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UTILISING MEASURABLE UNCONTROLLABLE FACTORS IN PARAMETER DESIGN TO OPTIMISE THE RESPONSE

A thesis presented in partial fulfilment
of the requirements for
the degree of Doctor of Philosophy in
Statistics at
Massey University

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Abstract

This thesis looks at issues in Industrial Experimental Design and Quality Control. The first part is a review of Parameter Design and its evolution into methods of modelling the mean and variance as one system.

The second part introduces the concepts of observable and unobservable factors as an extension to the ideas of controllable and uncontrollable factors of Parameter Design. Methods will be devised to show how to choose the best settings of the controllable factors and how to move to those settings once chosen. In the last section estimates for tracking the unobservable uncontrollable factors will be devised. These will be examined to see whether they can be used to improve the monitoring of the system via control charts.

■

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To Whom It May Concern.

This is to state that the research work carried out for my Ph.D. thesis entitled 'Utilising Measurable Uncontrollable Factors in Parameter Design to Optimise the Response' in the Statistics Department at Massey University is all my own work.

The review work in Chapter 1 was originally submitted for my Masters Degree and has been revised and updated for this thesis. Details can be found in Appendix A.

I certify that the remaining material in the thesis has not been used for any other degree.

M J Pledger
8/9/98

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