

Copyright is owned by the Author of the thesis. Permission is given for a copy to be downloaded by an individual for the purpose of research and private study only. The thesis may not be reproduced elsewhere without the permission of the Author.

Least-squares Optimal Interpolation for Direct Image Super-resolution

A thesis presented in partial fulfilment of the requirements for the degree of
Doctor of Philosophy
in
Engineering

At Massey University, Palmerston North,
New Zealand.

Andrew Gilman

2009

In Loving Memory
Of
Alexander Gilman



20 May 1955 – 5 February 2006

At Rest

Think of me as one at rest,
for me you should not weep
I have no pain no troubled thoughts
for I am just asleep
The living thinking me that was,
is now forever still
And life goes on without me now,
as time forever will.

If your heart is heavy now
because I've gone away
Dwell not long upon it friend
For none of us can stay
Those of you who liked me,
I sincerely thank you all
And those of you who loved me,
I thank you most of all.

And in my fleeting lifespan,
as time went rushing by
I found some time to hesitate,
to laugh, to love, to cry
Matters it now if time began
If time will ever cease?
I was here, I used it all,
and now I am at peace.

Abstract

Image super-resolution aims to produce a higher resolution representation of a scene from an ensemble of low-resolution images that may be warped, aliased, blurred and degraded by noise. There are a variety of methods for performing super-resolution described in the literature, and in general they consist of three major steps: image registration, fusion and deblurring. This thesis proposes a novel method of performing the first two of these steps.

The ultimate aim of image super-resolution is to produce a higher-quality image that is visually clearer, sharper and contains more detail than the individual input images. Machine algorithms can not assess images qualitatively and typically use a quantitative error criterion, often least-squares. This thesis aims to optimise least-squares directly using a fast method, in particular one that can be implemented using linear filters; hence, a closed-form solution is required.

The concepts of optimal interpolation and resampling are derived and demonstrated in practice. Optimal filters optimised on one image are shown to perform near-optimally on other images, suggesting that common image features, such as step-edges, can be used to optimise a near-optimal filter without requiring the knowledge of the ground-truth output. This leads to the construction of a pulse model, which is used to derive filters for resampling non-uniformly sampled images that result from the fusion of registered input images. An experimental comparison shows that a 10th order pulse model-based filter outperforms a number of methods common in the literature.

The use of optimal interpolation for image registration linearises an otherwise non-linear problem, resulting in a direct solution. Experimental analysis is used to show that optimal interpolation-based registration outperforms a number of existing methods, both iterative and direct, at a range of noise levels and for both heavily aliased images and images with a limited degree of aliasing. The proposed method offers flexibility in terms of the size of the region of support, offering a good trade-off in terms of computational complexity and accuracy of registration. Together, optimal interpolation-based registration and fusion are shown to perform fast, direct and effective super-resolution.

Author's Publications

A. Gilman, D. G. Bailey, and S. Marsland, "Image Models for Image Super-resolution," in Proceedings of IEEE International Symposium on Electronic Design, Test and Applications, Hong Kong, 2008, pp. 55-60.

A. Gilman, and D. G. Bailey, "Noise Characteristics of Higher Order Predictive Interpolation for Sub-Pixel Registration," in Proceedings of IEEE Symposium on Signal Processing and Information Technology, Cairo, Egypt, 2007, pp. 269-274.

D. G. Bailey, and **A. Gilman**, "Bias of Higher Order Predictive Interpolation for Sub-pixel Registration," in International Conference on Information, Communications and Signal Processing, Singapore, 2007.

A. Gilman, and D. G. Bailey, "Near optimal non-uniform interpolation for image super-resolution from multiple images," in Proceedings of the Image and Vision Computing New Zealand, Great Barrier Island, New Zealand, 2006.

D. G. Bailey, **A. Gilman**, and R. Browne, "Bias characteristics of bilinear interpolation based registration," in Proceedings of IEEE TENCON, Melbourne, Australia, 2005.

Acknowledgements

First and foremost, I would like to thank my supervisor Associate Professor Donald Bailey who has been in it from the start to the end. Thank you for letting me randomly pop up out of the blue and distract you with my nonsense for a good five years; thank you for all your support and guidance and correcting pages and pages of my drafts.

Secondly, I would like to thank my co-supervisor Dr Stephen Marsland for his help and friendship. Without his mathematical know-how and the ability and patience to transfer some of that knowledge onto me I would not be able to finish.

Next, I would like to thank my family – my Mom, Dad, Nastya, Grandparents and of course Larissa for all the love and support you have given me. I would not be here without you. A special thanks to Mom for taking over Dad's job of constantly phoning me up and pestering about how the work is going. I would also like to thank my friends Max and Chris for constant encouragement, support and advice.

Last, but by far not least I would like to thank my Love. Jess, thank you for being there for me twenty four-seven. I could not wish for a better partner in life.

Table of Contents

Abstract	iii
Author's Publications	iv
Acknowledgements	v
Table of Contents	vii
List of Figures	ix
Chapter 1. Introduction	1
1.1. Background Information	2
1.1.1. Imaging model	2
1.1.2. Super-resolution model	4
1.2. Super-resolution Techniques	6
1.3. Motivation for this Thesis	8
1.4. Thesis Aims	8
1.5. Proposed Approach	9
Chapter 2. Optimal Image Interpolation and Resampling	13
2.1. Image Interpolation	13
2.2. Linear Methods	15
2.2.1. Uniform image resampling	16
2.2.2. Traditional basis functions	17
2.3. Optimisation and Norms	20
2.4. Least-squares Optimisation	23
2.4.1. Experiment I – Region of support	24
2.5. Near-optimal Interpolation	26
2.5.1. Experiment II – Optimal kernel similarity	26
2.5.2. Experiment III – Step-edge dependence	27
2.6. Model-based Near-optimal Interpolation	30
2.6.1. Experiment IV – Performance assessment of rectangular pulse model	37
2.7. Summary and Conclusions	38
Chapter 3. Sub-pixel Image Registration	41
3.1. Image Registration using Optimal Interpolation	42
3.2. Performance Evaluation	46
3.2.1. Description of methods for comparison	47
3.2.2. Experiment V – Registration accuracy evaluation	52
3.3. Analysis of Bias in Registration using Optimal Interpolation	64
3.3.1. Sine wave image model	65
3.3.2. Step-edge image model	68

3.3.3. Effect of noise on bias	71
3.3.4. Effects of noise on bias with the step-edge model	73
3.3.5. Experiment VI – Model validation	75
3.4. Extension of Bias Analysis to Two Dimensions	78
3.5. Summary and Conclusions	81
Chapter 4. Image Fusion and Resampling.....	83
4.1. Non-uniform Interpolation and Resampling.....	84
4.2. Near-optimal Interpolation	84
4.2.1. Special case – global translational motion.....	87
4.2.2. Experiment VII – Optimal and near-optimal filters	90
4.3. Performance of Resampling Methods	96
4.3.1. Methods for comparison.....	96
4.3.2. Experiment VIII – Comparison to existing methods	101
4.4. Effect of Noise on Near-optimal Interpolation	103
4.4.1. Experiment IX – Effect of the smoothing parameter on the performance of model-based near-optimal filters	104
4.4.2. Experiment X – Effect of noise on performance	108
4.5. Summary and Conclusions	109
Chapter 5. Image Super-resolution	111
5.1.1. Experiment XI – Effects of registration error on image fusion	111
5.1.2. Experiment XII – Effect of misregistration on resampling filters of different order....	114
5.1.3. Experiment XIII – Comparison to existing methods	116
5.2. Summary and Conclusions	120
Chapter 6. Summary, Conclusions and Future Directions	121
6.1. Summary	121
6.2. Original Contributions.....	123
6.3. Conclusions	123
6.4. Future work directions.....	124
References.....	127

List of Figures

Figure 1.1	Image capturing process.	2
Figure 1.2	Observation model.....	3
Figure 1.3	(a) Pixel dimensions and (b) magnitude of the resulting sinc PSF.	3
Figure 1.4	Observation model.....	5
Figure 2.1	Two uniformly sampled images at the same resolution can differ by a sub-pixel offset. ...	16
Figure 2.2	Four of the images used in this thesis, referred to as ‘bird’, ‘cat’, ‘face’, and ‘text’.	24
Figure 2.3	Plot of the RMSE against filter window size (2 represents 2×2 window and so on) for images (a) ‘Bird’, (b) ‘Cat’, (c) ‘Face’, (d) ‘Text’.....	25
Figure 2.4	Results of near-optimal interpolation. Rows contain results from interpolating different images. Each column contains results of interpolating using a filter optimised on a particular image. The bar that represents the optimal filter is highlighted. The last column is the result of interpolating each image using Keys’ cubic convolution [96]......	27
Figure 2.5	Image ‘rings’.	28
Figure 2.6	Results of near-optimal interpolation using the best-performing near-optimal filter from figure 2.4, a filter optimised on image ‘rings’ and Keys’ cubic convolution.	29
Figure 2.7	Location of a step-edge inside the resampling filter window.	30
Figure 2.8	Area-sampling of a slanted step-edge.....	30
Figure 2.9	A vertical edge creates multiple linearly dependent constraints.....	31
Figure 2.10	Rectangular pulse model.	32
Figure 2.11	Area-sampling of the rectangular pulse model.	33
Figure 2.12	Three cases of equation (2.36). Refer to text for explanation.....	36
Figure 2.13	Comparison of near-optimal using rectangular pulse model (highlighted) to other popular methods.	38
Figure 3.1	Locating the correlation peak in 1D. (a) Pyramid fit. (b) Parabola fit.....	50
Figure 3.2	Images ‘beach’ and ‘text2’. Original high-resolution image on the left and a low-resolution image on the right.	53
Figure 3.3	Effect of noise contamination of images ‘beach’ and ‘text2’.....	54
Figure 3.4	Plots of bias (solid) and SD (dotted) vs noise SD resulted from applying Lucas-Kanade method to images ‘beach’ (left plot) and ‘text2’ (right plot). Colours represent results after different iterations of the algorithm.....	56
Figure 3.5	Plots of bias (solid) and SD (dotted) vs noise SD resulted from applying the Lucas-Kanade method to images ‘beach’ (left plot) and ‘text2’ (right plot). Colours represent results after different iterations of the algorithm. Results from 100 different image pairs with the same offset were averaged to show variability due to aliasing. See experimental method description for explanation of the difference to figure 3.4.	57
Figure 3.6	Plots of bias (solid) and SD (dotted) vs noise SD resulted from applying cross-correlation methods to images ‘beach’ (left plot) and ‘text2’ (right plot).....	58
Figure 3.7	Plots of bias (solid) and SD (dotted) vs noise SD resulted from applying cross-correlation methods to images ‘beach’ (left plot) and ‘text2’ (right plot). Results from 100 different image pairs with the same offset were averaged to show variability due to aliasing.	58
Figure 3.8	Plots of bias (solid) and SD (dotted) vs noise SD resulted from applying Fourier phase-based methods to images ‘beach’ (left plot) and ‘text2’ (right plot).	60
Figure 3.9	Plots of bias (solid) and SD (dotted) vs noise SD resulted from applying Fourier phase-based methods to images ‘beach’ (left plot) and ‘text2’ (right plot). Results from 100 different image pairs with the same offset were averaged to show variability due to aliasing.	61
Figure 3.10	Plots of bias (solid) and SD (dotted) vs noise SD resulted from applying optimal interpolation-based method to images ‘beach’ (left plot) and ‘text2’ (right plot). Colours represent results obtained using filters of different order.	62

Figure 3.11	Plots of bias (solid) and SD (dotted) vs noise SD resulted from optimal interpolation-based method to images ‘beach’ (left plot) and ‘text2’ (right plot). Colours represent results obtained using filters of different order. Results from 100 different image pairs with the same offset were averaged to show variability due to aliasing.	63
Figure 3.12	Plots of total RMS error vs noise SD for selected methods.	64
Figure 3.13	Bias in estimating the offset of sinusoids ($\omega=\pi/2$) as a function of the offset.	66
Figure 3.14	Frequency dependence of bias (sine-wave model).	68
Figure 3.15	Step-edge image model.	69
Figure 3.16	Bias characteristics of a blurred step-edge for different order interpolation filters (orders one to five).	71
Figure 3.17	Step-edge bias under different noise levels (a) filter order one, (b) filter order two.	74
Figure 3.18	The effect of noise on bias for sub-pixel registration (step-edge model).	75
Figure 3.19	Bias characteristics measured from the “Beach” test image for different order interpolation filters (first to fifth order).	77
Figure 3.20	Noise characteristics of the bias measured from the “Beach” test image for different order interpolation filters (first to fifth order).	78
Figure 3.21	Pincushion maps of bias resulted from the use of 2×2 optimal filter (left column), 3×3 filter (centre) and 4×4 filter (right column) at various noise levels. Bias resulted from the 4×4 filter was scaled up by a factor of 5 for display purposes. Standard deviation is reported in grayscale levels. See text for further description.	80
Figure 4.1	A non-uniformly sampled compound image formed by fusion of three images.	83
Figure 4.2	Resampling of a non-uniformly sampled image.	87
Figure 4.3	Fused image under global translational motion model. Output image pixels are located at intersections of dotted lines.	88
Figure 4.4	The output sampling grid can be broken down into four sub-grids, each with the same sample period as input images. Arrows indicate four nearest input pixels for each output point.	89
Figure 4.5	Cropped parts of low- and high-resolution versions of images ‘bird’, ‘cat’, ‘face’ and ‘text’.	91
Figure 4.6	The fused image does not require any interpolation, as the input samples fall on the positions of the output samples.	92
Figure 4.7	iCDF plots using image Bird and filter orders two to 14: (a) optimal (b) model-based.	93
Figure 4.8	iCDF plots using image Cat and filter orders two to 14: (a) optimal (b) model-based.	93
Figure 4.9	iCDF plots using image Face and filter orders two to 14: (a) optimal (b) model-based.	94
Figure 4.10	iCDF plots using image Text and filter orders two to 14: (a) optimal (b) model-based.	94
Figure 4.11	Median RMSE vs Filter order for images (a) ‘Bird’ (b) ‘Cat’ (c) ‘Face’ (d) ‘Text’.	95
Figure 4.12	Average region of support for filter orders 2 to 14 in the case of test offsets.	96
Figure 4.13	Some examples of inverse distance weighting functions.	97
Figure 4.14	The normal vector at vertex v is estimated as a weighted sum of normal vectors of patches around it, where the weight is determined by the triangle area [26].	99
Figure 4.15	iCDF curves for image Bird.	102
Figure 4.16	iCDF curves for image Cat.	102
Figure 4.17	iCDF curves for image Face.	102
Figure 4.18	iCDF curves for image Text.	102
Figure 4.19	The effect of the smoothing parameter on median RMSE resulted from the use of near-optimal interpolation on image ‘face’.	105
Figure 4.20	Crop of image ‘face’.	106
Figure 4.21	Cross section taken across image ‘face’, as indicated by a white line in figure 4.20.	106
Figure 4.22	Optimal value of the smoothing parameter versus noise level for the four test images.	107
Figure 4.23	Median RMSE versus noise standard deviation for optimal and near-optimal (dotted line) filters of orders 10.	109

Figure 4.24	Difference between median optimal and median model-based near-optimal RMSE as a percentage of median optimal value for filters of order 10.	109
Figure 5.1	The effects of misregistration on resampling error for different levels of image noise: (a) Bird, (b) Cat, (c) Face and (d) Text. Noise SD in the legend is stated in greyscale levels.	113
Figure 5.2	The effect of misregistration on resampling error for filters of different order for image (a) Bird, (b) Cat, (c) Face and (d) Text.	115
Figure 5.3	Plot of median RMSE versus noise standard deviation for a range of methods. Results obtained from image (a) Bird, (b) Cat, (c) Face and (d) Text.....	117
Figure 5.4	Example outputs: A – one low-resolution input image; B – nearest neighbour interpolation of A; C – pulse model-based near-optimal interpolation of the single low-resolution image in A; D – super-resolution output from four low-resolution images using the proposed method (order 10); E – high-resolution image; F – the result of deblurring D. Numbers under B, C and D indicate the RMSE between these images and E.	119