STATISTICAL METHODS FOR ESTIMATING TEPHRA SOURCE AND DISPERSAL

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Abstract

Tephra refers to any pyroclastic fragments ejected from a volcanic vent and its dispersal is one of the major hazards with explosive eruptions. The attenuation of tephra fall thickness is most commonly estimated after contouring field measurements into smooth isopachs. I explicitly describe the variability in thickness by using a semiempirical tephra attenuation relation as a link function. This opens the way to fitting models to actual tephra observations through maximum likelihood estimation (MLE). The method is illustrated using data published from the 1973 Heimaey eruption in Iceland.

Complex eruptions commonly produce several phases of tephra fall from multiple vents. When attempting to precisely reconstruct past eruptions from the geological record alone, separate phases are often indistinguishable. Augmented by a mixture framework, the MLE attenuation model was able to identify the sources and directions of tephra deposition for the 1977 Ukinrek Maars eruption in Alaska, US, from only the tephra thickness data. It was then applied to the unobserved 1256 AD Al-Madinah eruption in Saudi Arabia.

The estimation of the spatio-temporal hazard from a monogenetic volcanic field is critically dependent on a reconstruction of past events. The Auckland Volcanic Field (AVF) has produced about 50 volcanoes in the last 250,000 years. Although inconsistent, age data for many of these volcanoes exist from various dating methods with various reliabilities. The age order of some pairs is also known due to the overlaying of lavas (stratigraphy). A discussion is provided on how informative priors are obtained via expert elicitation, on both the individual volcano ages, and the reliabilities of the dating methods. A possible Bayesian model for reconciling the available inconsistent volcano age data to estimate the true eruption ages is also discussed.

To improve these eruption age estimates, some of the volcanoes can be correlated with the better dated tephra layers recovered from five maars in the field. The likelihood of any combination of volcano and tephra, incorporating the spatial variability based on the attenuation model and temporal components, is evaluated and is maximised numerically using linear programming. This statistical matching provides an improvement in the volcano age-order model and age estimates of the volcanoes in the AVF.

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- **Kawabata, E., Cronin, S., Bebbington, M., Moufti, M., El-Masry, N., and Wang, T.** (**2015**). *Identifying multiple eruption phases from a compound tephra blanket: an example of the AD1256 Al-Madinah eruption, Saudi Arabia. Bulletin of Volcanology*, 77(1): 6.

- **Kawabata, E., Bebbington, M., Cronin, S., and Wang, T.** (**under review**). *Optimal likelihood-based matching of volcanic sources and deposits in the Auckland Volcanic Field. Journal of Volcanology and Geothermal Research.*

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