The Linear Wave Response of a Single and a Periodic Line-Array of Floating Elastic Plates

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Abstract

We propose an improved technique to calculate the linear response of a single and multiple plates models due to ocean waves. The single plate model is the basis for the multiple plates model which we take to be a periodic array of identical plates. For the single plate model we solve the plate displacement by the Finite Element Method (FEM) and the water potential by the Boundary Element Method (BEM). The displacement is expanded in terms of the basis functions of the FEM. The boundary integral equation representing the potential is approximated by these basis functions. The resulting integral operator involving the free-surface Green's function is solved using an elementary integration scheme. Results are presented for the single plate model. We then use the same technique to solve for the periodic array of plates problem because the single and the periodic array plates model differ only in the expression of the Green's function. For the periodic array plate model the boundary integral equation for the potential involves a periodic Green's function which can be obtained by taking an infinite sum of the free-surface Green's function for the single plate model. The solution for the periodic array plate is derived in the same way as the single plate model. From this solution we then calculate the waves scattered by this periodic array.

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Nomenclature

Ω	water domain
Δ	plate-covered area
H	water depth
λ	wavelength
k	wavenumber
θ	wave-angle
ω	angular frequency
Φ	dimensional, time-dependent water potential
W	dimensional, time-dependent surface displacement
L	arbitrary length variable
ϕ	dimensionless, spatial-dependent water potential
ϕ^{In}	incident wave
ϕ^{s}	scattered wave
w	dimensionless, spatial-dependent water potential
α	dimensionless angular frequency
β	stiffness constant
γ	mass constant
A^{In}	incident amplitude
p	total number of panels
q	total number of nodes
a	half length of the side of a panel
$q_j^{(d)}$	index of the j-th node of panel d
$\mathbf{H}_0\left(x\right)$	Struve function of order zero
$J_0\left(x\right)$	first kind Bessel function of order zero
$Y_0\left(x\right)$	second kind Bessel function of order zero
$H_0\left(x\right)$	Hankel function of order zero
$P_j\left(x\right)$	Legendre polynomial of order j
P, Q	total number of integration point
l	width of a channel
b	width of a gap
σl	phase difference
σ_m	propagation constant in y-direction
μ_m	propagation constant in x-direction
c	smoothing parameter
$\psi_m^{\pm}$	angle of diffraction
$\tilde{\phi}_m^{\pm}$	diffracted wave
$A_m^{\pm}$	diffracted amplitude
M	lower diffraction order
N	upper diffraction order