



## Preface

## Preface to VSI: Advances in nonsmooth dynamics

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## ABSTRACT

This Special Issue on nonsmooth dynamics brings together recent developments in nonsmooth dynamics, from applications in control engineering and mechanics, economics, climate modelling, physiological modelling, medicine, ecology and epidemiology, and others, to theory of novel forms of unpredictability and nonlinearity, chaos and bifurcations, and the study of higher dimensions.

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## 1. Introduction

This Special Issue on nonsmooth dynamics brings together recent developments in nonsmooth dynamics, from applications in control engineering and mechanics, economics, climate modelling, physiological modelling, medicine, ecology and epidemiology, and others, to theory of novel forms of unpredictability and nonlinearity, chaos and bifurcations, and the study of higher dimensions.

Nonsmooth dynamics has made huge progress since the turn of the century. It has found a huge range of new applications, and in doing so raised problems that dynamical systems have not tackled before. It has also made unanticipated breakthroughs in its theory, with new phenomena, novel kinds of nonlinearity and chaos, new methods and quantitative concepts. This special issue brings contributions from a range of problems involving nonsmooth dynamics, showing the ongoing discussion between theory and applications bringing about new developments.

## 2. Applications

A sample of the growing applications to human and other living systems are seen here. Yan et al. [1] study the dynamics of therapeutic model for prostrate cancer as a Filippov system, and Yin et al. [2] present a vibro-impacting capsule robot with applications for endoscopy. Sayli et al. [3] discuss the nonsmooth dynamics of the two-process model of sleep-wake regulation, while

Kowalczyk et al. [4] give a pendulum model of human running gaits. Away from human applications, Doré-Hall et al. [5] show how ramp functions simplify the Michaelis–Menten kinetics in a model of plant metabolism.

The traditional applications of electronic and mechanical control continue to raise challenges and new phenomena. Here Chen et al. [6] present dynamics and bifurcations of an oscillator with a piecewise-constant forcing, and Costa et al. [7] study periodic orbit switching in an impact oscillator with delayed feedback. Also Osguei et al. [8] present an anti stick–slip tool for downhole drilling, and Zhusubaliyev et al. [9] show period adding bifurcations in a power inverter. Bifurcations at a codimension two discontinuity set are studied by Antali [10] with application to dry friction dynamics.

## 3. Theory

The other half of the issue concerns more theoretical techniques. Simpson [11] presents some dimension reduction for slow–fast piecewise linear ODEs with an example of ocean circulation, and Hill et al. [12] derive path integrals for SDEs with piecewise-smooth drift and derive most probable transition paths. Buzzi et al. [13] give stability conditions for refractive piecewise-smooth planar vector fields, and Novaes and Rondón [14] present limit cycles bifurcating from homoclinic connections in regularized Filippov systems. Several papers discuss piecewise linear systems: Novaes [15] derives the Hilbert number for piecewise linear systems, Medrado et al. [16] study limit cycles and invariants in piecewise-linear sewing systems in 3 dimensions, while Freire et al. [17] study limit cycles in a class of 3D piecewise-linear relay systems with a Hopf bifurcation at infinity. Carmona et al. [18] show how canards form in a piecewise-linear analogue of the van der Pol system.

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#### 4. Closing

A range of applications and approaches are on display here, but these still show only a small sample of the various research going on in the area of nonsmooth dynamical systems, and they raise numerous avenues of interest for future studies which we hope you will find stimulating.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### Data availability

No data was used for the research described in the article.

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