MODELLING OF VOLCANIC ASHFALL

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Abstract

Modelling of volcanic ashfall has been attempted by volcanologists but very little work has been done by mathematicians. In this thesis we show that mathematical models can accurately describe the distribution of particulate materials that fall to the ground following an eruption. We also report on the development and analysis of mathematical models to calculate the ash concentration in the atmosphere during ashfall after eruptions. Some of these models have analytical solutions.

The mathematical models reported on in this thesis not only describe the distribution of ashfall on the ground but are also able to take into account the effect of variation of wind direction with elevation. In order to model the complexity of the atmospheric flow, the atmosphere is divided into horizontal layers. Each layer moves steadily and parallel to the ground: the wind velocity components, particle settling speed and dispersion coefficients are assumed constant within each layer but may differ from layer to layer. This allows for elevation-dependent wind and turbulence profiles, as well as changing particle settling speeds, the last allowing the effects of the agglomeration of particles to be taken into account.

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Contents

Notation

c	particle mass concentration in the atmosphere	[kg m^{-3}]
C_s	drag constant	
D_c	cross-wind dispersion	[m^2 s^{-1}]
D_d	downwind dispersion	[m^2 s^{-1}]
D_h	horizontal dispersion in $x - y$ plane	[m^2 s^{-1}]
D_v	vertical dispersion	[m^2 s^{-1}]
D_x	horizontal dispersion in x direction	[m^2 s^{-1}]
D_y	horizontal dispersion in y direction	[m^2 s^{-1}]
D_z	vertical dispersion	[m^2 s^{-1}]
g	gravity	[m s^{-2}]
H	z-coordinate of the release point	[m]
L	turbulence length scale	[m]
M	source mass rate	[kg m^{-3} s^{-1}]
Q	mass release	[kg]
q	rate of mass release	[kg s^{-1}]
R	particle radius	[m]
Re	Reynolds number	
S	settling speed	[m s^{-1}]
U	mean horizontal wind speed in x direction	[m s^{-1}]
V	mean horizontal wind speed in y direction	[m s^{-1}]
t	time	[s]
X_0	x-coordinate of the release point	[m]
x	x-coordinate	[m]
Y_0	y-coordinate of the release point	[m]
y	y-coordinate	[m]
z	z-coordinate	[m]
Z_j	interface height	[m]
μ_a	dynamic viscosity of the air	[kg m^{-1} s^{-1}]
ν_a	kinematic viscosity of the air	[m^2 s^{-1}]
ρ_a	mass density of the air	[kg m^{-3}]
ρ_r	mass density of the particles	[kg m^{-3}]