Quantum description of dark solitons in one-dimensional quantum gases

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Abstract

The main objective of this thesis is to explain, from the quantum-mechanical point of view, the nature of dark solitons in one-dimensional cold-atom systems. Models of bosons and fermions with contact interactions on a ring are exactly solvable via the Bethe ansatz, and support so-called type-II elementary excitations. These have long been associated with dark solitons of the Gross-Pitaevskii equation due to the similarity of the dispersion relation, despite the completely different physical properties of the states. Fully understanding this connection is our primary aim.

We begin by reviewing the Gross-Pitaevskii equation and its dark soliton solutions. Next, we solve the mean-field problem of two coupled one-dimensional Bose-Einstein condensates, with special emphasis on Josephson vortices and their dispersion relation. Predictions are given for possible experimental detection. Then we give a derivation that justifies a method for the extraction of the so-called *missing particle number* from the dispersion relation of solitonic excitations.

A derivation of the finite Bethe ansatz equations for the Lieb-Liniger and Yang-Gaudin models follows. These describe a single species of bosons and two component fermions, respectively. We review the elementary excitations of the Lieb-Linger model, and carry out a comprehensive study of the (much richer) excitations of the Yang-Gaudin model. The thermodynamic limit Bethe ansatz equations for all states of interest in both models are derived, and the missing particle number and the closely-related *phase-step* are extracted from the dispersion relations. Next, we develop a method for approximating the finite-system dispersion relation of solitonic excitations from the thermodynamic limit results.

Finally, we show that the single particle density and phase profiles of appropriately-formed superpositions of type-II states with different momenta exhibit solitonic features. Through this idea, the missing particle number and phase step extracted from the dispersion relation gain physical meaning. Moreover, we use a convolution model to extract the fundamental quantum dark soliton length scale across the range of interactions and momenta. The insight gained in the bosonic case is used to make inferences about dark solitons in the fermionic case. Furthermore, we study the Hess-Fairbank effect in the repulsive Yang-Gaudin model and the fermionic super Tonks-Girardeau regime.

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Contents