Getting it Right and Getting it Finished: Mathematics in Year Five.

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Abstract

This study examined the extent of inquiry and traditional classroom practices and attitudes during mathematics teaching and learning in two year 5 classrooms (8-9 years old). The cultures of the classrooms were examined in the light of recent research into the social, affective and task environments of students learning mathematics.

The study was designed as an ethnographic case study, with the intention of providing a rich description of the classroom interactions and environment. Data collection was carried out during the 3rd and 4th terms of 1998, during which classroom observations were triangulated by focus group interviews, teacher interviews and questionnaires. The resulting data was analysed using the theoretical framework of Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human (1997) and related to recent theories of teaching and learning of mathematics.

The results of the study indicate that the cultures of the two classrooms were partly conducive to linked, thoughtful and contextual mathematics as envisaged by the Mathematics in the New Zealand Curriculum document and encouraged by many recent researchers in mathematics education. On the other hand many of the student and teacher practices and attitudes were reminiscent of traditional classrooms. The most important concern of the students was to complete their work and to be seen to get their answers right. The interplay between the different paradigms resulted in a mixture of rote learning and understanding of mathematics. The students showed some ability to move between paradigms depending on the classroom environment established by the teacher, but usually pressed the teacher towards traditional practices. The power environment in the classroom and the way tasks were constructed and used by the teacher were of critical importance to the quality of mathematical outcomes experienced by the students.

The study suggests that effective inquiry based teaching and learning is possible in the New Zealand context, especially where the teacher is able to link together a system of classroom practices that offer the students an alternative paradigm to traditional 'mathematics in school'. There needs to be a long-term campaign of information and professional development for both pre-service and in-service teachers, focussing on the potential and effectiveness of such linked systems of classroom practice.

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1. INTRODUCTION

> *Change is coming, but its exact nature is not entirely clear. ...This means that students must learn mathematics with understanding. Understanding is crucial because things learned with understanding can be used flexibly, adapted to new situations, and used to learn new things. Things learned with understanding are the most useful things to know in a changing and unpredictable world.* (Hiebert et al., 1997, p.1)

Mathematics teaching and learning in New Zealand is in a state of change. Until recently most New Zealand teachers and students experienced mathematics education under traditional ways of teaching and learning where the main goal was often to encourage procedural capability rather than learning with understanding. Recent changes in educational philosophies, however, have affected the ways mathematics teachers think about education. Goals and approaches, which were the norm even 20 years ago, are now being questioned and often seriously criticised. Alternative approaches are being promoted by curriculum writers and educational researchers as more appropriate for effective teaching and learning.

In particular, constructivist understanding about learning and research results in many areas of education has influenced many teachers to change their teaching practices. Teachers are now expected to take a holistic and coherent approach to learning, with the goal of developing both procedural capability and conceptual understanding in their students (Ministry of Education, 1992).

In the mathematics classroom the process of change is not straightforward. Influences from outside the classroom impact on what is possible inside. Parents and caregivers were often conditioned by their past experiences of schooling to expect mathematics to be rote learning of symbolic algorithms, often with little meaning or connection to their lives. Today they understandably challenge new ideologies of schooling and pass their own beliefs about mathematics teaching and learning to their children. However, mathematics nowadays is likely to involve a great deal of practical work closely connected to everyday life, the algorithms being learnt in context. Faced with different sets of expectations, today's children must try to satisfy both teachers and parents as they make sense of the classroom world.

Primary teachers may also experience some difficulties with mathematics education. Many of them have an incomplete and uncertain understanding of the subject (Garden, 1997) and are often acutely aware of their shortcomings. Add the pressures of teaching all the subjects in primary school with the attendant workload, and it is clear that innovation is not easy for teachers who would like to explore alternative approaches to teaching and learning mathematics in New Zealand (Stigler & Hiebert, 1999). In order to implement change, teachers need to be able to create a classroom culture that supports learning with understanding (Boaler, 1998; Cobb, 1990, 1994; Cobb & Bauersfeld, 1995; Hiebert et al., 1997; Raymond, 1997). Like all cultures, this cannot be imposed on the students but must be built interactively and with their consent.

This study documents the teaching practice and classroom cultures of two year five classes whose teachers were moving towards inquiry-based learning and understanding of mathematics. The features of the classroom environments that did or did not support learning with understanding were investigated. Conclusions are drawn as to what steps might be taken by teachers, school managers and teacher educators to further improve mathematics education in New Zealand.

1.1 Mathematics in New Zealand classrooms

The curriculum document "Mathematics in the New Zealand Curriculum" (MINZC) (Ministry of Education, 1992) is set out in 8 levels to cover year 1 to year 13. MINZC includes many teaching goals and suggestions that are in harmony with constructivist philosophies. It advocates the content and teaching practices expected to be enacted in New Zealand classrooms but its implementation is open to interpretation by teachers (Begg, 1994; Neyland, 1995a; Walshaw, 1994).

New Zealand teachers are in widely differing schools and communities and have varying degrees of familiarity with these approaches to mathematics teaching and learning (Openshaw, 1992). The implementation of the curriculum is likely to be influenced by how closely the philosophies of the school and teacher approximate to the philosophies inherent in the curriculum, and by the culture of the local community. The interplay between the expectations and beliefs of the teacher, and those of the children therefore become critically important in determining what mathematics is experienced by the children in any given classroom.

Children enter school with a wide range of mathematical skills and understanding. The first two years of schooling in New Zealand are characterised by a practical environment that emphasises a holistic approach to mathematical learning (Young-Loveridge, 1999). Children need to develop all facets of number concepts before they leave this to go into year three where there is more emphasis on symbolic manipulation. Some children need much more help than others to cope with the change. Children who have early difficulty learning the mathematics taught in school may well find their problems compounding as the years go by. Failure to keep up with the pace of the class may occur for a number of social and cognitive reasons (Hoard, Geary & Hamson, 1999) but the repeated failure may well result in learning avoidance or learned helplessness (Perkins, 1995).

By the time students enter year five at primary school in New Zealand at about 9 years old, the emphasis for number is on knowledge of the basic facts as well as procedural competence and conceptual understanding of algorithms up to the multi-unit level of understanding (Young-Loveridge, 1999). These are expected to be applied to a range of contexts from the students' daily lives. The mathematics prescriptions for the other strands of MINZC (Ministry of Education, 1992) also emphasise both following the procedures and understanding their applications.

The recent 'Third International Mathematics and Science Study' (TIMMS) shows the relatively low international standing of New Zealand students at mathematics at 9 years old (Garden 1997). Learning mathematics with understanding may contribute to the raising of New Zealand's standing, but little is known of what mathematics teaching and learning is like in New Zealand's classrooms (Walshaw, 1994). A deeper understanding of how mathematics teaching and learning in New Zealand classrooms actually happens might inform professional development efforts towards more effective teaching practices and better remediation of those who fall behind.

1.2 Design of the study.

The conceptual background.

Constructivism comes in several forms depending on the emphasis on social or cognitive processes. At the two ends of the spectrum are the radical constructivist (Von Glaserfeld, 1991; Anderson, Reder, & Simon, 1996, 1997) and social constructivist theories (Greeno, 1997; Neyland, 1995c). While fully accepting the importance of cognitive processes in building of knowledge, social constructivism asserts that the socio-cultural environment of a learner is also very important in the process of 'coming to know'. This is closely related to the systemic approach to change in modern psychology (Friedman & Combs, 1996).

The analysis of the classroom culture in this study relies heavily on a social constructivist model of how mathematics can be learnt with understanding, developed over many years by Hiebert and colleagues (Hiebert et al., 1997). It also uses the concepts of power (Manke 1997, Tauber 1993), and culture (Bishop, 1988; Brinton, & Nee, 1998; Copp, 1995; Cobb & Bauersfeld, 1995) to facilitate a systematic analysis of mathematics teaching and learning in two New Zealand classrooms.

Much research in mathematics education has looked at the mathematics itself in order to clarify the ways in which children learn mathematics, or at particular parts of the environment of mathematics learning: children's strategies; motivation; cognitive processes; the psychology of learning; emotional factors and other areas. For New Zealand primary schools a range of reports and studies have been presented on various aspects of how mathematics is taught and learnt (e.g. Higgins, 1994; Neyland, 1994, 1995; Nuthall & Alton-Lee, 1990; Thomas, 1994; Young-Loveridge, 1992, 1999). There is also a need for case studies on the whole classroom environment and how it affects mathematics teaching and learning (Nuthall & Alton-Lee, 1990; Walshaw, 1994). Such studies can explore the variety of factors, which affect children's learning of mathematics. "... classroom research can play a central role in integrating many other areas of educational research." (Nuthall & Alton-Lee, 1990, p. 566).

Research methods background.

Increasingly, qualitative research methods such as ethnographic case studies have been used to examine of the culture of the classroom. This qualitative study explores the extent to which teachers are basing their practice on the constructivist paradigm implicit in MINZC (Ministry of Education, 1992).

Over the last twenty years there has been a gradual change in research paradigms for mathematics education. Quantitative research looking for statistical correlations between factors has been edged aside by qualitative research (Nuthall & Alton-Lee, 1990), often with situated constructivist – interactionist philosophies (Cobb & Bauersfeld, 1995; Hiebert et al., 1997; Steffe and Wood, 1990; Voigt, 1995). The result has been an explosion of new perspectives on how children learn mathematics and on what environments help them to do so. These perspectives focus on children building their mathematical understanding and knowledge, rather than absorbing it ready made.

Along with the understanding that children build new knowledge on prior experiences and knowledge, the classroom environment is also assumed to be important for effective learning. This environment includes the beliefs and expectations the children and teacher bring to the classroom; the cultures of the school and its community; and the physical environment. Quantitative methods cannot hope to investigate such a complex system of interacting factors, the acceptance of research methods from anthropology, sociology and psychology have made such investigations possible

This study uses the methods of ethnographic case studies to present a 'rich description' (Eisenhart, 1988) of two working classrooms, seeking to provide information on what mathematics teaching and learning is actually like for the children and teachers involved. The results of this study will therefore be a contribution to the baseline of data on the changing practices of mathematical teaching and learning in New Zealand.

1.3 Research objectives.

This study explores the way two teachers have put into practice their beliefs about what mathematics teaching and learning should be, and how their students experience mathematics learning. The actions and expectations of the teachers and the children are examined to find out how the classroom cultures reflect the mathematics teaching and learning promoted by MINZC (Ministry of Education, 1992). Social, affective and mathematical aspects of classroom interaction are examined, since it is now widely believed that the whole environment of the classroom influences mathematics teaching and learning (Appelbaum, 1995; Cobb & Bauersfeld, 1995; Hiebert et al., 1997; Perkins, 1995). The theoretical model proposed by Hiebert et al. (1997) is used to provide a structure for the systematic analysis of the classroom interactions and environment.

The study specifically asks:

- What are the normative actions and expectations of the teachers and students during mathematics time; how do these relate to 'traditional' and 'inquiry' classrooms?
- What sorts of power relationships are operating in these classrooms and how do these affect teaching and learning?
- How do the student and teacher perspectives interact?

1.4 Definitions of terms, notes and conventions.

- In all the data reporting which follows, arithmetic skill work is known as '*sums*', word problems are '*problems*', and higher order thinking or multi-skill exercises are '*problem solving*'. '*Projects*' can be investigations or set work, but involve the planned application and development of skills and strategies.
- The young people who took part in the study are called 'children' where the context is outside the classroom and 'students' where the context is inside the classroom. This is to highlight the existence of the classroom cultures, and the roles associated with being a student or a teacher.
- The term 'parent(s)' has been used to refer to the caregivers of the students. It may be that 'caregiver', 'family', 'whanau' or other terms might be more appropriate in various cases. 'Parent' is intended to be read as inclusive of these terms.

1.5 Overview of the report.

Chapter 2 reviews the literature on relevant aspects of mathematics education and classroom cultures. It includes notes on the theoretical constructs used and sets out the theory behind the analysis. In chapter 3 the methodology for this study is discussed, with particular attention to triangulation of results.

Chapters 4 and 5 report on the results of the study. Chapter 4 presents the cultures promoted by the teachers, with their attendant beliefs and expectations. Chapter 5 examines the student culture in each classroom, noting the range of ways each dimension is expressed. Chapter 6 analyses and discusses the results while chapter 7 presents the conclusions and implications of the study. Three Appendices provide the research instruments used and the detailed results of the questionnaire.

2. THE CASE BACKGROUND: LITERATURE REVIEW

2.1 Overview

The literature review begins with a broad theoretical background to the study and a picture of the historical situation of mathematics teaching and learning. It then examines the conçepts of culture, norms, roles and power, followed by beliefs and motivation, and the relevant paradigms of social constructivism and interactionism.

The following section applies the general theory to the mathematics classroom by giving a theoretical basis for the analysis of the data. Firstly there is an historical section on the place of understanding that places the theory in context, then some notes on the theoretical frameworks that were used to examine the data. These are based on Hiebert et al.'s (1997) five dimensional model of teaching and learning mathematics with understanding, augmented by Manke's (1997) and Tauber's (1993) analyses of power relationships.

2.2 Theoretical background to the study.

THE HISTORICAL BACKGROUND.

The philosophies behind our present system of schooling have changed a great deal over the past century. As society has changed in its socio-economic and political structures, schooling, from the perspective of some teachers and parents, has changed to try to prepare the students for the challenges they are likely to meet after leaving school. For other educators, the changes have been towards helping all students to reach their potential in as many curriculum areas as possible. From the perspective of politicians and the business world, schooling has changed to produce people with the skills needed for the workplace. For the people concerned with democracy and citizenship, schooling has changed to equip young people to take their place in society. All these agendas and others have played a part in the changes that are in progress.

Mathematical schooling has changed from what is commonly known as traditional classroom teaching, towards modes of learning based on reflective thinking, investigations and understanding. In traditional classrooms 'doing maths' was cut and dried. Methods of solving ever more complex problems were taught in carefully graded sequence, with the assumption that what was needed was facility in 'doing sums'. Teachers set tasks to lead their classes towards being able to get their sums right, and to finish their work before the bell went. Tests and exams measured progress, and the reports graded the students on their ability to get marks in these tests. This paradigm of schooling developed over the first half of the century and served the society reasonably well, and many students very badly.

Recent educational research has showed that mathematics teaching and learning could be quite different, leading to deep understanding of mathematics for most learners (Hiebert et al., 1997; Perkins, 1995). Inquiry based classroom practices have emerged as viable ways to both meet the needs of society and to provide nearly all learners with mathematical understanding and capability. Keys to the development of effective inquiry based practices have been new concepts of how people develop new knowledge and about interaction and communication within classroom cultures.

CULTURE, NORMS, ROLES AND POWER.

There is a growing body of research that highlights the importance of the culture of the classroom as a factor in effective mathematics teaching and learning (e.g., Bishop, 1988, 1991; Nickson, 1992; Perkins, 1995; Verschaffel & De Corte, 1996; Yackel & Cobb, 1996). This research is part of the cognitive/constructivist revolution (Thagard, 1992) in psychology and education that has changed the way schooling is conceptualised by many teachers.

In this new tradition, which can be termed 'inquiry classrooms'; the students come to understand not only the means of mathematics – the computation skills and strategies – but also to develop what Verschaffel and De Corte (1996) call a mathematical disposition. This is an integration of understanding, ability, inclination, and sensitivity to mathematical opportunities. Each student grows in understanding and this individual 'coming to know' (Helme, Clarke & Kessel, 1996) happens in the social and cultural environment of the classroom in a reflexive manner. Each student builds their own understanding of mathematics and the norms of the classroom culture and by doing so they contribute to the development and maintenance of that classroom culture (Bishop, 1988).

Bishop developed the notion that the mathematics classroom operates as a cultural microcosm, with norms of behaviour and attitudes established either by fiat or by negotiation (Bishop, 1988, 1991). These norms cover what is to be taken as mathematically true, different, and acceptable as well as the behavioural and organisational aspects of the setting. Yackel and Cobb (1996) extended this and introduced the term 'socio-mathematical norms' – aspects of mathematical activity that are seen as right and proper in a given social setting. These socio-mathematical norms operate alongside norms of the wider cultures of the classroom and the community and influence students' attitudes to life, learning and knowledge.

Voigt (1995) investigated socio-mathematical norms using an interactionist approach. He was particularly concerned with the ways teachers and students modified each other's actions and attitudes in the classroom.

> The interactionist approach views negotiation of meaning as the mediator between cognition and culture. A main assumption is that the objects of the classroom discourse are ambiguous, that is, open to various interpretations. The participants gain a taken-as-shared understanding of the objects when they negotiate mathematical meaning. (p.163)

An interactionist perspective could provide new insights into the ways New Zealand classrooms operate as cultural entities.

Culture.

The concept of culture was originally used by sociologists and anthropologists to describe ways of living of ethnic or geographic groups of people (Brinton & Nee, 1998; Copp, 1995). It is now used more widely to denote the ways of acting and speaking of many diverse groups of people. The key notions are that the actions of the group are related to the setting and are distinguishable from those of other groups in other settings. This study follows Stenhouse (1967, cited in Bishop, 1988, p.5) in regarding culture as " a complex of shared understandings that serves as a medium through which individual human minds interact in communication with one another."

The classroom culture exists alongside the cultures of the school, community and country. Each of these has a sphere of action and they interact in complex ways. Mathematics education in New Zealand today exists in relation to these wider cultures, and each member of a classroom culture is likely to have a unique combination of external cultures. The classroom culture is expected to help prepare students to participate in their outside cultures, thus the skills of communication, social interaction, competition, self-management, and co-operation are essential elements of a New Zealand education (Ministry of Education, 1992).

Bishop (1988) notes that cultures grow and develop as they are lived. A classroom can be seen as a place where the participants share a culture with agreed ways of doing things and an active process of development of those ways.

Norms.

"Norms are explicit or implicit rules of expected behaviour that embody the interests and preferences of members of a close-knit group or a community" (Nee, 1998). Norms of a classroom community are developed and reinforced through communication and interaction between the students, and between students and teacher.

There are three features of norms that are useful in this context (Nee, 1998). Firstly, norms are usually bound to settings or contexts, although there may be norms, which are universal to the culture. Classroom norms are often quite specific in their context, for example what is acceptable while the teacher is looking might be quite different from what is acceptable at other times. Secondly, norms have a reflexive character. Changes in norms usually happen gradually as the participants reflect on the consequences of their actions and evolve new ways of acting. Thirdly, norms have a power to influence the actions of the participants in a culture. People who do not follow accepted norms often find the other participants will attempt to influence them to conform.

Roles.

Within every culture there are 'roles' which are held by the participants. These roles carry responsibilities, consequences and rewards for the holders. Roles have a static aspect, where they belong to individuals or groups, and a dynamic aspect where they are established and developed by the interactions between the people concerned. People in roles are expected by other participants to act according to established norms of the setting but also to interpret the roles according to changes in the setting. Classroom teachers often assign roles to the students as a way of getting things done, and also to enculturate them into the norms of the classroom or wider culture. The teacher may have several roles to play, for example; administrator, guide, mathematical expert, disciplinarian, mentor, and friend. These roles vary with the setting and each have their own sets of associated norms and power relationships.

Power.

One of the features of cultures is that they involve unequal shares of power among the participants (Appelbaum, 1995; Bishop, 1988; Manke, 1997; Tauber, 1993). Classroom power belongs to both the teacher and the students and is both taken and given in very complex ways. A teacher has power from their position as adult-in-charge to direct and guide what happens in the classroom. Some forms of power also derive from their personal qualities. They rely on students' acceptance of their power to be able to carry out their roles. The students have power to give or withhold co-operation, and also some power to interpret what the teacher requires.

There are two main ways to view power in the classroom (Appelbaum, 1995). Traditionally it was viewed as something held and exercised by a person, allowing them certain privileges and responsibilities. In this sense having power provides opportunities and privileges. This is a useful way to interpret some of the interaction in a classroom where the focus is on outcomes and results. Where the focus is on relationships and processes, however, it is more useful to interpret power as an indicator of the meanings attached to practices and interactions. In this sense power is the result of the ways people perceive themselves as acting in relationships. This interpretation was developed by Foucault (Appelbaum, 1995; Rabinow, 1984) and involves a shift in focus from the effects of exercising power, to the origins and process of a power relationship.

Some of the meanings that people attach to what they do can be inferred from how they participate in a power relationship. For example, the ways the students either co-operate with the teacher's directions or avoid that co-operation indicate the areas in which they are willing to assign power to the teacher (Manke, 1997). Teachers also assign power to the students in allowing them to make choices concerning what they will do and when they will do it. Teachers usually carefully control this devolution of power, and students concur by accepting the limits of that devolution. Where students choose to act in ways that they know would not be sanctioned by the teacher, such as being off-task, they are taking a form of power to themselves.

A student may be off-task because they do not understand the mathematics. If the power relationship operating is one of control rather than co-operation, the student may feel that their lack of understanding is in some way a fault and conceal it from the teacher. In such a case the teacher is deprived of some essential input and knowledge about mathematical understanding and misunderstanding.

Sometimes off-task behaviour is openly disruptive. Manke (1997) noted that the sort of actions by students that were seen as disruptive depended on the classroom culture promoted by the teacher. Strict teachers had more disruptive activity and more hidden non co-operation. Such situations resulted in reduced student learning due to the diversion of teacher effort from the task or lesson objectives. Teachers who were able to use power as influence rather than control were able to keep students on task more than stricter teachers (see also Tauber, 1993).

The construction of knowledge by students can only happen effectively when the student is willing to engage with the learning task. A major part of classroom research for many years has focused on ways of constructing the classroom so that students are willing participants. A high degree of co-operation for learning can be a feature of classroom culture, provided the power relationships operating are based on influencing rather than controlling the students (Manke, 1997; Middleton & Spanias, 1999; Tauber, 1993). The work by Hiebert et al. (1997), Davis (1997), Cobb et al. (1992) and Yackel and Cobb (1996) among others, shows how this becomes operational in classroom mathematical settings.

Much of the resistance to schooling in this co-operative mode can be attributed to a pervading view in the community that schools, as agents of the state, ought to operate in controlling ways. Children, therefore, need to be controlled before they can be taught (Appelbaum, 1995; Manke, 1997). Foucault (Interview with Foucault in Rabinow, 1984, p.63) contended that power, law and prohibition historically go together in western nations. This view of power as fundamentally disciplinarian in nature has the effect of conditioning our courts, legislature and schools to be run as institutions devoted to control rather than co-operation. Parents and administrators often have large stakes in the control theory, if only because they have been brought up with the effects of that theory as a major backdrop to their whole lives. Children come to school having lived in a (more or less) benevolent dictatorship all their lives, and often expect teachers as authority figures to act in the same way.

Any teacher or school trying to reconstruct schooling using influential rather than controlling power therefore has to deliberately re-educate the students and their parents in new ways of acting and speaking. They have the support of that part of human nature which values kindness and caring, and also the theories of learning such as constructivism and interactionism that emphasise the inter-relatedness of emotional, social and cognitive well-being. Teachers and students can develop a culture where the students see their growth in understanding as the goal of schooling if they are in a system that allows this option.

Tauber (1993) has provided a theory of power relationships that categorises five varieties of power exercised in the classroom (see table 2.1). Two forms of power are related to controlling students - power to reward and power to punish. Legitimate power, derived from the position of being the teacher, is seen as management related. Two further forms of power are related to influencing the students. They are referent power that derives from mutual reference to agreed norms, and expert power that derives from the mathematical knowledge of the teacher. While Tauber (1993) considers these five forms of power only as pertaining to student – teacher relationships, the Foucauldian analysis above indicates that they can just as easily apply to student- student relationships too.

Table 2-1Tauber's analysis of Power relationships.

Form of power	Type of relationship	Origin of the power.
Reward power	Controlling	Position in the culture.
Coercive power		
Legitimate power	Managing	
Referent power	Influencing	Personal qualities
Expert power.		

Reward power belongs to people as a result of their having control over extrinsic rewards which are valued by others. Like the other forms, reward power is both taken and given interactively according to the cultural norms of the classroom. Coercive power is the teacher's control over punishment. This may also be a feature of some student – student interactions, particularly where social positioning is in question. Reward and coercion operate differently depending on the motivation of the people being controlled. Legitimate power is the result of the teacher being assigned the right to direct learning experiences and the classroom culture. Students assign this power to the teacher by virtue of their acceptance of the teacher – student relationship. These three forms of power are characterised by Tauber as "*positional powers* – you hold that power while you hold that position." (p1).

These first three forms of power seem to be expedient and effective but tend to be short-term and difficult to use well. Teachers who use them a lot "slide into endless, as well as escalating, reprimands and rewards and repeated reference to positional authority." (Tauber, 1993, p.2) Because they are based on control or management they need the teacher to be watching for when the students are complying with or evading instructions, taking up much valuable teacher time and energy.

Referent and expert power are personal powers, attaching to people because of their personalities and skills. Referent power is the "ability to create a sense of oneness, common purpose or identity" with another person. (Tauber, 1993, p.1). The norms of the situation become the referents for behaviour and attitudes, releasing the teacher from the monitoring role. People with this power have charisma or leadership qualities, whether natural or learnt. Expert power attaches to people who have knowledge, understanding or skills respected by others. Teachers using these forms of power relationship are able to channel their energies more into guiding student learning, relying on the students' internal motivation to order the classroom interaction and communication.

THE AFFECTIVE DOMAIN.

Beliefs and motivation.

"If research on learning and instruction is to maximise its impact on students and teachers, affective issues need to occupy a more central position in the minds of researchers." (McLeod, 1992, p.575). Current psychological theory proposes that people are motivated to act by a complex interaction between environmental and internal factors (Pressley & McCormick, 1995). The environment is perceived through the senses and the input is processed by reference to cognitive schemata of previous experiences. These schemata contain beliefs that are invoked to motivate and direct the individual in their responses.

Beliefs are part of the affective domain of our human nature, along with motivations, emotions and attitudes. McLeod (1992) offers a theory of affect that places beliefs, attitudes and emotions on an increasing scale of affective involvement, decreasing cognitive involvement, increasing intensity of response, and decreasing levels of response stability. Since beliefs are the most stable of the three, they are more amenable to research, and can be thought of as directing the other two.

"Students construct beliefs about mathematics, often ones based on the experiences they have as part of mathematics instruction. These beliefs ... (often) undermine the development of sophisticated mathematical behaviours" (Pressley & McCormick, 1995, p415). Common beliefs are: 'Mathematics problems have one and only right answer', 'There is only one correct way to solve any mathematics problem – usually the rule the teacher has most recently demonstrated to the class', 'maths is mostly facts and procedures that have to be memorised' (Perkins, 1995).

McClelland, Koestner, and Weinberger (1992) link motivation to beliefs. Motivation is an observable consequence of belief, where that is taken as a broad concept that can include some conscious cognitive involvement. They postulate levels of motivation, from the conscious self-attributed level to implicit motivation, and suggest that different research means are needed to access the levels. McClelland et al. report that motivations identified from self-attributions in questionnaires and interviews are correlated with beliefs which have more cognitive involvement in their expression. Test instruments that encourage free expression or fantasy are more likely to access a deeper (implicit) level of belief.

Middleton & Spanias (1999) provide an overview of the current state of research on motivation. They provide five points that are consistent across the studies reviewed. Firstly, that "students' perceptions of success in mathematics are highly influential in forming their motivational attitudes" (p79). The difficulty of being sure of what constitutes 'success' for a given student underlines the necessity for observational triangulation when reporting on this factor. Students of different mathematical capability are motivated by different levels of challenge in traditional classrooms. Higher achievers enjoy being extended, while those who perceive themselves to be struggling are put off by task difficulty. In inquiry classrooms, where understanding rather than task completion was the goal, the difference in motivation between capability levels was less.

Pressley & McCormick (1995) point out that most new entrants believe that they can succeed at school tasks, but by the end of year 3 most students are more aware of their failures than their successes. Young-Loveridge (1992) suggests that this reaction may vary across ability levels.

The second point concerned the onset of positive or negative attitudes towards mathematics. Middleton and Spanias (1999) found that teacher patience and support, plus a classroom environment focussed on understanding were the key factors in developing positive attitudes towards mathematics during primary school. Positive attitudes were strongly linked to observed persistence at mathematical tasks and to self-reports of motivation. Where students had good experiences of mathematics in their primary schooling they tended to maintain these over time and to weather pockets of difficulty.

Middleton and Spanias' (1999) third point was that extrinsic motivation needed to be used sparingly and phased out quickly in favour of building students' intrinsic motivation. The way this intrinsic motivation was built depended on the level of challenge that was effective for a student. In addition, students needed tasks that were both real, open-ended, and helped them uncover knowledge in other fields of interest.

The fourth point concerned student perceptions of mathematics. In general, girls and students from minority groups have tended to expect less success than students from dominant sections of society, and boys. In New Zealand Young-Loveridge (1999) found that this expectation came in very early in schooling, and disadvantaged these students before they began to develop well-linked mathematical concepts. Wood (1992) linked such early problems and low expectations of success to the failure to learn or understand mathematics at secondary school.

The last point is a hopeful one. By designing instruction carefully, motivation can be rebuilt. When students learn that their efforts increase their understanding and that they can take more control of their learning they begin to view mathematics in a new light. The key is that instructional patterns need to be supportive of understanding rather than just computational success; they need to be stable over a long time for attitudes to change significantly; and the whole classroom environment needs to be emotionally supportive and culturally respectful (see also Hiebert et al., 1997). Middleton and Spanias (1999) make a plea for in-service and pre-service programmes to show teachers how to influence motivational attitudes as a key to improving mathematical achievement. However, they also claim that there is a great deal yet to be understood in this area.

Perceptions of mathematics and mathematical learning.

Young-Loveridge (1992) studied several aspects of perceptions about mathematics. She found that mathematics ranked highly in the popularity stakes of school subjects among 9 year olds. Boys liked it more than girls (76% to 47%) did, and relatively high achievers liked it noticeably more (65%) than lower achievers (50%). Interestingly, only 45% of the high achieving girls chose mathematics as one of their three most liked subjects compared to 92% of the high achieving boys. These rankings were similar to the results of another study by Elley (1985) quoted by Young-Loveridge.

When asked about what mathematics they liked or disliked all the children responses concerned operations and textbook work. There was little agreement on the aspects disliked (division topped the list at 16%) and addition (35%) and subtraction (21%) led the list for the aspects the children enjoyed. None of the children in the classes surveyed mentioned problem solving or investigative work. Another question asked about why mathematics was important: "There was nothing in the children's answers to this particular question which indicated that they regarded school mathematics as being relevant to real-life problems or in any way useful (to them at the time)" (Young-Loveridge, 1999, p.94). Mathematics was seen to be important mostly for jobs in the future, however, with a few responses mentioning future schoolwork or home related life situations such as shopping.

When asked why they did well at mathematics, the students mentioned effort (50%), ability (19%) and difficulty (12%) as the reasons. When asked about doing poorly, they mentioned effort again (56%), difficulty (13%) and ability (7%). When queried about how they tackled difficult work, most said they would keep trying, and ask the teacher if necessary. Some were likely to miss out the hard ones, especially the lower achievers (35% cf. 24% high group). Girls were more likely than boys to ask their friends for help. Most of the children believed it was wrong to look at a friend's work for help (81% of boys: 68% of girls.) In several of these classrooms Young-Loveridge reported that there was little encouragement for children to work together, while in a few there were systems of peer tutoring available.

MATHEMATICAL KNOWLEDGE AND UNDERSTANDING.

One of the more popular paradigms for describing how people acquire knowledge is known as constructivism (Leder & Gunstone, 1990; Perkins, 1991). The essence of constructivism is that people build their ideas rather than take them in ready made from an external authority. They become able to use their knowledge and understanding to solve problems which are real to them in their worlds (Clements & Battista, 1990; Pressley & McCormick, 1995). People use what they already know and build on it as they are able. Real contexts for the new ideas are important because knowledge is linked in the mind with the social and physical environments in which the understanding developed. This situated view of knowledge can explain why a great deal of what is learnt by rote is simply unavailable to the learner.

This study is based on a variety of constructivist thought known as Social constructivism. In this paradigm personal knowledge construction has been linked to an appreciation of how the social world around a person affects how they can learn (Good & Brophy, 1995). The Interactionist approach to education emphasises the way teachers and students combine to create the world and culture of the classroom (Voigt, 1995; Yackel & Cobb, 1996). "It should be emphasised that learning is an interactive as well as a constructive activity" (Cobb, 1990, p.209). Voigt (1995) interprets interaction in the mathematics classroom in terms of people working together to build culturally acceptable views of experienced reality. When a student asks for help they can be regarded as revealing what they don't know or understand, and also trying to find a socially acceptable explanation of the situation. Perkins (1995) points out that such help seeking reveals a readiness to learn. The teacher's attitude and the classroom social environment need to be supportive of such requests if the student is to construe the situation as an opportunity for growth in understanding rather than as evidence of a fault or lack of effort.

Social constructivism.

Social constructivism involves a view of reality quite different from that of the scientific world. Physical objects of course are real, but their objective reality is filtered through what people believe about the objects in order to make sense of them (Von Glasersfeld 1991). It follows that different people will experience the world differently and objective knowledge will be what is agreed on within the culture one lives in. The objects that children interact with, both people and other physical objects, therefore carry primary meaning for the child within the culture they know. That is not to say that there is no objective reality that everybody experiences, but only that everybody filters their experience through what they already believe, and naturally come to regard their interpretations as fully objective and real. "Mathematical objects are, for all intents and purposes, practically real for the experiencing subject" (Cobb 1990, p 203).

Adoption of a social constructivist perspective has considerable consequences for mathematics education. Mathematics itself cannot be a system of immutable truths, but a system of acceptable ways of representing the world of experience. For example, the laws of number become viewed as culturally acceptable ways of writing and speaking of operations on objects. These objects are either in the world, or symbols that have come to have agreed meaning in the world. That 2+2=4 all over the world simply means that it is agreed that that is how a common experience of humankind will be represented. It is possible to disagree as to whether there are two, or four, but once those matters are dealt with, the notation follows.

Within each mathematical system there must be consistency of notation, but once this is taken-as-shared in a culture, educators are freed from any need to prove or defend the truths of mathematics. They can treat mathematical education as a process of introducing young people into the ways of their culture. This removes any need for learners to always reproduce the notation exactly before they can see themselves as successful. It rather opens up the chance for education to become a progressive process of learners coming closer to the socially acceptable ways of representing mathematical aspects of their reality, while retaining their own self-esteem. Mathematical schooling under the traditional paradigm often makes the mistake of glossing over the individual logical reasoning which links mental objects to their physical or experiential counterparts, prematurely emphasising symbolic notation over the spoken or mental reasoning the symbols represent. Students feel pressed to conform to conventions rather than develop their own understanding.

A social environment for 'coming to know mathematics' (Helme, Clarke & Kessel, 1996) provides a learner with linkages and contexts for the knowledge. Society and social interaction in general are an essential factor in the constructivist view of knowledge. There is no direct way of checking the correctness or truth of our knowledge against reality itself (Sfard, 2000). This is because everything we believe we know has been subject to assimilation and accommodation – fitting our perceptions to our prior knowledge and then testing it against its social acceptability as well as how well its fits our experience of the world.

Assimilation is where a person changes the outside world to fit their beliefs about it, and accommodation is where the person changes their understanding to fit their experience of the world. Education is about facilitating both these processes. Such 'active learning' involves students using a range of cognitive, affective, resource management and metacognitive strategies in their quest for understanding and knowledge construction (Anthony, 1996, 1997). Moreover, being aware of one's thinking and learning enables a learner to be in control of how and what they learn.

Reflection and communication are key processes of active learning. "The process of reflection is central for cognitive psychology, and the process of communication is central for social cognition." (Hiebert et al., 1997, p.5). Reflection on the links made with a mental object, and communication with others about it are essential parts of several models of understanding (Davis, 1992; Carpenter & Lehrer, 1999; Hiebert et al., 1997; Pirie and Kieren, 1994). Reflection on experiences and prior knowledge helps a learner to integrate new ideas and understandings into existing mental schemas. Communication allows the learner to check their growing understandings with others and to modify them, if necessary, towards a taken-as-shared reality. When there is such a reality, groups of people can learn more quickly because they effectively share each other's prior knowledge – the group knows more than the individuals in it. For such learning to become the norm in a classroom the social environment needs to support and value understanding (Cobb, 1994; Hiebert et al., 1997).

In this study, the way the teachers and students view mathematical teaching and learning will be examined, in order to gauge the extent to which the culture of teaching and learning in these classrooms has moved from traditional notions of transmission of knowledge towards the constructivist paradigm. In particular the extent to which the students choose to value success at symbolic manipulation over success at developing their understanding will be explored.

2.3 Teaching and learning with understanding

THE HISTORICAL CONTEXT

The nature of mathematics teaching and learning in New Zealand schools today is the result of centuries of social, political and economic pressures impinging on education. Educational philosophies have developed out of a cauldron of competing and evolving paradigms. The current emphasis on learning with understanding is not new, but a restatement of ideas that have been around for millennia: "The value of wisdom is far above rubies; nothing can be compared with it. Wisdom and good judgement live together, for wisdom knows where to discover knowledge and understanding." (Proverbs 8 v11, The Bible, New Living Translation).

Understanding was certainly a goal for some mathematics educators in the middle of the 1800's. For example Warren Colburn in 1849 wrote a textbook dedicated to children making sense of arithmetic (Lindquist, 1997). Although there have been advocates of mathematics with understanding such as Dewey in this century, there was for a time a more dominant focus on computational fluency. Thorndike's principles of mathematical training were reinforced in mid-century by the ascendancy of the behavioural paradigm in psychology and education (Nuthall & Alton-Lee, 1990; Lindquist, 1997). Behaviourism in education was part of the scientific approach to knowledge and encouraged teachers to see learning in mechanistic terms. In such as an environment affective, social and relationship factors of teaching and learning were seen as either unimportant or impossible to study (McLeod, 1992).

NEW APPROACHES TO UNDERSTANDING EDUCATION.

The swing back towards learning with understanding in the 1970's to 1990's has been part of a much wider movement in western society. Structuralism and modernism, philosophies related to a scientific rational approach to knowledge, were succeeded by post-structuralism and post-modernism which emphasised the inter-related nature of the world. There are many aspects to these, including the feminist movement and new forms of art, music and architecture. In education social constructivism has developed, with many of its principles being in harmony with post-modernist movements (Lather, 1992).

Contributing to the development of social constructivism was a growing realisation that the new maths and a purely behaviourist approach were not producing mathematics learnt with understanding. Mathematics teaching and learning as an enterprise gradually changed its focus from being "a set of rules and formalisms invented by experts, which everyone else is to memorise and use to obtain unique correct answers" (Romberg, 1992, p. 453) towards "a view that learning mathematics involves processes of abstraction, inference and logical reasoning." ... it should instead emphasise construction mathematical meaning." (Wood, 1996, p.85).

Constructivism, and social constructivism in particular, offered something that had been missing – a holistic view of how people learn (Carpenter & Lehrer, 1999; Leder & Gunstone, 1990). The idea grew that people build their knowledge from what they already know, and that the social and emotional environment of the learner mattered a great deal (Neyland 1995). Allied to the insights of social constructivism was a parallel set of research results from cognitive psychology (Good & Brophy, 1995; Pressley & McCormick, 1995) that gave insights into how memory and cognition were linked with perception and motivation to produce effective learning.

In New Zealand *Mathematics in the New Zealand Curriculum,* (MINZC), (Ministry of Education, 1992) provided a "fundamentally new approach to the teaching and learning of mathematics, challenging historical understandings of the nature of mathematics education and contemporary practice" (Walshaw, 1994, p.2).

> The key outcome of mathematics education is the development of the ability to apply certain of the essential skills described in the *New Zealand curriculum Framework*. ... A balanced mathematics programme includes concept learning, developing and maintaining skills and learning to tackle applications. These should be taught in such a way that students develop the ability to think mathematically." (Ministry of Education, 1992, p.10-11)

No longer was mathematics education officially aimed at procedural facility but at application of skills and mathematical thinking.

THE PLACE OF UNDERSTANDING.

"In the era of behavioural objectives, we heard less about meaning, thinking, and understanding, partly because they were difficult to measure. If we wrote an objective saying that students should understand, we were in trouble" (Lindquist, 1997, p.xi). In the era of inquiry classrooms, learning with understanding, communicating and reflecting on learning are among the objects of teaching, rather than assumed by-products. "Learning to communicate about and through mathematics is part of learning to become a mathematical problem solver and learning to think mathematically" (Ministry of Education, 1992, p.11).

In the past understanding as a goal has suffered from the lack of a theory of what it is and how to promote it in a mathematics classroom.

> Most teachers would say that they want their students to understand mathematics, and in fact that they teach for understanding. Teachers generally believe that understanding is a good thing. However, we have not always had a clear idea of what it meant to learn mathematics with understanding, and we have had even less of an idea about how to tell whether a classroom was designed to facilitate understanding. (Hiebert et al., 1997, p.3)

Several researchers have recently put forward theories of mathematical understanding, including Davis (1992) and Pirie and Kieren (1994). A key idea in these theories is that understanding develops recursively, with reflection as the major strategy behind its growth. Pirie and Kieren (1994) suggest that a person attends first to the surface features of something new and tries to relate those to something that is already known. If links can be made, then this first level of understanding develops by exploring the surface features, forming a tentative mental object or image of what ever it is. This mental object becomes steadily more complex and better linked into existing mental schemata as the person reflects on and works with the new thing. As understanding develops the new thing becomes reified in the mental schemata of the person, contributing to their whole worldview. The new thing can be an experience, physical object, notation or symbol, or a part of a language. This model can explain a great deal of what people do as they come to know something.

Lindquist (1997) enlarges the theoretical position of understanding by referring to Skemp's (1987) distinction between relational and instrumental understanding. Skemp "defined relational understanding as knowing what to do and why, and instrumental understanding as knowing what to do or the possession of a rule and ability to use it." (Lindquist 1997 p5). This is close to Sfard's (1998) distinction between possessive and relational views of learning. It is sometimes useful to think of learning, understanding or knowledge as something possessed by a person: as mental objects or processes within their mind. On the other hand a person can relate their knowledge to other areas of life in a dynamic way, seeing knowledge as growing and developing, and interacting with other people in the use of the knowledge. This is another aspect of the dynamic interplay between the cognitive and social sides of a social constructivist theory of understanding.

Understanding in the classroom.

As Hiebert et al. (1997) pointed out above, most teachers want their students to understand mathematics. Skemp's (1987) distinction allows us to see how understanding in a traditional classroom is not likely to be the same thing as in an inquiry classroom. Instrumental understanding was the main focus of traditional teaching, where the ability to execute an appropriate algorithm was the criterion of success. Examinations and curricula up until 1992 in New Zealand emphasised instrumental understanding. All people needed to know was how to do the examples since it was assumed that students would be able to apply them outside the classroom. Most parents and teachers of present day students were schooled under that unquestioning culture. Even today the cry of many secondary students is 'Just tell me how to do it!'

Relational understanding takes a lot more initial effort on the part of the learners and the teacher. Pirie and Kieren's (1994) model of understanding locates relational understanding in the higher levels of their model, requiring reflection and discussion with others to be developed. Several other recent research reports also identify relational understanding as a key goal of teaching and learning (Anthony, 1998; Anthony & Knight, 1999; Middleton & Spanias, 1999). New Zealand's curriculum for mathematics promotes relational understanding strongly since communication and problem solving skills are among the eight essential skill areas to be interwoven into all parts of the curriculum. (Ministry of Education, 1992).

How far New Zealand teachers have come in working towards relational understanding is uncertain. Walshaw (1994) reporting on the implementation of the new curriculum by a group of thirteen secondary teachers, concluded that there was a very wide range of ways in which the curriculum had been implemented. "The reality of the curriculum document for each teacher is essentially unique" (p. ii). Many pressures on the teachers combined to reduce the extent to which they adopted the constructivist notions of building understanding through practical contextual mathematical experiences. In the early years of the curriculum implementation Walshaw noted that: "Whilst maintaining a sense of traditional practice, the case study teachers are beginning to create environments conducive to students thinking about mathematics, are selecting instructional materials and activities that encourage students learning and are beginning to engage students in a discourse which enhances their mathematical learning" (p. ii).

Changes such as these cannot happen quickly in the classroom. Lindquist (1997) and Walshaw (1994) both point out that there are many influences on what a teacher does in the classroom, the curriculum being only one. A teacher's personal expectations, the influences of their past, the cultures of the school and the community, the resources available and the policies of the school all must be weighed together and filtered through the teacher's convictions and beliefs. It may be that the most potent influences are those the teacher is often least aware of – their beliefs about the nature of mathematics and education. In the daily rush of teaching there is little time or encouragement for teachers to reflect on their practice, and even less time for both reflection and communication between teachers, nor easy access to expert practice. There are many forces for keeping the status quo in spite of a wide recognition that the situation with mathematics teaching and learning is not satisfactory (Stigler & Hiebert, 1999).

There are however potent forces pressing for change. The curriculum is one, but, however worthy its aims, it is imposed more from above than from the teachers themselves. (Walshaw, 1994). The impetus for better teaching comes largely from the integrity of the teachers who constantly want their students to learn more effectively. The new philosophies and principles of learning with understanding offer hope and structure to teachers looking for these better ways. There is still a problem to be overcome in finding ways to widely disseminate the new practices. When such ways are developed and linked to socio-economic and political forces pressing for better outcomes from schooling, the stage will be set for the gradual development of more effective teaching and learning (Stigler & Hiebert 1999).

TOWARDS A THEORY OF INQUIRY TEACHING AND LEARNING.

Mathematics is part of the classroom culture and there are norms and roles for mathematics time that form the cultural backdrop within which mathematical meaning can be negotiated. The teacher establishes standards of interaction that they believe are likely to facilitate learning. These are likely to be internalised by the students as they accept the teacher's power, and the standards will recede into the background as classroom norms.

In order to look at changes in the practice of teachers it is necessary to have a theoretical framework for inquiry-based teaching. Considerable progress has been made on this in recent years (see Cobb et al., 1992; Davis, 1997; Hiebert et al., 1997; among others). Hiebert et al.'s (1997) analysis of learning with understanding, supported by Manke's (1997) and Tauber's (1993) analyses of power relationships forms the basis for the current study.

Based on findings from four international reform projects ('Cognitively Guided Instruction', 'Cognitively Based Instruction', 'Problem Centered Learning' and 'Supporting Ten-Structured Thinking'), Hiebert et al. (1997) propose five dimensions of classroom culture which are important during mathematics teaching and learning:

- the nature of classroom tasks,
- the role of the teacher,
- the social climate of the classroom,

- the use of mathematical tools, and
- equity and accessibility of mathematics.

These dimensions link together as a system to form the culture of the classroom and each have a number of core features. Hiebert et al. (1997) suggest that their model could provide a "set of guidelines that teachers can use to move their instruction towards the goal of understanding. ... The dimensions and the core features within each dimension, provide guidelines and benchmarks that teachers can use as they reflect on their own practice" (p7). Some of these features are essential and some optional for optimum learning, and together they form an interlinked system of practices associated with inquiry-based learning.

Hiebert et al. (1997) make three assumptions to guide their theory:

> First, the primary goal of mathematics instruction is the development of conceptual understanding and progress towards this goal is possible for all students. Second, understanding is developed in the same way for all in the social community of the classroom. ... Third, the dimensions ... work together to shape a system of instruction that enables all students to learn with understanding (p66).

THE NATURE OF CLASSROOM TASKS

Classroom tasks provide students with experiences on which to form their beliefs about mathematics. It is part of the craft of teaching to select tasks that will suit the class since different tasks lead to different learning outcomes. Daily routine computations that must be finished give students quite a different message from situations where mathematics time focuses on problem solving and investigations. Both approaches lead to facility in computation, but in the inquiry classroom the students also learn to think mathematically (Boaler, 1998; Cobb et al., 1992; Davis, 1997; Wood & Sellers, 1997). Anecdotal evidence indicates that most mathematics teachers in New Zealand today use a mixture of those approaches but are also concerned that they do not focus enough on problem solving (discussions at NZAMT conferences and local Mathematics Association meetings.)

Hiebert et al. describe four core features of classroom tasks which lead to inquiry based learning with understanding. The first is that the tasks must be more than routine. They must be problematic for the students and must interest and challenge them. That is not to say they cannot be algorithmic. For students who are coming to grips with notation, successful practice exercises of a method can be very rewarding. The key point is that the process of solution is not the mathematics, but a tool for mathematical thinking. When the mathematics in question has been embedded into some familiar contexts, and the student becomes aware that there are some similarities in approach and strategy, then they will be ready to internalise their own version of a useful algorithm. This will help them to interpret the situation in mathematical terms and solve it usefully. By embedding the mathematics in familiar situations, linking in with the prior knowledge of the students and capitalising on their interests, students become open to understanding algorithms in a relational way. If the mathematics is within the capability of the student but also provide a way to extend them; they are likely find the challenge within their grasp and persevere.

The second feature is that the mathematics must be the problematic part of the situation. There is little point in trying to teach students how to use some equipment and to engage them in mathematical problem solving with it at the same time. Place value rods are a classic case. Unless a child has a near-automatic understanding that each rod carries place value meaning, they will be unlikely to connect to ten-structured thinking, re-naming or carrying. Instead they focus on pleasing the teacher by learning to manipulate the rods mechanically. "Students must be able to us the knowledge and skills they already have to begin developing a method for completing the task" (Hiebert et al., 1997, p.8).

The third core feature is that the mathematics behind the tasks needs to be important to the students. That is, the mathematics must be related to the student's everyday lives, and must extend their current concepts, allowing space for productive reflection. This is how relational understanding grows in students – engaging in a task which is in a familiar context, finding it challenging and interesting, and pushing the boundaries of their current grasp of mathematics. Vygotsky's 'zone of proximal development' carries the same idea, that people have an area of experience which has become more or less internalised and a zone beyond that which is at the front of their consciousness (Pressley & McCormick, 1995). Being available for thinking, as it were, mental objects in this zone are easily reflected on and linked to other objects in the student's mental schemata (Yates & Chandler, 1994).

Above all, tasks used in mathematics teaching need to encourage the students to think deeply and to discuss their thoughts with others. This is more likely to happen if the tasks are problems to be solved rather than procedures to be practised and if the tasks are closely related to the real world of the students. Henningsen and Stein (1997) agree with Hiebert et al.(1997) in their analysis of what a classroom instructional task should be like. They emphasise the need for the classroom to be a place where high level thinking is the norm[1], and where, the tasks and the social culture reinforce each other reflexively in pressing the students towards actively engaging with the mathematical features of problematic tasks.

THE ROLE OF THE TEACHER

Hiebert at al.(1997) point out that in traditional teaching the teacher acted as "the main source of mathematical information and the evaluator of correctness." (p.8). In inquiry teaching, the teacher selects tasks, engineers learning situations to be within the culture and experience of the students, facilitates the establishment of an inquiry culture in the classroom, and mentors and guides the students as they discuss and reflect on their experiences and methods. Informing all these parts of the teacher's craft is the belief that the students will drive their own learning by their curiosity and desire to master their world.

Vygotsky suggested that anything a child learnt needed to first 'appear' in a social setting, then become internalised through practice and familiarity (Bliss, Askew & Macrae, 1996). Social relationships and their setting were thus seen as essential to successful learning. The teacher's role thus includes providing an emotionally safe environment; being sensitive to their home and community circumstances, and creating a culture where students can put aside their other concerns during class time. Some students who are disaffected from the culture represented by the school would most likely be unwilling to put effort into 'succeeding' in the terms of the school (Perkins, 1995; Pierce, 1994).

The sorts of questions and answers expected in a classroom can be critically important (Boaler, 1998; Cobb, Wood, Yackel & McNeal, 1992; Davis, 1997). Cobb et al. identified types of responses as a key feature of an inquiry classroom climate that could

[1] Stevenson (1998) provides a questionnaire for classroom use that examines the prevalence of high and low level thinking in the classroom.

be used to distinguish between that and a more traditional climate. In Cobb et al.'s inquiry classroom the teacher established a classroom climate where the students expected to justify and explain their reasoning. Relational understanding was the norm and the students felt safe to make mistakes and to develop understanding over time. The teacher would initiate a situation that invited the students to import their own cultural reality into it, guiding them to mathematise aspects of the situation in ways that were real to them. She would respond to student questions and explanations with invitations to the class to discuss possible answers, and repeatedly encourage them to justify their responses. The mathematics constructed in this classroom was interactive, linked to cultural reality, and always in a state of guided growth. Seatwork had two functions, firstly as an aid to thinking, and secondly as a useful notation for summarising solutions. The symbols and algorithms of mathematics were the tools rather than the objects of mathematical understanding.

Cobb et al. (1992) also examined a classroom where the teacher was following traditional ways of questioning. Here the teacher set tasks that practised the use of algorithms; learning the steps of the algorithms was the primary objective of the lessons. Contexts and social reality were the tools here, rather than the background environment of the teaching. Questions from the students were couched in 'how do I ..." terms, and were answered in procedural ways. The mathematics constructed in this classroom was focussed on the processes of mathematics, with correct answers valued and celebrated. Discussion in this classroom was centered on processes rather than relationships, and justification was by appeal to 'correct' procedures rather than reasonable mathematical logic. The type of understanding expected by the teacher differentiated the type of mathematical understanding constructed in the two classrooms.

Other researchers have noticed similar differences between traditional and inquiry classrooms (e.g., Boaler, 1998; Cowie, 1995; Davis, 1997). Davis (1997) identified the way the teacher listens as a critical factor, suggesting that a teacher who listens for whether a student is doing an algorithm 'properly' will be focussing on process, while a teacher who listens for whether a student is making progress towards understanding the reasons for the algorithm is likely to be focussing on relational understanding. Davis contrasts a teacher who listens for particular responses and prompts for them, with the same teacher later in the year listening for evidence of a student's current understanding and prompting them to think more deeply. Cowie (1995) describes a number of ways a teacher can focus on student thinking. "When students are working it is our role to facilitate their thinking. The challenge in this is to help without telling because telling often involves doing the thinking for the student." (p.56). Jacobson and Lehrer (2000) describe a study of year 2 classrooms where student talk was used as a window into their thinking during a topic on 'geometry through design'. The classroom conversations became more productive of deep thinking and knowledge construction as the teachers became more aware of how student talk could serve to inform their pedagogy. They encouraged the students to share their ideas and helped the other students to critique and develop those ideas. That this could happen in year two indicates strongly that deep thinking can occur in suitable social climates at any level of schooling. These authors all concur with Hiebert et al.'s two core features of the teacher's role – firstly to guide the mathematical activity of the students, and secondly to establish a classroom culture in which communication and reflection are expected and rewarded.

Teachers have to balance the demands of the learners, themselves, the environment (school and community) and the subject matter when designing a learning experience. The official curriculum and the teachers' own historically determined attitudes constitute other dimensions to be taken into account. Sosniak, Ethington and Varelas (1994) found that teachers did not have a single coherent viewpoint on teaching and learning, but moved from progressive to traditional as the matter under question moved from teachers considering the theory of teaching and learning, towards the actual practice of the classroom. In other words, the reflections and intentions of the teachers led their practice by some distance.

Knight and Meyer (1996) investigated the espoused needs of mathematics teachers as they implemented the 1992 curriculum. The most common needs were for time and resources rather than for professional development. Teachers felt that they were familiar with the intent and goals of the curriculum and in sympathy with those goals. Consequently there was resistance to attending courses that focussed on raising awareness of the intent and conceptual background to the new curriculum. Such attitudes could be expected to make the introduction of investigative or inquiry learning rather difficult.

THE SOCIAL CULTURE OF THE CLASSROOM.

Hiebert et al.'s third dimension – the social culture of the classroom – sees the teacher as creating the classroom climate that promotes student expectations of discussion, explanation and justification as the normal activities of mathematics time. Hiebert et al.(1997) identify four key features of the social climate needed to facilitate reflection and communication.

The first feature is that "ideas are the currency of the classroom" (Hiebert et al.,1997, p9). Student's ideas have the potential to contribute to everyone's learning and consequently warrant respect and a response. This means that thinking rather than answers are valued and celebrated. By sharing ideas, the students are communicating about their own worlds, and will generally be more engaged because they have a stake in what is happening. Rather than a few students providing the solutions to sums and others writing them down and kidding themselves that they 'could have done it', the results in the classroom are methods not answers. The thinking of all students is valued because the general expectation is that everyone will come to understand.

The second feature of the social culture of the classroom is that students come to recognise that there is more than one way to solve a problem. Several students are likely to have slightly different approaches to solving a problem. The respect that students develop for each other's thinking has an important side effect – that of reducing the academic hierarchy associated with traditional classrooms and improving the social equality of the students.

Thirdly where answers are not the object, mistakes cease to be wrong, but are reframed successfully as simple dead-ends. Students are less inclined to hide their mistakes. Other researchers have noted the critical importance of errors in mathematical learning. Cobb et al. (1992) emphasise that in an inquiry environment the search is for truth rather than for procedural accuracy. They note that errors in an inquiry classroom don't carry the emotional baggage of 'being wrong'. The result is seen as ineffective rather than wrong. The consequence is that a further search for truth is needed, and the students go on without feeling that they are personally at fault.

Bishop (1988) points out that where students come to regard the obtaining of correct answers as the object of doing mathematics, their learning becomes impersonal and separate from the rest of their lives. If they are struggling with the mathematical level of the work, they are likely to focus on their need for self-esteem rather than mathematical goals, accepting any way of getting right answers as reasonable and right in their culture. In the traditional classroom the search is for correct answers by correctly following procedures, and being wrong is personal – emotionally negative. How the teacher treats errors is therefore critically important (Anthony, 1998; Booker, 1997). In an inquiry classroom the teacher leads the students to see their own errors by asking for justifications, and allows them the space to feel good about the progress they have made. The classroom becomes a place where it is safe to be in error, since the object of the enterprise is improvement over time.

Booker (1997) offer a five step process for intervening successfully in generative ways where the students will be led towards understanding through re-examining their ineffective solutions. His process starts with the student or teacher identifying an error by noting logical inconsistencies, then the teacher looking for the source of the error and leading the learner to see the error and accepting it. By providing opportunities for re-examining the language, process or strategy the learner is encouraged to reconstruct the thinking in more logical ways. Finally the teacher helps the learner to generalise the new learning into related areas.

Hiebert et al.'s fourth feature of the social culture of the classroom is that the logic and structure of the mathematics must be the criterion of error or truth. In the traditional classroom error is what the teacher says is wrong, regardless of what the student thinks. This happens because the teacher is focussed on teaching process rather than on the growth of student understanding. When the internal logic a student uses while moving towards understanding an accepted solution is not valued as a 'step in the right direction', the likely outcome is that the student gets the message that their thinking is not important. Their basic but flawed understanding is supported by sensitive questioning so that it can grow towards scientific or logical intuition and understanding. In an inquiry classroom error is determined interactively by the evident inadequacy of a solution (Anthony, 1998). Where the teacher expects the students to explain their reasoning the students come to see that justification of their answers is a normal part of mathematical thinking..

The teacher also has to take into account that students might interpret the tasks in ways the teacher had not intended (Anthony, 1994). "For some it might be a matter of trying to recall what the teacher told them to do, while for others the focus could be on mathematical sense making." (Cobb, 1990, p.201).

Voigt (1995) regards the mathematics classroom as a microculture that is developed over time. "In time, the negotiation of meaning forms commitments between the participants and stable expectations for each individual. Through mutual accommodation, the participants form the impression that they know what mathematics teaching and learning is." (p.176). Thus the classroom is a place where interaction between the participants creates and constitutes the culture of the situation. What comes to be taken-as-shared moves from being considered explicitly to being a tacit part of the everyday culture.

However, there are situations where students do not accept the norms promoted by the teacher, or where the students are able to act largely outside the norms pressed on them by the teacher. If these situations are regular features of the classroom and the students are able to avoid the teacher's cultural interpretations, it is likely that some ways of acting and speaking which do not help mathematical learning will develop and become normative. These norms will necessarily be hidden from the teacher but will operate where the goals and beliefs of the students differ from those of the teacher. "The pattern of interaction is a structure that is not necessarily intended by the teacher. In the dynamics of human interaction, the pattern is constituted turn-by-turn" (Voigt, 1995, p.179).

Cobb (1995) has suggested that where mathematical discussion can take place between students of comparable understanding, learning is likely to be greater than in situations where there is an imbalance of understanding. Situations where the participants accepted each other's understanding as comparable to their own led to genuine listening and thinking. In his research of junior classrooms, where groups of students contained a variety of abilities it was usual for the students of lesser ability to accept the answers of their mathematical superiors without question. In this situation the negotiation of meaning was one sided and consisted of assertions which were written down without thought on the part of the other students, resulting in little learning on either side. A complicating factor in this socio-mathematical situation was the possible existence of an imbalance in social standing that went against the imbalance in mathematical standing. "Learning opportunities do not necessarily arise for children in small group work merely because they explain their solutions to others; it seems crucial to consider the social situations within which they develop their explanations." (Cobb, 1995, p.73). Cobb suggested that while the mathematical standing could influence social standing, the power derived from social standing would be the stronger influence on whose answers were taken as true.

MATHEMATICAL TOOLS.

The fourth dimension of the inquiry mathematics classroom concerns the mathematical tools used to support learning. For Hiebert et al. (1997) these include written and oral language, procedural skills; symbols; notation; and physical equipment. When these things are thought of as tools of learning rather than the objects to be learnt about, the focus of teaching and learning can shift away from getting correct answers on paper towards the reasons why they are seen as correct. As the students use the tools they develop meaning based on a relational understanding of mathematical ideas. Written mathematics becomes an aid to learning and understanding by recording progress towards a solution, and the equipment of the classroom can be used as a support for the building of well-integrated ideas.

Hiebert et al.(1997) contend that tools shape the way people think. Anyone who has some facility in a foreign language is aware of ideas that can be expressed far better in one language than another, and it is the same with mathematical language tools. The type of tools used influence the type of mathematical ideas that can be understood. With certain kinds of tools it is possible to clarify ideas which are otherwise unclear. Thus the selection of appropriate equipment, language and notation is part of the everyday craft of a teacher.

Symbolic notation

When these principles are applied to the use of symbolic notation some important features of an inquiry classroom emerge. Exploring what notation is the best to represent a situation becomes an integral part of the process of mathematising. Writing down the steps in a process becomes a journey of unfolding the truths of the situation represented rather than one of conforming to a rule of the process.

This can be illustrated by the example of a student using repeated addition to represent a '5 lollies in each of 3 bags' situation. A student who uses repeated 5's to stand for the lollies is directly modelling the situation. If another student suggests they could use multiplication for the same purpose, they might be led to consider the lollies in terms of groups rather than repeated ones. Their understanding of how the symbolic tools of digits and operations can work for them is likely to deepen. In comparison, in a traditional classroom the student would be likely to expect to learn the new process but not to generalise or link it to a wider group of problems.

Physical equipment

The use of physical tool is part of the stock in trade of the teacher, but recent research has uncovered some significant pitfalls. Tools such as place value rods and blocks don't carry implicit meaning with them; their meaning needs to be learnt by the students before they can be of use in representing mathematical situations.

Cobb (1995) argues this case with regard to the hundreds board. His analysis indicated that "children's use of the hundreds board did not automatically support the construction of increasingly sophisticated concepts of ten. However, children's use of the hundreds board did appear to support their ability to reflect on their mathematical activity once they had made this conceptual advance" (p.362). Students who were counting by ones on the hundreds board saw the tens as place keepers in the counting process, useful reference points to see how far they had got. They could use the board to keep track of the tens, and even count from e.g. 31, 41, 42, 43; to add 12 to 31. When asked to explain their reasoning they would represent tens by a full set of fingers. When they had some experience with strips and singles to make images of tens as single units they were able to do the same process in a subtly different way. They put up single fingers to stand for the tens in their sum, using the same fingers for ones when it suited them. This indicated that their concepts of tens had progressed from what Young-Loveridge (1999) called single unit understanding, to multi-unit understanding. The hundreds board could represent either concept but it took other tools to move the students along the conceptual path.

Ritchie (1991) also looked at the circumstances needed for the use of mathematics equipment to help learning in mathematics. He noted that there are three ways equipment can be useful: as external memory; to keep students on a solution track where the equipment directs the use of strategies; and as conceptual models for number or spatial ideas. An important distinction was between 'using mathematics when working with equipment', compared with 'using equipment to do mathematics' (p.82). The first focussed the learner on the mathematics rather than the equipment and left them free to bring in other ideas, while the second limited the learner to the possibilities of the equipment itself. Ritchie identified five problems associated with using equipment:

- Competent use of equipment is not a prerequisite for being able to do the associated mathematics, though teachers may believe that it is. On the other hand, if students are to focus on the mathematics, they need to be able to recognise the ways in which the equipment represents the concepts involved. This takes time and guidance.
- Students often fail to transfer mathematical concepts between logically equivalent tasks without specific help to recognise the links.
- Sometimes the very familiarity with equipment can prevent it being useful as an aid to introduce or illustrate fresh ideas. Existing mental schemata may not easily accommodate the newer concepts or associations.
- Using equipment may discourage children from using higher-level strategies. The equipment, in its ease of use, may hold children at lower levels of abstraction rather than encourage them to transfer their reasoning towards symbolic manipulations. However, Pirie and Kieren (1994) note that students who are given encouragement to investigate ideas seem to move back and forth between concrete and symbolic manipulation as their concepts become more complex. The learning environment established by the teacher may well be the key to which effect is dominant.
- The cognitive space demands of using the equipment may reduce learning. Working memory may be quickly overloaded in unfamiliar situations.

Language

In an inquiry classroom language is seen as a tool of communication between peers as they search for effective ways to solve their problems. Language has dual purposes, being a vehicle for thinking and a vehicle for reporting to the group, class or teacher on what has been thought (Adler, 1999). Mathematical notation becomes something to think with and something to put on paper so others can see the way you are reasoning through a problem. When there is an agreement it becomes a way of representing a consensus on the result.

Where the everyday language of the students is different from that of the teacher, or where students have an incomplete grasp of the meanings of mathematical terms in use in a classroom, there is considerable potential for misunderstandings between the participants. By understanding mathematical language, the students become able to take part in mathematical discourse, and grow in both mathematical efficacy and confidence. Adler (1999) suggests that teachers should take care to teach notation and language in small bites, so they become usable mathematical tools in their own right – mathematical language should be made visible so that it can become invisible to the classroom participants.

The discussion on the role of tools illustrates the variety of tools available to teachers and students. However, according to Hiebert et al.(1997) the choice is not the central factor:

> What seems to be important is not which tool a teacher chooses to introduce into the classroom, but rather that the teacher thinks carefully about the way in which students' understanding might be shaped by using particular tools. This kind of thinking is fruitful because it requires a thoughtful analysis of the mathematics in which the students will be engaged and the kinds of understandings that might be left behind (p.63).

EQUITY AND ACCESSIBILITY

Equity means that all the students are able to build deep understandings about mathematics: "Every learner – bilingual students, handicapped students, students of all ethnic groups, students who live in poverty, girls, boys – can learn mathematics with understanding." (Hiebert et al., 1997, p.65). Hiebert et al. cite several studies that support the idea that all students are able to reach well-integrated understandings of mathematics and claim that the lower achievers particularly need to learn with understanding if they are to break the failure cycle.

Equity resides in the intentions and expectations of the teacher and in the established interactions and norms of the classroom. Power relationships within the classroom need to be positive and supportive for the equity goal to be reached. As reviewed earlier, Tauber (1993) says that power has five forms, three of which can tend to promote negative feelings and relationships, while the other two are positive for all concerned. For a classroom to be operating equitably the power environment needs to be based on respect and shared purpose.

Equity does not mean that all students can or should learn the same mathematics. Rather, every learner should be provided with the particular mathematical experiences that might enhance their world and expand their understanding. The culturally situated socio-political nature of schooling means that every school and teacher will make choices as to the contexts and mathematical notation deemed appropriate for their students (Foster & Tall, 1996). Every student needs access to the learning environments that are likely to assist in developing the understandings most appropriate to their lives and cultures. That this doesn't happen yet in New Zealand is evident in the results of the regular surveys by the National Education Monitoring Project team (Flockton & Crooks, 1997).

The central problem of inequity stems from schooling having a political side, in that the dominant political group in the country determines the curriculum (Appelbaum, 1995). In New Zealand that has been the business oriented middle class of society. Until recently the values of this group have resonated with a view of mathematics as skills based, and of schooling as outcomes oriented. Children who were not raised with the values of this group were likely to find schooling in general, and the mathematics in particular, somewhat foreign to them. They are inevitably lost and confused until they learn enough of what is expected of them to survive and learn effectively. Generally people seriously underestimate the importance of cultural security and familiarity as a background to effective learning (Barton, 1993; Ohia, 1993; Wood, 1992).

Where there are differences in the cultural backgrounds of some of the students and the teacher, there is the possibility that the teacher might not notice the problems being experienced by some of their students. "Teachers and students bring their own agendas to the classroom – agendas with potential for significant conflict. For their own reasons they often conceal these agendas beneath a public shared agenda of co-operation, or perhaps beneath some other shared agenda." (Manke, 1997, p.7). In a classroom that is committed to equal access to mathematics for all students it might become safe for those concealed agendas to be acknowledged and perhaps harmonised.

Skovsmose and Nielsen (1996) in a description of the Danish project called Critical Mathematics Education, argue that culture and conflicts raise basic questions about discrimination: "Does mathematics education reproduce inequalities which might be established by factors outside education but, nevertheless, are reinforced by educational practice?" (p1257). Critical Mathematics Education tries to bring the political facet of education out into the open. It tries to make the values and goals of classroom practice transparent to the teacher and the parents, and to encourage the students to reflect on the socio-political context of the mathematics.

Most of the features described under the other dimensions have consequences for equity and accessibility in that they encourage participation by every student. Where students feel their ideas are going to be respected and used, they are more likely to share them. Where students feel they are expected to think and reflect on the mathematical aspects of their own lives, they have the opportunity to build their own culturally appropriate mathematics. These conditions are likely to occur where the teacher encourages rich discussions of mathematical ideas and values the emerging understanding.

Tasks for equity need to be accessible to the students, both culturally and mathematically. They need to "invite the students to problematise the mathematics of the situation, and invite the students to use the mathematical knowledge they already have" (Hiebert et al., 1997, p.69). This knowledge is from both their community and the school cultures. Thus teachers are more likely to be effective if they can draw from a knowledge of the culture of their students' communities. Reflection and discussion about the tasks and results is more likely to be free and effective if the language of the classroom is similar to the language of the community. In contrast, if the classroom language and experiences have little to do with the world in which the student moves, the classroom mathematics is unlikely to transfer to that world.

The teacher's role in equity is clearly vital. In an inquiry classroom "the teacher knows each of their children well, understands the mathematics that should be learned, selects tasks that enables each student to engage in problematic mathematics, and orchestrates the complex world of the classroom so that children reflect about their thinking and participate in mathematical discussions" (Hiebert et al., 1997, p.72.). Accordingly, one of the key roles is to train the students to interact effectively, especially in group work and discussions.

2.4 Summary.

This study is concerned with mathematics teaching and learning in year five (about nine year olds) in two New Zealand classrooms. As noted above, New Zealand is in a process of change in teaching paradigms. The beliefs and expectations of the teachers might be expected to be composed of both traditional and inquiry based approaches to teaching and learning.

Hiebert at al.'s (1997) five dimensions of how an inquiry classroom operates can serve as a blueprint for analysis of classroom interaction. This is enhanced by insights into the importance of the teacher's beliefs and expectations for mathematics teaching and learning (Boaler, 1998; Cobb, Wood et al., 1992; Davis, 1997), and the role of power (Tauber, 1993; Manke, 1997).

Teachers moving between the paradigms of traditional and inquiry classrooms have the problem that their past experience of being schooled and the expectations of the community press them towards traditional roles, while the curriculum, recent research, and their training and professional development programmes all press them towards the roles described in this review. Teachers are only human, and change cannot be expected to happen overnight. The world of western education is in the middle of a conceptual revolution, and this will inevitably take time to happen (Thagard, 1992).

3. METHODOLOGY

3.1 The research design.

Research in mathematics education has been described by Schoenfeld (1994) as "theoretically based, disciplined ways of enhancing our understanding of mathematical thinking, learning, and teaching". Within those parameters, research designs can take many forms and are generally classified as either quantitative or qualitative. Until the 1970's almost all education research was quantitative in nature. As the limitations of the empiricist/quantitative approach to educational research became apparent (Freudenthal, 1991; Nuthall & Alton-Lee, 1990; Schoenfeld, 1994), many researchers reframed mathematics education in terms of cultural systems (Appelbaum, 1995; Bishop, 1988; Lester, 1996; Steffe & Wood, 1990). They called on research methods from anthropology and sociology to test the new insights (Eisenhart, 1988; Constas, 1998).

> The experimental paradigm is both theoretically and practically inadequate for understanding the complex interactions that take place between students and their teachers and the ways student's lives are affected by their school experience.... We have made substantial progress ... in our understanding of the questions that need to be asked, the research methods that are needed to answer these questions, and the kinds of answers that will be most useful to teachers. (Nuthall & Alton-Lee, 1990, p.548)

The sort of activities going on in a classroom setting do not easily lend themselves to being controlled in the scientific sense, since the number of variables involved is immense. Consequently any properly scientific study of a classroom was likely to run into problems with the volume of data, assuming it could even be measured and collected objectively (Burns, 1997; Freudenthal, 1991; Merriam, 1998). This doesn't rule out quantitative designs altogether, but indicates the need for careful consideration of whether a quantitative design can produce useful results.

Research methods that take the unpredictability of people into account might be more productive (Bogdan & Biklen, 1992; Burns, 1997; Cohen & Manion, 1994; Merriam, 1998). Qualitative designs try to operate in natural settings, disturbing them as little as possible. Methods are used which describe the setting and the activities in it, which includes the individual characteristics of other people, the physical setting, the shared or private expectations of the participants and the tasks being done. Burns (1997), writing of the "multiple realities and socially constructed meanings" of educational settings (p.12), points out that since each person in a classroom has a unique history and personality, so their view of the classroom is likely to be unique too. Qualitative designs recognise that in human cultures there are cultural norms which show what is acceptable to the participants, and that there is a range of variation around the norms. By describing both the norms and the variability the lived reality of the classroom can be illuminated.

Ethnographic case study.

There are many types of qualitative research designs and the choice of research design depends on the researcher's intentions for the study. The object of this study is to examine how teachers and students experience mathematical teaching and learning. Case studies allow the researcher to focus on the roles, norms and interactions of the participants. A picture of the whole classroom environment can be built up, giving space for the community based cultures of the participants as well as the culture they develop together. "Case study ... provides thick description, is grounded, is holistic and life-like, simplifies data to be considered by the reader, illuminates meanings, and can communicate tacit knowledge." (Merriam, 1998, p.39)

Merriam (1998) distinguishes between case studies with sociological, descriptive, interpretive and evaluative purposes. This study is descriptive since the purpose is to describe the extent and nature of inquiry and traditionally based approaches to mathematics education in the classrooms. Including two classrooms in this study also improves the value of the description. "The inclusion of multiple cases is a common strategy for enhancing the external validity or generalisability of your findings." (Merriam, 1998, p.40)

Theoretical perspectives are needed for the classroom setting. Classrooms can be viewed as including the participants, the environment, and the interactions and practices within it. The environment and practices can be investigated from a phenomenological perspective, while the interactions between the participants can be regarded as narrative. Ethnographic case studies place prime importance on the meaning of events to the participants in a cultural setting (Bogdan & Biklen, 1992; Vandenburg, 1997). Each person has their own understanding of their world, constrained by the social world they live in. The job of the researcher is to find and describe that world:

> Qualitative forms of investigation tend to be based on a recognition of the importance of the subjective, experiential 'life world' of human beings. Such reflection is the province of phenomenology. The phenomenological field of educational action embraces the host of personal meanings that are derived from the context of direct experiencing. Perceptions and interpretations of reality are linked with these meaning structures. Thus the 'reality' of a given educational setting may not be seen as a fixed and stable entity but as a type of variable that might be discerned only through an analysis of these multiple forms of understanding. (Burns, 1997, p.11)

The qualitative researcher tries to find the similarities and differences between the viewpoints of all the participants. To do so they must try to use the original statements or actions of the participants as their evidence, and find in them some regularities which would point to the shared meanings of the participants. By finding these, the qualitative researcher can hope to find predictable order in the complexity of a social setting such as a classroom. Of particular interest are those practices and expectations that the participants take for granted. These form the background for the choices made during teaching and learning, and often invisibly limit what is acceptable in the setting. Brown (1996) argues that children have their own private phenomenological world that grows in response to their meeting and making sense of their world. Better understanding of those worlds is possible with a phenomenological approach to ethnographic case studies.

In addition to seeing classroom practices as phenomena, the interactions of the classroom can be viewed as having the quality of narrative – stories told by the participants through their actions and statements as they go about the business of teaching and learning. Narratives can be analysed by looking for the beliefs and attitudes revealed in the stories. "In any classroom there are at least three different narratives: the learner's, the teacher's, and that of mathematics itself. As such, these narratives ... reflect alternative ways of knowing" (Baker, Clay & Fox, 1995, p.3). Respecting the different ways each child sees reality helps a teacher or researcher to understand or reconstruct the shared reality of the classroom (Bauersfeld, 1990).

Applicability of the study.

The constructs of truth and applicability need to be reframed and redefined to suit the type of evidence provided by qualitative research (Merriam, 1998; Bishop, 1992; Wagemaker, 1992). Naturalistic classroom research, by its nature, is not expected to involve large sample sizes or repeatable trials with controls. Classroom research is more suited to small scale testing of plausible ideas, or to observations of actual teaching and learning to see if given features occur naturally.

Merriam (1998) lists characteristics of case study research which enable claims to be made about the generalisability of the results. Firstly, case studies provide data that is both concrete and contextual - drawn directly from the worlds of the participants. The generalisable features of those worlds could be expected to be recognised by a reader who is familiar with similar environments. Secondly, the theoretical setting around the description provides the reader with a framework for reflection on how their own environment and practice resonate with research findings from the area. Thirdly, the reader is not a passive receiver of the research findings. Merriam argues that the way the reader uses the research findings constitutes the only real criterion of generalisability.

In this study the classes and teachers concerned have been described so that their characteristics become transparent to the reader. All findings have also been triangulated. "Because the phenomena we seek to explain are subtle and elusive, 'triangulation' – looking at an issue from multiple perspectives to make sure you've really got a handle on it, is critically important." (Schoenfeld, 1994, p.5).

The case study classrooms were selected to represent part of the current state of mathematics teaching and learning in New Zealand. "The case might be selected because it is an instance of some concern, issue, or hypothesis" (Merriam, 1998, p.28). These two classrooms provide instances of mathematics teaching and learning in New

Zealand in 1998 that one would expect to find in many other year 5 classrooms in New Zealand. There are several reasons for supposing that these classrooms might be representative: the New Zealand education system is fairly consistent in its training, so the teaching approaches described later are probably similar to those being used elsewhere in the country; the students came from a fairly wide range of backgrounds, as do the children in many New Zealand classrooms; the mathematics curriculum has national currency and the teachers in the study set a high priority on following its direction, so the content at least of the observed lessons would have been similar across the country at that level.

There is a danger that a report such as this might be read as a full description of the classrooms studied. It is important to realise that this is neither intended nor possible. Any report can only be of a slice of the life observed, selected by the researcher and subsequently filtered through their understanding and concerns. This report is therefore the researcher's view of part of a complex reality (Burns, 1998; Merriam, 1998). Thus, within a qualitative study the researcher needs to set out their own interests and concerns, plus the assumptions made during the study, so the reader can enter the researcher's world and judge for themselves whether the interpretations and evaluations are reasonable (Merriam, 1998). Merriam recommends replacing the traditional concern with reliability with a concern for consistency and dependability between the data and the findings.

The questionnaire used in this study derives its reliability from the work of its author (Stevenson, 1998), and from whether the results from it triangulate with the other data. The focus group interviews have been conducted according to accepted standards and procedures (Krueger, 1994) and the observations have been carried out with minimal influence from the researcher.

Merriam (1998) provides three strategies for use by researchers wishing to provide acceptable reliability, generalisability and validity. Firstly, by the evident quality of the rich, thick description from the data; secondly by providing evidence of the characteristics of the study population so it can be compared to the other groups in the experience of the readers; and thirdly through purposeful or random multi-site sampling to increase the range of the phenomena described. All these strategies have been used in this study.

3.2 Conduct of the study.

The participants.

The classes used in the study were chosen as a convenience sample from a small range of available classes in a small rural town in New Zealand. The criteria for selection were that they were composed of mainly year five students (nine to ten years old); that the teacher and their school were willing to have the researcher in the room observing mathematics time and doing interviews during the year; and that the schools were physically accessible to the researcher. Year five was chosen as the stage at which children were beginning to be able to reflect verbally on their learning, and at which some attitudes and expectations of schooling might have become stable.

Mrs K's class were mostly year five students with some year fours. Mrs K had 30 years of teaching experience with years three to six, and was keen to examine and improve her practice. One of her principal concerns was that students would develop their autonomy and sense of responsibility while in her class. Mathematics was her speciality subject and she saw the research as an opportunity for professional development. The students in the class were drawn from a wide range of ethnic and socio-economic backgrounds, from both town and country areas.

There were 28 students in Mr J's class, mostly year 5 with a few year six students. Mr J had nine years of teaching experience at the year five to six level. He had a keen interest in group work, and also believed in building student responsibility and autonomy. This teacher joined the study to gain some fresh input into his teaching practice. Many of the students in Mr J's class were from an affluent area; there was a mixture of ethnic backgrounds in the class.

Ethics

Ethics approval was granted prior to the study. Permission was granted by the Boards of trustees of the schools, the teachers, and the parents and caregivers of the students (except three town students whose data was carefully deleted). The students were free to participate or to withdraw form the study at any time for any reason. One student in the suburban class withdrew from a focus group without offering a reason on one occasion and took part again on the following occasions. This was accepted as being normal.

The children were assured that they would not be 'told on'. The interviews and observations provided evidence that the students felt safe to express opinions and describe actions they hid from the teacher, and to act in ways that did not occur when the teacher was watching or nearby. The researcher did not offer value judgements on those confidences or actions, but made it clear that they would be simply recorded.

The teachers were given access to the collated results after the period of data collection, with the identities of the children masked. This enabled the teachers to comment on their perceptions of the realities that were evident in the data, without compromising confidentiality.

METHODS

Common methods used in ethnographic case studies are participant observations, interviews and document analysis, augmented by surveys and questionnaires as required (Burns, 1997; Eisenhart, 1988; Merriam, 1998). All these methods were used in this study, with the main data sources being participant observations and focus group interviews. These data collection strategies aimed to discover the ways in which the participants in the classrooms saw their own worlds.

Burns (1997, p.302) notes that "What people do and what they ought to do are very often different. Because of this, there is frequently a discrepancy between what people do and what they say they do. Therefore one must look behind the 'public' and 'official' versions of reality, in order to examine the unacknowledged or tacit understandings as well." By becoming part of the classroom environment without being an authority figure the researcher was able to obtain a privileged view of the worlds of the students as well as of the teachers. Overall, the methods used gave access to some of both the public and private views of the participants.

The study was conducted in four phases. Firstly there was an extended period of weekly visits to each classroom as a teacher aide. This role enabled the researcher to become familiar to the students and for the culture of each classroom to be observed. Secondly a pilot study was conducted consisting of focus group interviews with a group from each class, to test the emerging direction of the study and the focus group technique. At this time the attitudes and beliefs of the students towards mathematics were outlined, giving a purpose and direction for further questioning and observation. Observations and focus group interviews with groups of students followed, with informal teacher interviews to check their positions concerning the emerging themes. As with many ethnographic case studies, the focus of the study was refined as data was collected, transcribed and analysed.

Observations.

Observation methods followed those used by Jaworski (1994). Audiotapes were made on several occasions in each classroom. Transcriptions were augmented by field notes taken during the year and annotated by reflective notes identifying themes and regularities. Both small group and whole class observation were used to provide alternate views on the situation and the thinking of the participants. Jaworski notes that there is a range of roles possible for an observer, ranging from 'participant as observer' to 'observer as participant'. In this study the role of the researcher moved from participant as observer in the teacher aide phase towards observer as participant in the main study. This change was similar to Jaworski's experience of learning to be an ethnographic researcher: as the study progressed the focus moved from becoming familiar with the culture of the classroom towards testing of ideas about the meaning of the interactions being observed. In the early phase there was more to be learnt from being part of the culture, but in the later phase it was appropriate to be as unobtrusive as possible, participating only so as to maintain the familiar role.

The teacher aide phase allowed the researcher to learn the acceptable ways of acting expected of a teacher aide in those environments, and gaining the respect and confidence of the participants. At no time did the researcher act in a disciplinary role, but only provided help with the work. During this time notes were taken of the classroom environment and any features which seemed worthwhile. During this period it became apparent that there were significant differences in the beliefs and expectations of the students and the teachers. These differences were explored during the pilot study and the results used to frame the questioning and observations of the main study.

Over the course of five weeks in term four focus groups were conducted for all the students and audiotapes were made of all the work groups in both classes, but there was no effort to ensure that every child was taped. This was because the focus was on the culture of the class rather than the mathematics of individuals, so the object was to record interactions between as many of the students as possible.

There were undoubtedly observer effects in the research in spite of the efforts made to become part of the scene. As expressed by Jaworski (1994) "It was incumbent on me to interpret what I experienced relative to this involvement" (p.64). In many cases the tenor of the discussion indicated whether the students were talking from their private world or one of their public worlds.

Focus groups.

One form of interview is the focus group (Krueger, 1994). Focus groups are a flexible data collection method, and although they are mainly suitable for groups of strangers, they can be successfully adapted to use with classes of students, with careful attention to the social world of the participants. Focus groups have the potential to open a window into the mathematical world of students by careful leading of the group discussion from general ideas common to the group, into the particular interests of the researcher. Efforts aimed to maintain a relaxed and friendly discussion format encourage the participants to open up their more private feelings in a safe environment. Questions were chosen to be non-threatening but interesting, and data collection was by audiotaping, which is less intrusive than note taking. Each student took part in only one focus group and the interviewees were chosen randomly from the remaining pool for each focus group. This ensured that the opinions and feelings of all the class members were obtained.

The teacher interviews

There were two semi-structured interviews with Mr J and three with Mrs K. These were taped and transcribed for analysis. Questions focused on teacher perceptions of their classroom organisation and on how they developed and managed the classroom culture. A semi-structured format enabled the researcher to keep the focus of the interviews on the classroom culture while allowing the teachers to expand on their ideas, thereby providing evidence of both espoused and implicit motivations and beliefs (Smith, 1992).

Following standard qualitative interview practice (Burns, 1997; Merriam, 1998), the initial questions were very general to establish a co-operative environment, then followed a flexible schedule which responded to the line the teacher wished to pursue as well as returning to the concerns of the researcher. In the first interview and at intervals through each one there was a period of time when the researcher and the teacher were seeking a common language and common ground on which to build some shared understanding (Jaworski, 1994). The style of the interviews was very much a matter of mutual searching for a clear description of practice that was understood by both parties. The role taken by the researcher was to reflect the thoughts of the teacher back to them, and to ask such questions as might prompt reflective thinking about aspects of practice.

Difficulties with participant observation and interview methods.

Garner (1988) has provided useful ways of understanding the effects of being an outsider while trying to make sense of an insider activity such as mathematics teaching and learning. The main advantage of interview methods is that they allow access to the worlds of the participants, in their own words. The words of the participants are therefore uniquely valuable as a window into their meanings, provided the researcher shares the meanings of the language used.

However, interviews with children can be problematic: Garner (1988) argued that children are unpractised at reporting what is in their minds. They often don't have the language to report their cognition. What is automatic for them is often hidden; as something becomes more routine the children are less aware of it happening. For these reasons, what they say may be only a rough approximation to what they think.

However, discussions may bring out ideas that an individual child might not be aware of until prompted. In this study discussion was encouraged during the focus groups and class times where appropriate, with specific prompts used to start the dialogue.

Garner (1988) also noted that self-reports often contained a mixture of 'we do', 'we should do (feel we ought to)', and 'we are going to do (we intend to)' types of reporting. McClelland, Koestner and Weinberger (1992) wrote of the same differences in their consideration of motivations for actions. The situation of students in classrooms having different mental worlds available depending on the setting and who is present suggests that these different self-reports could be of value in sorting out the levels of motivations and expectations of the students. McClelland et al noted that different types of data collection accessed different levels of motivations (see also Jaworski, 1994). Direct questioning tended to elicit the 'we ought to' responses, while subsequent discussions opened windows into what really happened in the student's private world. Only the context of the discussion could separate the 'we ought to do' and the 'we do now' statements. A third level of motivation, 'the teacher likes us to' was seen in some of the early focus groups and observations, and was separated out as a theme in the analysis. As a result of this early theorising, Stevenson's (1998) questionnaire and the paragraph writing were used to triangulate the other data.

Stevenson's (1998) questionnaire.

Stevenson's (1998) 'Cognitive Holding Power' questionnaire (Appendix B) was used in this study to look for regularities among the students answers to questions about how they think and act in classroom situations. The questionnaire probes how students approach classroom situations to gather evidence for underlying cultural 'presses' for higher or lower levels of thinking within a classroom culture. Stevenson (1998) suggests that the questionnaire could be used by teachers to monitor "their efforts to create environments that emphasise different levels of thinking." (p.407). It was used for this purpose in this study, providing useful data on the classroom cultures, to triangulate the interview and observation data. Lower order thinking could be expected to be more prevalent in traditional classroom environments while higher order thinking would be more prevalent in an inquiry classroom.

A 'press' in Stevenson's (1998) terms is the result of many small factors resulting in the students using lower or higher level thinking. There are two sources of press in Stevenson's work: press from the environment, and press from the teacher. In addition, Stevenson provides a reality check in the questionnaire by including questions on what the students actually do in the situations described. By doing this the strength of the presses can be measured against self-reports of actions. These in turn can be triangulated with classroom observations to provide more clear descriptions of the microcultures of the classroom.

There were 30 questions on presses for first and second order thinking. These included questions about the teacher's wishes, feelings of obligation, and actual student actions (see Appendix B). This study reports the means and standard deviations of the student responses for each question to show how the students viewed their classroom culture with regard to the press towards higher and lower order thinking.

The paragraphs.

With the questionnaire was a page of paragraph starters (appendix C). These were designed by the researcher to be semi-narrative, to be completed by the students in their own words. The phrases opening each paragraph used classroom language so they would prompt answers from the private worlds of the students. Most of the students filled the paragraphs in with short phrases rather than extended answers, perhaps reflecting the mathematical setting or perhaps their unfamiliarity with the technique. Analysis of the resultant writing assumed that the data reflected more private than public motivations (McClelland et al., 1992; Friedman & Combs, 1996). This provided validation and improved reliability for the questionnaire, observation and interview data for the parts of the students' worlds that were less public.

Documents

Documents obtained included the year, term and weekly mathematics planning schedules of the teachers; answers to questionnaires done by the students and teachers; a writing task completed by the students; and copies of mathematics tasks given to the students. These documents were examined to provide information on how the intentions and espoused beliefs of the teachers and students tallied with their observed actions.

3.3 Data analysis and interpretation.

Qualitative research in education assumes that there are regularities in any classroom that can be identified and analysed. Voigt (1995) calls these "thematic patterns of interaction ... produced when the teacher and the students routinely constitute a theme around some related issues" (p.185). His use of 'thematic patterns' as a term for recurring processes which are observed during mathematics teaching and learning are interpreted as 'norms' in this study. By treating classrooms as cultural systems and viewing the interactions between the participants as carrying meaning about the culture, it becomes possible to reconstruct what is important mathematically and socially within each classroom.

Hiebert et al. (1997) proposed five dimensions of classroom interaction that have been used to analyse the classroom cultures for the level of inquiry-based practice in the classrooms. These were:

- the nature of the tasks;
- the role of the teacher;
- the social culture of the classroom;
- the way the mathematics was used as tools;
- the equity of the classroom and accessibility of the mathematics for all.

These five dimensions were adopted as a theoretical framework for the reporting of the classroom cultures since they capture both the social and mathematical aspects of the classroom: "In mathematics communities the goals are problems to be solved and understandings to be developed. The ways of working together are norms for communicating and interacting with each other. Learning to be a member of a mathematical community means taking ownership of the goals and accepting the norms of social interaction" (p.43).

THE PRACTICE ADOPTED.

Analysis and interpretation of ethnographic case studies is not an exact or well-delineated process. There are several levels or analysis that can be attempted, from simple description of the cases in some organised fashion, through the development of theoretical categories which can be used to interpret the data, to the development of a coherent theory which might link cause and effect in particular situations. Merriam (1998) has set out the options for analysis of multiple case studies such as this one, and suggests the researcher find their own pathway, depending on what they have set out to do and the audience for whom the study was undertaken.

In the early stages, the constant comparative method (Merriam, 1998) was used to build conceptual categories and to refine these as the data accumulated. The intention was to develop a grounded theory from the data, and to report on this. As the study progressed, it became apparent that the categories being constructed had a great deal in common with Hiebert et al.'s (1997) five dimensions, to the extent of all being subsumed under them. In this study, therefore, the classrooms are described under the theoretical categories provided by Hiebert et al. (1997) (the case record), in order to provide a detailed account of the teachers' and students' beliefs and actions during mathematics teaching and learning.

It was clear from the beginning that the teacher's concerns beliefs and norms were not shared by all the students all the time. A second theme of analysis developed, which was to describe how the expectations and norms of the teacher and the students differed for mathematics education. The power relationships in the classrooms provided a further perspective on these differences, and have been analysed using Tauber's (1993) theory of five types of power, enhanced by Manke's (1997) theory of power relationships in the classroom.

The differences in expectations and norms were linked to how much of Hiebert et al.'s (1997) dimensions were apparent in the classrooms and the whole was analysed using Boaler's (1999) concept of 'constraints and affordances'. This helped to ascertain which aspects of the classroom cultures either enabled or inhibited learning of mathematics with understanding.

3.4 Summary

This study was conceived and carried out as an ethnographic case study of two mathematics classrooms. Research methods included interviews, observations and documentary analysis to provide a triangulated view of the cultures of the classrooms. The data was analysed in terms of classroom culture, power and interaction using a variety of congruent theoretical tools.

4. THE CASE RECORD: THE TEACHERS.

'My classroom works like this'.

In the reporting that follows, the data is from observations and notes taken from informal conversations, triangulated with questionnaires and interviews. These stories are the teachers' views of their classrooms; the stories from the students are in chapter five. The dimensions of Hiebert et al. (1997) have been used to provide a structure for the reporting and analysis that follows. Hiebert et al.'s five dimensions make up a system of instruction and "it is important to remember that they are fully mixed and entangled in practice. In fact, that is what a *system* means" (p.51, original emphasis).

4.1 Mr J.

Mr J worked from planning sheets that had been prepared with his syndicate of teachers. Weekly planning meetings discussed intentions for the current topics and student needs. On a normal classroom day Mr J began mathematics time with four mixed operation sums on a card. Following this maintenance work the students completed set work or Mr J would introduce new work. The day ran according to a flexible routine but mathematics time was almost always after morning break, and the students were expected to be at work by the time Mr J came in. They always had plenty of current work from texts, worksheets or investigations.

THE SOCIAL CULTURE OF THE CLASSROOM.

Mr J expected respectful treatment of everyone by everyone, with himself as judge and sergeant if necessary. He had established norms for general and mathematical behaviour early in the year.

> *The behavioural boundaries are pretty well set, and that gives the children opportunities to actually have a go, because they know they're not under any pressure anywhere else. They know they can get on with something and they have to behave in certain way. ... I think that's set out from the management structures of the classroom.*

He expected the students to do the work he set and to follow his instructions as to method and approach. Within those parameters the students were expected to use their initiative to do the work well and quickly.

I think their version is that I pretty much tell them what they have to do. But, I'm also aware that they know that I won't hold their hands. Like I tell them what and how, but I don't expect rubbish from them. They have to produce the goods. Their impression would be that they follow pretty much what is expected of them, (behaviourally), and they get on with what I say (process wise.)

Mr J wanted the classroom to be a place where the students would feel emotionally and physically safe where they would all be freer to focus on the tasks and on learning.

One of the important (things is), ... when you sit kids down to have a go, they actually do do it. And not give up on it.

The social culture of the classroom was based on the students showing respect for each other and getting on with the work set. Misbehaviour observed during the observation time was dealt with primarily by a warning note in Mr J's voice as he called the student's name, followed by a request to answer 'what should you be doing?' if he thought more was needed. This served to reinforce Mr J's version of the classroom culture and the referent power base that was established at the beginning of the year.

Mr J's responsibility norms fitted with the safety norms to encourage a climate of task involvement with individual and group activity towards set goals. He wanted the students to be able to use mathematical algorithms accurately and to understand how and when to use them. He commented that progress towards acceptance of these norms and goals had begun in February and was ongoing in December.

While the students were working on the sums Mr J walked around the room monitoring progress and giving help. This informed his planning and whom he worked with the rest of the lesson time. After the maintenance, there would be either some teaching of new content or a new task. The students then continued with some part of their current work. Task completion within specified periods was a dominant feature of the classroom norms.

We would set a contract saying 'this is how much you have to do; this is when it has to be finished by; this is what is expected. When it's finished, you tick this box, you colour this box in.'

The time management training supported the independence norm and was on-going during the study.

And so, on Wed this week we planned our interviews, talked about what the process was when we went away to do our interviews. They were told that by Friday their planning needed to be finished, so that on Monday at 8.45 we'd be going out to do them. Now they had to realise that if by Friday it wasn't done, it was their responsibility to find some way of getting it finished. I say 'You solve the problem!' ... It takes ages' (to get them to do this).

He wanted the students to become independent of him in completing their tasks.

Well, basically my thinking is that especially as they get older, they develop high levels of independence and learn to solve problems for themselves. There's a lot of putting the problem back to the child and saying, 'you know what the problem is, Rather than come to me, you think up some solutions that you know will work. What are some of the ways around getting from where you are to where you want to be'.

He didn't want the students to regard him as the source of mathematical correctness, but to agree among themselves before checking with him. He had developed a range of ways to promote this norm.

By using lots of self and peer evaluation. Conferencing with me too. I started out with me evaluating their work; ... I still do that. But I moved on to them evaluating each other's, on set criteria and stuff. We've got to the stage now where we use little 'post-its', and they go round and find things on each other's work and comment on it.

Additionally, Mr J allowed and encouraged a great deal of mutual helping among the students, relying on the culture he was promoting to reduce the copying and non-involvement of some students. He would use prompts such as 'where have you seen this sort of thing before?' to encourage reflective elaboration strategies.

Mr J had a long-term plan for the development of his classroom norms for behaviour, independence and perseverance. This helped establish the referent power base in the classroom.

The fact is that you set a standard of what you expect right from the start, and you keep that standard up. They know that's what you expect from them. Then you can move yourself away and go into other areas as well.

The classroom was generally a calm place in which the students were expected to work together quietly. Silence was only asked for once during mathematics time in the course of the study observations. It was while Mr J was absent and he had set a statistics worksheet to be done as a test. The reliever explained the task, gave out the sheets and called for quiet, restating the classroom norms as she did so:

> *Mr J has given me a worksheet on a statistics review. He wants you to do it on your own, in your books, so he can have an idea of how much you've learnt of what you've done with him. With this, I can help you read the questions; I can't help you answer them.... Put that other work aside, ... you shouldn't be talking to anyone, and you shouldn't be sharing your work.*

The students began quietly, and someone asked for help. The reliever went over, and got caught up re-teaching one of the graph skills, while the rest of the class began to quietly help each other, sharing ideas and answers as in a normal class situation. It was nearly ten minutes before the reliever noticed what was happening, and once again called for quiet. That lasted about three minutes. This incident indicates that the teacher expectation of quiet co-operative work was shared by the students to the extent of being a habit they returned to, in the absence of the 'test' atmosphere being enforced by the reliever.

The questionnaire.

Mr J and the students were asked to complete Stevenson's (1998) questionnaire (Appendix A) for their feelings and actions relating specifically to mathematics time. Mr J's view of the social culture of the classroom is illuminated by his responses to the questions beginning 'I feel I have to...'.[2] The results in the tables include the student mean responses for comparison.

[2] The scale was a Likert type, with 1= almost never, 2= seldom, 3= sometimes, 4= often, and 5= quite often. The * indicates where Mr J's assessments differ more than one standard deviation from the student responses.

Table 4-1 **'I feel I have to...'**

(The bracketed numbers are '1' for lower order and '2' for higher order thinking)	Mr J would like them to be	Mr J thinks they are ...	Class mean and standard deviation.
6. Copy from the teacher (1)	3	5 *	2.5 ±1.1
17. Do the work as shown (1)	4 *	5 *	2.9 ± 0.9
25. Do what the teacher tells me in mathematics time. (1)	4	5 *	3.5 ± 1.0
4. Find information myself. (2)	5 *	4	3.6 ± 1.0
22. Find links myself. (2)	5 *	3	3.0 ± 0.7
12. Use my own knowledge to check results. (2)	4 *	3	2.6 ± 1.1
10. Question the teacher to check my results. (2)	3	3	3.0 ± 1.0
2. Try out new ideas myself. (2)	5 *	4	2.9 ± 1.2

The table shows that Mr J wanted the classroom culture to be one of working as shown and following instructions (Q17, 25), and that he wanted the students to be investigative (Q4, 22), reflective (Q10, 12) and curious (Q2) in their approach to mathematics. This confirms the two aspects to Mr J's desired classroom culture: one where the students would accept direction when Mr J wanted them to; another where they would be guided to think for themselves. Illustrating this dichotomy, Mr J was often seen to value correct answers during daily maintenance, but to encourage students to check with each other if some part of their worksheets was not correct. However, few of the worksheets or textbook exercises included open-ended questions where the students had to explore and discuss their mathematical processes in order to solve the problems. The ones observed were used in the applications phases of the topics. This reflects his planning which typically involved learning about an area of mathematics before applying the knowledge in contextual problem solving.

Table 4-1 shows that Mr J's beliefs about how the students felt obliged to act were sometimes quite different from theirs for lower level thinking but a good match for higher-level thinking. He expected them to feel strongly obliged to follow his instructions and methods and moderately obliged to think creatively. Their responses indicate they felt only moderate levels of obligation for both levels of thinking. Mr J's preferred culture would have had the students feeling stronger obligations towards both thinking levels than their reports indicated.

THE ROLE OF THE TEACHER.

The interviews and observations indicated that Mr J had a definite view of his role as teacher.

> *Right from day one you have to say 'my job here is to guide and to coach you; it's not to hold your hand through everything. You have to go out and make decisions for yourself.*

He believed the students saw him more as a benevolent dictator.

> *I've got an idea that they perceive that, ... that I set out the rules and basically what I say goes, and that they operate within that. That I'm not ... not, quite, a dictator, but it's more ... yeah, I think they perceive me more as a chalk and talk type.*

Mr J chose the tasks, set boundaries for behaviour both socially and socio-mathematically, and chose when and how to provide help and guidance.

Early in the year he had begun a programme of time and resource management training for the students.

> *We talked a lot in the beginning about 'making decisions, about the quality of your decisions and about what you're producing' and a whole range of self-management. We did a whole range of units on time management.*

This programme was aimed at developing a strong culture of self-reliance, with personal and group responsibility being accepted by all students. He promoted norms which included students doing drafts of written and project work, groups critiquing their own work before doing a final version, groups and individuals setting timelines for tasks, and sharing the responsibility for parts of it. This applied mostly to the end-of-topic work during the study. The mathematics in these tasks was mostly on worksheets, rather than being integral to the completion of the main task. Time management during earlier parts of the topics was a matter of getting the worksheets finished on time.

Mr J's preferred role as benevolent dictator / expert fitted well with his attitude towards mathematics. He wanted the students to learn how to learn the mathematics themselves by learning to keep on task, to reflect on their work, and to get his help when there was a problem. In his topic planning he deliberately chose contexts that would help the students to see the mathematics as useful procedures that could be applied to a range of problems in the world. As the teacher he saw himself as helping the students to learn the procedures and to learn how to select and apply them.

Part of the questionnaire responses (from Stevenson 1998, Appendix A) give some insights into Mr J's desired and actual roles as a classroom teacher. Mr J filled out the questionnaire twice, once for where he would like the class to be, and then where he thought they actually were.

Table 4-2 ' We do...' and 'The teacher encourages...'

Question ..., (the numbers in the brackets refer to first or second order of thinking)	Mr J would like them to feel	Mr J thinks they really feel	Student mean and standard deviation.
We do this ...			
8. Getting information from teacher (1)	1	4 *	1.9 ± 0.9
18. Teacher showing links (1)	2	4 *	2.6 ± 1.2
28. Relying on teacher for ideas (1)	4	4 *	2.4 ± 0.9
The teacher encourages ...			
9. Copying from teachers (1)	3	5 *	2.8 ± 1.4
26. Doing work as shown (1)	4	5 *	3.4 ± 1.0
16. Doing what we are told. (1)	4	4	4.0 ± 1.1
The teacher encourages ...			
15. Finding information themselves. (2)	5 *	4	3.7 ± 1.1
3. Finding links themselves (2)	5 *	3	3.4 ± 1.0
29. Use own knowledge to check results. (2)	5 *	3.5	3.1 ± 1.3
21. Questions to teacher to check results. (2)	5 *	3	3.3 ± 1.1
11. Try out ideas themselves. (2)	4	5 *	3.1 ± 1.3

From the table it can be seen that Mr J wanted the students to accept his directions 'often' (Q9, 26, 16), but at the same time to be independent of him in areas of higher order thinking (Q8, 18, 15, 3, 29, 11). This agrees with his statements about him being in charge of the behavioural climate of the classroom in order that the students would be able to get on with their learning. However, Question 28's response indicates that he believes the students rely on him for information as well as for social stability. Many of Mr J's desired roles were in the direction of more independence for the students, however, his responses to "Where the class really are..." indicate that he sees his enacted role being more directive in practice. This could be seen in the classroom observations, where Mr J would normally provide worksheets with most of the necessary information on them. Investigations where the students had freedom to set the topic and find their own information were not seen during the observations, but had been done earlier in the year in association with a science contest.

Mr J's desired and enacted roles had an interesting relation to the roles as perceived by the students. This will be explored in further detail in the discussion, but it can be seen that for first order thinking Mr J's desired role was similar to his role as perceived by the students, while his perception of his enacted role was not generally shared by the students. For higher order thinking the match was reversed. The students by and large saw him as being less directive than he saw himself. He also thought that he was open to their innovative thinking, but they didn't agree.

Power.

Mr J had developed classroom norms that allowed him to use referent power (a common purpose) and expert power (pertaining to his knowledge) in most of his interactions with the students. He seldom punished students overtly but used references to the classroom norms to remind students of the expectations. He had constructed the classroom norms so that legitimate power (pertaining to his position) and controlling power were in the background as a backup where the students did not fully accept his directions. As a result the classroom environment was generally positive and focussed on the tasks.

THE NATURE OF THE TASKS.

Mr J believed in teaching mathematical skills through a mixture of textbook exercises, worksheets and projects. He tried to present new material within a context, and to teach the content of a topic through a range of tasks. For him, mathematics had several faces including basic facts, content from the curriculum strands, and extended problem situations. He would plan his teaching of a topic by referring to the curriculum requirements first, but regarded the mathematics as only part of what the students needed to learn.

> *The first things I look at are the objectives from the curriculum documents. What needs to be taught. Then I look at what's available to be used as texts for it. Then I take it from there, so basically it's based first on what needs to be done, then on what they need to know. You break it down into objectives, then also have a look at some skills, which may even be mathematical skills. They may be group skills or time management skills as well. And then the specific learning things.*

When he came in Mr J would put up the maintenance sums on a card, and the students were expected to stop their other work to do these in a set part of their books. Mr J believed firmly in regular revision of the basic skills.

> *That's to do with reviewing skills that have been used, and the constant maintaining of their ideas so it comes back to them. So it doesn't decompose – it's always being touched on.*

On the back of the card were the worked sums for the class to compare their work with. The students were expected to record their daily results on a bar chart in their exercise books.

Mr J chose tasks to suit the mathematical and social abilities of the class, paying attention to the task demands from both the content and context aspects. He was aware that many students held an instrumental view of mathematics and worked to broaden that:

> *I think the structure of the mathematics curriculum is that a lot of them have had skills first, so you've got to adapt the skills to what's going - the problems. Like, most kids have had or been told that 2+2=4. So we have to use that to focus in on a problem. But you never know what they've been taught ... They might have been taught 2 bananas +2 bananas = 4 bananas. So there's a problem anyway.*

Overall, Mr J saw the curriculum as determining what the students needed to know, with their gaps in content knowledge driving a lot of his planning. He put a lot of effort into making sure the students had secure basic facts knowledge.

> *What they feel good about would be their basic facts. Their recall and knowledge of the basic facts. That's something they've made real progress on and I've mentioned it quite often.*

He was happy with the students using their own invented algorithms, praising students when they offered such ways, but he also taught the standard ones. He expected students to set their sums out neatly and set short deadlines for tasks.

The class was divided into two groups for mathematics. Mr J's group worked in his classroom while about eight of the more capable students went to the library and did enrichment work with the principal. The tasks for each group were intended to be interesting and challenging for the mathematical level of the students.

> *I'll find stuff the less able students can do and make progress with, so they won't get all worried and give up. But you've got to have them doing the class work too, or they'll get teased and feel useless.*

The more capable students in each group were expected to complete more of the work. Mr J commented that as the year progressed the tasks he chose became more contextual, and that he brought in word problems and problem solving earlier in the topic. Most of the tasks were taken from a range of photocopy masters or a class set of texts.

In order to develop understanding and skills he went over some of the concepts of the topic each day, building them into different contexts and real life settings. This would steadily build more complete pictures of the applications. He usually taught this phase from the blackboard. During the observations most of his questions to the class were requests for students to repeat his instructions or to recall previous examples of the skill or concept being taught. He commented that he did this to prompt the students to pay careful attention and to link the work to earlier learning. Questions to prompt reflective thinking were rare in this class teaching phase. The students were carefully guided through the worksheets or textbook tasks.

The understanding Mr J wanted was both instrumental understanding and relational understanding (Lindquist, 1997), that is, he wanted the students to know what to do and why as well as to have rules they could use effectively. The style of questioning Mr J used in class (discussion and individual helping), was a mixture of prompting for instrumental understanding and relational understanding depending on the task. Mr J prompted students to work out 'what to do and why' in projects and word problems. Procedural facility was his goal in maintenance time and when students were doing practice exercises from the textbooks.

Strategies for helping the students understand the tasks included conferencing (students would put their names on the board and wait for a time to discuss their progress); asking the students why they had chosen to use a particular method, how they had completed a task, or what they thought they needed to do next; and peer evaluation, where the students had to write short critiques of the work of the whole group.

Group work was sometimes used for students who were struggling with particular skills or understanding. The type of help was specific to the purpose of the current task, whether procedural, for example graph skills or understanding, for example which graph to choose. Following the group help, the students would be set a mixture of skill exercises and worksheets designed to build the skills into some familiar contexts.

It was common for topics to finish with a project designed to bring together the skills, strategies and contexts of the topic. These tasks were designed to integrate the mathematics into other subjects or into areas of community life, such as the statistics project on house paint colours. The mathematics in these projects was intended to strengthen instrumental understanding and move the students towards relational understanding.

TOOLS.

Mathematical tools can be physical materials, oral language, written notation, thinking skills and other things the students use to help them solve problems. The three essential features of tool use for relational understanding were that tools needed to be used with a purpose, that their meaning needed to be constructed by the user, and that the tools were used for record keeping, communication and thinking (Hiebert et al., 1997).

Mr J's general enacted belief about problem solving strategies and thinking skills was that the students would practice them as they worked through the tasks. During the observation period he did not teach thinking skills as generic strategies to the class, but sometimes encouraged the students to reflect on the strategies they already had. Mr J commented that he liked to teach thinking skills and strategies mostly in context (as problem solving skills) but that he had only done a few exercises earlier in the year. He mentioned that he wanted to move more towards teaching thinking skills, but was not ready to do so at the time.

He saw thinking skills as a necessary part of the students' development of autonomy and self-responsibility.

> *It does depend on what the children come in with. They may come in with a whole range of problem solving skills, so they don't need to be told what to do – those ones can instantly think, 'well, I can solve this myself.' But kids being kids will take the option ' The teacher is there so he's going to tell me. I'll take his answer, heck, why not.' And so, you have to continually reinforce the fact that this is not going to happen.*

Tools can also be the procedures and algorithms of mathematics. In answer to a question about whether he taught mathematics through problem solving or procedures, he replied

> *Both ways. You give them the opportunity to say ...(I say to them) 'this is the problem, how are you going to solve it? Go away and think' and you also at some stage have to teach skills. Once you've given that you give them a practical everyday usage of it. You're not giving them something that's going to sit there and have no relevance to what they're doing. I think both ways work well and I actually have no opinion on which way works best. I use them both. ... Just how it comes up.*

In practice, Mr J moved from emphasising skills early in a topic to applications later on.

> *... at the end of it I expect them to produce a piece of work which will demonstrate those objectives, but also some particular parts to it, and that sets out exactly what I want them to get out of it*

Mr J aimed to have all the students proficient at the algorithms before transferring these skills to problem solving situations. He didn't spend much time developing the concepts behind the tasks, but expected relational understanding to develop in the classroom culture of being on-task and helping each other.

EQUITY AND ACCESSIBILITY.

Hiebert et al.'s (1997) first feature of equity is that the tasks should be accessible to all students. Mr J organised his mathematics teaching so that all the students would be doing work at their own levels. About a third of the students did extension mathematics with the principal most days. This group consisted of the students identified as having covered the content in any given topic. The membership of the group changed through the year.

The rest of the students ranged in understanding from those who were comfortable applying the earlier content levels of the topic to the contexts provided, through to those who were struggling to link even the earlier work into their existing knowledge. For these students Mr J chose a range of worksheets in each topic. Mr J would try to bring all the students up to the same level and set the final project so they would all be able to make some meaningful attempt at it. This was done by requiring the application of basic skills and setting other parts of the tasks to involve the new work. Group skills, time management and thinking skills were encouraged within these projects.

Mr J wanted the students to feel they could all contribute meaningfully in class discussions. He promoted a classroom culture that encouraged respect and fair play and valued what any child said during class time. However, there was little whole class discussion observed; mostly Mr J asked questions of named students as he revised or taught new material. Mr J was careful to ask questions which were within the capability of a given child, and accepted every answer. He did not often ask for explanations or justifications of those answers, preferring to rephrase questions to prompt the answers he was looking for. This is a characteristic of Cobb et al.'s (1992) procedure oriented teacher, and triangulates well with the data on tools, tasks and social environment reported above.

SUMMARY.

Mr J taught in a coherent and well-structured manner, with the various parts of his teaching practice linked together to support his goals. His espoused beliefs were somewhat in advance of his practice as judged by Hiebert et al.'s (1997) dimensions, showing that he was moving in the direction of more inquiry based teaching. During the year he maintained a similar routine for both the structure of the mathematics topics and the culture of the classroom, providing the students with a predictable and safe environment, emotionally, physically and culturally. This is known to be a most valuable part of an effective learning environment (Perkins, 1995). Mr J believed in the importance of knowing how to do mathematical procedures as the basis for successfully applying mathematics to problem solving, and structured his teaching accordingly.

4.2 Mrs K

Mrs K had 30 years teaching experience across most of the primary year levels. She gave her specialities as Language, Social studies and Mathematics and was in charge of the senior syndicate mathematics programme during the study. The senior syndicate shared information about students and teaching approaches each week, as well as co-ordinating resources and assessment.

Mrs K planned her year's programme around the curriculum requirements and included local situations and community happenings as themes for projects and investigations. She tended to teach skills before showing the students how to use them in contexts, often in small groups. Mrs K expected to re-teach skills and concepts several times, and would call together students who were unsure of something. She expected that understanding would follow if the students did the work.

THE SOCIAL CULTURE OF THE CLASSROOM.

Mrs K firmly believed that computation, self-esteem, and understanding were equally important classroom goals. She also encouraged individual responsibility for completing the set work, with sanctions and guidance for those who didn't manage well. She expected the students to do the work themselves and to grow to be independent of her direction. In practice these beliefs were strongly affected by the wide range of mathematical understanding in the class.

Mrs K also wanted to promote a culture that valued all the students. To this end tasks were chosen to provide a level of appropriate challenge resulting in individual work programmes for many. She expected the students to help each other but not to provide answers. The individual programmes and practical programmes helped with this important aim, although her emphasis on correct answers to sums sometimes worked against it.

Given that students were often on group or individual programmes (apart from maintenance time) whole class discussion was only used to introduce a new topic or project. Her style of questioning, which was mainly apparent in-group and individual situations, was aimed at procedural facility, in keeping with her strong emphasis on instrumental understanding. The students responded with attempts to show her that they could follow the procedures accurately, and this constituted for them and for her the appropriate mathematical response. She was occasionally observed to challenge a student to explain how they had arrived at a solution.

Mrs K believed that the maintenance and textbook work was necessary as a precursor to the applications. She wanted the students to use their mathematical skills as tools in problem solving and practical work but the students persisted in treating this work in the same way as the maintenance, with norms of completion and correctness being to the fore. The discussion and exploration norms that she wanted them to use for the applications did not seem to be familiar. Assessment and feedback for the problem solving and practical work was also partly related to numeric correctness, perhaps leading some students to see little difference in the two areas of mathematics. During the study Mrs K made repeated efforts to reframe the problem solving and practical work in inquiry terms.

Mrs K frequently said to the students that she wanted them to be independent learners and supported this with specific teaching and plenty of opportunities for individual organisation and creative effort.

> *I have said to the children, 'I want you to be independent, I want you to be responsible', and part of that is me teaching strategies to them so they can become independent.*

Her class routine was designed to encourage the students to choose their own order of tasks, provided they were each finished by the time set for checking or marking them. Every morning she put on the board the tasks that were to be done, and the resources needed for those.

> *Maths is programmed every day (after the start-of-day routine), and they know that's what it is. A lot of them come in first thing in the morning, and before school. They'll have their revision up on the table, finished, before school starts, and several of them will do that a couple of times a week, because they know that's part and parcel of the work. They also know that when they've finished their work they can get on with the other part of it. That might be a project, it might be problem solving, it might be a contract – they're wanting to get on with that.*

Mrs K expected the students to work together but to produce their own results.

> *It's important to me that they are showing initiative and responsibility. For me to be a good teacher, that's what I want to put through to them. I couldn't have a classroom where all the books are identical – that's not learning!*

Mrs K linked perseverance to independence and responsibility. She used 'contracts' to set performance goals for individual students; 'silence' as a short term sanction for those who weren't working steadily; repetition of similar tasks for a week or so to train students to accept the idea of steady improvement; and praise for tasks well done or improved performance.

> *I've been giving them the easier ones, and then, we haven't just done it once. Now, if you take the magic squares, lines adding up. The first day I gave them that, 2 or 3 got it right and we showed them on the blackboard and showed, ... one way of getting it. The next day half of them got it right, and on the 3rd day, at least three-quarters.*

She encouraged perseverance by all students but was aware that there were personality issues involved.

> *You might have a child who will plug away and say, 'no, it's impossible!' and I'll say 'no, it can be done', and they'll persevere. Some children have that perseverance, and some haven't. Those ones probably don't persevere in anything.*

Mrs K wanted the students to work neatly in their books, and to work quietly and steadily. She reminded the students of these requirements several times during the observations, indicating that they were perhaps not so important to the students as to her. These norms were built into the daily routines by setting deadlines, moving around the classroom praising neat and quick work; scolding where students were off task or sloppy; and requiring work to be repeated where it was not up to scratch.

> *'Anything that's worth doing is worth doing well'... You can't force it on a pupil, you can encourage it. For example, if the work's not good enough to, ... not presentable enough. I like them to do it again. You don't want a person to rush through things and get them all wrong.*

The 'press from the environment' questions of Stevenson's (1998) questionnaire provide corroborating evidence for the sort of classroom culture Mrs K wanted the students to desire for themselves.

Table 4-3 I feel I have to ...

('1'= lower level thinking; '2'= higher level thinking)	Mrs K would like them to be	Mrs K thinks they are	Student mean and S.D.
6. Copy from the teacher (1)	1 *	3	2.4 ± 1.4
17. Do the work as shown (1)	3	3	3.9 ± 1.2
25. Do what the teacher tells me. (1)	4	5	4.2 ± 1.5
4. Find information myself. (2)	4	2	3.8 ± 1.4
22. Find links myself. (2)	3	2 *	3.4 ± 0.8
12. Use my own knowledge to check results. (2)	4 *	3	2.9 ± 1.0
10. Question the teacher to check my results. (2)	4 *	2	2.5 ± 1.5
2. Try out new ideas myself. (2)	4	3	3.1 ± 1.3

Q 6, 17, 25 interpreted as behaviour in the classroom during mathematics time indicate that Mrs K wanted the students to feel they should do as told but to have more freedom in the way they do the work. They were not to copy her working (but to think about it). The student responses to Q 4 and 22 indicate that they did feel pressed by her towards higher order thinking though she was not aware of those feelings. For all the other questions Mrs K was realistic about the level of press from her that was felt by the students.

Her espoused desires are towards the 'seldom' end of the Likert scale compared to the class means for these questions but follow the same pattern. She wanted the students to explore mathematics themselves and to question their results. On Q12 and 10 the attitudes she wanted the students to have was some distance from those they reported.

Overall, the social culture Mrs K wanted in her classroom was of accurate, neat work; the students being on-task and engaged in completing set work or projects; of them taking some responsibility for the order and speed of completion; and of mathematics being both computation proficiency and practical, enjoyable activities which used those computation skills.

THE ROLE OF THE TEACHER.

Mrs K wanted every student to have success with the mathematics they did. She believed that every student could learn to use the mathematics accurately, and organised the classroom to promote these two goals. Socially she promoted perseverance and self-responsibility. Her preferred role was to provide each student with an environment in which they would gain enough success to feel they could do the work independently. Mrs K used a range of resources tailored to the mathematical capability of the students. It was very important to her that the students could easily understand the language and notation of those resources.

> *What's the point of giving them stuff they can't understand? They'd just give up and start turning off maths. We'd never do that in reading!*

The students were allowed to sit in friendship groups; these largely paralleled ability levels for mathematics. Students were required to work alone if Mrs K suspected they weren't doing the work themselves. Friendship grouping gave the opportunity for some off-task behaviours, but Mrs K usually moved around the room fairly quickly during group work, keeping the students on task and giving help. Most of the students, both those in groups and those working alone, were given individual programmes to follow through a topic, so the level of the work was tailored to their growing capabilities.

Mrs K constantly encouraged all the students towards procedural facility with 2-3 place operations, though she was aware that some of them would not attach any mathematical meaning to the methods, at least in the initial stages. For these students, she saw facility with the algorithms as a tool for self-esteem.

> *That's where W is. He's onto doing the strategy without really understanding. He's going through the format. T is the same, R is the same. They're learning that this is what you do, and it gets it right. ... It's really basic, but you see, if these kids need a strategy to increase their self confidence, then if that's going to do it for them, that's what I'll use.*

She believed that these students would eventually make sense of the procedures. This instrumental understanding (Lindquist, 1997) seemed to be the principal goal of maintenance time. The students often seemed to concentrate on instrumental understanding during their textbook and worksheet activities even when Mrs K had asked them to *'ask yourself why it works'* as she often did.

Mrs K used the problem solving, projects and investigations specifically for developing relational understanding in both mathematical and everyday contexts. All the students were encouraged to tackle the activities whether they enjoyed them or not, but Mrs K trusted them to make good choices and deliberately didn't notice the occasional off-task episodes. Students who repeatedly avoided the tasks were given short-term goals and extra help.

Mrs K believed in the importance of cumulative feedback on student achievement. She made sure the students knew where they had done well or poorly every day, asking them to record their improvements on a chart in their exercise books. In addition, she kept detailed records of student achievement in mathematics including daily records of mistakes in the basic facts maintenance. These records were used for parent reporting and provided the students with feedback on how they had improved. Data collection took a lot of time and was done while marking the mathematics books during a silent reading period later in the morning.

Mrs M has a checklist where she writes which ones you got wrong, and she knows which ones you're not very good at. (Student report)

Her records of practicals, textbook work and other mathematics was done in a similar way, with the recording done as each student made progress with a piece of work. This regular marking was a powerful factor in her classroom culture. She made sure the students were encouraged, and that they saw they were improving.

> *They need to be given that feedback, not to be left with, like, tomorrow you're going to do the same work whether you know it or not.*

She used her records extensively to guide her choice of work for each student.

Mrs K saw herself as a resource for the information the students needed but not the sole source. She wanted the students to look to the mathematical logic for whether a sum was correct and to agree among themselves before coming to her for help. This was the motivation behind her practice of checking all the mathematics work every day and requiring the students to revisit their mistakes with attention to the structure of the algorithm. In the project and practical work she wanted them to discover things for themselves, having been shown what they were looking for and where to look. She was observed linking their reading skills to their mathematics project work, reminding them that they knew how to find the important information from a resource. In project work, she used class discussion to make private knowledge available to the class, encouraging the students to see each other as resources. During the later stages of the study students were speaking freely at these times. Mrs K was seen to be valuing every contribution and encouraging the students to ask questions of each other. This style of interaction was not seen to happen when mathematical problems were the focus.

The questionnaire.

The Stevenson (1998) questionnaire provides some triangulation for the observation and interview data. Table 4-9 reports her responses to the 'the teacher encourages' questions, giving her responses and the student responses to the same questions for comparison. This data confirms the interview and observation data regarding her cultural roles.

Table 4-4 'We do...' and 'The teacher encourages...' (The scale was a Likert type, with 1= almost never, 2= seldom, 3= sometimes, 4= often, and 5= quite often. The * indicates where the student responses are more than a standard deviation away from Mrs K's assessments).

Activity... ('1'= lower level thinking; '2'= higher level thinking.)	Mrs K would like them to feel	Mrs K thinks they really feel	Class mean and standard deviation.
We do this ...			
8. Getting information from teacher (1)	2	3	2.9 ± 1.6
18. Teacher showing links (1)	3	3	3.3 ± 1.6
28. Relying on teacher for ideas (1)	2	4	3.0 ± 1.4
The teacher encourages ...			
9. Copying from teachers (1)	1	3	2.0 ± 1.1
26. Doing work as shown (1)	2 *	3.5	4.2 ± 0.9
16. Doing what we are told. (1)	4	4	4.8 ± 0.6
The teacher encourages ...			
15. Finding information themselves. (2)	5	4	4.5 ± 0.8
3. Finding links ourselves (2)	4	2 *	4.0 ± 1.2
29. Use their own knowledge to check results. (2)	4	2	3.3 ± 1.2
21. Asking questions of the teacher to check results. (2)	5 *	3	3.4 ± 1.4
11. Trying out ideas themselves. (2)	5	4	4.2 ± 1.0

Mrs K's level one desired responses indicate she wanted the students to rely on themselves in how they went about the tasks except for Q16, "the teacher encourages us to do as we are told." indicating that she saw herself as directing the students in what they did. She believed (Q18) that there would be times when she would show them how ideas and experiences were linked. Mrs K scored all the level two ('I would like them to feel this way about me') thinking activities as 'often' or 'very often', indicating a strong desire that the students did not see her as the only source of wisdom and information. Questions 29 and 21 show she wanted them to be reflective about their mathematics solutions.

The realistic nature of her beliefs about the students' views of her is indicated by how few of her responses were more than a standard deviation away from the students' responses. Questions 26 and 21 are the only areas in which her preferred role was not near to where the students saw her. She would have liked them to feel they could be innovative in the way they did their work, while the class generally felt she wanted them to work as shown. This indicates one of the areas where her practice has yet to produce the student beliefs she would like them to have.

The overall picture of Mrs K's role as a teacher is one of selecting a range of tasks to fit her understanding of how children learn mathematics. She aimed to promote a classroom culture of limited independence within set boundaries of behaviour towards other people. Mathematically she wanted an environment where the algorithms must be mastered both as an end in themselves and as tools for applications within mathematics and the students' lives.

Power

Mrs K was careful about the power relationships she established in her classroom. Using Tauber's (1993) schema for types of power, Mrs K used legitimate and referent power a great deal. She kept a close watch on the progress of every student and was the recognised authority in disputes and the direction of the work. She had, however, gone to some trouble to build norms of self-responsibility and perseverance into the classroom culture (referent power). Some of the students complied because she would notice if they didn't, indicating they were in a legitimate power relationship with her.

She built rewards into the system, with the students knowing when they were entitled to claim time for themselves or time on the computer. These rewards were usually contingent on completion of part of their contracts. Punishments (coercive power) were usually linked to failure to progress. This resulted in a rebuke or them working alone for a short period of time. Little overt disruption of the class routines was observed.

Mrs K shared the expert power between herself, physical resources and the students, refusing to be the sole source of information or authority for mathematics. Correctness for sums was her domain, but if she marked something wrong she required the student to find and correct the error. In the project or practical work she expected the students to refer to each other first in cases of uncertainty or disputes, then to come to her for guidance.

THE NATURE OF THE TASKS.

Mrs K focussed on two aspects of mathematics: algorithmic skills to ensure student competency in arithmetical operations, and a wide range of mathematical activities to locate the skills in the lives of the students and encourage deep understanding. Her mathematics class time typically began with five to eight sums. These were one of a variety of tasks or task titles that Mrs K put on the blackboard at the beginning of each day. The students had some freedom to choose the order in which to do these tasks.

She taught arithmetic processes in a sequence - 'single digit without carrying' at the beginning of the year up to 'multi-unit with carrying' by September. Each day's sums would have a range of levels, with some students not expected to do them all.

> *The revision is down to 5 now: 1 addition, subtraction, multiplication, division, and a long multiplication, and all with adjustment (carrying). At the beginning of the year, with year 4,5,and 6 in the class, we started with no adjustment, and they wouldn't do any division. So there would be addition, multiplication and subtraction - no carrying, and for some of them these carried on for, ... half a year!!, until they learnt.*

She expected that this would maintain and extend the more capable students and provide the others with the practice they needed. Mrs K tried to get a balance between helping the slower students to catch up and challenging the more able students. To do this she taught the standard algorithm methods to all the students using steadily increasing levels of difficulty and also worked with the slower students several times a week re-teaching both the methods and the ideas behind them. When teaching she was careful to relate the methods to understanding the place value reasoning behind them, but believed that many students would need frequent repetition of the explanations before they would make sense.

She was happy to accept a variety of formal and informal solution methods if the students used them.

> *I teach a way, but if they do another way and get it right, it doesn't matter. I don't mind how they get the answer, as long as they get it done. As long as the answer is correct, it really doesn't matter if you start from the 1's or the 10's. (pause) For a lot of children they need the structure of 'six and what is zero, you can't do it', rather than '0 take away 6'. I think for those children who are slower at it they need the structure, they need to know 'if I do it this way, I will get the right answer'.*

Mrs K believed that the best way for students to make sense of the methods she taught was to have regular practice, with explanations and the use of equipment when they needed it.

> *I used place blocks for them; I used them for everyone at the beginning. But if I did that... W is year 4, T and R are year 5, if I said to them 'you're going to use the blocks' every time we did mathematics, what would that do to their self-esteem? Now they're not going to understand, for sure. What I'm hoping is that, maybe, at some stage, they'll <u>get</u> it. Because it does happen! All at once it comes together.*

Mrs K used a lot of practical work in context to connect the skills and concepts together.

> *We just happen to be doing problem solving now. Last term it was measurement, with equipment. ... And the money one where we had shops. It's just, ... trying; ... I love maths, ... trying to make mathematics more meaningful for them.*

Mrs K would usually choose activities that related to the children's activities outside school. Other activities were aimed at linking to art, science, language or the social studies curriculum. Purely mathematical activities had their place too, such as pre-algebra pattern making. Mrs K made a point of making the activities multi-sensory.

Well, some children love hands on stuff. The water, the volumes, was specifically hands on. The problem solving, the drawing patterns have been hands on. You should have seen it on Friday; the classroom was just smothered in equipment.

Mrs K designed work so the students could apply the skills she had taught. For the practical sessions she would make sure the earlier work in any topic used straightforward mathematics, and the later work needed more thought. The less capable students were thereby able to do what they could cope with while the more able students were extended. It did mean that only the more capable students did a lot of problem solving.

For her, independence and responsibility seemed to be most easily exercised when the students had the background skills to be used. Mrs K expressed an awareness of the different natures of the open-ended creative work and the fixed results of mathematics computation.

Now, with mathematics it's a little bit different, because maths is either right or wrong. There are different ways of getting there, but the answers are the same, every way. Where the individuality comes in is, when you get that over with, get on with your project, problem solving, or extension work. There are several pairs who have got the extra book and are doing extension work.

Overall, Mrs K had a two pronged approach to mathematical tasks. Firstly, she put a lot of time into teaching and drilling the standard algorithms so the students would have automatic tools at their disposal. Those who struggled to understand the algorithms would at least be able to get sums right if they were told what to do. Secondly, she encouraged creativity and fun with activities. These had the purpose of linking the mathematical algorithms into everyday mathematical contexts; developing both instrumental and relational understanding; and maintaining student interest in mathematics as a whole.

MATHEMATICAL TOOLS.

Mathematical tools for Mrs K included a wide range of physical objects and mathematical concepts. The physical tools included counters, fingers, equipment and books that Mrs K used to help the students to build mathematical concepts.

> *That's why they use their fingers. I say to the kids, 'use your fingers, God gave you ten fingers, there are ten numbers'. Generally speaking, I think if you need your fingers, you can go to ten, 'if you need more, use your toes'.*

A major aim of her mathematics teaching was to get the students to a stage where they could operate on multi-digit numbers easily. She assumed that place-value understanding would develop with procedural facility. When a student could 'do them' she inferred understanding from the correctness of the solutions. In conversation she demonstrated a well developed sense of how students developed number understanding and used that to guide her judgements as to the assistance needed by any given child.[3] She was very aware of the need for numeracy with understanding and the central role of 10's place understanding to underpin the standard algorithms:

> *Do you remember when the new maths books came in? A lot of the work was, if it was 365 plus 123, it would be 300 plus 60 plus 5, add them up separately, and then add the whole lot up together. It was great for the children who were average and good, it was absolutely hopeless for those slow children, because they still couldn't understand – it's numeration!*
>
> *I think if you gave them the number 365, they would know that the 3 was 300... but you see, ...they wouldn't be able to work with that in any way, ... because of that problem of 5 into 27* (how to do grouping with divisors). *If I told them to go and do that, I don't think they'd know where to start. Although I've done it with them more simply, I've taken 27 counters and asked them to put them into groups, they can't do it.*
>
> *I would say that the majority of the kids in my room would be at the, ... the majority would be at multi-level concepts, but not securely. You've got to*

---

[3]Mrs K readily understood Young-Loveridge's (1999) model of numeracy and found a great deal in it that was congruent with her practice.

remember that most of them are only year 4, and they're working on concepts that are actually level 3. They're well up.

Mrs K said that she had spent a considerable amount of class time earlier in the year on metacognitive skills: thinking strategies, problem solving strategies, and reflective thinking. She had taught these in language contexts and reinforced them regularly in other subject areas except mathematics. It was a surprise to her to become aware of this omission in the course of a conversation. She said that she saw thinking skills as tools of mathematical understanding and commented that she had expected the students to use them without specific teaching to transfer them to mathematical contexts. She intended to redress the omission forthwith.

She believed that thinking was helped by good content and general knowledge, but that a student could think well with whatever level of mathematical capability they had. She did not, however, provide the slower students with very much problem solving during the study although they participated in the practical work and associated group assignments.

EQUITY AND ACCESSIBILITY.

Mrs K believed that the students were all individuals with their own needs and strengths. This attitude lay behind many of her teaching practices, which were aimed at starting from where the students were and developing their abilities with experiences appropriate to their learning needs.

"*They're all different, that's why I work on individual learning programmes*"
"*Where the individuality comes in is, when you get that over with (the daily maintenance), get on with your project, problem solving, or extension work.*"

This is in harmony with constructivist ideas about learning, especially the importance of prior knowledge and situated knowledge, but does have the potential to restrict communication and interaction. Mrs K tried to set similar work for groups of students where appropriate and to find extension work for her more capable students. Mrs K frequently provided individual help:

"*We've just kept working at it and working at it. He's year 5! He still needs the one to one. He's not too bad, ... he's Ok with division and adding.*"

She was careful to match the task demands with both the mathematical level and the emotional needs of the students, tailoring the support to match student needs.

I had a boy, HK, he couldn't seem to add or subtract any way, but give him a problem, and he was absolutely fantastic. He would do what you were talking about. But if you (gave a problem) to W, he'd clam up, so would T. Now these students haven't got the vocalising, they haven't got the verbal background that gives this ability to them. I used place blocks for them - I used them for everyone at the beginning. But if I did that... W is year 4, T and R are year 5, if I said to them you're going to use the blocks every time we did mathematics, what would that do to their self-esteem. Now they're not going to understand, for sure. What I'm hoping is that, maybe, at some stage, they'll get it. Because it does happen! All at once it comes together.

... Some children start straight away, so they will get it finished. Then you've got the procrastinators, who waste time no matter what they are doing. Whether they like the subject or not, doesn't matter what it is. You've got the talkers, who sit and talk about anything. So there's lots of reasons – the children are different. If some children are not checked regularly they won't bother handing their books up. It's just that inborn thing.

4.3 Summary

Mrs K had a coherent and well tested approach to mathematics education that worked for her and for her class. She had two main mathematical goals for the students: to become thoroughly familiar with the standard algorithms up to the multi-unit level, and to be able to use those algorithms confidently in practical situations. Socially, she promoted self-responsibility, self-esteem and a caring classroom environment. She saw her role as guiding and directing the development of these norms using her experience and beliefs as a guide for striking a balance between the legitimate and referent power relationships involved in her role.

5. THE CASE RECORD: LEARNING.

'THIS IS OUR CLASSROOM'.

The cultures of the two classes were somewhat different from each other although each had elements of both inquiry and traditional approaches to mathematics education. The data from each class is reported under Hiebert et al.'s (1997) five dimensions in the same way as the teachers' data.

Mathematics teaching and learning is seldom the sole focus of students during mathematics time. Because of this the reports in this chapter cover actions and statements that are peripheral to mathematics but which are a very real part of classroom culture. As will be seen, these peripheral concerns have the potential to greatly modify the mathematics teaching and learning.

5.1 Mr J's Class

THE SOCIAL CULTURE OF THE CLASSROOM.

The students expected to be doing mathematics every day at a set time. They expected to follow the routine set out by the teacher, which was to do the maintenance or revision questions, then move on to other work. Mathematics was seen as being mostly 'doing sums', with projects and problem solving as minor aspects of mathematical work.

There was generally a positive attitude to the mathematics.

> *S6:*[4] *I kinda like it (mathematics in general), but I don't like doing maths sums and such, but otherwise it's all right.*
>
> *S2: I'm from Canada, and I'm finding we do the same topics but in different order. I like pretty much everything.*
>
> *S3: I like mathematics, because I like adding up sums and such, times tables.*

[4] The students are identified within each focus group as S1, S2, etc. Since the quotes are from several different focus groups and observation periods, 'S1', for example, may refer to several different students.

S7: I like the numbers, and doing those things on the blackboard – problems on the cards. Usually, it's just that, we usually work independently at our own pace – I prefer it that way.

This interview evidence is confirmed by the written responses[5], which focused on being right and finished and general enjoyment of mathematics. Sixty nine percent of the responses reported positive feelings about mathematics.

Table 5-1 Affective aspects of mathematics time.

Most of the time in maths I feel...	Number of responses.
... like doing the work.	21
... happy because I'll finish soon.	3
... happy / I enjoy it.	5
... I don't want to do it.	7
... bored.	2
... worried.	4
In maths time I feel good about...	
... getting them right.	10
... getting finished.	6
... knowing the maths.	11
... working.	7
... not being growled at.	1
... I don't feel good!	4

It is noticeable that understanding was not directly mentioned, perhaps indicating that it was not an explicit goal of the culture. From the observations 'knowing the maths' could refer to instrumental understanding or to low-level relational understanding. Table 5-2 reinforces the students' focus on task completion and correctness.

Table 5-2 Some things I worry about in maths time...

	Number of responses.
... not being right.	15
... not knowing what to do.	5
... having to do it again.	3
... not getting finished.	4
I don't worry.	4

[5] The open-ended written responses have been summarised and coded into the categories in the tables. The students had free choice in their responses, and could put down as many as they wished (the complete questions are available in Appendix C).

The type of mathematical understanding the students valued was illuminated by how they responded to difficulties. The students were observed to spend a lot of time seeking help from each other and the teacher. This was confirmed by the written responses.

Table 5-3 When I don't understand I ...

	Number of responses.
… ask the teacher	17
… ask my friends.	14
… keep trying.	4
… work it out myself	5
… leave it out.	1
… ask the teacher only if it's really hard.	4

Discussion in the focus groups revealed more views on help seeking and understanding:

R: Who do you get help from?

S4: A friend... not always, sometimes.

S1: I try to understand it first. I have another go and double check it. Then if I still don't understand it I'll go and ask another person. If they don't understand it I'll go and ask Mr J. If he doesn't understand it then I can't either.

S3: I'll go back and have another go. I don't give up....

Some students preferred to work through the tasks and wait for them to be marked before thinking about their answers. If the answers were right they were forgotten about but if they were wrong the students looked for errors in the working. It was unusual for them to question their choice of strategy or algorithm.

S3: Well, I just see what I think. I just say the answer I think.

S6: I go over the question a number of times and get an answer. If that doesn't work (doesn't make sense?) and I can't find anyone who understands what it meant, I just use that answer and see what happens, then I'll come back to it later.

These quotes reinforce the impression from the written responses and observations that the students valued instrumental understanding above deeper relational understanding. Getting answers and being finished was the principal goal and understanding what to do and how to do it promoted those goals. Knowing 'why and when' were very minor features of the classroom environment.

How students treat mistakes is an important indicator of the classroom climate. When asked in the focus groups "what do you do when you get something wrong?", the students replied:

S5: Me, I get in a shitty (laughter).

S2: I get annoyed with myself, ... sometimes, then go... (pause) And then I go and work it out again if it's not too hard ... (long pause).

S5: I go and write the right answer, and put a tick next to it so I get it right (General laughter)

The coping strategy of S5 was often seen in class, especially among the less able students.

S6: Um, when we're doing our maintenance, and I get something wrong, I always just, ... rub it out and put in the correct answer, and then put a tick (giggles).

R: How do you know that it's correct?

S6: Because Mr J puts the answers up.

S1: (Responding to the initial question) I just kill myself. Then after I kill myself I, um, ... um, rub out my answer and look on the board and I really check my answer (with 'no I don't really' body language), and (more confidently) when no one's looking I give it a tick.

More able students reported more effective learning strategies:

S4: Well I go Grrrr and get annoyed, and then I keep on trying to work it out and get it right ...

R: Do you always do that, or just when someone's watching?

S4: (definitely) Nup. Most of the time.

It was also observed that the more able students frequently used the answers to confirm their written solutions, while the less able ones looked them up before they put an answer down. All students said they wanted to be able to do them without looking, and were seen to do so with more straightforward problems.

When the answers were available in the textbook, the students had mixed beliefs about using them.

S2: Oh that would be too easy

S4: Not very good, because if you didn't know anything you could just go to the back of the book and copy the answer.

R: That'd be Ok? (mmm, yeah, nope)

S6: But then Mr J would ask you ' how did you get that answer?, and you can't say and you stay in. You'd have to say 'the back of the book'.,

S4: Yeah, I'm honest. I'd say ''B' told me', or someone.

S1: Mr J says you're allowed to look in the book for the answer. I reckon it's good for the answer to be in the book, then if you don't understand something you can look to see the answer and work it out that way.

S3: I wouldn't mind the answer to like proof read them, to mark it. That's what they're there for.

When there was pressure on students to finish the most capable student in the group would do the exercises, while others would try to do them, then call out for an answer. If a student believed to be capable had a different answer, the others would rub theirs out and put in the 'correct' one. Some students would not even try, asking around for the answers anyway. The students had mixed feelings about this copying. Several variations of 'I could have done it' were heard in class.

S2: I reckon you should actually have a go at working out the sum before you go and ask someone else. (why?) cos you'll go, 'oh yeah, I knew how to do that. I just couldn't be stuffed doing it.'

These coping strategies were carefully hidden from the teacher, and the students would hand their work up separately if there had been a lot of copying. Students wanted to fit the norm of 'doing the work yourself' but frequently modified this practice when under pressure.

While many students in Mr J's class worked steadily at their mathematics others tended to spend a lot of time chatting, colouring in borders, or watching their neighbours work. Observations indicated that these off-task students were often those who reported negative feelings about mathematics. The written responses indicated that the students attributed mathematical success to ability, enjoyment of mathematics, and good work habits (summarised in Table 5-4).

Table 5-4 Some people in our class learn mathematics better than others because...

	Number of responses
... they are more çapable / they know more.	8
... they are faster.	10
... they like maths.	8
... they don't talk	2
... they don't get distracted.	6
... they work together.	1
... they are scared of the teacher.	1

Focus groups provided similar responses to this question:

S5: They've got, like, a quicker brain. If they hear something it sticks in their brain. Like me, I can run real good, but other people can't and they're real brainy at something else. I'm not that brainy.

S6: Some people get something into their brains easy, and it keeps working.

S5: Some people can hear the question and respond quicker than others. Some people who don't understand, it takes them just ages to get it.

S1: They're brainy and can get it.

Additionally, some students felt that 'outside help' could help students to be good at mathematics.

S3: Some people's mums might go to maths classes, and they might teach them.

S2: They might get taught at home, and have flash cards and stuff.

S3: I do times tables at home, but I'm only good at that properly.

S7: Some people do Kip McGrath or something after school.

Interestingly, no one talked of being able to improve their ways of doing things themselves, indicating (negatively) that there may not have been an established norm of reflection on their classroom practice.

The norms reported here were triangulated by the responses to Stevenson's (1998) questionnaire. The questionnaire (Appendix A) asked the students to report their general feelings of obligation; how they felt the teacher wanted them to act; and what they actually did in such circumstances. Stevenson (1998) uses the term 'press' to denote the normative pressure felt by the students as an obligation to act in particular ways. Two presses were investigated by the questionnaire, the press from the environment (home, friends and community), and the press from the teacher. These presses were further classified as either first order (towards instrumental understanding and lower order thinking) or second order (towards relational understanding and higher order thinking).

When examining the press for lower order thinking (table 5-5), the means for most of the class responses are near the middle of the Likert scale and the standard deviations indicate few strong responses.

Table 5-5 Presses for lower level thinking[6].

Mr J's class. (question numbers are given before each question).		Class means and standard deviations
Copying from the teacher.	6. 'I feel I should' (general obligation.)	2.5± 1.1
	9. 'The teacher expects'	2.8± 1.4
	20. 'I do' (Self reported actions)	2.7± 1.1
Working exactly as shown.	17. 'I feel I should' (general obligation.)	2.9± 0.9
	26. 'The teacher expects'	3.4± 1.0
	30. 'I do' (Self reported actions)	3.1± 1.1
Doing as the teacher says.	25. 'I feel I should' (general obligation.)	3.5± 1.0
	16. 'The teacher expects'	4.0± 1.1
	5. 'I do' (Self reported actions)	3.5± 1.0

The students made it clear afterwards that they had read the first two questions as referring to mathematics time in particular while the third question ('Doing as the teacher says') was interpreted by the students as being more about general classroom behaviour.

The presses felt by the students match their reported actions quite closely. Their responses support the impression that the classroom culture included some pressure to follow instructions and to complete tasks without active inquiry into the ideas behind the work, although 'copying' was expected slightly less than 'working as shown'. The responses to 'doing as the teacher says' indicate that classroom behaviour was firmly under Mr J's control – a legitimate rather than referent power base from the students' perspective.

[6] Student responses were: 1=very seldom, 2=seldom, 3=sometimes, 4=often, 5=very often.

Table 5-6 Presses for higher order thinking.

Mr J's class (question numbers are given before each question).		Class mean and standard deviation.
Finding things out ourselves	4. 'I feel I should'	3.6 ± 1.0
	15. 'Teacher expects us'	3.7 ± 1.1
	27. 'I do'	3.3 ± 1.2
Finding links between ideas.	22. 'I feel I should'	3.0 ± 0.7
	3. 'Teacher expects '	3.4 ± 1.0
	13. 'I do'	2.6 ± 1.3
Check results against own knowledge.	12. 'I feel I should'	2.6 ± 1.1
	29. 'Teacher expects '	3.1 ± 1.3
	7. 'I do'	2.9 ± 1.4
Ask the teacher to check results.	10. 'I feel I should'	3.0 ± 1.0
	21. 'Teacher expects '	3.3 ± 1.1
	1. 'I do'	3.0 ± 0.7
Trying out new ideas	2. 'I feel I should'	2.9 ± 1.2
	11. 'Teacher expects'	3.1 ± 1.3
	19. 'I do'	3.5 ± 1.0

The differences between the means is small both for the different presses and between the same presses for different questions and most of the results are very close to the middle of the Likert scale (3= sometimes). While some of the students felt some press for higher order thinking there is little evidence for a strong classroom culture of exploring mathematical ideas or self-checking the results of computations or problems.

Interaction and communication

Most students preferred to work together during mathematics time. Reasons varied, with 'being able to get answers or help more easily' a common response.

S1: We work with partners, so you can get better answers then. The more answers you get; the more people who are in the group, quite often you get the answer quicker.

S3: Because if you're stuck, you can get help from the others.

S5: It's good to work with your partner cos if you're really stuck you can do it together.

S4: If you get it wrong you have to think in your mind and it might take forever to get the right answer.

Other students felt it was better to work alone, at least some of the time. This was particularly noticeable among the more mathematically capable students.

S6: If you work together you talk a lot.

S4: We work in groups a lot. I like to work independently cos you don't get an urge to copy off the other person.

S1: If you work with a partner all the time they might give you all the answers, but you need to do some independently so you know ..., you might get a little bit of help and then go back to working on your own.

THE ROLE OF THE TEACHER.

(Hiebert et al., 1997) suggest that there are three main classroom roles for an inquiry-oriented teacher: selecting mathematical experiences and guiding the students through them; sharing information appropriately; and facilitating the development of an inquiry oriented classroom culture. These teacher roles enable the students to build mathematical understanding through social interaction and engagement with the tasks: "The teacher relies on the reflective and conversational problem solving activities of the students to drive their learning" (p.8).

The student view of Mr J in mathematics time was that he wanted them to complete the maintenance, textbook work, and worksheets and that he was there to explain to them how to 'do the maths'. Observations indicated that the assistance the students gave each other was mostly aimed at following the procedures correctly and completing the tasks rather than developing understanding. Peer explanations also varied in their content: students who knew the work well were much more likely to give reasons for the procedures along with the methods; less capable students often gave the answer only, or told their peer what steps to follow.

Often the student or teacher explanations were given several times without the student understanding the points being made:

S1: He writes stuff up on the blackboard, and goes over it in class. Then he'll tell us again. Then you have to work it out yourself. He used to keep going over and over it, and he got sick of it, so now he tells you twice. You can't be sick or you'll miss it!

S1: (If you still haven't got it) You have to get someone else to get it off. We work with partners, so you can get better answers then. The more answers you *get; the more people who are in the group, quite often you get the answer quicker.*

S3: I like working in groups because you get more (unintelligible) In our class Mr J will just give you the answer. Like, some people don't know it, and he doesn't ask you before he gives it out. Then at the end you won't know what he did because he just gives you the answer, straight down on the blackboard.

S6: He does for everyone a couple of times, then he'll say, 'no, no, no, I've taught you that' (nods of agreement).

When talking about how Mr J responded to their requests for help, students indicated that he would explain what to do, mark the work right or wrong, then get them to redo the wrong ones. For quite a large group of the class this did not translate to him wanting understanding, just correctness and completion. Observations indicated that Mr J clearly thought the strategy would press the students to understand the methods as well as to become proficient with them.

Several students found Mr J's explanations difficult to understand:

S1: I go 'Please Mr J, I don't understand this.' He explains it and you still don't get it.

S3: Yeah, that's like I usually go ...

S4: Yeah. (other agreement.)

S7: I was trying to do this worksheet, I asked a friend and I asked Mr J, and I couldn't understand him. I just put down this answer.

S2: I, um, ask him for help, and it happens all the time (not understanding). I ask at home, and just write any answer if ...

When re-teaching a section or concept Mr J would normally explain things himself rather than elicit explanations from the students.

When looking for information the students expected to be told what they were to do or find, and that it was up to them to help each other to complete the task. They were supposed to use the textbooks, library and other sources to answer the worksheet questions.

... we've got the information in our books. I get information from the library, from my parents, I get it from my brain or my textbook.

However, the term 'information' seemed to be linked to investigation contexts, while 'answers' were what they got for 'sums'.

The students believed that Mr J wanted them to do the mathematics his way mostly, although several of them had alternative solution methods for the standard algorithms. These were not generally revealed to Mr J but were readily shown to the researcher on asking. The students did not seem to distinguish between instrumental understanding (how to do the sum) and relational understanding (why they should use that method); their goal was to be right and to be finished and any sort of understanding that would serve this goal was sufficient.

Mr J's role in setting classroom culture.

Negotiation of responsibilities and consequences for misbehaviour or being off task had occurred early in the year. These had resulted in a set of taken-as-shared norms for classroom behaviour that were reinforced by Mr J directing the classroom interactions when needed. The students certainly saw Mr J as the authority in terms of behaviour (see table 5-5). They all tried to behave as expected when Mr J was near and accepted his dictums without public murmur. They also acted in accordance with most of the behavioural norms when Mr J was out of the room. The norms that were sometimes not adhered to when Mr J was absent were related to keeping on task. Very few incidents of bullying or teasing were seen, and these were always hidden from Mr J.

Praise and printed certificates for completion of work, high scores or sustained effort was the main reward structure in the classroom. In addition, students who finished early were normally allowed to do their own choice of activity. Improvement in understanding was seldom seen to be rewarded formally during the study although Mr J looked for such evidence in his preparation for parent interviews.

There was a perception among the slower students that Mr J rushed them too much and required them complete work in the same time as the faster students.

> *S3: He goes, 'you guys hurry up' when the first person finishes. They're normally the brainiest, and all the not very good people are staying in at lunchtime, like me and (list followed). You feel all the time you're ...*

S6: Yeah, and one time we had a worksheet with like 55 questions and ½ an hour!!! If you don't get finished you have to stay in at lunch.

S4: It takes too long, cos Mr J gives us too little time to do it. I want to get it finished (plaintively).

Student responses and classroom observations concerning Mr J's role are also supported by the written responses summarised below (tables 5-7, 5-8, 5-9).

Table 5-7 The teacher likes us to work ...

The teacher likes us to work ...	Number of responses
... quietly	18
... quickly	4
To 'get on with it'.	17
To listen and concentrate.	3
... neatly.	1
Other responses	1

While quick and quiet work of itself does not indicate either a procedural or inquiry focus it is certainly a common feature of procedurally based work.

As the 'boss' of the classroom Mr J was seen to direct, approve or sanction the actions of the students. In answer to *What sorts of things does Mr J say?* the students responded with laughter and a babble of voices:

S6: (cheerfully) He says 'Get on with your math's (growly tone).

S1: Go back to your seat (laughter).

S3: He goes 'Shut up and get on with it'.

(Chorus) Noooo, He's never said that to me.

They accepted this disciplinary role and his growls. Mr J very seldom growled during the observation period, though he frequently spoke directively (legitimate rather than controlling power).

Table 5-8 When we start our maths time the teacher says ...

When we start our maths time the teacher ...	Number of responses
... says to do our revision/maintenance work.	12
... says 'get out your maths book.	15
... explains about the work	4
... says to 'be quiet'.	1
Other responses	3

The bulk of the students expected to be sitting at their desks, doing bookwork of some sort. They were almost always all sitting down and working by the time Mr J came in after morning break.

Table 5-9 When we start mathematics my attitude is...

When we start mathematics my attitude is...	Number of responses
... sort of happy.	11
... I want to listen/learn	3
.... sort of ok / ready to get on with it.	10
... lets just do it/ get it finished.	5
... this is boring	2
... oh no, not again.	5
Other responses	0

Mr J's role and approach was acceptable to most of the students, showing they shared the classroom norms of steady work on the set tasks.

Mr J's power base was therefore mostly experienced by the students as legitimate and referent power, except in the case of rewarding the early finishers and where the students felt their reasons for non-conforming had not been listened to. In these cases there were perceived elements of reward and coercion. The students relied on Mr J's role as expert to help with their mathematics difficulties although they sometimes felt frustrated with a perceived lack of clarity of his explanations.

NATURE OF THE TASKS.

The tasks observed included textbook exercises, algorithm practice from the blackboard, problem-solving exercises from worksheets, and practical activities involving recently taught concepts and skills. There was no comment from the students on the appropriateness of the tasks that Mr J assigned to be done. The students generally got on with what was set and the tasks appeared to be at the right level for most of the students. There was not a great range of abilities in the class so even the slowest students at mathematics were able to attempt most of the work. There was little appreciation from the students that the tasks might have a developmental function, although a few students commented that they could see some progress in their understanding or capability over the course of a topic.

During the focus group interviews the students gave a range of responses to the question "what mathematics do you do?", probably reflecting their most recent mathematical activities. Many students commented on the daily maintenance, which focussed on mathematical skills:

S4: Oh, just number stuff, you know, times and ... such.

S2: We do adding and numbers.

S7: We have to do twenty of them at the start each day.

S1: First, after play, we go in, he's normally got, um, some maintenance, (agreement) that's on the board, otherwise we have to work on our book...

S5: We work on anything we have to do, and then when he comes in, we do our basic facts, and then we do, um....

Practical work and problem solving was also mentioned.

S5: We get to make shapes out of paper, and , sort of doing a little bit in our book but we get constructions and see how they work out. We work out the dimensions for the right volume and so on.

S6: Like you'd count how many golf scores, in a sports game or something ....

S4: Yeah, right, graphs and everything in algebra. And you count how many balls, like you can in algebra.

In the written responses there were two items that indicated the range of tasks considered to be mathematical. Responses included maintenance, the curriculum strands, content areas within the strands, games, problem solving and activities.

Table 5-10 'What do we do in mathematics time?', and 'What do we like?'

Mr J's class: Mathematical activity	"We have done these this year" Number of responses.	"The mathematics I like best is" Number of responses.
Curriculum strands[7]	18	8
Maintenance /revision of algorithms.	10	6
Content within a strand.	6	1
Basic facts revision	6	1
Textbooks	6	0
"heaps of things"	5	-
Problem solving	2	4
Worksheets.	2	4
Mathematical games	1	0
Practical activities	1	3
Hard work/new work	-	11
No response /non-task response	1	8
Totals	58	46

The 'Hard work/ new work' response indicates that many students felt competent and capable with the level of mathematics. Similar responses were given in the focus group interviews.

S1: Oh we like maths, it's good

S2: Except maintenance, I don't like that

S2: I like maths because I'm good at algebra.

S4: I like maths, cos I'm the first finished.

During class time the students were seen to spend a lot of time with textbooks and worksheets. The end-of-topic projects took about a week every six weeks.

How we do the mathematics.

There was a common view that mathematics consisted of doing the work set, that their role as mathematics student was of fulfilling the obligation to complete the work correctly. There was some discussion about the work but it was mainly directed towards the mathematical procedures involved.

When working with or talking of the times tables, daily maintenance or textbook work the students regarded the mathematics as some thing they had to 'get right'. Their

[7] Any number of strands or content areas counted as a single entry.

reasons ranged from wanting to move on, wanting to be finished, not having to do more of the same work, and wanting to know where they had gone wrong.

> *S5: If you get it right, you feel good about yourself, and if you never got it right before, you won't have to worry about struggling with that again.*
>
> *S3: You don't need to learn them again*
>
> *S2: You get it over and done with!*
>
> *S1: I want to get it finished.*
>
> *S4: I'm happy when I'm finished and I'm marking it cause then I know how many I got wrong.*

During the projects there was plenty of discussion and mutual helping. However, much of the interaction was mathematical only in that it involved organising the display of numeric or graphical information. Specific word problems on the sheet were interpreted as isolated events to be done like textbook exercises and finishing in time was a major concern.

Although Mr J regarded the end-of-topic project as an important part of the mathematics programme the students did not include 'projects' or 'investigations' as being mathematics in the written responses. Few students recalled a 3-4 week topic that focused on the Australian mathematics contest earlier in the year.

A statistics project, set at the end of the statistics topic, required students to collect data on local traffic patterns, to display and explain what they had found about the situation. Most students chose to collect data the same as they had already done for practice and relied on the worksheet to format their responses, only answering the specific questions from it. Most of the students treated the task as one of showing that they could draw bar graphs and write the standard statements about them that were prompted by the worksheet. There was a great deal of looking at what other students had done, to 'make sure we're doing it right'. In spite of Mr J's encouragement only a few of the students used the opportunity to investigate something they were interested in, or to use the graphs to answer their own questions about the situation. The discussions observed in the class were focussed very much on management and completion rather than using the mathematics to understand the situation.

The general impression is that the students were looking for closure and correctness in any given episode of mathematics, so that they could leave a topic or technique behind and get on with the next requirement. The problematic nature of the work was largely restricted to discussion of how to do tasks to Mr J's requirements and the mathematics itself was treated as problematic for most students only to the extent of being able to do the required processes.

MATHEMATICAL TOOLS.

Mathematical tools "include oral language, written notation, and any other tools with which students can think about mathematics" (Hiebert et al., 1997, p.10). Most students did not seem to be aware of their language being a mathematical tool; there was very little evidence of deliberate choice of oral or written language to improve an explanation. Some students were aware of how communication could go awry:

> *S2: You get into trouble with the teacher if he wants it done like this, and you understood it differently (general agreement.).*
>
> *S7: Sometimes I feel like Grrrr! I did this cos I thought he said to!*
>
> *S2: I find that's where kids say a teacher is mean, where you misunderstand instructions and the teacher doesn't listen to you.*

Written notation was mostly used to provide evidence for the teacher that the work had been completed. Scrapbooks were used and drafts were done when required in the projects, and some progress in awareness of their value was apparent during the study, from 'Mr J said we had to', to 'this is useful'.

Standard physical equipment, such as felts, cardboard, and glue, was readily available. These were used more during the project phase of a topic or where a worksheet called for it. No students were seen using equipment (except for their fingers) to model sums or problems and there was no mention in the focus group interviews of doing so.

The skills of mathematics were seldom spoken of by the students as tools for use in mathematical situations. They did not seem to be reflective about their use of processes or algorithms. Stevenson's (1998) questionnaire results described above indicate that there was not a strong inquiry culture in the class, and the rather vague understanding of the processes of problem solving was congruent with the teacher directed culture that was observed. In the written responses only four students included 'problem solving' as one of their favourite parts of mathematics, while three wrote 'activities' (Table 5-10). Likewise, in the written responses to *In mathematics time we do different sorts of things. This year we've done ...*, only two students mentioned 'problem solving' and one other student wrote 'activities'. The implication is that problem solving and investigations were not perceived to be an important part of their mathematics experience.

In an interview discussion of what mathematics was, the students did not offer problem solving as an option, so one group was asked *What about problems and investigations?* Two capable students responded:

> *S2: They're different sorts of mathematics. In geometry it could be building something, or gardening to do number – you write the numbers down.*
>
> *S1: Problem solving; we're supposed to do it every Friday but we don't get around to it. Problems, ... you work out if you've got to divide or multiply to get the answer. Investigations, ... well sometimes you work by yourself.*

When asked how they went about solving problems their responses did not indicate strong problem solving strategy awareness:

> *S2: I read the problem, think of which way to solve it, guess it and check, or try times and divide, and so on.*
>
> *S4: I look at the question and work it out.*
>
> *S7: Read the question, read it again, think, work it out.*
>
> *S1: With a problem, you read it, you can't do it unless you read it, then (long pause) you work it out, sort of.*

Most students shrugged or gave no response.

Overall, these responses indicate that using skills as tools was embedded in their other practice rather than being something they had identified as a specific part of their mathematical repertoire.

EQUITY AND ACCESSIBILITY.

According to Hiebert et al. (1997) equity is the opportunity for all students to learn mathematics with understanding. In this class the students primarily construed understanding as knowing how to do the algorithms and using them to complete the set work. Most students were able to achieve this.

The students perceived themselves as all being expected to take part in all the mathematical activities. Mr J's classroom management processes made sure that every student's progress was regularly monitored. All the students shared the class norms of doing the work to Mr J's requirements, even though it took some of them a lot more effort, time and help than others and some work was copied. The impression is that the students shared a perception that they all had access to the mathematics of the classroom.

SUMMARY.

The students in Mr J's class expected to follow his instructions most of the time, but to carry them out in their own ways. They accepted the norms of completing the work in a set time frame and of taking care over the work. The norms relating to doing the work themselves were modified considerably in practice by many students. Most students were willing to copy answers when they were under pressure to finish or to be correct, while simultaneously believing that it was important to learn how to do it themselves. The mathematics was seen by the students as mainly procedures and algorithms. They were largely unaware of applying these in problem solving or investigative work. These practices would inevitably result in instrumental or low-level relational understanding for many students even though Mr J tried to plan the topics to provide opportunities to enhance deeper relational understanding.

5.2 Mrs K's class.

There were 30 students in the class. Two students and their caregivers declined permission for them to take part in the study. Their data has been deleted. All the other 28 students were observed, took part in the focus groups, filled in sets of questionnaires and wrote the written responses. The quotes below are representative of those given by the class.

The students were of a wide range of mathematical and general academic capability, seated in friendship groups around the room. Mrs K commented that students of each ability level tended to sit together, creating a natural streaming effect. The least able and slower students generally sat themselves close to the teacher's desk.

THE SOCIAL CULTURE OF THE CLASSROOM.

The social culture for doing mathematics in Mrs K's classroom was of following routines of getting the maintenance done and moving on to the other work. The student goals for the seatwork and maintenance were correctness and completion while practical and applications work was seen to involve wider mathematical goals as well.

There seemed to be a public acceptance by the students of Mrs K's classroom norms: independence, self-responsibility, perseverance with the work until it made sense, and learning to both do and understand mathematical procedures. Most students tried to follow these norms most of the time, only departing from them when they were short of time to correctly finish the work. The students often resorted to copying to get the work finished, either because they had been off-task or were genuinely slow.

The students were expected to do the daily maintenance before beginning textbook or worksheet tasks. Students who were still learning to pace their own work were put on contracts to complete certain amounts of work within given times. These contracts were negotiated individually between the students and Mrs K and consisted mainly of tasks designed to provide quick success to bolster student motivation. The less able students and some others spent most of their mathematics time on contract but over half the class were able to satisfy Mrs K that they were keeping up. When asked which they preferred, the students' answers were evenly shared between contracts and problem solving.

A useful indicator of the student culture was their attitudes to the mathematical tasks. Two questions in the written responses covered this (tables 5-11 and 5-12):

Table 5-11 Most of the time in maths I feel ...	Number of responses
... like doing the work.	15
... happy because I'll finish soon.	4
... happy / I enjoy it.	3
... Ok.	2
... worried.	3
... bored.	4
... I don't want to do it.	4
Other responses	2

The feelings ranged across the emotional spectrum with sixty-five percent of students reporting generally positive feelings. The attitudes to the revision at the beginning of the day were less positive:

Table 5-12 When we start our mathematics time I feel ...	Number of responses
... sort of happy.	7
... I want to listen/learn.	3
... sort of ok / ready to get on with it.	9
... let's just do it/ get it finished.	2
... this is boring	4
... oh no, not again.	9
Other responses	3

Only 51% of responses (table 5-12) were positive towards the revision. Many students were seen taking their time to do the work and talking where they could get away with.

Comments in the focus group interviews confirmed the range of attitudes to the mathematics.

S7: I don't like bookwork.

S3: I prefer the sums, they're easy, I can do them.

S6: I like the sums, but when I finish them it's like, I'm glad they're over.

S3: Sometimes I think I might not know what to do.

The students were asked when they felt bad or worried in mathematics time:

S5: When I get them wrong.

S3: If I make a mistake, in a really easy one.

S1: When I get something wrong and everyone laughs at me!

S4: I feel sad when I get something wrong.

S1: Not getting it right, because then you have to do it again.

S2: I worry that I'll get something wrong or put the wrong number.

S4: Not understanding it. Getting it wrong or having to do it again. Or she's going to rip my page out or give me a growling.

These feelings were confirmed by the written responses summarised in table 5-13:

Table 5-13 Some things I worry about in maths time ...	Number of responses
... not being right.	22
... not knowing what to do.	5
... having to do it again.	2
... not getting finished.	1
I don't worry.	2
... someone copying.	2
Other responses	4

The student focus group responses about *what made them pleased with themselves* confirmed the importance of 'being right and finished':

S1: Just getting it done

S2: Knowing my mathematics and being quite fast.

S3: Doing things right or doing what I'm supposed to. Or when I like doing it.

S4: Not getting a growling, understanding it and getting it right.

S9: Getting things right and expanding my mind, because mathematics is fun.

S5: I take it up to Mrs K and she ticks it and I feel happy.

Table 5-14 summarises the written responses to this question:

Table 5-14 In maths time I feel good about ...	Number of responses
... getting them right.	11
... getting finished.	9
... knowing the maths.	9
... working.	1
... not being growled at.	3
Other responses.	2

The students were very aware of Mrs K's emphasis on perseverance. The students knew that their work should be within their capability, but on occasions felt that Mrs K pushed them too hard.

S5: When it gets too hard you start to struggle and you can't do it. You just get sick and tired of it. I know I can't do it. ...

S2: But you've got to say 'I can do it', but ...

S4: Once you've started it and got it wrong, you think, 'Oh I can't do this' but you've got to do it, and you're not allowed to give up. You've got to do it and do it right.

S6: Yes. I had to keep at it again and again (learning times tables) - months. Mum used to check me and I had to get them right or she'd say 'go back and do them again.

S1: It's like, once you've started it and you've got it wrong, you think 'oh I can't do this', but you've got into it. You think 'I'm going to give up' but you're not allowed to give up.

S2: It's like in class she gives us (a project or some practice work) and only gives us a week to do it. And after that she gives us more, and then more ...you don't have enough time to finish the first one.

In class, several students also spoke of wanting to work things out for themselves and to become competent at the mathematical work. During the practical work especially the students showed a commitment to doing the work themselves and to making sense of the methods (low-level relational understanding).

During bookwork, students had a range of strategies to make sure their work was correct and finished. They would seldom check their work, but if they found a wrong answer they might rework the sum; get a friend to do it; or copy from a more capable student.

S3: I cross it out and do it again (Disbelief from others).

S5: I'll work it out again and if I don't get it right that time I'll ask for help.

S2: I go and change the number that's wrong and put the right answer in.

S6: When I've made a mistake I work it out again, and write it out.

S5: Sometimes in maths the children copy you as well. You sit next to them and they copy out of your maths book.

The students were asked if it was OK to copy:

S3: No, because you won't learn for yourself (agreement from several).

S2: Because they'll get the answer and they won't learn for themselves.

One student related not copying to future needs.

S3: It's very important, so that when you grow up you can choose what you want to be. ... It's also important that you learn how to do it because when you grow up and when you have children, and they ask you, you need to be able to help them, because otherwise they don't know anything and it goes on and on for generations.

In spite of such responses, which may have been representative of 'what we should say', the strategy of copying from someone who was believed to have 'got it right', was very common. Mrs K's strategies of encouraging students of similar ability to sit together and encouraging discussion appeared to allow some students to 'cruise' through the mathematics doing little thinking. Practical work involving group effort usually produced much more on-task behaviour and individual effort than the maintenance or textbook work.

In informal discussions many students would laugh shamefacedly, admitting that they copied a lot. They often said something like 'we could have done it, because we know it anyway'. Their failure to do well in subsequent tests did not seem to be connected in their minds with their avoidance behaviour, nor did it seem to bother them. Students who would earnestly avow in an interview that it was important to 'do the work yourself because you had to have it for your future' were among the ones observed to copy the work of the most capable student in their group. There were, however, some students of all capability levels who did not copy, and several of these said in private that they didn't want the others to copy from them. In practice they allowed it to happen, although several of the most capable students would go to some lengths to hide their work.

The students also felt a mixture of public annoyance and for many of them, private acceptance, when someone called out an answer during class discussion or a game (e.g. geo-housie).

> *S5: It annoys you when the children, ... they call out the maths answer. Like, we play housie, and they call out, like it's two fives, and you work out what that is ...*
> *S2: And they call out the answer, that's really annoying because you want to work it out yourself.*

When asked how they knew if their answers were right or wrong the responses indicated that Mrs K was seen as the arbiter of correctness but also that she wouldn't just tell them the answers.

> *S6: You go up and the teacher says 'you've got this revision wrong' – like she says, 'one and what makes two?' (pointing out the mistake).*
> *S1: Well, you hand it up (and leave the book with the teacher) and she ticks the rest. She calls you up (while the class has gone on to other work) and you do it on her gray table (the one that wheels around). She has a tables card, or you say what's wrong.*
> *S6: She'll call you up, and she'll say 'You got your times wrong, work on the tables card.' And you have to do it there and she'll help you. You have to do it yourself.*
> *S7: She helps you but she doesn't tell you. I'm happy, because I want to learn.*

Social interaction.

There was quite a bit of interaction quite unrelated to the mathematics, but forming the social background to the mathematics. Talking while working was common in the classroom. Groups of capable students tended to talk a little about the task and get on with the work, leaving the social chat until they were finished. Middle ability groups often chatted when unsupervised.

The students regarded not being distracted or not talking as the prime reasons for some students getting more work done than others, closely followed by ability and interest in mathematics.

Table 5-15 Some people in our class work more than others because…	Number of Responses
… they are more capable / they know more.	7
… they are faster.	8
… they like maths.	5
… they don't talk	12
… they don't get distracted.	5
… they learn from their parents.	2

The social climate with regard to the level of thinking and reasoning expected of the students can be inferred from the results of Stevenson's (1998) questionnaire. Table 5-16 reports on the first order presses and actions, and table 5-17 on the second order situation.

Table 5-16 Presses for lower order thinking.		Class Mean and Standard deviation
Copying from the teacher.	6. 'I should'	2.4 ± 1.4
	9. 'The teacher expects'	2.1 ± 1.5
	20. 'I do'	2.3 ± 1.3
Working exactly as shown.	17. 'I should'	3.9 ± 1.2
	26. 'The teacher expects'	4.2 ± 0.9
	30. 'I do'	3.8 ± 1.4
Doing as the teacher says.	25. 'I should'	4.2 ± 1.5
	16. 'The teacher expects'	4.8 ± 0.6
	5. 'I do'	2.5 ± 1.6

Copying from the teacher was a minor part of the classroom culture while working as shown was very important. Following the teacher's instructions was part of the espoused culture but was actually done a lot less than the students believed they were expected to. In later questioning the students made it clear that they had interpreted the first two sets of questions as applying to the mathematical procedures while the third set of questions was about behaviour while doing mathematics.

Table 5-17 Presses for higher order thinking:		Class mean and standard deviation.
Finding things out ourselves	4. 'I feel I should'	3.8 ± 1.4
	15. 'Teacher expects us'	4.5 ± 0.8
	27. 'I do'	3.5 ± 1.1
Finding links between ideas.	22. 'I feel I should'	3.4 ± 0.8
	3. 'Teacher expects us'	4.0 ± 1.2
	13. 'I do'	3.9 ± 1.1
Trying out new ideas	2. 'I feel I should'	3.1 ± 1.3
	11. 'Teacher expects us'	4.2 ± 1.0
	19. 'I do'	3.5 ± 1.1
Check results against own knowledge.	12. 'I feel I should'	2.9 ± 1.0
	29. 'Teacher expects us'	3.3 ± 1.2
	7. 'I do'	2.8 ± 1.3
Ask the teacher to check results.	10. 'I feel I should'	2.5 ± 1.5
	21. 'Teacher expects us'	3.4 ± 1.4
	1. 'I do'	2.2 ± 1.3

There was a pattern of medium to strong obligation from themselves and the teacher for most students on the first three questions that suggests there was some support for an inquiry culture in the classroom. A few students did not share the norms either in belief or action, but many were supportive of the teacher's norms. The weakest part of the inquiry culture was related to checking answers. Many students reported they didn't often check their work, nor did they feel much obligation to do so. The students later reported that they saw the inquiry behaviour (table 5-17) as expected during practical and project work but not so much during seatwork.

THE ROLE OF THE TEACHER.

Mrs K was seen by the students as very much in charge of the classroom. The students saw Mrs K as expecting them to behave as she said during mathematics time and to follow her methods, but not to copy her (tables 5-16 and 5-17). In interviews they clarified this as referring to not copying the examples she did on the blackboard while explaining the methods. The students felt she expected them to think for themselves, find links between ideas and try out new ideas. However, their reported actions lagged a little behind their perceptions of Mrs K's presses in each case.

The work set was carefully matched to each student's abilities, and this was noticed and often remarked on by the students. Mrs K was also seen as wanting the students to persevere, using growlings and encouragement, and requiring them to repeat their wrong sums until they got them right.

S1: Mrs K makes us do it, she goes (growl).

S2: Oh, Mrs K wants us to keep at it.

S1: You've got to do it and do it until it's right. ... Whenever you do it wrong she always marks them and just sends them back to do again.

S2: It's the way to learn.

Alongside her emphasis on perseverance, the students reported Mrs K trying to get them to believe in themselves. They commented that she would encourage them to tell themselves that they could do the work, and had given them a practical demonstration of the power of positive thinking.

S2: Mrs K, what she did was, she got W to stand up and say 'I can't do it, I can't do it', and she got her hand and he held out his and she pushed it down easily. Then she got him to say 'I can do it, I can do it', and she couldn't push his hand down ...

S4: ... and she was putting all her weight on it!

S6: When someone believes in themselves ...

S5: ... they can do what they want to.

The students responded well to Mrs K wanting them to become independent thinkers. This was seen repeatedly during the observations, when students would ask for help, and Mrs K would ask them to describe what they had done so far. Often they would see for themselves what the mistake was, and she would tell them they could have done the check for themselves. Mrs K normally didn't give answers to sums when asked 'how do I do this?'. If her encouragement didn't help the student find the mistake she would rehearse the method with them.

S5: If you ask for help Mrs K will come and she won't tell you the answer, she'll tell you how to do the sum, and then you have to do the sum by yourself.

From the written responses Mrs K was seen as requiring quiet, quick and steady work during mathematics time:

Table 5-18 The teacher likes us to work ...	Number of responses.
... quietly	17
... quickly	10
To 'get on with it'.	13
to listen and concentrate.	2
... neatly.	2

The students experienced the power relationship as a mixture of legitimate and referent power, with some of them getting on with the work without any prompting and others waiting until they got the instruction before starting:

Table 5-19 When we start our maths time the teacher says ...	Number of responses.
... says to do our revision/maintenance work.	11
... says "get out your maths book.	6
... explains about the work	6
... growls 'do your maths'.	2
... says to 'be quiet'.	3
Other responses	2

Being put on 'silence' for 10-15 minutes was the main disciplinary measure for this class. During silence a student had to sit alone, avoid all communication with other class members, and do set work (usually skill practice exercises). Silence was often followed by a contract that required them to work on set tasks in a specified order.

THE NATURE OF THE TASKS.

Tasks mentioned by the students in the focus group interviews were maintenance, problem solving, worksheets, textbooks, tests and homework. These were all observed during the study. The worksheets were usually written by Mrs K and emphasised applications rather than skills practice. The students also did whole class exercises such as practical measurement with jars and water and class projects like 'creating the Barrier Reef' that involved using their geometry, measurement and number skills in cross-curricular contexts. The students enjoyed these projects whereas they did the daily maintenance willingly but without noticeable enthusiasm (table 5-20).

The written responses provided confirmation of the wide range of mathematics done in the class. All the strands were mentioned several times each, the algebra being known as 'patterns and graphs'[8].

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[8]It is not certain whether 'measurement' and 'geometry' denoted just the practical activities associated with it or the whole strand. The strand names have been coded as strands unless other evidence showed that practical activities were meant.

Table 5-20 What do we do in mathematics time? [9]	"We have done these this year"	"The mathematics I like best is"
Problem solving	23 (23%)	9 (24%)
Various curriculum strands	20 (20%)	4 (11%)
Content within a strand.	19 (19%)	2 (5%)
Maintenance /revision of algorithms.	14 (14%)	6 (16%)
Basic facts revision	10 (10%)	1 (3%)
Mathematical games	6 (6%)	10 (26%)
Practical activities and projects	5 (4%)	5 (12%)
Worksheets.	1 (1%)	1 (3%)
Textbooks	0 (0%)	0 (0%)
Totals	98	38

The students referred to the mathematics content both by the curriculum strand name and by the name of the particular concepts and skills making up the strand content. This indicates a good familiarity with mathematical language. The nil responses for 'textbooks' is noteworthy, showing that these were not seen as mathematics in themselves and none of the students mentioned they enjoyed using them.

The daily revision of algorithm skills (maintenance) occupied a large part of their mathematical world.

S1: We learn maths by, um, we do revision every day...

Various: That's like, one to one, division, things like that.

S2: Some people can do the long times.

S3: No, we've got, pluses, then minuses, then times, division, and a long times.

S3: And, last term we got these maths books, and we work out of them, and it's like, money and, time, ...

S4: and there's the blue books.

S2: Yes, and we do them, after the revision, and you just, ...

S6: The time, it asks, what's the time on the clock,...

Problem solving was a regular part of mathematics, separate from the word problems and textbook work. It included mathematical puzzles and group exercises that were taken from a range of resources, mainly 'maths challenge' booklets. The mathematical level of the problem solving questions was quite high for the class level, so several of the students found them difficult. Opinion about them was divided:

[9] Many students wrote a list of items for the first question, and only one for the second.

S4: I like times and adding, but problem solving, it's yuk!

S1: I love them.

S6: I reckon it's boring and I'm not interested in them.

S3: Sometimes the problem solving is boring, and sometimes it can be interesting.

S5: If you don't do the problem solving you have to do the bookwork, contract stuff.

There was very little comment from the students about the book and paper resources used for mathematics, perhaps indicating that they were very much background parts of the scene, and taken for granted. It could also indicate that the students were focussed more on the content of the work than on the form in which they encountered it.

Many students mentioned mathematics homework. They identified times tables practice and worksheets as the usual form of homework. For times tables Mrs K accepted them doing homework on their home computers rather than on the sheets.

S1: When we go home, we have to do tables at home. I do all my tables on the computer, cause I've got this game, '3 great adventures' like, ...(description of the maths game).

S1: Mum reads the tables out, like, one at a time. If she says, like, 3x2, we just say, 6, and we have to do it as fast as we can. We have to do it fast, and we have to learn it every night.

S4: We sometimes get homework sheets that have revision, and family of facts and pluses, and times on them, and things like that, and then they have problems on them too.

In some families homework was valued, and not so in others.

S1: Mum comes to me and looks over the work and says 'that one is wrong because you haven't done this', and she actually helps me do things. We've got these books at home, and every day after school we have to do them, she'll look over them with me and sit with me and tick them off.

S2: I don't even ask my mum for help, I don't even do them!

S3: I used to have trouble with my maths, so mummy bought me this book, but I don't do it any more because it's too easy.

S4: I don't do any.

S6: My parents don't like telling me the answers and they don't like giving me help on every one. They don't mind I ask for help on some of them.

Tests were a regular part of classroom life. The students expected the tests to contain questions in real contexts rather than just algorithm skills.

S6: usually it's, ... sometimes it's about money, and they ask you, what, when you go to the shop, if you spent, like, $1.50, and gave them $5.00, how much change.

S2: How much would you get if you bought a certain item.

Mrs K sometimes reacted strongly to inadequate work or effort.

S3: With maths tests, the teacher doesn't like it if you've got the whole page wrong, she sometimes rips it out and you have to do the whole page again.

S2: She gets really mad.

S1: ... or if it's messy.

Test time also elicited quite strong feelings from some of the students.

S3: With maths, if you've got lots wrong you feel bad about it, from your parents.

S5: I hate it when it goes on your reports. That's what the teacher does.

S4: ... What happened, we were tested on the Wednesday, and I didn't want to go to school. I didn't want to sit the test and I told mum about it, ohh ... but it was quite easy!

S2: There was one time, there was this maths test, and there were like, 3 pages to do ... I pretended to Mum that I had a headache and a sore tummy. She didn't believe me – she knew I hate maths – she sent me to school anyway.

How we do the mathematics.

There were two competing sets of norms observed during mathematics time. Under the first set of norms the important things about doing mathematics were 'getting it right' and 'getting finished'. Students were often observed doing the work and frequently comparing answers. The more able students checked after they had done the questions while the others preferred to look at the work of the most able student in the group, then try to get that answer. The least able students usually sat together and worked fairly co-operatively. Most of the mathematical communication during the text and worksheet tasks was 'what did you get for that one?' Queries such as 'why did you do it that way?' were only heard among the most capable students.

The second set of norms came into play when Mrs K suspended the bookwork in favour of project or investigation work, usually practical and very interactive in nature. Although the important thing during these activities was still to be able to answer the questions, Mrs K took care to make these tasks problematic by setting questions that needed thoughtful answers. The students really enjoyed these episodes and would take a lot of care to get their own answers and to satisfy the requirements to explain their answers. Finishing was much less important as Mrs K would encourage the students to 'do at least part of it really well'.

MATHEMATICAL TOOLS.

The tools available to the students were the mathematical equipment, notation, language, algorithms and procedures, and the concepts behind them. The students were able to use mathematical algorithms and procedures in their sums and problem solving but did not seem to be aware that they were using them as tools. There seemed to be a similar lack of awareness with regard to physical materials, mathematical language and written notation.

Some of the more able students were seen to be using notation to help them discuss the methods in the problem solving and word problems. However, most students were unable to explain why they had chosen a particular method, perhaps indicating that they were not often asked for explanations. For example, most students were aware of using times tables to solve times and division problems but could not say why they had chosen to use them.

While mathematical tool use was not explicit in the culture some of the more able students seemed to expect to understand the algorithms as well as use them. This fits with Mrs K's frequently repeated statement that she taught the skills to all the students so they would at least get the success of getting the sums right. The students had been trained earlier in the year to use generic thinking skills as tools but Mrs K had not used these tools during mathematics time. Students were not heard to talk about thinking strategies in problem solving, nor did they mention them in the focus groups or written responses.

Mathematics itself was sometimes talked of as being needed to get jobs.

S2: If you don't get ... if you get the mathematics wrong you won't get jobs when you grow up. Most of the jobs need it.

S5: Most jobs need maths.

S4: I don't really want to miss a test, cos I really do want to be a lawyer.

S2: I want to be a pilot when I grow up, and you've got to have good maths.

EQUITY AND ACCESSIBILITY.

The students were aware of the range of abilities and backgrounds in the class, but these were very much background givens – parts of the classroom environment which were taken for granted. They accepted Mrs K's strategy of working with groups of students according to their needs.

S7: Lots of children don't know times. So she sits them down on the mat and teaches them times.

The main management strategy for ensuring all the students had equitable access to mathematics during Mrs K's class times were the use of contracts. Students were noticed to be working at these contracts for 2-3 weeks before rejoining their groups for textbook work. The level of their work was both challenging and achievable for them, and the other students regarded this as part of Mrs K's good teaching. The students talked in class of contracts as helping them to understand and to catch up with their peers. The slowest students were almost permanently on contracts. They frequently asked for and received help from her and from nearby students.

During class discussions the students were encouraged to share their ideas, which were received respectfully no matter who the speaker was. Not many of the students contributed to these discussions but there was lively group talk during project and investigation work. It seemed that all the students felt free to collaborate on the tasks in these situations.

5.3 Summary

This classroom was a place where mathematics was constructed to be both procedural and problematic, where understanding was important, but procedures had to be mastered first. Mrs K's class saw mathematics as having two sides – the sums (revision and text work) and the practical (investigations and projects). The least able students saw mathematics in largely procedural ways, but they were able to use their limited skills for simple problem solving.

Socially, the class worked very smoothly on the surface, with a high level of public acceptance of Mrs K's norms. A lively subculture existed during mathematics time where students followed the norms relating to individual achievement but also followed their own norms of chatting and copying, especially during revision and seatwork times. The over-riding purpose of mathematics time was to get the work correct and complete.

6. THE CASE STUDY: ANALYSIS AND DISCUSSION.

"Understanding the parts of a whole, which is the kind of understanding we gain while analysing use, does not automatically translate into an understanding of the whole. The collection of pieces is not enough to reconstruct the 'living creature'." (Sfard, 2000, p47)

6.1 Introduction.

A classroom is very much like a 'living creature' during mathematics time. The norms and expectations of the teacher and students knit together into an organic interactive whole that depends on all the parts for its functioning. Just as biological entities can be analysed in parts so they can be seen as integrated into a growing and changing whole, the life in the classroom can be seen as a system of interacting systems that have the potential for growth and change throughout their lives. It is in order to identify the potentials for valuable change that analyses such as this one are undertaken.

In keeping with the structure of the data reports, the analysis is set out mainly under the five dimensions of Hiebert et al's (1997) model. The key features of the dimensions are provided and the situations in the two classrooms are summarised and discussed to provide a picture of the extent of inquiry teaching and learning in them. The approach to analysis is in harmony with Boaler's (1999) focus on 'constraints and affordances': the study highlights the aspects of the classroom environment and teacher practice that either inhibit or enhance mathematics learning with understanding. Because the dimensions of the inquiry classroom describe an interlocking system, many classroom practices fall under more than one dimension, so have been assigned to the dimension they most clearly relate to. Further features of the classroom are then examined as they relate to other research concerning teaching and learning mathematics with understanding.

While the cultures of the two classes were different in many respects they each contained many of the key features of Hiebert et al's (1997) five dimensions of an inquiry classroom. Both the teachers were reflective practitioners who had deliberately set out to establish classroom cultures to support their beliefs about effective mathematics education. The norms and power relationships they established formed a base for their teaching practice and for classroom learning.

Both teachers were very effective in teaching the mathematics they valued. The teachers reported that their students did well at their syndicate tests and the PAT scores improved each year. Parents of both classes were very supportive of the teaching their children received and both teachers were highly regarded by their colleagues and the school managers (personal comments from both principals). The students were all able to move into their year 6 classes and learn well relative to their capability.

6.2 The Social Culture of the Mathematics Classroom.

Hiebert et al. (1997) propose four key features of the social environment of an enquiry classroom. Most important is that *ideas are the 'currency' of the classroom.* This implies that students are focussed on the meaning of the mathematics they do as well as on the content. Second, *interaction and collaboration are vital to the development of robust individual understanding*. Students are expected to share ideas and methods and to allow the exchange of ideas to refine their own understanding. The collaboration needs to be focussed on the methods rather than the answers. Third, *mistakes are regarded as indicators of flawed understanding rather than of failure.* This re-frames errors as things to be investigated and discussed. Fourth, *the correctness of a problem solution is determined by its mathematical logic, not the teacher's approbation.* To introduce these features this discussion first considers the *social norms that contribute to the smooth running of a classroom.*

THE SOCIAL NORMS OF THE CLASSROOM.

In Mr J's classroom the students were expected to get on with the tasks during mathematics time and most of them accepted this. Mr J encouraged respectful behaviour by everyone in the classroom and there were very few put-downs or hassling of others in the class. Mr J expected the students to share ideas and to work together on the tasks, but also to complete the work by themselves to a high standard. These two norms were followed by many of the students as long as the work was seen as achievable and they were not under time pressure to complete the work. Under pressure many of the students would copy answers from more able students in order to be 'right and finished'. Several other student strategies were focussed on finding answers rather than understanding them: imitation of Mr J's methods; expectations for Mr J to explain methods repeatedly –which he did for individuals; and not expecting to question their answers. The students believed that higher level thinking was mainly for the project work and optional for less able students.

Sixty nine percent of the student responses indicated liking mathematics in general (table 5-1) although only thirty nine percent showed a liking for the daily maintenance (table 5-10). The sixty nine percent figure is similar to that reported by Young-Loveridge (1992).

Mrs K promoted goals of perseverance, independence, self-esteem and understanding in her classroom. Sixty five percent of her student responses reported a liking for mathematics in general and fifty one percent showed enjoyment of the daily revision (tables 5-11 and 5-12). Some of the students found it easy to avoid doing the daily revision themselves due to the seating arrangements in the room and Mrs K's practice of allowing them some freedom in choice of work, but all espoused the norm of individual achievement quite strongly. Most of the students did the seatwork components of mathematics as exercises in procedural facility without reflective thinking or expecting understanding to develop. Correctness and completion were the prime goals for this work but the practical activities were done with an eye to both completion and understanding of the mathematical ideas involved.

In both classes most of students' positive feelings about mathematics were related to getting the right answers and being finished. Students with negative feelings about mathematics were also those who did the least amount of work and were most likely to copy others. The student attributions concerning mathematical ability and performance were mostly related to greater knowledge, liking mathematics, getting extra help outside of class and not talking or getting distracted. The students normally worked together during mathematics time except for some of the more able students who preferred to work alone and check their answers afterwards.

The grouping of the students sometimes contributed to reduced inquiry into mathematical ideas. Cobb (1995) described how a group of students of similar capability would discuss mathematics more fully than groups of mixed ability. This effect was seen repeatedly in this study. Mixed capability groups of students in both classes would sometimes wait for the student perceived to be most capable to finish some work and then largely copy their working and answers. They felt an obligation not to do so and when asked about it they would justify their actions with comments such as 'I could have done it but I just didn't want to'. Groups of similar capability students were seen to be doing the work themselves rather more. This was most marked with very capable students. There was a gradation of intensity to this strategy: it was most marked in Mrs K's class during seatwork, less so in Mr J's class at any time and seen least in Mrs K's class during practical or project work.

The students could avoid the work partly because the teachers had established some of the inquiry norms (group work and co-operation) for mathematics time although they were less successful in establishing reflective thinking or problem solving strategies. The group work and co-operation norms worked well for Mrs K within the environment of investigative or practical work but were not useful for the routine practice times. It was interesting that the students seemed to accept the norms of group work and co-operation but not the associated ones. Mrs K certainly spent considerable effort to link these together by her stated expectations and enacted practices during the revisions time. It seems likely that group co-operation fitted the students' social needs and was easily adapted to satisfying social goals, while the norms that supported thoughtfulness, understanding and problem solving were less easily assimilated and accommodated into existing student discourses. After the study Mrs K chose to move the revision sums towards more context-based problems in order to make a better match between the environment and these tasks. She also tightened the requirement to demonstrate progress in both revision and the other tasks. She reported that these changes had increased both student commitment to all types of tasks and also student enjoyment of mathematics time.

IDEAS OR CONTENT?

Mr J structured the classroom culture to encourage the students to be independent learners. He believed there was more personal motivation for understanding than the students' actions and comments indicated (tables 4-1, 4-2). The students knew they weren't supposed to concentrate on the answers and did much of the seatwork thoughtfully unless under pressure (subject to the group effect noted above). They clearly focussed on instrumental understanding, however, and open discussions of methods and ideas was not observed or talked about. Although Mr J often explained the ideas and connections behind the methods he did not stress the importance of reflective thinking. Consequently, interaction among the students was largely about algorithms and answers.

Mrs K believed deeply in providing every student with a toolbox of fail-safe standard algorithms to make sure they had methods that would give correct answers. Mrs K also wanted her students to explore their own methods of solving problems to encourage deep understanding of mathematical concepts. She gave plenty of opportunities for this during practical and applications work in each topic, setting tasks which were designed to both practice the procedures and link them to the applications. The few whole class discussions were about how to tackle the project work. Mrs K managed them to rehearse useful ideas and methods that Mrs K wanted the students to use, and for students to propose their own methods. During seatwork Mrs K often encouraged the students to discuss their methods but they returned to talking about answers and how to do the standard procedures as soon as she moved on.

The students strongly believed that seatwork mathematics was all about being right and finished (tables 5-14 & 5-15), however that was achieved. Off-task behaviour was acceptable to the students as long as the closure norms were observed and they weren't caught. Mrs K's intentions for seatwork included both correctness and understanding and she encouraged these with only limited success. Mrs K also strongly believed in the importance of thinking skills but had not yet extended her teaching of problem solving strategies into the wider mathematical arena. The students felt pressed to use higher order thinking as well as to follow directions closely (tables 5-17 & 5-18). In practice, lower order thinking was characteristic of revision and seatwork time and higher order thinking more prevalent during practicals and problem solving time.

MISTAKES.

Mistakes were regarded emotionally by students in both classes. They reported negative feelings about them and took pains to erase the evidence of mistakes if they could, using pencil where possible for their mathematics so they could rub out the offending work. It was common for them to copy the correct ones without working through the sums again to find their mistakes. Mr J was not aware of the extent of this as the students carefully hid it from him. Discussion about the sources of the mistakes was not a regular feature of the classroom environment. It was Mr J's practice to give the answers to worksheets and encourage the students to mark their own textbook exercises from the back of the book. This seemed to reinforce the importance of answers over methods in the students' minds.

Mrs K's students used pencil erasing and removal and redoing of whole pages to promote the appearance of complete correctness. Interestingly, this behaviour was rare during the practical activities. This suggests that Mrs K had successfully cast practical mathematics as being about understanding rather than completion and correctness. The emotional treatment of mistakes is characteristic of traditional classrooms and indicates a key area where a change in teacher expectations could effect a large change in student practice and attitudes.

THE SOURCE OF TRUTH.

Mr J was seen by most students as deciding if results were right or wrong and there was consternation when he was discovered to be wrong. Only the more capable students were willing to challenge him in such situations. During class discussions Mr J would carefully orchestrate the giving of answers to his prompts as a way to bring out the points he wanted the students to be remembering. This practice emphasised answers rather than understanding as the product of mathematical thinking.

In Mrs K's classroom mathematical correctness during seatwork was warranted by Mrs K while mutual agreement was the norm for the practical work, probably because the questions on the worksheets were worded to encourage discussion and agreement. For the practical activities Mrs K wrote worksheets that encouraged linking mathematical ideas to physical situations (such as comparing the capacities of jam-jars and kitchen measuring equipment). The students enjoyed these tasks and the worksheets directed them towards relational understanding as much as applying the standard algorithms.

INQUIRY OR TRADITIONAL?

The two classroom cultures in this case study were in transition between traditional and inquiry paradigms of mathematics teaching and learning. Thagard (1994) points out, in his theory of conceptual revolutions, that during such transition phases there will be aspects of both paradigms operating in everyday cultures, competing for the allegiance of the participants. It is not until a considerable amount of the cultural environment can be understood in terms of the new paradigm that the participants will change their ways of thinking. In Mr J's classroom the traditional paradigm kept the upper hand due to the continued importance of traditional expectations. Interestingly, both paradigms operated in Mrs K's class but in different mathematical environments. Mrs K's students had made the shift for the mathematical environment that was coherently of an inquiry nature but retained the traditional attitudes and beliefs in the other work that featured a mixture of expectations. This illustrates the strongly situated nature of the socio-mathematical norms and belief structures of the classroom.

Sosniak et al. (1994) noted that most teachers espoused a range of beliefs covering the spectrum from traditional to inquiry based teaching and learning and that their enacted beliefs were usually more traditional than inquiry based. This was certainly true of Mr J and Mrs K. The students in this study also showed a mixture of beliefs. Both teachers believed that the students came to them with traditional beliefs deeply ingrained and both talked about their struggle to reframe mathematics as investigative and collaborative. Not surprisingly, the students' expectations of their teachers were closer to the teachers' enacted roles than their espoused ones, for example Mr J's students saw him in the role of benevolent dictator and didactic teacher far more than as guide and coach. They expected the teachers to control the classroom environment and students who weren't working expected to be 'growled'. These attitudes were very similar to those found by many researchers observing traditional classrooms.

The students brought to the classrooms their previous experiences of mathematics and mathematics education, plus their family and community backgrounds of attitudes and beliefs about mathematics and mathematical schooling. These backgrounds meant that the students' recognised their teacher's goals for the procedures and algorithms of mathematics but seemed to have little prior experience to support their acceptance of the inquiry or understanding based goals. In this study most students worked in harmony with their teacher in completing the set work and learning the procedures and algorithms of the mathematics. These goals fitted well with what the students said of their parents' desires for their mathematics and also with the type of mathematics they reported as having done in previous years.

6.3 The role of the teacher.

The basic principle behind Hiebert et al.'s (1997) key features of an inquiry teacher's role is that the mathematical goal of the classroom should be relational understanding. The teacher firstly *establishes and supports the social norms that enhance an inquiry culture*. The teacher manages a classroom culture where discussion and reflective thinking by all students can flourish. This requires attention to mutual respect and support as well as norms for thoughtful engagement with the mathematical tasks. Secondly they *provide direction for mathematical activities*. The teacher should select and manage sequences of mathematical situations that are problematic for the students, and encourage the students to see the logic and structure of a mathematical solution as the source of mathematical truth.

ESTABLISHING THE SOCIAL NORMS.

The social norms of Mr J's classroom had been established at the beginning of the year. They included practices supporting traditional and inquiry mathematics. The students had been trained to be self reliant and responsible for their own work and progress, and were becoming responsible for critiquing their work and that of others during project work. They expected to explain their reasoning when asked and to ask questions when stuck or uncertain how to proceed. There were also some equally strong expectations that led to the inquiry norms being avoided at times. Progress was measured by the amount of correct and completed work and assessment for seatwork was by the teacher or the textbook. There was little incentive to check or rework wrong answers and it was easy to let someone else do the thinking if a student was confused or behind in the work. That these more traditional expectations were part of the student culture made them almost invisible to the teacher most of the time. It is likely that these norms came from the students' previous experience of mathematics teaching and learning and from home and community cultural norms.

Mr J said he saw his social role as that of a benevolent dictator, directing the interactions between the students towards co-operation and mutual respect, with personal norms of self-responsibility, good time management and perseverance. He wanted to work as guide and coach and believed that by acting as a benevolent dictator to establish the classroom environment that the students would be able to treat him as guide during the mathematical activities. The students did not seem to notice this and related to him principally as boss and expert. This was reinforced by some of his practices, for example he often taught from the blackboard to present the mathematical methods and algorithms for the current topic, expecting the students to copy the forms rather than interact with the ideas being presented.

Mrs K's classroom roles were well defined. She had established a classroom environment where perseverance, self-responsibility, high self-esteem and personal time management were accepted norms. The daily routine was of a class set of revision questions followed by an individual or group based programme of tasks. Students expected to work at procedural facility with the standard algorithms every day. Alternative methods were accepted but not celebrated. Mrs K had carefully designed these norms to support her dual goals and seemed to have succeeded very well in that[10]. Mrs K also consistently promoted norms of automatic facility with basic facts and standard methods.

THE POWER ENVIRONMENT.

Applebaum (1995), Manke (1997) and Tauber (1993) all stress the importance of positive power relationships as a way to release teachers from classroom stress and focus students on learning. Tauber pointed out that traditional classrooms operate mainly on positional power relationships – 'You are the teacher so you can direct, reward and discipline us'. He noted the unending repetition of controlling and managing practices that positional power requires and the extrinsic motivation they tend to produce. These were consistently seen in these two classrooms during much of the study. However, since some of the classroom practices were inquiry based, both teachers were able to tap into influential power relationships part of the time and for

[10]The study was not designed to measure the extent of student progress but Mrs K reported that her class results were consistently ahead of national levels although the school was decile three. She reported that her students made strong progress compared to their standing in previous years

some activities. These activities tended to be linked with intrinsic motivation and a focus on understanding and agreement between students for solutions. Where the tasks were closed or the goals of an activity were based on correct answers and completion the students tended to press the teachers to use positional power relationships.

The power environment of Mrs K's classroom was based on referent power but had some coercive and reward elements – Mrs K would restore on-task behaviour and motivation with rewards such as praise and certificates and also occasional growling and giving of 'silence'. There was some legitimate power operating but Mrs K preferred to appeal to the accepted norms of the classroom routine when establishing task procedures. Mrs K used an expert power relationship in many of her teaching interactions to help direct the students to the mathematical logic and away from herself as a mathematical authority. The students resisted this quite strongly but Mrs K was able to continue to refuse to be the warrantor of correctness.

The power environment in Mr J's classroom was more uniform. He used some controlling power to keep students on task but wanted to be operating mainly in the influential mode to fit his espoused inquiry goals. Several of his enacted classroom goals were traditional, however. His students seemed to have reconciled the conflicting norms and expectations of the inquiry and traditional goals in by seeming to accept Mr J's referent power but really avoiding it. Referent power operated on the surface of many interactions, linked to Mr J's espoused inquiry goals for mathematical learning, but the other norms (described elsewhere) were apparent under the surface. These more traditional norms led the students towards a reality of a legitimate relationship with Mr J that he did not often perceive. This explains the high level of extrinsic motivation and the unwillingness of the students to work towards Mr J's expected outcomes of relational understanding during the end of topic project work.

Manke (1997) makes it clear that teachers can only operate in the power relationships their students will allow. For example, once the students of both classes had accepted that their peers and teachers should not readily give out answers they acted to maintain that by complaining when someone transgressed the norm, so Mr J's practice of marking worksheets by calling out the answers was not popular. Where the teacher wants influential power they need to structure the classroom environment and tasks to support it. During seatwork Mrs K's students sometimes pushed her towards coercive power from her preferred influential base (referent and expert power) by their exploitation of the freedoms she had built into the daily timetable. Mrs K's achievement of influential power relationships and intrinsic motivation during practical work indicates that the importance of influential power in the system of classroom features related to learning with understanding.

DIRECTING THE TASKS.

Mr J planned and set tasks according to his mathematical goals of procedural facility with methods, application of those methods in everyday contexts, and the development of understanding. He believed that students who were focussed on the tasks he set would be learning mathematics although this study indicates that many of them were learning only procedures rather than developing the deeper understanding he valued. Mr J planned each topic to teach algorithms and procedures first and then encouraged the students to explore methods for problem solving during the later stages of each topic. The tasks in the projects were designed to prompt more integrated thinking than those in the earlier phases of a topic. Observations and focus group interviews both suggest that the goal of understanding was implicit rather than explicit in Mr J's teaching and was not really noticed by the students.

Some of Mr J's strategies worked against his espoused goal of understanding: he would give detailed instructions for projects rather than leaving them open-ended to encourage creativity; when giving answers to set work he rarely discussed the working or reasoning; when explaining ideas he would usually give all the reasoning himself rather than inviting student participation; and when teaching new work he expected exact copying of his methods and procedures, devaluing any alternative methods the students may have had.

Only the most able students were observed to treat the applications and problem solving towards the end of each topic in the way Mr J wanted. The rest of the class treated them as isolated sums to be completed correctly, just like most of the seatwork. Many of the students therefore had little success with the problem solving, being unprepared to accept the increased cognitive demands. Mrs K designed tasks with the intention of providing both practice at standard algorithms and opportunities for developing both instrumental and relational understanding. She encouraged discussion about methods and insisted on perseverance when students were ready to give up. She also marked student work every day and called the students up to revisit their mistakes, refusing to give answers but directing the students' attention towards the source of their errors. She also encouraged discussion of methods and approaches to problem solving and projects. Mrs K expected these practices and norms to lead to independence and high intrinsic motivation for mathematical learning and believed she had achieved this goal for the practical work.

6.4 The nature of the tasks.

Hiebert et al.'s (1997) key features of classroom tasks were that they be *problematic for the students* to encourage reflective communication; that they *relate to the world of the students*; that they direct the students to the mathematics to *leave a residue of mathematical meaning* when they have been solved; and that the students become *aware of using mathematical tools to solve the tasks*. The last two will be covered under **tool use** in the next section.

TASKS SHOULD BE PROBLEMATIC.

The tasks provided in Mr J's classroom were largely computational at the beginning of a topic, getting more contextual as time passed. The tasks were well suited to the capability of the class with an enrichment group being given more problem solving based work by another teacher. The students often found the tasks problematic in the sense that they found it challenging to think carefully enough to solve them, but they often acted to reduce the challenge and the problematic features as previously described.

Mr J's intention to press the students towards inquiry mathematics seemed to be largely negated by features of his tasks. Most tasks were mainly designed to develop the ability to apply basic skills to simple situations. Mr J expected understanding of the ideas behind the skills to follow but the classroom features which supported this (e.g. collaboration, conferencing, peer review and assessment) were not strongly linked together and promoted as means to that end. Reflective communication about the methods became a very minor feature of classroom interaction in practice.

Furthermore, since the inquiry features were only weakly linked, it was easy for the students to reinterpret them as allowing other goals from their dominant discourse of being correct and finished. The overall effect was that the students chose the extent to which the tasks were problematic for them.

The tasks Mrs K used in her classroom were of two types, each well matched to the type of learning she wanted them to produce. Procedural facility with arithmetic was produced in most students by the daily repetition of revision sums, graded to the increasing competence of the students as the year passed. Mrs K's students also treated the seatwork tasks as requiring routine computation and resisted seeing them as problematic although most of them saw the project work in this light. It is notable that these activities were far more popular with the students than the seatwork.

The high learning value of the practical tasks was due to the combination of inquiry features present in those tasks: the worksheets asked for both numeric answers and explanations; they encouraged discussion and joint decisions; they were graded to the zone of development of the students and allowed for movement through the grades according to growth in understanding; the assessments were based on the explanations rather than the answers; Mrs K frequently reiterated that these tasks were intended for understanding and pointed out how the use of basic skills was embedded in the tasks. Some of these norms seemed to have carried over to the routine work for the more able students.

Open-ended tasks that are problematic for the students and are rooted in the everyday lives of students are promoted by Yates and Chandler (1994) and Henningsen and Stein (1997). Such tasks provide the opportunity for intricate linking of experiences to mental objects in cognitive schemata. Contexts that are familiar take up little cognitive processing space, allowing greater focus on the mathematical and logical aspects of the task. Regardless of the open-endedness of the tasks, as long as the students both practice the procedures and use them in applications they are likely to promote some cognitive linkages (Boaler, 1998; Cobb et al., 1992).

In Mr J's class most of the students had also accepted the norms of co-operation and group work but not reflective thinking or the use of problem solving strategies. Mr J set short time limits for the revision work and most of the students would try to do the sums but would copy if they felt they were going over time. Since the answers were on the back of the question cards it was easy for them to replace their wrong answers with correct ones when the cards were turned over, and so appear to have been correct. If they weren't seen to be wrong they could avoid being challenged to reflect on the reasons for their revision errors. While Mr J encouraged the students to value the errors as indicating misunderstanding, few of them indicated any more than a surface compliance with that expectation. It is likely that the practice of giving four revision sums every day linked so powerfully into the traditional mathematics discourses developed from family memories and experiences in earlier classes, that Mr J's attempts to reframe the discourses were not sufficient to dislodge the dominant theme of 'getting them right and finished'.

RELATING TO THE STUDENTS' WORLD.

Mr J wanted the students to first become familiar with the procedures so they could use them in context later on. The daily revision was context free but the skills were later used in worksheet tasks. The students often did not seem to be aware of the connections between the skills and the tasks. Mrs K made sure the projects were very closely related to the everyday worlds of the students but relied on published textbooks and blackboard revision sums for developing instrumental understanding.

6.5 Mathematical tools as learning supports.

The key features of mathematical tool use proposed by Hiebert et al. (1997) were that *notation and equipment should aid mathematical thinking and communication* and that *individual mathematical meaning should develop as a result of using mathematical tools.*

NOTATION AND EQUIPMENT AIDS MATHEMATICAL THINKING.

Mr J and the students regarded notation and procedures as things to be mastered and equipment and worksheets were seen as tools to support that goal. There was little awareness of items of mathematical notation becoming things to be used in various ways, for example the different uses for the addition sign. Thinking skills were seen by Mr J as tools but the students showed little awareness of using thinking strategies to understand the mathematical ideas. The lack of metacognitive awareness was a further barrier to the goal of fostering deeper understanding. However, the students were well conversant with time and task management strategies that were used to facilitate task completion. After the study both teachers spent time fostering such awareness and reported improved levels of thoughtful problem solving in several students. Mrs K especially targeted awareness of the choice of algorithms and notation in an action research project.

Mrs K relied on equipment to support student learning in many circumstances. Her use of equipment was largely in sympathy with the criteria of Richie (1991). She introduced the equipment before expecting it to be used for developing mathematical concepts; encouraged the students to view it as an aid to understanding; and promoted a learning environment that was focussed on collaboration and perseverance. Mr J's use of equipment was more structured but he also took care to introduce new materials before their mathematical use. Cobb (1995) points out that equipment of itself can support either traditional or inquiry learning, the difference in outcomes residing in the intentions of the teacher and students while it is in use.

6.6 Equity and Accessibility.

Hiebert et al.'s (1997) key features include that *tasks are accessible to all students*; that *all students can be heard and are respected*; that *there is a classroom culture focussed on improvement rather than performance*; and that *the students feel safe and accepted.*

The social environment of both classrooms was supportive and encouraging of all levels of mathematical capability. Students in both classrooms were cheerful about working at their own levels and peer pressure because of having 'easy work' was rare. The key features were present in both classrooms though Mrs K's classroom environment involved more supportive discussion than Mr J's. Mrs K used individual or group programmes to cater for the wider mathematical capability of her students while Mr J had an enrichment group for the more capable students. It was noticeable that in both classrooms there were many students who took advantage of the supportive environment to avoid being challenged by the harder tasks. That this didn't happen in Mrs K's practical activities indicates that the solution does not lie in the reintroduction of coercion but in the nature of the mathematical environment.

6.7 What sort of understanding is important?

Most teachers want their students to understand the mathematics they are doing. 'Understanding', however, has two complementary and often confused meanings. Instrumental understanding is what most of the students in this study aimed for – knowing what to do when asked. Instrumental understanding implies that a student has understood how to read and answer a question that has little or no context but where the method is given. Few of them tried for relational understanding - relating the methods to why the method worked and when it should be used. There are many levels of relational understanding and much of the progression through levels of mathematics involves learning how to relate and connect one step skills into multi-step skills. The problem with instrumental understanding is that students don't get the connections that let them make sense of what they are doing, so they treat mathematics as a set of isolated rules. Motivation to improve suffers very quickly because the students don't achieve the personal satisfaction that goes with taking control of the world for themselves.

A lot of information in the world is mathematical in form, appearing as graphs, tables, formulae and numbers in combination with oral or written language information. The more adept a person can be at recognising and interpreting the signs, symbols and information, the more chance they have of participating in and shaping their world. The attitudes and skills needed for such abilities in the world are what De Corte (1995) termed a *mathematical disposition*.

The comments and actions of the two teachers showed that they wanted to encourage both types of understanding but structured their classroom culture and tasks to emphasise instrumental and low level relational understanding. This was not deliberate but seemed to be a result of incomplete understanding as to the linkages between the tasks and the way the students received them. Students in this study sometimes meant low-level relational understanding when they spoke of understanding some mathematics. Most just wanted to be told what to do – they were able to do the sums in the worksheets or textbooks if someone told them whether it was times or plus. There was little evidence of students seeing their task as interpreting the questions to find the appropriate tool.

It may be that the strong emphasis on correct answers to sums during the revision and seat work times, plus the absence of organised and open discussion of methods and ideas allowed the students to put low value on the inquiry features the classrooms. The strong community support for traditional approaches would also be likely to have militated against an inquiry tradition taking hold.

The greater expectation of high level thinking in Mrs K's classroom can be traced to her refusal to emphasise correctness plus her practice of asking for explanations, especially during the practical activities and projects. That emphasis was supported by her desired norms of self-responsibility and perseverance that she took such pains to engender. These very features allowed off-task behaviour during seatwork, since many students didn't accept those norms in that situation. An inquiry classroom environment is a linked system – each part is valuable but they work best together.

Mr J's practice was effective in developing the type of understanding he valued at the time. The more capable students were able to treat the projects as requiring relational understanding perhaps because of their extension programme that was based on problem solving. Perhaps the lack of explicit expectations for explanations of methods, plus the closed nature of most of Mr J's tasks led to the inquiry based features of his practice failing to link together as a system. Mr J had hopes of deeper understanding resulting from his teaching and was in the process of including more inquiry features into his practice. In conversation after the study he said he intended using more open-ended questioning the following year. Thoughtful listening and questioning by the teacher can transform student expectations and discussions, leading to deep understanding of underlying concepts as well as skills and procedures. Concepts are more likely to be internalised and integrated in such settings.

Perkins' (1995) concerns over the premature use of symbolic notation are supported by the results of this study. In both classrooms the stress placed on mastery of algorithms and on procedural facility appeared to contribute greatly to the prevailing climates of 'get it right and get it finished' so inimical to an inquiry culture. Students with lower capability were encouraged to learn to reproduce the algorithms by rote and had generally negative attitudes to mathematics and little interest or facility at problem solving. Mrs K's belief that procedural facility would lead to higher self-esteem may have been borne out (it was not investigated in this study) but did not appear to produce an inquiry frame of mind in those students, even with problems matched to their capability. Hiebert et al. (1997) point out that the way mathematical tools are constructed in a classroom shapes the thinking of the students. In these classrooms mathematical skills and procedures were usually ends rather than means and this probably had a large effect on the observed classroom cultures.

Perhaps the most notable feature of an inquiry culture that was yet to be established in these classrooms was metacognitive awareness and control of learning among the students. While metacognition is only mentioned in passing by Hiebert et al. (1997) as a component of reflective communication and tool use, several other researchers have pointed out the value of it for student learning. Stevenson's (1998) questionnaire results indicate that thinking about ideas or reflecting on errors or methods was not a strong component of either classroom culture. The interview comments and classroom practices showed that few of the students were aware of either their problem solving strategies or of using mathematical skills as tools. Without this awareness and control of their practice the students could be expected to be largely blind to the linkages between the topics and skills or to the direction and overall picture of their mathematical education. This was indeed the case in these classrooms.

7. THE CASE STUDY: CONCLUSIONS AND IMPLICATIONS.

The results of the study highlight the importance of classroom culture; its interlinked dimensions; the way it is managed and moulded by the teacher and the ways the students accept, modify or avoid the norms and expectations of the teacher. The teachers in this study were knowledgeable about mathematics and committed to their students learning with understanding. They had structured their classroom environments to support that goal. Even so, their students spent much of mathematics time avoiding deep learning and pursuing goals of completion and correctness. Many of them often didn't try to do the work themselves even though they wanted to and believed they should. The students did learn mathematics reasonably well and passed their tests, but the story of New Zealand mathematics achievement is one of unfulfilled potential and poor international standing.

Deep understanding and thoughtful application of school mathematics are among the goals of MINZC (Ministry of Education, 1992) and are important for empowering people to take a full role in society and to learn the skills needed in the workplace, as well as contributing to strong self esteem and a sense of personal control within the real world. This study shows that students have a good chance of attaining those goals if teachers implement the dimensions and features of Hiebert et al.'s (1997) model of teaching. Tauber's (1994) model for power relationships provides a further tool to understand the social culture of the classroom. Using these as tools teachers would be able to see which features of their practice already support learning with deep understanding and how their practice might be moved in fruitful directions.

7.1 Whose mathematics, what beliefs, whose power?

The conditions for deep understanding of mathematics are now agreed on by many researchers. Hiebert et al's (1997) report summarised the consensus reached by four groups of researchers, and the literature review in this study has linked their work to that of several others. The norms supporting an inquiry classroom have been discussed in chapter six and the extent of them in the study classrooms has been reported. In these classrooms at least, traditional norms co-existed alongside inquiry norms and in many situations were more powerful, leading to shallow learning and reinforcing procedural rather than conceptual based attitudes towards mathematics. This is understandable, since students who believe that mathematics consists mainly of completing exercises correctly would be unlikely to develop well-linked mathematical concepts. The teachers in the study believed that their students had come to them with such beliefs already well developed, probably largely at home reinforced by teacher attitudes and approaches in the earlier primary years. This needs to be investigated in depth in New Zealand before remediation can be effective. The issues concerning parental attitudes to mathematics education and how they might become involved in school mathematics have been surveyed by Peressini (1998), who suggests that a gulf has grown between parents and teachers during the moves towards modernising mathematics education. This gulf needs to be understood and ways found to bridge it before parental attitudes to reform mathematics could be expected to change. Both the schools involved in this study found that the Family Maths programme was one way to influence parental attitudes in ways that increased parental acceptance of current school mathematics programmes. The research by Savell (1998) into parent newsletters indicates another way to bring parents closer to an understanding of school mathematics teaching and learning.

The types of behaviours and underlying beliefs that contributed to the ineffective learning have been described in this report. Most of the students were aware that their teachers wanted them to understand the work as well as to be proficient at the procedures and algorithms and many of the students said they wanted to do the work themselves. These goals were often overcome by a combination of social goals and mathematical goals from the traditional paradigm. The immediate rewards of social chat and well established habits of letting more able students do the work combined to make 'right and finished' quite an attractive goal involving little personal effort. This goal clashed with the responsibility and self-achievement goals, so students could only choose it where they chose to reject or ignore their own longer term mathematical interests in favour of short term advantage. In turn, this was facilitated by classroom norms that emphasised short-term 'answer focussed' goals. The complex web of traditional goals and the behaviours that could exist under that paradigm all made surface learning the reality for many students in this study. The power relationships that supported lots of context free practice and skills focussed revision were clearly of legitimate and controlling types, since the teacher needed to use control to keep the students on task, and legitimate authority to get them to do it in the first place – not surprising when the students were doing it just because it was 'school mathematics'. They saw little profit in it for themselves.

That traditional environment contrasts strongly with the web of influential power relationships, self- motivation and sustained on-task behaviour that was apparent among all students during Mrs K's project work. This was also seen among the more able students quite a lot of the time, but the open tasks and the value put on explanations and relational understanding led to real mathematical exploration, sustained co-operation and argument, and joy amongst the students as they came to understand the underlying concepts as well as the methods of computation. In Mr J's class this also happened during the project work but for fewer students. Mr J had a higher level of inquiry norms operating all the time than did Mrs K, but didn't reach the level of complete change-over that was apparent in Mrs K's class.

Thagard's (1994) analysis of conceptual change and paradigm shifts clearly applies to this situation. Only when many factors combine to support a new paradigm can it take the place of an established one. The new paradigm also needs to offer a better or more effective explanation of observed reality before it will be adopted. In this study the students of Mrs K's classroom adopted the inquiry paradigm when many factors combined: Mrs K's inquiry norms; interesting and challenging tasks; believable potential for both understanding and skills mastery; a power environment that emphasised co-operation and support; and minimal emotional consequences for mistakes. Although it came close, Mr J's classroom environment didn't offer enough inquiry features for the synergy to occur across the class.

The implication is that mathematics teaching and learning can produce deep understanding and fruitful collaboration provided most of the environment is tuned to inquiry norms and practices: being on task is rewarding to the students; they have tasks which engage and challenge them without being perceived as too hard; the teacher acts and is seen as a guide and resource rather than boss and taskmaster; collaboration and communication about the mathematics are normal and rewarding; and the task goals are to do with development of understanding rather than answers.

7.2 The tasks and the task goals.

Both of the study teachers tried to promote both traditional procedural facility and inquiry based relational understanding. Both found that the students chose to take far more notice of the traditional task environment than the inquiry environment. Mr J set tasks that were largely closed and found that the students also treated the open tasks of the projects as closed. In spite of his constructing the classroom as valuing perseverance, self-responsibility and co-operation his students persisted with their more traditional associations because the tasks carried traditional implications for them. His mistake may have been to construct the topics as continuums of tasks and try to change the goals without changing the rest of the environment. Mrs K succeeded in changing the goal environment by having separate inquiry tasks from the seatwork tasks. She set up two mathematical environments and so could have two sets of goals. She didn't actually want to do so, but was so constrained by the student expectations that she could only establish some of the inquiry norms during seatwork and revision time.

7.3 More about power.

The power analysis used in this study has great potential as a tool for teachers to understand their classroom interactions. It fits beside the inquiry features that are tools for designing teacher practices, since power relationships must affect whether the teacher will be able to establish their desired norms. The evidence of this study is that a teacher must be very careful to act as a guide and coach all the time, if they want the students to see them in this light. They also need to find out how and when to use controlling power so that it doesn't trigger off the strong associations with coercion and powerlessness seen in this study. The students came to the classrooms expecting to go through the motions of doing mathematics and to somehow absorb something along the way. This speaks strongly of past experiences of legitimate and controlling power in mathematical environments. With such negative associations to contend with an inquiry oriented teacher would need to carefully avoid using growling or punishment except for deliberate disruption. Off-task episodes would be better dealt with like mistakes – as things that were ineffective for understanding. Referent power along the lines of 'what do we do when we don't understand?' would likely trigger reference to norms of asking, questioning, persevering and peer collaboration. Such responses keep the student focussed on the task rather than the unhappy feelings or the social associations of being off-task.

Legitimate power also needs to be used carefully. The teacher has the right to set the tasks and the classroom environment – that is very necessary and connects to expert power. What they should avoid is implying that the students should do the tasks because the teacher said so. Tasks need to have meaning and purpose in the lived world of the students. Where the teacher lets the students structure the tasks they avoid legitimate authority and tap into the students' ways of understanding the world.

Referent power and expert power relationships offer the teacher effective tools to let the mathematics take center stage rather than themselves. Scaffolding and apprenticeship systems of mathematics teaching thrive on teachers constructing themselves as guides and coaches. How to do so is a journey of discovery that takes time for most teachers, trained as most are under the traditional didactic systems of the past. Mr J wanted to be guide and coach but ruefully accepted that his students didn't treat him that way. Mrs K achieved it in the project work to some extent. The reasons lay in the extent to which the whole classroom environment was coherently based on inquiry principles.

7.4 Making it work.

An inter-linked system of inquiry norms and practices cannot be established quickly even when a teacher is fully conversant with the theory and practice of inquiry learning. In these classrooms the teachers were in the process of investigating and developing their ideas and practice and had constructed their classrooms to fit their current beliefs. Both teachers expected to further develop their ideas and actively sought the involvement in this study as a way to explore recent research. The evidence of this study is that any teacher who felt dissatisfied with the level of mathematical involvement or progress of their students and set out to learn more would be able to make gradual and progressive changes to their practice. They could expect to see their students move along a pathway of thinking more deeply, trying out ideas, discussing ideas as well as methods, believing in themselves and enjoying their learning more and more.

If many of the students in these classrooms could effectively avoid learning with understanding, how much more so could the students in classrooms where the teachers were less sure of their mathematics, their educational philosophies or their classroom practice? The students in the study classes came to these classes from several different schools and teachers. Few of them reported previous mathematical experience based on inquiry and understanding. Is it true that most New Zealand teachers concentrate on teaching their students to use specified algorithms without introducing them to the power of mathematics to explain and illuminate their worlds? Is problem solving confined to short word problems designed to apply recently learnt methods? Or are there lots of teachers out there showing their students that mathematics is interesting, achievable, intriguing and powerful? My experience as a secondary school teacher is that most students finish primary school uninterested in mathematics, expecting to do things without thinking, and resigned to never understanding what they are doing. No wonder they don't expect to learn anything!

7.5 Where to next?

This study suggests that many primary teachers may hold very traditional views of mathematics teaching and learning and of power relationships in the classroom, and that they concentrate on procedural facility and controlling/ legitimate power rather than on developing a co-operative inquiry culture in their classrooms. If most teachers are constructing such traditional environments for mathematics teaching and learning then New Zealand students are unlikely to develop inquiry-based beliefs. There is therefore a need is to extend this case study research to the whole range of year levels in New Zealand schools; to further investigate classroom environments and teacher beliefs and attitudes towards mathematics education. Other research is needed to discern the extent to which the student beliefs and expectations described in this case study represent those of New Zealand students and how those beliefs link to the teaching practices they experience over the primary years.

Since this study has strongly affirmed the efficacy of linked systems of inquiry based mathematics teaching and learning, an important practical step would be to step up the provision of pre-service and in-service professional development programmes designed to share such inquiry approaches and effective understanding of classroom power relationships (see also Middleton and Spanias, 1999). Some courses in Colleges of Education and among in-service agencies already have such a focus, but the Ministry of Education and its agencies need to promote them more strongly, especially among practising Primary teachers who see themselves as weak in mathematics.

This suggestion for a focus on conceptual change may seem to be in conflict with the recommendations of Knight and Meyer (1996), but this study indicates that teachers are slow to understand new teaching approaches or to change current practices, so they want support to keep on doing what they already understand. While they approve of the 1992 curriculum and identify the need for resources to implement the practices they already believe in, this study shows that there is still a need for conceptual change so that more effective practices can be implemented. Given the expressed desire of teachers for support in the shape of resources and time, it may be more effective to provide the conceptual change elements as the background to resource based professional development programmes, and to deliver these as ready made lesson sequences with modelling of their use by advisers in the classrooms. This is being done with resources such as the www.nzmaths.co.nz website, which is reported among local maths associations to be an effective tool for raising teacher awareness of investigatory approaches to mathematics.

New Zealand mathematics teaching has a chance to change its direction to take on the range of proven practices described in this study and elsewhere. Since teachers such as Mr J and Mrs K are already exploring such practices, it is likely that there exists a readiness to accept soundly based professional development. The sooner this is undertaken, the sooner New Zealand children might shed the misconceptions of the past and learn to view mathematics as it really is – a wonderful collection of tools for working with the world.

FINIS

APPENDIX A

Research Questionnaire: Making Sense of Mathematics, 1998. (From Stevenson, 1994)
There are ten different statements asked in three ways each – "I do this", " I feel I have to do this" and " the teacher encourages us to do this."

You may feel the same or differently about each way of asking each question. Please take each one as it comes, without trying to be consistent – the purpose is to find out how you feel, not how you think about these things. If you feel you have to go back and change something, please put numbers on your different answers, to show the order in which you did them.

Please put a tick, cross or small circle on the line to show how often each statement would be true for you.

1. When I have done some maths, I ask questions to check my answers or results.

   1.Almost never 2. 3.sometimes 4. 5. quite often

2. I feel I am supposed to try out new ideas.

   1.Almost never 2. 3.sometimes 4. 5. quite often

3. The teacher likes us to find links or connections between the different things we learn.

   1.Almost never 2. 3.sometimes 4. 5. quite often

4. I feel I am supposed to find information for myself.

   1.Almost never 2. 3.sometimes 4. 5. quite often

5. I let the teacher tell me what to do.

   1.Almost never 2. 3.sometimes 4. 5. quite often

6. I feel I am supposed to copy what the teacher does.

   1.Almost never 2. 3.sometimes 4. 5. quite often

7. I check my answers or results against things I already know.

   1.Almost never 2. 3.sometimes 4. 5. quite often

8. I get all my information from the teacher.

   1.Almost never 2. 3.sometimes 4. 5. quite often

9. The teacher encourages us to copy what they do.

   1.Almost never 2. 3.sometimes 4. 5. quite often

10. I feel I am supposed to ask questions to check my answers or results.

1.Almost never 2. 3.sometimes 4. 5. quite often

11. The teacher likes us to try out new ideas.

1.Almost never 2. 3.sometimes 4. 5. quite often

12. I feel I am supposed to check my results against things I already know.

1.Almost never 2. 3.sometimes 4. 5. quite often

13. I find connections or links between the things I learn.

1.Almost never 2. 3.sometimes 4. 5. quite often

14. I do what I want to do in class.

1.Almost never 2. 3.sometimes 4. 5. quite often

15. The teacher likes us to find out things for ourselves.

1.Almost never 2. 3.sometimes 4. 5. quite often

16. The teacher likes us to do what we are told.

1.Almost never 2. 3.sometimes 4. 5. quite often

17. I feel I am supposed to work exactly as I am shown

1.Almost never 2. 3.sometimes 4. 5. quite often

18. I rely on the teacher to show me the links or connections between things.

1.Almost never 2. 3.sometimes 4. 5. quite often

19. I try out new ideas.

1.Almost never 2. 3.sometimes 4. 5. quite often

20. I copy what the teacher does.

1.Almost never 2. 3.sometimes 4. 5. quite often

21. The teacher likes us to ask questions to check our answers or results.

1.Almost never 2. 3.sometimes 4. 5. quite often

22. I feel I am supposed to find the links or connections between things.

1.Almost never 2. 3.sometimes 4. 5. quite often

23. I accept my answers or results without checking them or wondering if they are correct.

1.Almost never 2. 3.sometimes 4. 5. quite often

24. I do things my way.

1.Almost never 2. 3.sometimes 4. 5. quite often

25. I feel I am supposed to do what the teacher tells me.

1.Almost never 2. 3.sometimes 4. 5. quite often

26. The teacher likes us to do our own work exactly as we are shown.

1.Almost never 2. 3.sometimes 4. 5. quite often

27. I find out information by myself.

1.Almost never 2. 3.sometimes 4. 5. quite often

28. I rely on the teacher for new ideas.

1.Almost never 2. 3.sometimes 4. 5. quite often

29. The teacher likes us to check their results against things they know.

1.Almost never 2. 3.sometimes 4. 5. quite often

30. I work exactly as I am shown.

1.Almost never 2. 3.sometimes 4. 5. quite often

APPENDIX B.

Detailed results of the Questionnaires

Research Questionnaire 1998 Mrs. K's class

Question # to the right

Student # down	1	2	3	4	5	6	7	8	9	10	11
1	5	3	3	2	1	3	3	3	3	3	5
2	1	1	3	2	4	3	1	2	1	2	4
3	1	3	3	3	1	1	3	1	1	1	3
4	3	3	5	1	1	3	1	1	5	3	3
5	5	3	1	3	5	3	5	5	1	5	5
6	3	3	5	5	3	3	3	5	0	5	5
7	1	1	5	5	3	3	5	5	1	1	3
8	1	3	5	5	5	1	3	3	1	3	3
9	3	5	5	4	1	1	3	1	2	3	5
10	1	3	5	3	1	5	3	5	5	1	5
11	1	5	3	5	0	1	3	1	1	1	5
12	1	1	5	5	5	1	3	5	3	1	3
13	3	4	5	5	3	1	1	3	4	5	5
14	3	2	3	4	1	1	3	1	3	4	5
15	2	4	3	3	2	2	3	2	1	3	3
16	1	3	5	1	1	3	3	3	1	0	3
17	3	5	5	5	3	5	1	3	1	1	5
18	3	2	2	5	5	5	4	5	4	3	5
19	1	3	4	4	3	2	2	3	3	2	5
20	1	3	5	4	3	2	1	3	2	2	3
21	3	5	3	5	1	1	5	1	1	3	5
Not answered	0	0	0	0	1	0	0	0	1	1	0
Almost never	10	3	1	2	8	8	5	6	10	6	0
Hardly ever	1	2	1	2	1	3	1	2	2	3	0
Sometimes	8	10	7	4	6	7	11	7	4	7	8
Often	0	2	1	4	1	0	1	0	2	1	1
Quite often	2	4	11	9	4	3	3	6	2	3	12
	21	21	21	21	21	21	21	21	21	21	21
Mean	2.19	3.10	3.95	3.76	2.48	2.38	2.81	2.90	2.10	2.48	4.19
Standard deviation	1.33	1.26	1.24	1.37	1.63	1.40	1.29	1.58	1.48	1.47	0.98
Median	2.00	3.00	5.00	4.00	3.00	2.00	3.00	3.00	1.00	3.00	5.00

	12	13	14	15	16	17	18	19	20	21	22	23
1	3	3	2	4	5	4	5	3	3	5	3	3
2	2	3	3	5	5	3	1	2	1	3	4	4
3	1	3	1	5	5	3	5	5	1	3	3	3
4	3	1	5	5	5	5	1	1	5	3	3	3
5	3	5	1	4	5	4	5	3	2	5	3	2
6	3	3	3	5	5	5	3	3	3	5	3	3
7	3	5	1	5	5	5	3	3	3	3	3	3
8	1	5	3	3	5	5	5	3	1	5	3	5
9	4	5	2	4	5	4	4	5	2	4	5	2
10	3	5	3	5	5	3	5	3	3	5	3	5
11	3	5	1	3	5	3	1	3	1	3	3	3
12	3	5	1	3	5	5	5	3	3	1	3	3
13	3	4	2	5	5	5	4	3	2	5	5	4
14	3	4	1	5	5	4	3	4	1	5	4	3
15	3	3	2	4	3	4	2	3	2	3	3	2
16	5	3	1	5	3	3	3	5	1	1	3	3
17	1	5	3	5	5	3	1	5	1	1	3	1
18	4	3	1	5	5	4	5	4	5	4	3	4
19	3	4	2	4	5	4	4	4	3	3	5	3
20	3	4	2	5	5	5	3	3	4	2	4	3
21	3	3	1	5	5	0	1	5	1	3	3	3
Not answered	0	0	0	0	0	1	0	0	0	0	0	0
Almost never	3	1	9	0	0	0	5	1	8	3	0	1
Hardly ever	1	0	6	0	0	0	1	1	4	1	0	3
Sometimes	14	8	5	3	2	6	5	11	6	8	15	12
Often	2	4	0	5	0	7	3	3	1	2	3	3
Quite often	1	8	1	13	19	7	7	5	2	7	3	2
	21	21	21	21	21	21	21	21	21	21	21	21
Mean	2.86	3.86	1.95	4.48	4.81	3.86	3.29	3.48	2.29	3.43	3.43	3.10
Standard deviation	0.96	1.11	1.07	0.75	0.60	1.20	1.59	1.08	1.31	1.40	0.75	0.94
Median	3.00	4.00	2.00	5.00	5.00	4.00	3.00	3.00	2.00	3.00	3.00	3.00

Student #	24	25	26	27	28	29	30
1	3	4	5	3	5	4	3
2	4	3	4	4	2	3	1
3	1	5	5	3	3	3	5
4	5	3	5	5	3	5	0
5	1	5	4	3	3	2	4
6	3	5	3	3	3	3	3
7	1	5	5	1	3	3	5
8	0	1	3	3	3	5	5
9	1	5	5	5	4	4	3
10	3	3	3	3	5	3	3
11	3	5	5	3	1	1	5
12	1	5	5	3	5	5	3
13	2	5	5	3	4	4	5
14	1	5	5	4	1	3	4
15	2	0	3	2	2	2	4
16	5	5	3	5	5	5	5
17	3	5	5	3	1	3	3
18	5	5	4	5	2	2	5
19	2	5	4	3	4	2	4
20	3	5	5	4	3	3	5
21	1	5	3	5	1	5	5
Not answered	1	1	0	0	0	0	1
Almost never	7	1	0	1	4	1	1
Hardly ever	3	0	0	1	3	4	0
Sometimes	6	3	6	11	7	8	6
Often	1	1	4	3	3	3	4
Quite often	3	15	11	5	4	5	9
Total # students	21	21	21	21	21	21	21
Mean	2.38	4.24	4.24	3.48	3.00	3.33	3.81
Standard deviation	1.50	1.45	0.89	1.08	1.38	1.20	1.40
Median	2.00	5.00	5.00	3.00	3.00	3.00	4.00

Research Questionnaire 1998 Mr. J's class

Question # to the right

Student # down	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	2	3	4	4	4	1	0	0	2	2	3	0	0	2
2	2	3	4	1	2	1	2	4	1	1	2	2	2	1
3	3	1	5	5	3	1	5	1	1	3	3	3	5	3
4	3	3	3	5	5	3	3	1	3	3	5	3	3	1
5	2	2	3	4	3	1	4	2	3	3	2	1	3	2
6	1	2	3	4	2	1	2	2	2	1	2	2	3	4
7	4	4	4	5	4	4	4	3	3	4	4	3	4	2
8	4	4	3	4	4	2	3	2	2	4	4	4	4	3
9	3	4	3	5	3	2	3	1	3	4	5	3	4	3
10	3	1	5	3	5	3	3	1	5	3	5	3	3	1
11	3	3	2	2	4	4	2	2	3	3	3	3	1	3
12	3	5	3	5	3	1	1	1	1	3	3	1	3	3
13	3	4	4	4	3	4	5	2	4	3	3	4	2	3
14	3	3	5	3	2	2	2	1	4	3	2	4	1	4
15	3	2	3	3	4	3	2	3	3	3	4	2	2	3
16	3	4	3	4	2	2	5	3	2	3	0	3	4	3
17	2	3	4	3	5	3	3	2	2	3	2	3	4	2
18	3	3	1	4	3	3	2	2	4	4	2	4	2	2
19	3	3	3	3	5	3	3	3	3	1	3	3	3	1
20	2	3	2	3	4	2	1	3	5	3	2	1	3	3
21	3	2	3	3	4	2	4	2	5	3	4	3	3	2
22	4	4	4	3	3	2	5	3	3	5	5	3	2	2
23	4	5	4	3	2	3	4	1	3	2	2	2	3	2
24	4	2	2	2	4	4	3	3	0	3	2	3	0	2
25	3	3	4	3	4	3	2	1	3	3	2	2	3	1
26	4	4	4	5	5	3	4	2	1	5	5	3	4	3
27	3	0	3	4	2	5	0	1	1	4	3	0	0	3
28	3	2	3	3	4	2	4	2	5	3	4	4	3	2
Not answered	0	1	0	0	0	0	2	1	1	0	1	2	3	0
Almost never	1	2	1	1	0	6	2	9	5	3	0	3	2	5
Hardly ever	5	6	3	2	6	8	7	10	5	2	10	5	5	10
Sometimes	16	10	12	11	7	9	7	7	10	16	7	13	11	11
Often	6	7	9	8	10	4	6	1	3	5	5	5	6	2
Quite often	0	2	3	6	5	1	4	0	4	2	5	0	1	0
	28	28	28	28	28	28	28	28	28	28	28	28	28	28
Mean	###	2.93	3.36	3.57	3.50	2.50	2.89	1.93	2.75	3.04	3.07	2.57	2.64	2.36
Standard deviation	###	1.18	0.95	1.03	1.04	1.11	1.42	0.94	1.38	1.00	1.27	1.14	1.31	0.87
Median	###	3.00	3.00	3.50	4.00	2.50	3.00	2.00	3.00	3.00	3.00	3.00	3.00	2.00

1	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
2	4	4	3	2	4	1	3	3	3	3	4	4	4	2	4	3
3	4	4	3	5	4	5	3	2	1	4	5	5	1	4	5	5
4	3	5	1	1	4	1	2	3	1	5	3	2	5	1	5	1
5	5	3	5	3	5	3	5	3	3	3	3	5	3	1	3	3
6	5	5	3	0	2	3	3	3	3	2	3	3	3	3	1	4
7	3	4	3	2	5	1	1	2	3	4	3	3	5	3	2	2
8	4	3	3	2	4	3	4	4	4	5	4	3	4	1	4	3
9	4	4	3	4	3	2	4	4	4	2	4	4	3	4	4	2
10	4	4	2	3	4	2	3	3	3	2	2	3	3	2	3	3
11	3	5	3	3	1	3	3	3	3	3	5	3	3	3	5	3
12	2	3	4	2	4	3	4	2	2	4	4	4	4	3	2	4
13	3	5	3	1	5	1	3	3	3	3	1	3	5	1	3	3
14	5	4	3	3	4	3	5	4	4	3	3	4	5	2	4	3
15	4	5	3	3	4	4	4	4	1	5	4	3	2	3	3	0
16	4	5	3	3	3	3	4	3	4	4	5	5	4	3	3	3
17	4	5	3	2	4	2	3	3	2	4	3	3	3	3	4	3
18	5	5	4	3	3	3	5	2	3	3	5	5	3	2	5	5
19	4	4	4	2	3	4	4	2	2	2	4	3	5	1	2	4
20	0	3	3	3	2	3	2	3	3	2	3	3	3	3	3	3
21	5	5	3	4	4	2	1	2	5	4	5	5	3	1	1	5
22	3	0	2	3	3	3	4	3	3	2	3	4	0	3	4	4
23	5	3	4	4	3	3	2	3	3	2	3	2	3	3	3	2
24	3	3	2	1	3	2	3	3	2	3	3	2	3	2	2	3
25	3	4	2	4	4	4	3	3	3	3	3	2	3	2	2	3
26	4	4	1	1	3	2	3	3	4	0	4	3	3	3	2	4
27	4	5	3	3	4	4	5	5	5	3	3	3	3	3	3	3
28	3	4	3	5	3	4	3	3	3	2	3	4	3	3	0	2
	4	3	2	2	2	2	4	3	2	4	3	3	3	2	4	4
Not answered	1	1	0	1	0	0	0	0	0	1	0	0	1	0	1	1
Almost never	0	0	2	4	1	4	2	0	3	0	1	0	1	6	2	1
Hardly ever	1	0	5	7	3	7	3	6	5	8	1	4	1	7	6	4
Sometimes	8	7	16	10	9	11	11	17	13	9	14	13	16	13	8	13
Often	12	10	4	4	12	5	8	4	5	7	7	6	4	2	7	6
Quite often	6	10	1	2	3	1	4	1	2	3	5	5	5	0	4	3
	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28
Mean	3.71	3.96	2.89	2.64	3.46	2.71	3.32	3.00	2.93	3.07	3.50	3.43	3.29	2.39	3.07	3.11
Standard deviation	1.08	1.10	0.88	1.22	0.96	1.05	1.09	0.72	1.05	1.15	0.96	0.96	1.15	0.92	1.30	1.13
Median	4.00	4.00	3.00	3.00	4.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00

APPENDIX C.

The students were encouraged to write as many responses as they wished. The responses were anonymous.

Research: Making sense of Mathematics. Story writing.
Please finish each paragraph. If you want to write more, use another page and write the number of the question you are writing about.

1. When we start our maths time, the teacher ______

When this happens my attitude is ______

2. Some people in our class learn mathematics better than others because ______

3. In maths time, we do different sorts of things. This year we've done ______

4. When I don't understand something I ______

5. I know how we're supposed to work in maths time. The teacher likes us to ______

6. Most of the time, when it's maths time I feel ______________________________

7. I remember what maths was like last year, we used to ______________________________

8. Some things I worry about during maths time are ______________________________

9. In maths time I feel good about ______________________________

10. The thing I liked best about maths this year was ______________________________

It was good because ______________________________

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