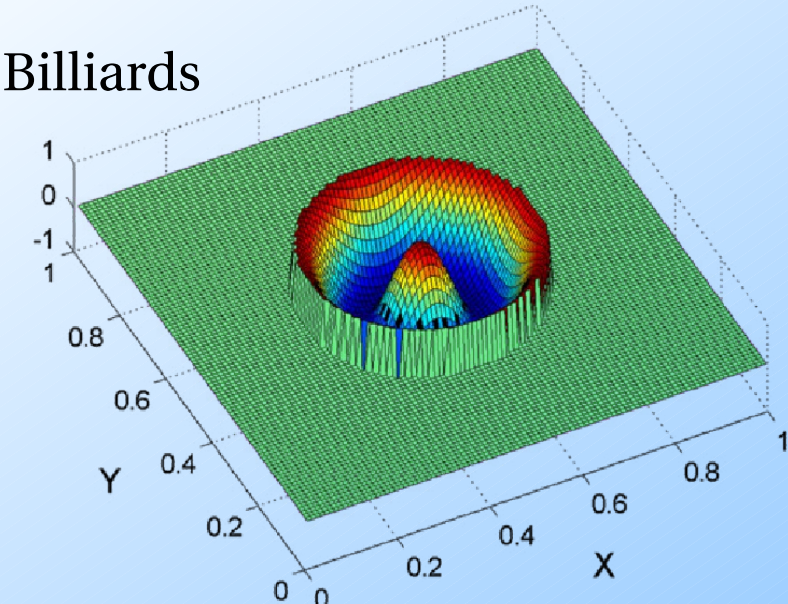
$$\alpha = \begin{cases} \dfrac{1}{1+d\,\sigma}, & \text{Weak Chaos} \\ \dfrac{2}{2+d\,\sigma}, & \text{Strong Chaos} \end{cases}$$

- "Exotic" Models?



Chirikov Typical Map

Kicked Billiards



Frahm & Shepelyansky, Phys.Rev. E ***80****, 016210 (2009)*

Matrasulov et al., Physica D ***240****, 470 (2011)*