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PORTFOLIO SELECTION BY HOMOGENEOUS PROGRAMMING:

A NEW ZEALAND CASE STUDY

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ABSTRACT

For investment managers through to the individual the task of solving their particular portfolio problems remains a principal objective.

It has been shown that an efficient portfolio can be specified in terms of its expected return and profit variance, (risk), that is the first two moments of the investor's subjective yield distribution. Selection of an efficient portfolio can always be achieved by quadratic or homogeneous programming.

An integral part of the efficient portfolio selection process by homogeneous programming lies in the use of the truncated minimax criterion which gives a measure of risk preference, m . Varying m will give the complete set of efficient portfolios from all possible ones. This is detailed in chapter one where it is also shown that an optimal portfolio which allocates the budget with maximal caution can be selected from among the efficient ones under the additional criterion that the marginal value of the investment dollar is not exceeded by its marginal cost.

Using a specified algorithm an optimal portfolio is selected from stocks qualifying as Trustee Investments under the Trustee Amendment Act 1974 listed on the New Zealand Stock Exchange.

Chapter two details the manner in which a five year data base of weekly observations, 1979 to 1983 inclusive, was developed for this operation and gives the preliminary results of expected return and profit variance for the stocks selected. A printout of the complete data file is included in the appendix.

Chapter three of this thesis shows in detail the manner in which the algorithm is applied and gives a final result using weekly data over the four year period, 1980 - 1983 inclusive. The characteristics of this optimal portfolio are shown together with details of its performance over the twelve month period Jan - Dec 1984.

Finally consideration is given to the robust nature of the portfolio selection system by looking both at a range of efficient portfolios selected from the four year data and also an optimal result from the full five year data.

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CHAPTER 1

EFFICIENT AND OPTIMAL PORTFOLIOS

INTRODUCTION

The manner in which an individual distributes his wealth gives his portfolio and for any individual the set of portfolios from which this choice is to be made is extremely vast. From this vast selection of possible portfolios the individual is faced with the task of first separating out those that are efficient from the inefficient and then determining that efficient portfolio which provides the most suitable combination of risk and return given his particular circumstances. The same problem is of course faced by the investment managers of pension funds and the like.

Pioneering work in the area of portfolio analysis was carried out by H. Markowitz¹ who characterized a portfolio by its expected profit or return and profit variance, a measure of risk.

"The portfolio is Markowitz efficient if no portfolio with at least equal profit expectation has similar profit variance and no portfolio with at most equal variance has a larger expected profit."²

Markowitz went on to show that, given any particular set of candidates for the portfolio, a complete set of efficient portfolios can be generated by quadratic programming. The validity of the results of course being dependent on the validity of the estimated values of expected return and risk.

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1. Markowitz, H.M., Portfolio Selection, Wiley, New York, 1959
 2. Moeseke, P.v. and Hohenbalken, v.B., "Efficient and Optimal Portfolios by Homogeneous Programming", Zeitschrift fur Operations Res. 18., pp. 205.

P. v. Moeseke³ further showed that efficient portfolios can always be selected by homogeneous programming and that the duality theorem of homogeneous programming allows for an optimal portfolio to be selected from among the efficient ones. A summary of the major principles behind this approach developed by P. v. Moeseke [op.cit.] is now given with readers being referred to this work for all relevant proofs.

EFFICIENT PORTFOLIOS

To generate the complete set of efficient portfolios by homogeneous programming a risk criterion is required that will yield exact results. The problem as stated is, given the expectation, E and variation, σ of outcomes, to separate out all those portfolios which give the maximum expected return for any level of risk or which are the least risk portfolios for any level of expected return.

It is assumed that the outcome distribution is approximately normal, the normal distribution being completely characterized by its first two moments E and σ . This renders the minimax and related rules irrelevant for a possible risk criterion as the minimum yield of every portfolio is $-\infty$ so that the minimax is indeterminate.

The truncated minimax criterion which attaches relative weights l and m to the expected return and standard deviation of an outcome was therefore developed having a confidence interval interpretation. Here the tail of the distribution is cut off, truncated, so that extreme outcomes are ignored and exact results can be obtained.

Under the stated assumption that the outcome distribution is normal for all x in X the risk preference parameter, m can be constructed in terms of expected returns and variance.

3. Moeseke, P.v., "Stochastic Linear Programming", Yale Economic Essays, 5, 1965, pp. 197 - 253

$$\phi(x) = \bar{c}x - m(xVx)^{\frac{1}{2}}$$

$$\bar{c} = [\bar{c}_i]$$

$$\text{where } \bar{c}_i = \frac{1}{T} \sum_{t=1}^T c_{it} \quad i = 1, \dots, n$$

and c_{it} is the net return per dollar of security i for time period t .

$$V = [\sigma_{ij}]$$

$$\text{where } \sigma_{ij} = \frac{1}{T-1} \sum_{t=1}^T \sum_{t=1}^T (c_{it} - \bar{c}_i)(c_{jt} - \bar{c}_j) \\ ij = 1, \dots, n$$

Say $m = 1.645$, then from the table of normal distribution $\alpha = .05$. Maximizing $\phi(x) = \bar{c}x - 1.645(xVx)^{\frac{1}{2}}$ corresponds to comparing the competing distributions at the .05 confidence limit.

Note that as well as having an interpretation in terms of confidence limits, m also expresses the investors risk aversion as it represents a linear weighting of expectation and standard deviation.

The following table now shows the range of possible values for m together with appropriate risk attitude interpretations.

TABLE 1
The Truncated Minimax (and Maximin) Criterion

Value of Risk Preference m	Character of max $\phi f(x)$ $x \in X$	Risk Attitude
> 0	Truncated minimax criterion	Risk averting
0	Borderline case	Expectational
< 0	Truncated maximin criterion	Risk seeking

Source: Moeseke, P.v., op.cit. pp. 210

By varying m and maximizing $\phi(x) = \bar{c}x - m(xVx)^{\frac{1}{2}}$ one can obtain the complete set of efficient portfolios.

OPTIMAL PORTFOLIOS

Having generated the set of efficient portfolios it would now be desirable to select from these an optimal portfolio. v. Moeske defined a portfolio as optimal if it satisfied the following optimality criterion.

$$\text{Max}_x \phi(x|m^+) \quad (\text{the optimum of all } \phi(x) \text{ solutions})$$

where $m^+ = \max m \text{ on } M = \{m \geq 0 | \max \phi(x|m) \geq r\}$

and r is the relevant rate of interest on the capital market.

Now by the duality theorem of homogeneous programming the primal problem

$\max \phi(x|m) \text{ on } X$ implies the dual problem

$$\min \lambda \text{ on } L = \{\lambda \geq 0 | \lambda q \geq \phi x; x \in X\}$$

where ϕx is the gradient vector $[\partial \phi / \partial x_1, \dots, \partial \phi / \partial x_n]$

Also for any dual solution λ^*

$$\lambda^* = \phi(x^*|m) = \bar{c}x^* - m \sigma x^*$$

λ measures the marginal value of the budget dollar to the investor using criterion ϕ .

This optimality criterion therefore selects that portfolio which allocates the budget with maximum caution under the additional criterion that the marginal value of the investment dollar is not exceeded by its marginal cost.

The optimal portfolio x^+ satisfies

$$\text{Max}_{x \in X} \phi(x|m^+) = \phi(x^+|m^+) = \lambda^+ = r$$

Using an algorithm to be details in Chapter 3, v. Moeseke⁴ selected an optimal portfolio from a selection of nine U.S. stocks as an example of Portfolio Selection by Homogeneous Programming.

In further work in this area v. Moeseke and v. Hohenbalken⁵ generated an optimal portfolio from a set of 54 Canadian and U.S. Stocks using weekly data from May 3 1968 to May 30 1969.

The optimal portfolio was arrived at where $m = .5$ for an interest rate of 8.5%. As the weight attached to the expected return under the truncated minimax criterion is one the NYSE investor valued expected return twice as highly as he tries to avoid risk. This was a very acceptable value for m .

4. Moeseke, P.v., op.cit., pp. 248 - 253.

5. Moeseke, P.v. and Hohenbalken, v.B., op.cit., pp. 205 - 214.