# Bose-Einstein Condensates in Coupled Co-planar Double-Ring Traps

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Tania J. Haigh

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**Abstract**

This thesis presents a theoretical study of Bose-Einstein condensates in a double-ring trap. In particular, we determine the ground states of the condensate in the double-ring trap that arise from the interplay of quantum tunnelling and the trap's rotation.

The trap geometry is a concentric ring system, where the inner ring is of smaller radius than the outer ring and both lie in the same two-dimensional plane. Due to the difference in radii between the inner and outer rings, the angular momentum that minimises the kinetic energy of a condensate when confined in the individual rings is different at most frequencies. This preference is in direct competition with the tunnel coupling of the rings which favours the same angular momentum states being occupied in both rings.

Our calculations show that at low tunnel coupling ground state solutions exist where the expectation value of angular momentum per atom in each ring differs by approximately an integer multiple. The energy of these solutions is minimised by maintaining a uniform phase difference around most of the ring, and introducing a Josephson vortex between the inner and outer rings. A Josephson vortex is identified by a $2\pi$ step in the relative phase between the two rings, and accounts for one quantum of circulation. We discuss similarities and differences between Josephson vortices in cold-atom systems and in superconducting Josephson junctions.

Josephson vortices are actuated by a sudden change in the trapping potential. After this change Josephson vortices rotate around the double-ring system at a different frequency to the rotation of the double-ring potential. Numerical studies of the dependence of the velocity on the ground state tunnel coupling and interaction strength are presented. An analytical theory of the Josephson vortex dynamics is also presented which is consistent with our numerical results.

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