MODELLING OF THE DRYING SECTION OF A CONTINUOUS PAPER MACHINE

A thesis presented in partial fulfilment of the requirement for the degree of Master in Production Technology at Massey University

HUITING MA, B.E. (East China Chem. Tech.)

SUMMARY

This thesis sought to develop a suitable dynamical model of the PM3 paper machine drying section at Tasman Pulp and Paper Ltd. which could be used for subsequent control system simulations.

An extensive literature search uncovered only two suitable mathematical models, that of Lemaitre *et al.* (1980) and Knight and Kirk (1975). Both models were simulated but only the Lemaitre's model was capable of reaching the observed data by suitable choice of parameters, and the Knight and Kirk's model was abandoned.

The Lemaitre's model was fitted to steady state data from the PM3 machine by finding empirical or theoretical values for some parameters and optimizing the remaining ones.

The model gave reasonable responses in simulations under varying conditions. It predicted:

1. an insensitivity to inlet air conditions between -4 and 23°C and 30 and 80% RH,
2. a relative insensitivity to steam temperature in the first section as compared with the final section,
3. that the cylinder (drum) speed could be increased if matched by an increase in steam temperature.

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1. INTRODUCTION

1.1 Introduction

The invention of paper in 105 A.D. was a milestone in the history of civilization and demand for paper has been increasing steadily ever since. Although it has become more and more popular to store, process and transfer information in electronic forms, paper is to date still the most common means for recording information.

According to Storat (1993), production in the last twenty years has increased by more than 60 percent, while capital expenditures in the industry have grown to almost 12 percent of sales, or double the average expenditures of other manufacturing industries. This capital investment has gone towards capacity expansion and extensive rebuilds of existing mills - almost 60 percent of the existing capacity comes from modern facilities containing machines either newly installed or rebuilt in the past ten years. As a result, fossil fuel and energy consumption in this industry fell by 46 percent in the last two decades.

However, even with modern technology, the pulp and paper industry remains extremely energy intensive. It ranks fourth in total energy consumption by U.S. manufacturers, behind chemicals, steel and petroleum. The paper industry is the leading industry in total energy used for drying operations (Salama *et al.* 1987). Within the paper industry the drying process is the major consumer of energy. The dryer section of a typical paper machine removes water, via evaporation, to less than 0.4 percent of that originally present in the fiber/water suspension, and requires 7.5-14.5 MJ/kg of energy per ton of paper produced. This accounts for nearly one-third of the total energy requirement of an integrated mill and an even greater fraction of the energy used in a nonintegrated mill (Chiogioji 1979). Therefore, significant scope exists to reduce total mill energy consumption through reductions in the energy expended within the paper machine dryer sections.

Moreover the drying section has a strong influence on the quality of the paper. Paper drying is the last part of the water removal phase and the one in which most of its mechanical properties are conferred to the paper sheet. Reducing the moisture content variations of the sheet at the end of the machine will improve the quality of the paper.

1.2 Research Objectives

Although both the energy consumption and investment cost required by the paper drying process are high, the design of the drying section has so far largely been based on empirical rules and the optimum performances, as well as the corresponding working conditions, are not very well known. Improving the drying section running condition is of very great interest and this subject has been much studied (Seyed-Yagoobi *et al.* 1992b), but few studies have been done to develop methods for systematic dynamic analysis of paper machine drying sections. The present research has been performed in this area.

The conventional multicylinder (drum) dryer is by far the most common in the U.S. paper industry, representing 95 percent of the dryers in paper and board applications and over 82 percent of all pulp and paper dryers (McConnell 1980). It is also the most common in the New Zealand paper industry, and it provides the basis for this research.

The overall objectives of this thesis are to:

(1) perform a literature search to find dynamical, analytical models of conventional multicylinder drying section paper machine,
(2) select one or more of these models,
(3) optimize the parameters of the model based on plant data,
(4) perform a sensitivity analysis to gain insight into the paper machine drying section operation.

The research was carried out on a paper machine at Tasman Pulp and Paper, and is supported by the Pulp and Paper Research Organisation of New Zealand (PAPRONZ).

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2. LITERATURE REVIEW

2.1 Physical Description of Paper Drying Section

Paper is most commonly dried in industry by threading a continuous wet web of paper around each of a series of dryer drums which successively dry the paper to an acceptable moisture content. At the entrance to the dryer section, the paper sheet moisture content is about 60% and ranges from 4 to 8% at the exit, depending on the type of drying system and grade of paper (Bell *et al.*, 1992).

The individual dryer drums consist of large, hollow, cast iron cylinders, essentially pressure vessels. The cylinders range from about 1 to 2 m in diameter and up to 10 m in length, with a shell thickness of about 25 mm. The outer surface of the cylinders must be highly finished and free of imperfections to avoid marking the paper. The typical number of cylinders in a dryer section ranges from 40-70, depending on the machine speed and the type of paper produced (Polat and Mujumdar 1987; Bell *et al.*, 1992). Fewer cylinders will be used for light grades of paper such as tissue.

Part of a typical dryer section is shown in Fig. 2.1-1. The dryer felt (clothing) is a highly porous material whose main purpose is to hold the paper sheet in close contact with the dryer shell to increase heat transfer between the paper and the dryer and to help prevent shrinkage and deformation of the paper sheet.

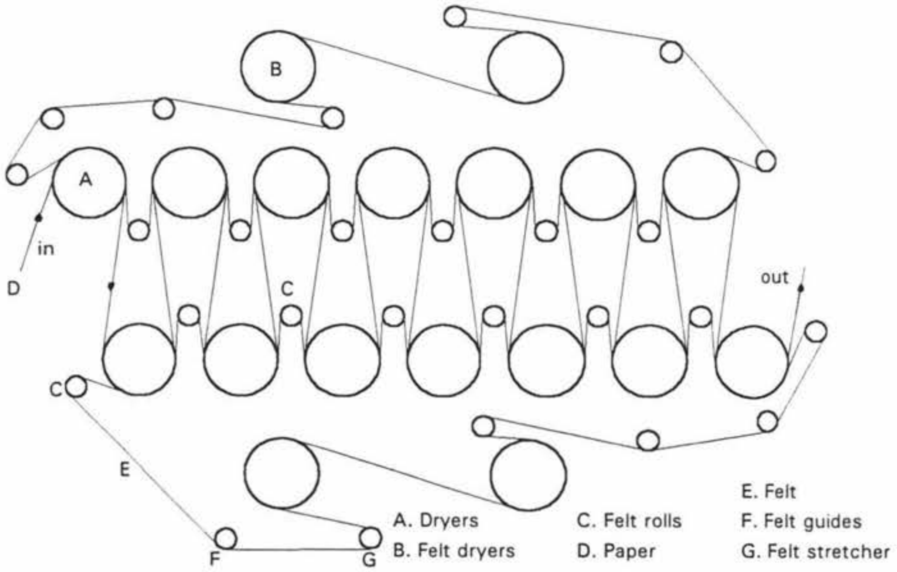

Figure 2.1-1 A typical dryer section (Polat and Mujumdar 1987)

The energy for the drying process comes from saturated steam injected under pressure into the cylinders. Steam pressures in drum dryers can reach up to 1035kPa (Bell *et al.*, 1992). As the steam condenses on the inside of the dryer shell, the latent heat of vaporization is released. The heat is transferred through the condensate layer and dryer shell to the paper on the outside surface.

Fig. 2.1-2 shows that the heat transfer through the condensate is dependent on the behaviour of the condensate. At low dryer speeds, the condensate forms a puddle at the bottom of the cylinder, and the steam condenses directly on the inner wall. As the speed increases, a thin film of water is carried up the dryer wall. The bulk of the water remains in a puddle at the bottom of the dryer. The thickness of this film increases with increasing dryer speed. As the thick layer of condensate approaches the top, some of the condensate layer falls back to the bottom of the dryer in a motion called cascading, accompanied by large-scale turbulence and improved heat transfer. As the dryer speed increases, the centrifugal force pushes all the condensate into a relatively uniform layer around the inside of the dryer. This condition, called rimming, has decreased turbulence and heat transfer. At low rimming speeds, the condensate may still "slosh" slightly with respect to the cylinder wall, creating better heat transfer. As the rimming speeds increase, this sloshing effect virtually ceases, and the heat transfer takes place through an almost stagnant water film (Chance 1989).

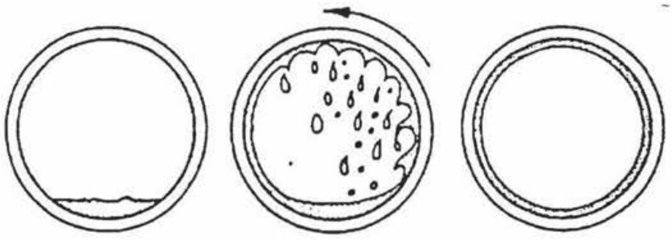

Figure 2.1-2 Condensate behaviour in a dryer (Chance 1989)

As shown in Fig. 2.1-3, the heat released by the condensing steam travels through several resistances before reaching the outside air. The heat first travels through the condensate layer, which creates high heat transfer in the ponding or cascading conditions, but low heat transfer in the rimming condition. Rust or scale on the inside dryer wall is the next resistance, followed by the dryer shell. Chemicals from the wet end form a coating buildup on the dryer's outer surface, which with the entrained air between the shell and paper sheet causes a thermal resistance between the dryer and paper. This thermal or contact resistance is also affected by the finish on the outer surface of the dryer cylinder. A smooth finish gives the paper a greater area of contact with the shell, increasing the heat transfer. The paper sheet creates a resistance to heat transfer that is difficult to calculate because the conditions of the paper such as moisture content, temperature, and thickness constantly change as the paper is dried. The felt and the air boundary layer over the felt are the final resistances to heat transfer.

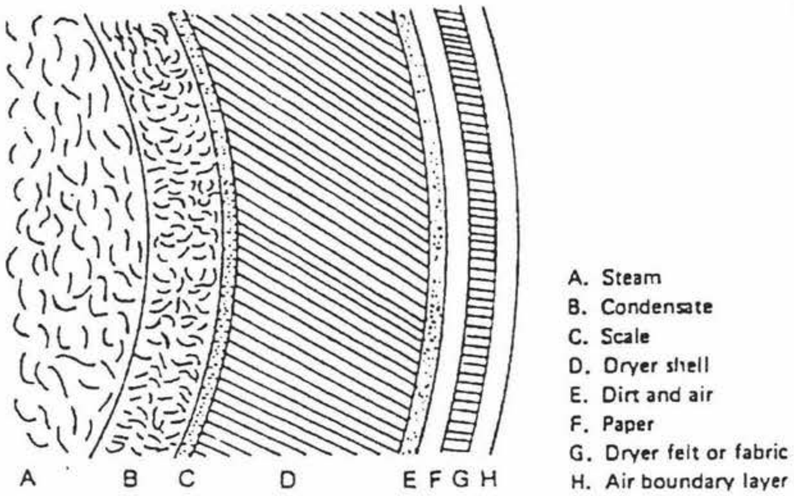

Figure 2.1-3 Resistances to heat transfer on a cylinder dryer (Polat and Mujumdar 1987)

Most paper drying machines have three to five independently felted sections, each with variable speed control to maintain sheet tension between sections and adjust for any sheet shrinkage that occurs. Usually, three to five sections are also grouped for independent steam pressure control; these may be the same or different from the felt groupings (Smook, 1982). The whole drying section is set in a hood so that the atmosphere in which the evaporation takes place can be controlled by action on the inlet and extracted air flows (Lemaitre *et al.*, 1980).

The paper sheet, with a known solids and water content per unit area and of known temperature, approaches a cylinder of known surface temperature. One side of the sheet begins to heat up while the other side, being still open to the atmosphere, cools by evaporation, radiation and conduction. Thus, immediately temperature and moisture gradients are set up in the direction of the thickness of the sheet. Similarly, gradients are set up along the length of the sheet. After a short time the paper reaches the felt. It covers the outer side of the sheet and a new system of temperature and water content dynamics is established: this, in its turn changes as the felt leaves the cylinder and is free to dry on both sides in air.

Fig. 2.1-4 shows the whole drying process. Paper was dried through three periods; a heatup period, a constant-rate period and a falling-rate period.

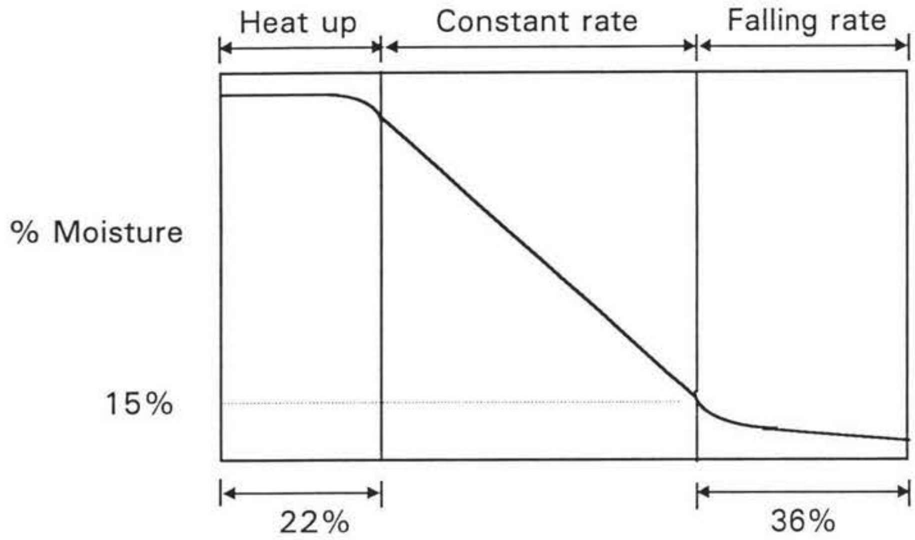

Figure 2.1-4 Conventional drying curve (Sharma 1989)

The drying operation can be divided into two areas: (1) heat transfer from the steam cylinders through the paper into the air, (2) mass transfer of the water evaporating from the paper sheet. Heat and mass transfer through the paper sheet cause the properties of the paper to vary continuously. Formulae can be derived from the physical laws governing the heat and mass transfer through the paper sheet to determine the evaporation rate. Temperature and moisture content are two of the most important variables, but others are: paper thickness, density, porosity, permeability, thermal conductivity, specific heat capacity, contact time, ambient air conditions, air motion, felt and paper tension, etc.

Including all these parameters as variables in the mathematical formulae would result in a set of partial differential equations very difficult to solve analytically or numerically. Approximations and assumptions, along with empirical relations, are necessary to reduce these equations to a solvable form (Bell *et al.*,1992).

2.2 Development of Mathematical Models

Several researchers have attempted to develop detailed mathematical models of the drying process which could be applied to industrial paper machines. The mathematical models could be divided in to two categories. In the first the major assumption for the mathematical models is that there is no temperature or moisture variation through the paper sheet thickness. In the second, mathematical models take into account heat and mass transfer within the paper sheet. Representative of researchers of the former method are Nissan and Kaye (1955) and the latter one is Han (1970).

One of the earliest mathematical simulations of paper dryers was by Nissan and Kaye (1955). In this model the dryer cylinder be divided into cycles of four phases (Fig. 2.2-1), which are repeated on each dryer cylinder.

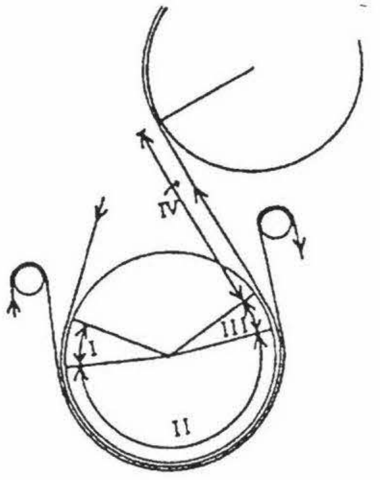

Figure 2.2-1 Nissan's nomenclature for four phase of drying (Kirk 1984)

Phase I: Begins when one side of the sheet first touches a cylinder and ends when the felt first touches the "outer" side of the sheet;

Phase II: Begins when the felt first touches the sheet and ends when the felt leaves the sheet;

Phase III: Begins when the felt leaves the sheet and ends when the sheet leaves the cylinder;

Phase IV: Begins when the sheet leaves the cylinder and ends when it next touches a cylinder.

Certain parameters were assumed constant throughout the entire drying operation (Bell *et al.* 1992):

1. Moisture-free weight of the solid per unit area
2. Speed of the paper sheet
3. Air conditions
4. Steam temperature

5. Length of each phase
6. Thermal conductivity of the paper sheet
7. Specific heat capacity
8. Overall heat transfer coefficient

Other arbitrary assumptions in their model were:

9. No temperature or moisture variation through the paper thickness
10. Arbitrary value of the heat transfer coefficient
11. Zero effect from cross-machine air currents
12. Zero pressure gradient across the sheet thickness
13. Small enough heat and moisture losses in phases I, II, and III, so that all the heat input raises the paper temperature
14. Constant conditions through each phase or subphase
15. Negligible radiation heat losses in phases I, II, and III
16. Conduction heat transfer only in the paper sheet
17. Mechanical pressure from the felt in phase II increases the heat transfer coefficient, but takes no part in water removal

The initial conditions at phase I of drum 1 are the known conditions of the paper at the entrance of the paper drying section. The conditions at the entrance to each phase are assumed to be the same as the conditions leaving the previous phase.

The mathematical formulae for the model can be summarized as follows:
For phases I, II, and III:

$$T_{\Delta t} = u - \left[(u - T_0) \exp\left(\frac{h_{vp} \Delta t}{W(c_f + c_w M)} \right) \right] \qquad (2.2\text{-}1)$$

where,

$T_{\Delta t}$: paper temperature change;
u: steam temperature;
T_0: initial sheet temperature;
h_{vp}: overall heat transfer coefficient;
Δt: time interval in phase or in subphase;
W: weight of moisture-free solids;
c_f: fiber specific heat capacity;
c_w: water specific heat capacity;
M: moisture content.

$$\frac{dm}{dt} = -2.12 \times 10^{-7} L^{0.77} b(P_f - P_a)(1 + 0.121 V^{0.85}) \qquad (2.2\text{-}2)$$

(Powell and Griffiths 1935)

where,

m: mass of water in sheet;

t: time;

L: length;

b: width;

P_f: partial pressure of water vapour at the surface of web;

P_a: partial pressure of water vapour in the air;

V: velocity.

The calculations of paper temperature and the rate of evaporation will be made for each phase separately since conditions change abruptly as the sheet enters the different phases. But, it is sometimes found that even within one phase, conditions at the beginning are substantially different from those at the end. In such a case, it is necessary to divide one phase into several subphases so that the approximation is made as close as is reasonably acceptable.

Since the over-all evaporation of water in these phases is small, it is neglected when calculating temperature, but it must be included to calculate the moisture loss.

For phase IV:

$$\frac{dT}{dt} = 2q\left[W\left(\frac{(\rho_f c_f + \rho_w M)}{(1+M)}\right)\left(\frac{1}{\rho_f} + \frac{M}{\rho_w}\right)\right]^{-1} \qquad (2.2\text{-}3)$$

where,

T: paper temperature;

q: heat;

ρ_f: fiber density;

ρ_w: water density.

In Eq. (2.2-3) q is the sum of the heat due to convection, radiation, and evaporation. The rate of evaporation of phase IV is calculated from Eq. (2.2-2) and is doubled to account for the evaporation from both surfaces.

The Nissan and Kaye's method was used to evaluate a specific paper machine dryer section consisting of 37 total drying cylinders. The only predicted value compared with observed values is moisture content (the ratio of the weight of water to the weight of the dry solid). Despite the many assumptions and approximations, the overall calculated results compared reasonably with the observed values. The calculated predictions are adequate for the entrance to the drying section but deteriorate as the process continues.

Depoy (1972) followed up the work by Nissan and obtained a numerical solution for a dryer configuration with one felted cylinder and one unfelted cylinder. He deemed the most important observation offered by the simulation is that applying air directly to the dryer is more efficient than applying it to the paper web in the draw between dryers. He divided two steam dryer drums into 5 repeating boundary configurations which are similar to but not quite the same as those of Nissan. He reduced the Fourier one-dimensional heat transfer equation to the finite difference equation.

Han (1970) compiled the work of several researchers to develop a model for cylinder dryers based on partial differential equations representing the heat and mass transfer in the paper sheet.

Major assumptions of his model were (Bell *et al.* 1992):

1. Four phase system developed by Nissan and Kaye
2. Two dimensions (machine direction and paper thickness)
3. Moving coordinate system
4. Temperature at the end of draw equal to ambient air temperature
5. Steady state heat conduction through the cylinder shell
6. Negligible radiation heat losses
7. Liquid flow important only in the early drying stages
8. Contact resistance modeled by a stagnant film of air
9. Condensate thickness varies with position around the inside of cylinder
11. Diffusivity found by an experimental correlation
12. Linear reduction of paper thickness

The mass balance equation based on mass fluxes within the paper is:

$$\frac{\partial m}{\partial t} = -\frac{\partial J_w}{\partial z} - \frac{\partial J_v}{\partial z} \qquad (2.2\text{-}4)$$

where,

J_w: water mass flux;
J_v: vapour mass flux;
z: paper thickness direction.

The heat balance equation is:

$$[c_w \rho_w s \varepsilon + c_f \rho_f (1-\varepsilon)] \frac{\partial T}{\partial t} = -\frac{\partial Q}{\partial z} - c_w (J_w + J_v) \frac{\partial T}{\partial z} - \lambda \frac{\partial J_v}{\partial z} \qquad (2.2\text{-}5)$$

where,

s: saturation (volume of liquid/volume of voids);
ε: porosity (volume of voids / total volume);
Q: heat flux.

This equation represents the rate of heat accumulation per unit volume in the element resulting from the net heat input by conduction, associated with the flow of water and vapour, and heat loss due to evaporation (on the left and right hand sides respectively).

For heat conduction, the Fourier law applies:

$$Q = -k \frac{\partial T}{\partial z} \qquad (2.2\text{-}6)$$

where k is thermal conductivity

While the capillary flow of water follows Darcy's law:

$$J_w = \left(\frac{K}{\nu_w}\right) \frac{\partial P_{ca}}{\partial z} \qquad (2.2\text{-}7)$$

where,

K: permeability;
ν_w: water kinematic viscosity;
P_{ca}: capillary pressure

For vapour transport, Fick's law applies:

$$J_v = -\left[\frac{D_a w_v}{(1-y_v)}\right] \frac{\partial C_v}{\partial z} \qquad (2.2\text{-}8)$$

where,

D_a: air diffusivity;

w_v: vapour molecular weight;

y_v: vapour mole fraction;

C_v: vapour molar concentration.

Han gave a preliminary analysis of the energy balance for the first cylinder, the constant-rate drying period and the falling-rate drying period. He suggested the analysis may be further simplified or refined, depending on the purpose and information on hand. He gave no method of solution or any results based on his analysis.

Donner and Renk (1982) developed a system of partial differential equations which describe the paper drying process for pinpointing the source of poor dryer performance. The equations were similar to Han's with a few notable exceptions to further simplify the model and allow a numerical solution.

The assumptions that are different from Han's are:

1. The presence of felt was ignored
2. A fixed coordinate system was used
3. The temperature at the end of the draw was found by setting heat flux at the surface equal to the convective losses
4. A uniform condensate heat transfer coefficient was used
5. Contact resistance was neglected
6. A constant paper thickness was used and the liquid flow was neglected

A finite difference technique was used to solve the transfer equation. Web moisture and temperature distributions were calculated at virtually every point in the dryer section. The Technical Association of Pulp and Paper Industry (TAPPI) drying and heat transfer rate were computed for each dryer cylinder. The simulation was designed mainly as a diagnostic tool, and most of the results presented dealt with ways to improve dryer performance. Simulation results fitted the expected behaviour.

Seyed-Yagoobi *et al.* (1992a) gave the cylinder drying system a theoretical analysis which is partially based on Han's model, though the governing differential equations were improved and a set of realistic boundary conditions were derived.

They adopted Han's mass and heat balance equations and defined the porosity as:

$$\varepsilon = 1 - \frac{W}{Z\rho_f} \qquad (2.2\text{-}9)$$

where Z is web thickness

and sheet caliper as follows:

$$Z = Z_{fin}\left(1 + \frac{M\,\rho_{p\,fin}}{\rho_w}\right) \qquad (2.2\text{-}10)$$

where,

Z_{fin}: web thickness at exit of dryer section;

$\rho_{p\,fin}$: paper density at exit of dryer section.

The finite difference technique was used to obtain the numerical solution. The surface nodes in the model were assumed to be located slightly (one-half step size in the z direction) inside the paper sheet, and not exactly on the surfaces. This approximation simplifies the numerical solution technique. Average sheet moisture content and temperature along the length of the dryer section as well as moisture content and temperature profiles within the sheet at cylinders 1, 30, and 65 are predicted by the model for certain test conditions.

Asensio and Seyed-Yagoobi (1992) followed Seyed-Yagoobi *et al.*'s (1992a) work to develop a theoretical model for simulation of conventional steam-heated cylinder dryers. An empirical correlation for the thermal contact conductance between the cast iron dryer surface and paper web is incorporated into the drying simulation model to reflect reductions in heat input to the sheet during drying. Expressions for the reduction in sheet porosity were developed for inclusion in the model.

The dryer-paper web interface contact conductance was given by:

$$h_i = -20.13 + 19.87\ln(P) + \frac{20999.78}{W} + 97.70\omega^2\ln(P) - \frac{8543.43\omega^{\frac{5}{4}}}{W} + 188.65\omega^{\frac{3}{2}} \qquad (2.2\text{-}11)$$

when,

$0.68 < P <= 328.81$ kPa, $0.084 <= W <= 0.3129$ kg m^{-2}, $4.8\% <= \omega <= 60\%$,

T=85°C, $\sigma = 16.62$ W m^{-2} °K^{-1} (4.6%), and $R^2 = 0.98$

where,

h_i: dryer-paper web interface contact conductance;

P: pressure;
ω: moisture content, wet basis (weight of water / (weight of water + weight of dry fibers));
σ: standard deviation;
R^2: adjusted coefficient of determination.

Porosity:

$$\varepsilon_{\text{fin}} = 1 - \frac{\rho_{\text{p fin}}(1 - M_{\text{fin}}) - \rho_a}{\rho_f - \rho_a} \tag{2.2-12}$$

ε_{fin}: porosity (volume of voids / total volume) at exit of dryer section;
M_{fin}: moisture content at exit of dryer section;
ρ_a: air density.

$$\varepsilon = 1 - \frac{1 - \varepsilon_{\text{fin}}}{1 + \dfrac{MW}{\rho_w Z_{\text{fin}}}} \tag{2.2-13}$$

Average sheet moisture content and temperature along the length of the dryer section as well as average evaporation rates per cylinder are predicted by the model. The three stages of drying, *i.e.*, initial sheet warm up, a relatively constant rate and a falling rate, were all predicted by the model without any external correction factors. Consideration of the internal dynamics of the drying process allows profiles of sheet moisture content, temperature, liquid flux and vapour flux through the sheet thickness to be predicted by the simulation model throughout the dryer section.

Asensio and Seyed-Yagoobi (1993) presented a theoretical model which was similar to their previous work (1992). Liquid and vapour transport, evaporation from a porous surface, sheet shrinkage and porosity variation were all considered during model development. They studied the effect of drum/paper thermal contact conductance on the average sheet moisture content and evaporation and found that doubling the contact conductance decreases the required number of dryers from 95 to 58 by increasing the average evaporation rate from 13.7 kg $m^{-2}h^{-1}$ to 22.4 kg $m^{-2}h^{-1}$ for certain operating conditions. This produced a 64 percent improvement in the drying rate (see Table 1 in their paper). Hence, the drying efficiency is highly dependent on the contact conductance between the dryer drum and the paper web.

Baines (1973) developed a mathematical model of paper drying by considering paper sheet finite thickness and neglecting capillary transport of liquid and convection of vapour through the sheet.

For a sheet in contact with a cylinder (phases I, II, III):

$$\frac{\Delta T}{\Delta T_1} = z' + \frac{2}{\Pi}\sum_{\eta=1}^{\infty}\frac{(-1)^{\eta}}{\eta}\exp(-\eta^2\Pi^2 t')\sin\eta\Pi z' \qquad (2.2\text{-}14)$$

where,

ΔT: $= T - T_o$;
T_o: temperature of wet-dry interface;
ΔT_1: $= T_1 - T_o$;
T_1: temperature of dry side;
t': $= \frac{(\alpha_p + \alpha_v)t}{Z^2}$;
$\alpha_{.p}$: paper thermal diffusivity;
α_v: vapour thermal diffusivity;
z': $= \frac{z}{Z}$.

$$E = \frac{D\varepsilon A\rho_v\Delta T_1}{Z}\left(1 + 2\sum_{\eta=1}^{\infty}(-1)^{\eta}\exp(-\eta^2\Pi^2 t')\right) \qquad (2.2\text{-}15)$$

where,

E: evaporation rate;
D: diffusivity;
A: a constant;
ρ_v: vapour density.

For a sheet in the draw phase (phase IV):

$$\frac{\Delta T}{\Delta T_1} = \frac{2}{\Pi}\sum_{\eta=1}^{\infty}\frac{(-1)^{\eta}}{\eta}\exp(-\eta^2\Pi^2 t')\sin\eta\Pi z' \qquad (2.2\text{-}16)$$

$$E = \frac{D\varepsilon\rho_v\Delta T_1}{Z}\sum_{\eta=1}^{\infty}\exp(-(2\eta-1)^2\Pi^2 t')) \qquad (2.2\text{-}17)$$

Baines considered this model complex enough to account for all aspects of sheet drying but, if it is used for the design and operation of a paper dryer, it must be extended in several ways:

1. More complex but realistic boundary conditions used in obtaining solutions
2. Numerical values for the physical properties of wet paper obtained
3. Experimental results used to verify solutions.

Powell and Strong (1974) presented a mathematical model reliant on several main assumptions as follows:

1. Two phases (drum phase which begins when the sheet first touches a cylinder and ends when the sheet leaves the cylinder and draw phase which begins when the sheet leaves the cylinder and ends when it next touches a cylinder)
2. Two dimensions (machine direction and paper thickness)
3. Values of the thermal conductivity and the contact conductance are simply estimated from the available experimental data
4. No evaporation takes place from the sheet while it is on the drum
5. Temperature at the end of draw equal to ambient air temperature

In addition, dimensionless variables are introduced by them to build the model.

For the drum phase:

$$T_{p\,fin} = T_a + \theta(T_c - T_a) \qquad (2.2\text{-}18)$$

where,

$T_{p\,fin}$: temperature of the paper sheet as it leaves a dryer drum phase;
T_a: temperature of the air in the dryer section;
θ: dimensionless temperature;
T_c: cylinder tenperature.

For the draw phase:

$$\frac{dM}{dt} = \frac{3.6C_{mpa}}{WRT_a}(P_S - P_a) \qquad \text{if } M >= M_c \qquad (2.2\text{-}19)$$

where,

C_{mpa}: mass transfer coefficient between paper and air;
R: gas constant;
P_S: saturated pressure;

M_c: critical moisture.

$$\frac{dM}{dt} = \frac{3.6C_{mpa}}{WRT_a}\left(\frac{M - M_e}{M_c}\right)(P_S - P_a) \qquad \text{if } M < M_c \tag{2.2-20}$$

where M_e is equilibrium moisture.

$$\frac{dT}{dt} = \frac{3.6fC_{mpa}(P_s - P_a)}{RT_a c_p (W + MW)} \tag{2.2-21}$$

where,

f: $= 1$, if $M >= M_C$;

f: $= \left(\frac{M - M_e}{M_c}\right)$ if $M < M_C$;

c_p: paper specific heat capacity.

Powell and Strong admitted that the model was simplified to a very large extent, but the analysis was cast in a form where many of their assumptions can be removed in a systematic way as more detailed experimental information about the process becomes available.

Knight and Kirk (1975) derived a theoretical analysis of the operation of a conventional drying section which emphasised the operation of the drying section itself rather than the heat and mass transfer within the web. The model shows that an analysis of drying in the machine direction cannot be complete without consideration of cross direction effects. A basic assumption for the model is no temperature or moisture variation through the paper thickness.

Surface-air heat transfer coefficients:

$$h = (6.48 - 0.0098T_a)L^{-0.2}V^{0.8} \tag{2.2-22}$$

Mass transfer coefficients:

$$C_m = 1.09h \tag{2.2-23}$$

$$\frac{1}{C_{mpa}} = \frac{1}{C_{mpfe}} + \frac{1}{C_{mfea}} \tag{2.2-24}$$

where,

C_{mpfe}: mass transfer coefficient between paper and felt;

C_{mfea}: mass transfer coefficient between felt and air.

$$C_{mpfe} = \frac{\varepsilon D}{Z_{fe}} \qquad (2.2\text{-}25)$$

where Z_{fe} is felt thickness.

Mass transfer:

$$E = C_m(P_S - P_a)\frac{M}{M_c} \qquad \text{if } M < M_c \qquad (2.2\text{-}26)$$

$$E = C_m(P_S - P_a) \qquad \text{if } M >= M_c \qquad (2.2\text{-}27)$$

For phases I, II, III:

$$\frac{dT}{dt} = \frac{1}{Wc_f + 0.01MWc_w}\{h_{cp}(T_c - T) - h_{pa}(T - T_a) - \lambda E\} \qquad (2.2\text{-}28)$$

where,

h_{cp}: heat transfer coefficient between cylinder and paper;

h_{pa}: heat transfer coefficient between paper and air;

T_a: the air temperature over the clothing;

λ: latent heat of evaporation of water.

For phase IV:

$$\frac{dT}{dt} = \frac{-1}{Wc_f + 0.01MWc_w}\{h_{pa}[(T - T_{a1}) + (T - T_{a2})] + \lambda(E_1 + E_2)\} \qquad (2.2\text{-}29)$$

where,

T_{a1} or T_{a2}: air temperature of each side of the paper;

E_1 or E_2: evaporation rate of each side of the paper.

The results of comparison of the modeling with practical data have been reasonably good which suggests that the simplifications that have been made in the analyses do not detract from their ability to describe dryer section operations. Knight and Kirk deem the main limitations are due to errors introduced by the lack of theoretical or empirical knowledge of the drying process and these limitations will decrease with greater understanding and can be overcome to a certain extent by the fitting of experimental data to provide values for unknown parameters.

Soininen (1980) derived two simultaneous differential equations from the heat and mass balance to model the contact drying process which can be applied especially for the study of the drying intensity in a conventional paper machine. The paper thickness was considered. He also discussed the nature and boundary conditions of the through-drying process. The model of the contact drying process cannot be solved analytically. He developed a program for the numerical solution, but didn't give any detail about the program. He showed a couple of worked examples for main parameters, but gives no comparison with practical data.

Lee and Hinds (1981) developed a laboratory technique for measuring the transport phenomena which occurs within moist sheets of paper or board dried under laboratory controlled conditions. Generalised constitutive relationships describing the internal movement of liquid, vapour, and conducted heat can be deduced from these laboratory data. The empirical expressions for the internal transport coefficients of liquid, vapour, and heat are specific to the composition and structure of the sheet used in these experiments. These empirical expressions will vary depending upon the furnish and formation of the sheet tested. It is important that the appropriate empirical expressions for the particular sheet of interest be determined in order to formulate a reliable model. They derived a semiempirical mathematical analogue for the thermodynamic behaviour of the moist sheet based on these empirical expressions and on elemental heat and mass balances. The model is capable of predicting the movement of moisture and heat throughout the sheet exposed to a variety of boundary conditions.

Hinds and Neogi (1983) developed a mathematical model of the drying process that took into account the heat and mass transfer both within and outside the paper sheet. The equations governing the elemental mass and energy balance are highly nonlinear, second-order, partial differential equations. Therefore a VAX computer with a VMS operating system was used to solve the equations using a finite difference technique. The results of computer simulation were compared with machine data for several paper types.

Lemaitre *et al.* (1980) described a method giving systematic analysis of industrial drying sections during their run and based on a mathematical model of paper cylinder drying.

The main simplifying hypotheses used to construct this mathematical model are as follows:

1. Two typical positions of the paper web are distinguished: web on the cylinder and web in the draw
2. Temperature and moisture content are supposed to be uniform through the web
3. All the internal phenomena (vapour diffusion, capillary transport and *etc.*) are considered only with global relationships
4. The drying section is in a steady state

For the web on a cylinder:

$$W(c_f + Mc_w)\frac{dT}{dt} - h_{vp}(T_v - T) - h_{pfe}(T_{fe} - T) + E\lambda(T) = 0 \tag{2.2-30}$$

where,

h_{vp}: heat transfer coefficient between steam vapour and paper;

T_v: steam temperature;

h_{pfe}: heat transfer coefficient between paper and felt;

T_{fe}: felt temperature;

$$E = W\frac{dM}{dt} = -\frac{C_{mpfe}}{R(273+T)}P\log\left(\frac{P - P_a}{P - p_f}\right) \tag{2.2-31}$$

where P_a is vapour pressure in the air over the felt.

For the web in the draw phase:

$$W(c_f + Mc_w)\frac{dT}{dt} - 2h_{pa}(T_a - T) + E\lambda(T) = 0 \tag{2.2-32}$$

$$E = W\frac{dM}{dt} = -\frac{2C_{mpa}}{R(273+T)}P\log\left(\frac{P - P_a}{P - p_f}\right) \tag{2.2-33}$$

where P_a is vapour pressure in the pocket air.

For the web on a cylinder and in draw phase:

$$P_f = P_s(T) \qquad \text{if } M < M_c \tag{2.2-34}$$

$$P_f = P_s(T)e^{-k_f\left(\frac{1}{M} - \frac{1}{M_c}\right)} \qquad \text{if } M < M_c \tag{2.2-35}$$

where k_f is (affinity) coefficient of the water for the sheet

The author applied a parameter estimation method to evaluate the heat and mass transfer coefficients of industrial processes to make the mathematical model workable. The model has been used for an industrial printing paper machine. Computer simulations allowed them to find the optimal size-press location in the drying section and the machine speed variations obtained after on increase in the number of cylinders and all this for three different basis weights.

Videau and Lemaitre (1982) described a mathematical model based on a theoretical analysis of the physical phenomena that occur in a multicylinder paper machine drying section. This model takes into account the main design and working parameters: the different evaporation phases, the heat and mass transfer conditions, the main ventilation variables and also the steam system constitution. The major approximations for the model of the sheet are the same as Lemaitre *et al.* (1980), such as, no temperature or moisture variation through the sheet thickness. The equations of the model of the sheet are almost similar to Lemaitre *et al.* (1980). The heat transfer equation (Eq. (2.2-36)) for web on the cylinder is considered only from hot cylinder surface.

$$W(c_p + Mc_w)\frac{dT}{dt} = h_{cp}(T_c - T) + h_{pfe}(T_{fe} - T) - E\lambda(T) \tag{2.2-36}$$

The heat and mass coefficients, as well as some unknown coefficients for the sheet model, the cylinder model, the steam system and ventilation system models, are determined for a given operating point by using an identification method. This identification method was tested on a pilot paper machine and on different industrial drying sections. The authors studied the influence of some working conditions like basis weight and machine speed and obtain results such that it is possible to keep constant the numerical values of the estimated parameters, for simulations around the selected operating point. But, the authors give no detail about the identification method.

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