Frontiers of Decision Theory

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I would like to dedicate this thesis to:
My parents;
My main Ph.D. Supervisor Dr. Simona Fabrizi; and
Researchers who, with their seminal studies on decision theory,
have made it possible for me to base my own research on theirs.
I also wish to dedicate this thesis to:
All scholars who will find it useful when pursuing their research.
It is for those people that this thesis has been written.

Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and acknowledgements.

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Abstract

The well-known jury paradox – the more demanding the hurdle for conviction is, the more likely it is that a jury will convict an innocent defendant – heavily relies on Bayesian updating. However, with ambiguous information (e.g., a forensic test with accuracy of 60%, or more), standard Bayesian updating becomes invalid, challenging the existence of this paradox. By developing novel theoretical models and by testing their predictions in laboratory settings, this thesis advances our understanding of how individuals process more realistically imprecise measures of information reliability and how this impacts on information aggregation for the group decision-making. Hence, our findings inform the institutional design of collective deliberation, from small to large group decision-making.

Preface

"...two factors [are] commonly used to determine a choice situation, the relative desirability of the possible pay-offs and the relative likelihood of the events affecting them, but in a third dimension of the problem of choice: the nature of one's information concerning the relative likelihood of events. What is at issue might be called the ambiguity of this information, a quality depending on the amount, type, reliability and 'unanimity' of information, and giving rise to one's degree of confidence in an estimate of relative likelihoods."

Daniel Ellsberg, *The Quarterly Journal of Economics*, 1961, 75(4), p. 657-659.

Much real world negotiation and decision-making takes place in small groups. Members within groups are either chosen by public voting or by authorised nomination. They gather together to deliver their opinions or cast votes for determining an authoritative decision. For example, congressmen are chosen by national voting to join Congress to decide whether certain policies are allowed to take place or whether outdated codes need to be abolished. Similarly, corporation board members need to deliberate during seasonal meetings regarding business strategies, innovation projects, and so forth. Also, medical teams, judicial trials (up to the Court of Appeal level), policy offices, and any expert teams are typical sources of authoritative decisions ultimately impacting either treated patients, defendants, suspected offenders, and/or the general public. Hence, small-group deliberations often determine final outcomes (or consequences) mattering to – and affecting to various degrees – a multitude of agents, from single individuals, households, businesses, and organisations, to communities and the entire society.

The underlying mandate of those small groups and their related – implicit or explicit – obligation, when e.g., acting either as expert teams or committees within institutions, is to reach 'the' right/optimal/best decision for 'the' given case at hand. However, a final recommendation/decision by a group of individuals will likely be affected by the processes, the regulations as well as by the voting rules under which their deliberation(s) occurs. In turns, the processes, the regulations, the voting rules, combined with the quality of available information to the decision-partakers, all contribute to whether the outcome of a collective deliberation, beyond being in line with the 'declared' goal set out to be achieved by the deliberation group in the first instance, ultimately best reflects and is consistent with the 'true' nature of the case at hand.

Despite the fact that for any given selected process and voting rule combination it is always possible to characterise/conjecture which decision(s) could be reached by a small decision-group under specified preferences and information structure (whether the information is common knowledge to all parties involved or not), it is far from obvious to anticipate what the possible decision(s) would be in the presence of information which is inherently *ambiguous*.

Ambiguity exists not only in the inability to assign well-defined, numerical probabilities to specific events, as in Ellsberg (1961). It is also embedded in the language, the signals, and even in many among well-recognised social norms used by agents to communicate with one another within any given decision-making context. The language used in the regulations may be interpreted differently by different individuals. The probability of reaching the expected payoff of implementing a new government policy might be difficult to assess. Similarly, decisions whether to grant a patent to an invention by a given patent office, which rely to some extent on the assessment of the quality/novelty of the idea/innovation within a given jurisdiction, could appear to be inconsistent when one patent office grants a patent on an innovation and another denies it. Yet, not all information related to an innovation can be perfectly and unambiguously codified, possibly explaining the apparent contradiction. Another example of when ambiguity matters, is within jury trials. It is not difficult to conceive that jurors often need to form a verdict, based on the evidence submitted to a court of law, the accuracy of which cannot though be assessed perfectly. In other words, jurors are required to cast their votes in favour or against (acquit or convict) a defendant, despite the potential source of ambiguity in the quality of the information provided to them. Thus, even if the very same set of information, ambiguous information, is given to all individuals within agencies/committees/jury in charge of making a decision, that information may still be responsible for generating differing priors among those individuals. And, if more individuals need to agree on the votes they cast, in favour or against, a given choice at hand, the presence of ambiguity could alter the way consensus will be reached, and, potentially, the outcome of such consensus, as opposed to predictions under canonical Bayesian settings. Ambiguity might lead to misunderstanding, sub-optimal choice, and ambiguity-avoiding strategies.

Other Motivating Examples

Below we provide a list of cases, to name but a few examples of other small group decision-making situations where a decision has to be reached for a binary choice under information ambiguity.

The Court of Criminal Appeals This Court responds to defendants who require a review of any adjudications made by the lower court during the original trials. When cases are to be reviewed, lawyers prepare material based on all relevant past cases, including the decisions as they were reached in these cases, and present them to the appeals court, consisting of three or five judges in total. There is no hearing or debating process during the appeal: the judges only read the briefs and the legal documents of the trial court and decide whether to dismiss an appeal. The appeal will be rejected whenever the majority of judges agree with the trial court, and vice versa. Although it does not require an unanimous agreement among the judges, the fact is that different judges might respond differently to similar cases given in the briefs. And, thus, they will hold different opinions regarding the decision of the lower court, which might explain why they eventually fail to reach an agreement even when faced with the same materials.

The Surgical Team Assume a surgical team, consisting of one chief surgeon and a few attending surgeons, has to come up with a solution about how to treat a cancer patient. The surgeons could either opt to operate on the patient to try to remove the lesion, or conduct chemotherapy and hope the cancer cells will shrink. The surgical team has to reach a decision on the treatment before taking any further actions. Although surgeons are able to come up with the probability of success if surgery is chosen, based on a large database of similar cases, this ability alone does not guarantee that all surgeons will ex ante all agree on the same treatment decision, as not all past cases are exactly the same as the case at hand, due to the uniqueness of the human body, as well as the personal history and idiosyncratic characteristics of the patient[1]. Thus, surgeons may fail to unanimously agree on a particular course of action (decision regarding the treatment) at the state of the surgical consultation, due to different priors/beliefs about which treatment has better odds of success, if undertaken, for this particular patient. Based on those priors, they will most likely have to mediate their positions, to reach an agreement (whether an unanimous one or not) and to be able to treat the patient accordingly.

Organ Allocation Two heart failure patients are waiting for a heart transplant. Whether one of them or the other makes it to the top of the transplant list will determine whether a donor heart will go to the patient who needs it the most and can make the most out of the transplant. When there is a donor heart, the organ allocation center will have to decide to whom they will allocate this heart. Suppose there is a small medical team within the organ allocation center which has to analyse these two patients' cases and vote for who gets first on that list. Whenever an unanimous vote is reached, it decides the receiver of the donor heart. Although there are strict rules for evaluating who should be the receiver, there is still some chance that the two patients' medical conditions are extremely similar, and, thus, there is no obvious way of choosing whom the heart should go to. For example, these two candidate patients for a transplant have the same physical tissue and blood type matching, severity of the disease, recovery potential, etc. It could even be the case that twins are waiting for the heart, and, unfortunately, there is only one suitable donor organ available. Then, it will be

[1]The patients might have different ages or weights; their tumours might be of different sizes, or located in different organs, benign or malignant. Based on past cases, a surgeon clearly knows whether this type of surgery has succeeded or failed in the past. However, each patient treated before is different from the present one. If the surgeon thinks this way, the adequate past cases seem impractical to him and he will end up with nothing plausible to which to resort (Gilboa and Schmeidler, 2010). That is why the patient will be asked to sign an 'informed consent' form to capture his understanding of all the potential risks that might happen in the treatment, including death.

a very hard decision of who gets the heart and who does not.[2] Having to come up with a choice, agreed by the team may be affected by each expert's belief regarding who is more likely to react positively to a transplant, and the final allocation may be affected by possible differing priors about those chances of successful transplant (such decision needs to be a swift one, as the clock is ticking, determining the chances of any transplant to succeed at all).

The Innovation Funding Programme Suppose that the government offers a fund which is only sufficient to promote one innovation project. All major universities have the opportunity to submit their projects to the funding committee. The funding committee has to come up with only one recipient from among all the candidates and their research projects. The judgement standard includes the novelty, promise, feasibility of reproduction, and the potential social contributions of the research project, as well as the project proposer's academic background, publishing record, his/her network of the relevant experts in the field, their co-authors' backgrounds, research reputation, etc. After a few rounds of pre-selections, only two final projects remain in the final round of assessment. In order to minimise the potential dissent in the final decision, the fund will only be given to the project receiving some degree of consensus from the members of the committee. However, members of the committee are likely to each have their own idiosyncratic prior as to the merits of each project, based on their own subjective assessment of its chances of success, say, up to the commercialisation (an innovation may function, technically, but not be successful in the final market, for example, due to how the market receives it – e.g., consumer taste for something really new cannot be anticipated for certain, as there are no other innovations in use which resemble any of those proposed new ones). Obviously, if the committee cannot find one project that all or at least most of its members agree on, the fund will be lost, putting pressure on the committee to find the best possible agreeable allocation of those funds, obeying the idea of the government to promote the most promising innovation project. The intuition is that in this case there is not enough statistical evidence about the distribution of 'good projects' versus 'bad projects' in the economy such that all members of the committee will be able to necessarily all share the same belief about the exact chances of each project submitted to their attention to be of either type. The members' differing priors are likely to impact on the final selection of the recipient of the fund.

[2]It could also happen for the parents of the twins to decide which kid they want to save by agreeing to the heart transplant.

In the above examples, medical surgeons, organ allocation officers, and funding committee members are the decision makers who are faced with some sort of ambiguous information within each small group. Such small group decision-making could also extend to legislatures, expert panels[3] and other judicial bodies. Decision makers are asked to make a choice between possible alternatives. Provided with the very same pieces of information, such as the medical data of the patients, surgical history, project proposal, and the merit of a project, decision makers will then generate their own ideas/opinions or beliefs independently. According to Ellsberg (1961), ambiguity stems from such information, which exposes decision makers to a potential dissent from their initial positions/judgements/beliefs. Decision makers need to be aware of the fact that someone will have to vote against their received information/signal, 'aligning their minds' to eliminate any dissent to reach a decision (whether unanimously or not) and that the quality of the information received matters in determining how such alignment may be reached.

In the remainder of this thesis, we take the jury trial as the leading example, as the metaphor for other small group decision-making examples, to study whether an ambiguous information structure could affect collective deliberation processes, and if so how, in order to gain a better understanding of the effects of different institutions on collective decision outcomes.

To advance our understanding of how ambiguity can play a role in a jury trial setting, we embed identical, but – at least partially – ambiguous information into the canonical jury decision-making model of Feddersen and Pesendorfer (1998). Our main goal is to study the effects of introducing different forms of ambiguity on the probabilities of convicting the innocent (type I error) and acquitting the guilty (type II error), compared to the canonical jury trial case.

To that end, in chapter 1 we begin by exploring a model in which jurors may distrust the precision of the information given to them, leading to jurors adopting potentially differing priors and altering the formation of their posteriors, used when casting votes to convict or

[3]The case of a legislature shares some similarity with the jury trial, in which there exists a default option, in the case a consensus fails to be reached, which is the acquittal. Although legislatures might still be making a binary choice, in general, they are choosing between whether to dismiss a proposal or accept it, a 'Yes' or 'No' question. That is the same for the expert panel, if they are considering whether to adopt a new technology. The main difference between them is that the final choice indicates different results. When an unanimous decision is not accepted, the status quo remains instead (similarly to the jury trial); then, the decision of 'rejecting the legislative proposal' is the same as not voting. A binary choice in these cases is not to choose one option out of two; it becomes whether to maintain the status quo or not. However, in the surgery case, either the patient receives the surgery or he/she will have chemotherapy, neither of which are the status quo.

to acquit a defendant. Within this model, we can summarise the following findings. As the size of the jury grows sufficiently large, when voters share the same 'trusting' level of belief, voting according to their private signals leads to a smaller probability of convicting an innocent defendant. This suggests that if there were ways of framing all voters to believe that the quality of the private information is the highest among alternative ones provided to them, and that belief is wrong (jurors trust the precision to be higher than its 'true' underlying level), type I errors would be reduced, if not even eliminated. Therefore, asymptotically, being trusting of the information received or framing the information to induce more trust in it, makes the unanimity voting rule less unappealing. However, for a small jury size, distrusting the information provided would be best to reduce type I errors and to improve the performance of the unanimity rule.

In chapter 2, we report results from an array of experiments designed to capture the collective voting behaviour under the two-point non-common prior model introduced in chapter 1 and to contrast them against results of canonical collective voting behaviour models. Our aim is to investigate the collective decision-making outcomes under different voting rules when the quality of the private information given to voters when casting their votes is unmeasurable, triggering voters to adopt potentially differing beliefs about it. The results of these experiments validate the theoretical predictions of voting under the two-point non-common prior model, suggesting the importance of the quality of the information structure in determining the collective deliberation outcomes. These results help establish when, in the finite case, the unanimity voting rule can outperform majority voting rule if voters adopts two-point non-common priors.

In chapter 3 we generalise the jury voting model of Feddersen and Pesendorfer (1998) by embedding ambiguity into the private signal structure and considering voters who, being ambiguity averse, adopt a Maxmin approach to form subjective beliefs. The Maxmin Expected Utility Theorem (MMEU) of Gilboa and Schmeidler (1989) helps capture the voter's attitude towards ambiguity to analyse how this impacts the collective voting outcomes under both the majority rule and the unanimity rule. According to MMEU, voters assign their priors in an act-contingent manner, that is, ambiguity averse voters assign the prior, which gives them the best among the worst expected utility levels when evaluating alternatives choices (in this context, voting choices, namely whether to vote to convict or to acquit). Within this framework we prove the existence of an informative voting equilibrium and of strategic voting equilibria. Moreover, we find that if ambiguity exists in the precision of the private information, it is easier to sustain informative voting as an equilibrium strategy, that is, there exists a larger set of reasonable doubt levels for the unanimity voting rule to prevail as an equilibrium of the voting game. This is an important result as voting informatively, especially under unanimity helps maintain the efficiency of information aggregation.

Our theoretical and experimental results call into question preconceived results about the performance of different institutional designs and voting rules for collective deliberation under differing information structures. When the objective probability of the information is imprecisely measured, that is when the common-prior assumption is relaxed, novel results arise which deserve further exploration, challenging our views about the virtues of adopting, say, majority voting, as opposed to unanimity voting, to avoid the bad outcome of exacerbating the odds of convicting an innocent defendant (jury paradox).

Table of contents

List of figures

List of tables

Chapter 1

Collective Deliberation Under Non-common Prior: Theory

1.1 Introduction

Much decision-making takes place in small groups. Members gather together to cast votes over alternatives proposed to them, be it for a bill to be passed in a Congress, a project to be selected for financing, a patient to be put on a transplant list, or a defendant's fate in a court of law.

The way in which decision-making occurs, its process and the voting rule adopted, as well as the information used by members of a committee/jury to finalise a decision, are all important elements in the quality of that decision. Although, it is always possible to conjecture which decision(s) would be reached by a small decision-group under specified preferences and information structure, that is, whenever a common prior is shared by all parties involved, it is far from obvious to anticipate what the final decision could be in the presence of information which is inherently *ambiguous*.

As anticipated, in order to advance our understanding of how ambiguity can play a role in a jury trial setting, we embed identical, but – at least partially – ambiguous information into the canonical jury decision-making model of Feddersen and Pesendorfer (1998). Our main goal is to provide more realistically imprecise measures of information reliability and to study how those impact on information aggregation for the group decision-making. Specifically, we are interested in studying the effects of introducing ambiguity on the probabilities of convicting the innocent (type I error) and acquitting the guilty (type II error), compared to the canonical jury trial case.

We begin by considering the case in which jurors/voters do not necessarily share the same trust in the precision of the information provided to them. This raises the possibility for multiple priors to coexist, altering the formation of posteriors, hence, the casting of the votes, ultimately affecting the final collective decision. In this environment, we demonstrate that as the size of the jury grows sufficiently large, when voters share the same 'trusting' level of belief, voting according to their private signals leads to a smaller probability of convicting an innocent defendant. This suggests that if there were ways of framing all voters to believe that the quality of the private information is the highest among alternative ones provided to them, and that belief is wrong (jurors trust the precision to be higher than its 'true' underlying level), type I errors would be reduced, if not even eliminated. Therefore, asymptotically, being trusting of the information received or framing the information to induce more trust in it, makes the unanimity voting rule less unappealing. However, for a small jury size, distrusting the information provided would be best to reduce type I errors and to improve the performance of the unanimity rule.

The remainder of this chapter is organised as follows. Section 1.2 contains a review of the related literature on collective decision-making. Section 1.3 presents the jury trial model and the main findings as studied in the seminal paper by Feddersen and Pesendorfer (1998). Section 1.4 and section 1.5 are devoted to the analysis of a specific theoretical model of voting under non-common prior: the two-point prior model, under the unanimity voting rule and majority voting rule, respectively. Section 1.6 presents the simulation results of the two-point prior model. Section 1.7 concludes. Appendices A.1-A.4 contain the technical proofs for the derivation of the main results of this study, as well as the simulations conducted within this study.

1.2 Related Literature

The Condorcet Jury Theorem states that collective decision generated by majority voting has a higher probability of selecting the correct alternative than the decision made by a single individual. However, this result is derived under the assumption that voters vote in accordance with their information, hence it precludes studying other rational behaviours of decision makers, such as voting strategically, that is, potentially voting against one's own information (Austen-Smith and Banks, 1996). In other words, the Condorcet Jury Theorem assumes that the behaviour of a voter when he is a member of the group is identical with the behaviour when he makes the decision alone. However, voting sincerely and informatively may not always be rational. To explore this further, three simple jury voting models have been put forward in order to understand possible voter behaviour under majority voting rule from a game-theoretic perspective.

Consider variants of a model in which all voters have a common preference for selecting the better alternative. Also, each voter receives a private signal independently from a state-dependent distribution, which can be taken as a hint indicating the better alternative. However, there are three sorts of possible voting behaviour: *sincere voting*, in which each voter selects the alternative which gives him the highest expected payoff based on his own signal; *informative voting*, in which each voter votes for the alternative, which not only gives him the highest expected payoff, but also is consistent with the private signal he receives; and *rational voting*, in which each voter updates the posterior belief and considers being pivotal while taking into account others' behaviour, and which, thus, constitutes a Nash equilibrium of the Bayesian game. In the first model, sincere voting is informative and it is also rational if majority voting rule is selected to aggregate individuals' votes for determining the collective decision. The second model assumes that each voter receives two independent draws of private signals. The third model also allows each voter to receive two private signals, and it also assumes that after observing their private signals, individuals receive a public signal from a different state-dependent distribution. Sincere voting cannot be both informative and rational under these two model setups. This indicates that if the Condorcet Jury Theorem applies, voters will vote sincerely. Whereas, the sincere voting strategy profile is irrational. Or if the voting strategy profile is rational, which means it is an equilibrium of the Bayesian game, this voting strategy profile cannot be sincere, and thus, the Condorcet Jury Theorem fails to apply.

Similar results are found in the paper of Feddersen and Pesendorfer (1998), that is, voting informatively might not be a Nash equilibrium under unanimous voting rule, thus, precluding information aggregation. We will talk in more detail about this model and its findings in the next section.

In the Bayesian game the pivotality assumption of the rational voters is unlikely to be true. That is, rational voters expect others all vote informatively and conditioning on this unlikely event, they might have the incentive to vote against their private signals. Thus, comparing informative voting with the rational equilibrium concept of the Bayesian game is problematic.

Laslier and Weibull (2013) propose a randomised voting rule, which restores the informative voting incentive of each voter and makes the pivotality assumption hold. According to the randomised voting rule, the classical majority voting rule will be adopted with a high probability; and with a low probability, the collective decision will be determined by dictatorship. The latter means that one casted vote will be randomly selected with equal probability from all voters. Thus, there is an incentive for voters to cast their votes in accordance to their private information. Laslier and Weibull (2013) extend the model of Austen-Smith and Banks (1996) by allowing heterogeneity in the preferences for different collective decision outcomes. And they find the more general necessary and sufficient conditions for informative voting being a Nash equilibrium under the classical majority voting rule. However, if private information is state-dependent and one signal is more informative than another, informative voting precludes the Bayesian Nash equilibrium. If the randomised majority rule is applied instead, each voter has a strict incentive to vote informatively and informative voting is the unique Nash equilibrium. And asymptotically, the efficiency of the classical majority rule is obtained and the Condorcet Jury Theorem holds.

There is another implicit assumption of the Condorcet Jury Theorem, that is there is no cost of obtaining the private signals. If this assumption fails to hold, results of the Condorcet Jury Theorem cannot be established. If information acquisition is costly, given a small jury size, the probability for a voter to be pivotal is higher than the cost of getting the information. Therefore, a voter will incur the cost and cast his vote based on the information received. However, if the jury size is big, voters prefer to avoid the cost of obtaining the information and free-ride on the information of other voters. Then, a large committee leads to a lower social welfare than a small one (using less information than it would be desirable). Therefore, this suggests that selecting an optimal size of the committee is also vital in order to maintain the efficiency of a large group.

Koriyama and Szentes (2009) studied the jury voting model by implementing a cost for each jury member to obtain one private signal. To be specific, voters simultaneously decide whether to invest in getting the private signals, which are not always informative. Then, voters cast their votes conditional on the obtained signals and make the ex post efficient decisions. Koriyama and Szentes (2009) find that there exists an optimal committee size, which is bounded. This means that the Condorcet Jury Theorem fails to hold. When the given committee size is small, there is a unique equilibrium in which all voters acquire their private signals with probability one. If the given committee size is big, then some voters will randomise. Although the social welfare decreases with more voters randomising, the oversized committee generates higher social welfare than the undersized committee.

In reality, deliberation can improve the understanding of the information committee members get, which would be helpful in terms of reaching a more informative decision. However, there is a tradeoff in the increasing cost of delaying the voting process. Therefore, Chan and Suen (2012) include the deliberation process of the committee by allowing the acquisition of public information for each round as long as an alternative is not selected according to the given aggregation rule. They assume voters have heterogeneous preferences for the two alternatives and incur different time cost regarding the duration of the voting process. In each round, voters can vote for either of the two alternatives or abstain. If there is an insufficient amount of votes for either of the alternatives to be selected as the collective decision, the group receives a public information. Otherwise, the voting process ends. The results show that under super-majority rule, deliberation will not collapse unless there are sufficiently many impatient voters, who incur higher time cost and prefer quick decisions. However, under simple majority rule, the deliberation ends almost instantaneously as long as there is one impatient voter.

Moreover, one of the common assumptions of the previous studies is that voters have common values regarding the true state of the world and the voting strategies are defined by maximising subjective expected utility. However, equilibrium results under such setups do not hold if there is ambiguity regarding the payoff relevant state (Ellis, 2012). And information aggregation of the Condorcet Jury Theorem cannot be established, especially if the information precision is too low to overcome the uncertainty of the ex-ante prior, when voters exhibit Ellsberg-type ambiguity averse attitude, that is voters conform to maxmin expected utility (Gilboa and Schmeidler, 1989). This leads voters to strictly prefer randomising as compared to adopting pure strategies. Thus, information aggregation and the efficiency of the election are not realised.

In this paper, we assume that voters are not confronted with any ambiguity regarding the payoff relevant state. Rather, the ambiguity arises from the state-independent private information with respect to the precision of the signal voters receive before casting their votes contributing to the group's decision. We assume the distribution of the precision of the information is unknown, and, therefore, unmeasured.

Binmore (2015) uses an interesting example to illustrate an ambiguous scenario, in which the objective probability measure is unmeasured. He states that decision-makers are not able to formulate subjective probabilities for some events for which they are not well informed about. In other words, individuals do not get sufficiently many empirical frequencies of the occurrence of these events to be able to form measurable probabilities for them. Binmore postulates that a blind anthropologist from Mars, who were to come to earth and join the roulette, without knowing what the roulette wheel really does, and with nobody around to explain to him/her what the rules of this gamble are, would face an ambiguous scenario/world. What the Martian hears is the croupier saying 'high' or 'low'; and then, the clink of chips being transferred. After many times of the roulette being played, the Martian would form his/her own subjective probabilities for the 'high' and the 'low' event, respectively. If, suddenly, a new player were to join the casino and to start betting on 'odd' and 'even', the Martian would update his knowledge partition of the state space from $\{Low, High\}$ to $\{Odd \cap Low, Odd \cap High, Even \cap Low, Even \cap High\}$. Because the Martian would not get a chance to observe any occurrence of events such as $\{Odd \cap Low\}$ and $\{Even \cap Low \cup Even \cap High\}$, these events would remain unmeasured. Of course, he/she would not distinguish whether the state space of the roulette wheel is $\{1, 2, \cdots, 36\}$ and thus, would not be able to recognise all subsets of the true state space. Unless the Martian were to sit at the roulette table and to keep playing for millions of times, he/she might never have the chance to hear all the events which could have happened; and, thus, might never be able to form an explicit and exact probability measure for the roulette wheel game.

Now, assume that another blind Martian were to land at the same casino, but to sit at a different roulette wheel table from the previous Martian. After sitting there and hearing the gamble repeat again and again for 5000 times, both Martians, upon flying back to Mars, would likely teach their fellow Martians different ways of how to play the same gamble. We could expect they would have developed different rules of the roulette wheel because the knowledge partition of the state space would likely not be the same for those two Martians, who were each not informed of all relevant events of the roulette. And their subjective probabilities/priors with respect to the same event, for instance, $\{High\}$, might not be the same, depending on the realised frequency of this event as each experienced it in each of the tables they were sitting at.

Thus, when an event is unmeasurable, it is impossible to assign a specific objective probability measure to it. However, as Knight (1921) suggests in his book, individuals will eventually assign an estimate towards an event and feel toward this estimate as toward any other probabilities. He suggests that, even in the uncertain scenarios, decision makers will form subjective probabilities and act according to them. Also, Ellsberg pointed out in his paper that to apply total ignorance to the composition of the balls in the ambiguous urn is improper. It is because of the ambiguity of the information the individual receives, rather than his/her complete ignorance, that the individual is not able to pin down a unique likelihood of the composition of the balls.

Therefore, even though we assume that decision makers receive information regarding the true state of the world, and that the distribution of the precision of the information is unmeasured, decision makers will still assign their (non necessarily identical) subjective probability to the event of receiving the information which corresponds to the true state of the world. Note that for simplicity, in the remainder of this chapter, we will refer to this type of information as ‘ambiguous’. This is to avoid having to use different terminologies when referring to each of the possible different sources, which are responsible for probabilities to be unmeasurable (whether due to objective ambiguity or else).

1.3 The Canonical Jury Trial Model: Feddersen and Pesendorfer (1998)

In Feddersen and Pesendorfer (1998), results demonstrate that voters' strategic behaviour would lower the efficiency of a more conservative voting rule, compared to that of other voting rules, which require fewer consensual decisions. In other words, the efficiency of a jury trial, where the guilty defendants are sentenced to conviction and the innocent ones are declared acquitted, is lower if an unanimous guilty/innocent verdict is required from jurors. Compared to simple majoritarian rule, the unanimity verdict is proved to be inferior under strategic voting due to the higher probability of committing type I error: convicting an innocent defendant; and, also, type II error: acquitting a guilty one.

In the setting of Feddersen and Pesendorfer, it is assumed that there are n jurors, $j = 1,\dots,n$, gathered together to decide the fate of the defendant, who is believed to be guilty or innocent with equal ex-ante probabilities. Before casting their votes, each of the jurors receives a private and imperfect signal $s_j = \{i, g\}$, with precision $p = Pr(g|G) = Pr(i|I)$ and $p \in (1/2, 1)$, pointing toward the '$i = innocence$' or '$g = guilt$' of a defendant given the 'true' state of the world $\{G, I\}$ – that the defendant is either '$G = guilty$' or '$I = innocent$'. Take a voting rule requiring at least $\hat{k}$ jurors to agree on a verdict, with $\hat{k} \leq n$ – be it simple majority with $\hat{k} = (n+1)/2$ or unanimity with $\hat{k} = n$ – and the trial will result in a verdict $\{C, A\}$ either to '$C = Convict$' or to '$A = Acquit$' the defendant, reflecting all jurors trying to do the 'right' thing, that is, voting to convict whenever some threshold for reasonable doubt, $q \in (0, 1)$, has been reached, and voting to acquit otherwise. All jurors are thus assumed to have the same preferences with respect to the outcome (quality) of the verdict, so that their preferences can be represented as follows:

$$u(A, I) = u(C, G) = 0,$$

$$u(C, I) = -q,$$

$$u(A,G) = -(1-q).$$

In their model setup, Feddersen and Pesendorfer did not consider the case where jurors vote naïvely. Rather, they study the case where all voters are strategic in terms of each voter considering whether he can be pivotal and such that his vote makes a difference as to whether a defendant is convicted or acquitted. After receiving their private information (signal), each juror updates his posterior belief with respect to the guiltiness of the defendant according to the Bayes' rule, such that the posterior probability that the defendant is guilty, conditional on observing n signals, k of which are guilty is denoted as $\beta(k,n) = \frac{p^k(1-p)^{n-k}}{p^k(1-p)^{n-k}+(1-p)^k p^{n-k}}$. If $\beta(k,n) > q$, the defendant is guilty beyond any reasonable doubt. If $\beta(k,n) < q$, the guilt of the defendant cannot be established beyond reasonable doubt. Hence, for any given voting rule, $\hat{k}$, voters will vote informatively, as long as $\beta(\hat{k}-1,n) < q < \beta(\hat{k},n)$. '*Voting informatively*' means that voters' votes correspond with their private signals, that is those who receives innocent signal i will vote for acquittal A; and those who receives guilty signal g will vote for conviction C. However, if the condition above does not hold, voters may vote against their signals. Using the wording of Feddersen and Pesendorfer, we will refer to this second case as '*strategic voting*'. Below is a summary of their main findings, to be contrasted next with our own findings under ambiguity.

Result #1: If voting is informative, the type I error under unanimous voting is smaller than that under any other voting rules; although the type II error under unanimous voting is bigger than that under any other voting rules.

Result #2: When $\beta(\hat{k}-1,n) > q$, voters will not vote informatively; and, under strategic voting, there exists a unique symmetric responsive Nash equilibrium, where voters who receive signal g vote for conviction and those who receive signal i vote randomly (between voting for conviction and acquittal). Then, the probability of committing both types of errors under unanimous voting is bounded away from zero even when the size of the jury is sufficiently large. Contrarily, the probability of committing both types of errors under other types of voting rules approaches zero when the size of the jury approaches infinity.

Therefore, the unanimity rule, which requires a more demanding hurdle for a conviction verdict to be established, is a uniquely bad rule as opposed to other voting rules, when jurors fail to vote informatively, especially for large size juries. This contradicts our normal intuition that we would otherwise hold, that requiring everybody to agree to convict, for a conviction verdict to be pronounced, provides more protection against innocent defendants: a protection against finding themselves convicted by mistake. Instead, as the size of a jury gets larger, and as voters tend to vote strategically, the unanimity rule leads to more opportunities for the miscarriage of justice, hence the '*jury paradox*.'

This disappointing result is obtained by restricting attention to jurors who possess information, the accuracy of which is commonly known and uniquely defined. However, it is more than plausible to think of many realistic situations in which the quality of the private information jurors receive is not that precise, rather often ambiguous. There is evidence, such as hearsay evidence, witnesses' testimony, and any indirect evidence that requires induction, and for which it is not obvious to know what the exact reliability is, as the evidence such as a DNA report. Assessing the reliability of this evidence requires some logical reasoning and inference; and, also, it may be affected by the relative persuasion of the arguments made by the prosecutor and the defence lawyer, respectively. Ultimately, the objective probabilities measure of the evidence is unmeasurable and jurors are likely not to share the same prior (belief about how reliable the evidence produced is). Differing jurors' prior are also likely to impact on the final verdict, which is what we want to address next, by introducing ambiguity about the quality of information provided to voters within a voting model, and by relaxing the common prior assumption, allowing for different jurors to be of different types, by nature, with respect to their beliefs about the precision of the information they are provided with, in the process of voting for acquittal or conviction of a given defendant.

1.4 Non-common Prior: The Two-point Prior Model

1.4.1 Basic Settings

The main difference between our model and that of Feddersen and Pesendorfer is that we assume the quality of the signal to be ambiguous to jurors rather than unique and commonly known (measured). We assume that the correlation between the private signal and the true state of the world even though positive is nevertheless ambiguous or partially ambiguous as defined in Chew et al. (2013). Introducing ambiguity in the accuracy with which signals are obtained allows us to depart from Feddersen and Pesendorfer set-up and to consider far more common and realistic situations, where the objective probability of the reliability of the evidence is unmeasured, in which jurors cannot assign a specific level of reliability to the evidence they are provided with, say, during a trial, and, furthermore, even if they were able to do so with a certain degree of confidence they would not necessarily agree on what that level of reliability ought to be. Our next crucial assumption in this model, is to consider jurors who are different by nature, with respect to their beliefs about the accuracy of the information they are confronted with. We will provide more details on this, when describing jurors' types.

Here, we assume that the accuracy of the signal each juror receives about the defendant being guilty or innocent belongs to the set $\mathscr{P}$, where $\mathscr{P}$ needs not be a singleton – contrary to Feddersen and Pesendofer set-up. In this chapter, we concentrate on the case of an exogenous two-point non-common prior for the accuracy with which signals are believed to come about, captured by $\mathscr{P} = \{\underline{p}, \bar{p}\}$. Alternatively, in chapter 3 we consider the entire set of values this accuracy can take within a closed interval, thereby capturing the extent of its potentially continuous degree of ambiguity within that interval, with $\mathscr{P} = [\underline{p}, \bar{p}]$.

Hence, if the non-common prior about the quality of private information can take only one of two possibly distinct values, $\mathscr{P} = \{\underline{p}, \bar{p}\}$, we can concentrate on the following informative signal cases, for which $1/2 < \underline{p} < \bar{p} < 1$. For example, the evidence hints toward the defendant being guilty with either probability 60% or 90%. This could be so, because the accuracy of a test/evidence is not universally accepted to be at a particular level, so that in the absence of any extra information regarding the probability distribution of $\underline{p}$ and $\bar{p}$, jurors are free to believe either of those levels for the accuracy to be true. Only one would be, for a specific case, but jurors have no way to verify that information when making their decisions.

We retain the assumption as in Feddersen and Pesendorfer (1998) that jurors have the same preferences with respect to reaching a right verdict (convicting the guilty, or acquitting the innocent) or a wrong verdict (convicting the innocent, or acquitting the guilty).

We distinguish two types of jurors/voters - sceptic and trusting - according to which level of accuracy for their private information/signal they eventually adopt. We assume voters form their subjective belief regarding the probability measure as follows: $Pr_j(g|G) = Pr_j(i|I) = (1-\delta_j)\underline{p} + \delta_j\bar{p}$, where δ_j is the index of trust for voter j. Next, we concentrate attention to the extreme belief case, for which $\delta_j \in \{0,1\}$, which means that each of the jurors can either be of an *extreme sceptic* or a *fully trusting* type. The radical trusting juror believes $Pr(g|G) = Pr(i|I) = \bar{p}$, with $\delta_j = 1$; whereas the extreme sceptic believes $Pr(g|G) = Pr(i|I) = \underline{p}$, with $\delta_j = 0$. Assume there exists a proportion m of voters who belong to the extreme sceptic-type and a proportion $1-m$ of voters who belong to the fully trusting-type, with $m \in [0,1]$.

To be specific, our *ambiguous jury voting game* can be described as follows:

(i) Nature first chooses the true state of the world, either "Guilty"-G, or "Innocent"-I, with equal probability, 1/2.

(ii) Nature makes n independent random draws of signals $s \in \{g,i\}$ for n voters, $j = 1,\cdots,n$, from random variable with precision $\mathscr{P} = \{\underline{p},\bar{p}\}$, where $1/2 < \underline{p} < \bar{p} < 1$. No further probabilistic information about the signal precision is provided.

(iii) Nature assigns types randomly and independently to all voters, such that each voter has probability μ of being sceptic and $1-\mu$ of being trusting. And the realised shares of extreme sceptics and fully trusting jurors within a jury/committee are denoted by m and $1-m$, respectively.

(iv) Each voter observes his private signal and votes according to his strategy $(\sigma_j(i), \sigma_j(g))$, which represents the probability with which that voter j votes for conviction conditional on receiving a signal hinting toward innocence or guiltiness, respectively.

(v) After all voters simultaneously cast their votes, the collective decision is determined according to the given voting rule.

In the remainder of this chapter, we use this minimal set-up to explore the question of how collective deliberation, such as voting by jurors in a trial, is affected by the reality that jurors have multiple/distinct priors. We seek to address how the results obtained under the common prior assumption change when multiple priors are allowed for, and, more importantly, whether such changes can ever offset the negative impact that more demanding voting rules have in exacerbating the occurrence of mistakes in the judicial system. Do multiple priors help mitigate or even eliminate the paradoxical result that the more demanding the hurdle for conviction – on the spectrum from simple majority to unanimity – the more likely it is that a jury will convict an innocent defendant?

1.4.2 Informative versus Strategic Voting

In order to avoid any confusion with our notation, let us clarify that we use $\bar{s}$ ($\underline{s}$) to represent the case in which the extreme sceptic (fully trusting) juror has received a given signal, and, consistently, we use notation $Pr(g|G) = Pr(i|I) = \bar{p}$ (or, alternatively, $Pr(g|G) = Pr(i|I) = \underline{p}$) to represent that juror's held belief. Hence, we denote the fully trusting voter's strategy as $\bar{\sigma} = \{\sigma_j(\bar{i}), \sigma_j(\bar{g})\}$, whereas we denote the extreme sceptic voter's strategy as $\underline{\sigma} = \{\sigma_j(\underline{i}), \sigma_j(\underline{g})\}$. In the remainder of this chapter, we will simply describe a voter to be in short either sceptic or trusting, to represent the polar cases of an extreme sceptic or fully trusting type, in terms of beliefs about the precision of the signal received.

In our setup, the fundamental rule of behaviour of each juror is the same as that in the setup of Feddersen and Pesendorfer. Naïve voting is not considered in our analysis. Voters simply compare the posterior probability of the defendant being guilty, conditional on being pivotal under the given voting rule $\hat{k}$, to the level of reasonable doubt. Due to the heterogeneity across voters' types, the pivotal voter has to take all possible combinations of $\hat{k}-1$ voters into consideration. We denote all possible combinations of the share of sceptic and trusting types, given the true state of world G, for these $\hat{k}-1$ voters as $\mathscr{A}$; and all possible combinations of the share of sceptic and trusting types, given the true state of world I, for these $\hat{k}-1$ voters as $\mathscr{B}$. Moreover, having assumed that a voter believes the precision of the signal to be either $\underline{p}$ or $\bar{p}$ also implies that a pivotal voter only cares about the votes of others, whether to convict or to acquit, rather than their beliefs about the level of accuracy. Put differently, it is plausible to think that if beliefs are extreme (as we are considering them to be in this chapter) the pivotal voter will simply assign to signals received by others the same level of precision attributed to his own signal, that is, the one he believes to be the true one. Therefore,

$$\mathscr{A} = \sum_{j=0}^{\hat{k}-1} \left(\begin{array}{c} \hat{k}-1 \\ j \end{array} \right) (1-\mu)^j \mu^{\hat{k}-1-j} = 1,$$

and

$$\mathscr{B} = \sum_{j=0}^{\hat{k}-1} \begin{pmatrix} \hat{k}-1 \\ j \end{pmatrix} \mu^j (1-\mu)^{\hat{k}-1-j} = 1.$$

Thus, for example, to the pivotal sceptical voter the posterior probability that the defendant is guilty, conditional on observing n signals, $\hat{k}-1$ of which are guilty given all possible combinations of $\hat{k}-1$ voters, and given he believes in the precision of the signal to be $\underline{p}$, is simply

$$\beta(\hat{k}-1, n, 1-\underline{p}) = \frac{\underline{p}^{\hat{k}-1}(1-\underline{p})^{n-\hat{k}+1}\mathscr{A}}{\underline{p}^{\hat{k}-1}(1-\underline{p})^{n-\hat{k}+1}\mathscr{A} + (1-\underline{p})^{\hat{k}-1}\underline{p}^{n-\hat{k}+1}\mathscr{B}} < q,$$

$$\beta(\hat{k}, n, \underline{p}) = \frac{\underline{p}^{\hat{k}}(1-\underline{p})^{n-\hat{k}}\mathscr{A}}{\underline{p}^{\hat{k}}(1-\underline{p})^{n-\hat{k}}\mathscr{A} + (1-\underline{p})^{\hat{k}}\underline{p}^{n-\hat{k}}\mathscr{B}} > q.$$

If the condition $\beta(\hat{k}-1, n, 1-\underline{p}) < q < \beta(\hat{k}, n, \underline{p})$ is satisfied, a sceptical voter will vote informatively. Similarly, a condition can be obtained for trusting voter to vote informatively, namely that $\beta(n-1, n, 1-\bar{p}) < q < \beta(n, n, \bar{p})$.

If the unanimity rule is adopted, $\hat{k} = n$. In this case, we first consider the possibility that all voters in a jury happen to be sceptical, so that $m = 1$, leading to

$$\beta(n-1, n, 1-\underline{p}) = \frac{\underline{p}^{n-1}(1-\underline{p})}{\underline{p}^{n-1}(1-\underline{p}) + (1-\underline{p})^{n-1}\underline{p}},$$

$$\beta(n, n, \underline{p}) = \frac{\underline{p}^n}{\underline{p}^n + (1-\underline{p})^n}.$$

Thus, if all voters are sceptics, informative voting, $\underline{\sigma} = \{\sigma_j(\underline{i}) = 0, \sigma_j(\underline{g}) = 1\}$, is an equilibrium if $\beta(n-1, n, 1-\underline{p}) < q < \beta(n, n, \underline{p})$.

Similarly, under unanimity rule, if all voters are trusting, i.e., $m = 0$, we have

$$\beta(n-1,n,1-\bar{p}) = \frac{\bar{p}^{n-1}(1-\bar{p})}{\bar{p}^{n-1}(1-\bar{p})+(1-\bar{p})^{n-1}\bar{p}},$$

$$\beta(n,n,\bar{p}) = \frac{\bar{p}^{n}}{\bar{p}^{n}+(1-\bar{p})^{n}}.$$

Thus, if all voters are trusting, informative voting, $\bar{\sigma} = \{\sigma_j(\bar{i}) = 0, \sigma_j(\bar{g}) = 1\}$, is an equilibrium if $\beta(n-1,n,1-\bar{p}) < q < \beta(n,n,\bar{p})$.

If there exists heterogeneity in the type of voters, if informative voting would be an equilibrium under unanimity if and only if (i) $\beta(n-1,n,1-\underline{p}) < q < \beta(n,n,\underline{p})$ and also, (ii) $\beta(n-1,n,1-\bar{p}) < q < \beta(n,n,\bar{p})$.

Proposition 1.1. *For $\hat{k} = n$, given the two-point non-common prior $\mathscr{P} = \{\underline{p}, \bar{p}\}$, informative voting is an equilibrium,*

(i) for $m = 1$ if $\beta(n-1,n,1-\underline{p}) < q < \beta(n,n,\underline{p})$;

(ii) for $m = 0$ if $\beta(n-1,n,1-\bar{p}) < q < \beta(n,n,\bar{p})$;

(iii) for $0 < m < 1$ if $\beta(n-1,n,1-\underline{p}) < q < \beta(n,n,\underline{p})$ and $\beta(n-1,n,1-\bar{p}) < q < \beta(n,n,\bar{p})$, where $\underline{p} > \frac{(\frac{\bar{p}}{1-\bar{p}})^{(\frac{n-2}{n})}}{1+(\frac{\bar{p}}{1-\bar{p}})^{(\frac{n-2}{n})}}$.

Moreover, for the two-point non-common prior scenario, we next derive expressions for the probabilities of convicting an innocent and of acquitting a guilty for the unanimity voting, which is informative. For that, we need to remind ourselves that even though some voters believe in $\underline{p}$ and others believe in $\bar{p}$ to be true, the 'true' precision of the signal can only be one or the other. Voters, by each choosing to cast a vote to acquit or to convict – which in turn is based on their idiosyncratic belief (prior) and on their idiosyncratic posterior belief, that is, in light of all the evidence, based on their respective degree of confidence about a defendant being guilty – determine the collective decision whether to convict or to acquit a defendant. These votes to convict or to acquit can be translated in the probabilities of convicting an innocent or acquitting a guilty, when using some underlying 'true' precision of the signals (all evidence) obtained, which we can simply refer to as p.[1] Comparisons of those probabilities of committing errors in either directions to those obtained under the canonical case are summarised in Corollary 1.1.

Corollary 1.1. *For $\hat{k} = n$, given the two-point non-common prior $\mathscr{P} = \{\underline{p}, \bar{p}\}$, and the 'true' precision of the signal equals p, if informative voting is an equilibrium,*

1. *Type I error, $Pr(C|I) = (1-p)^n$, is smaller than that for $\hat{k} \neq n$;*
2. *Type II error, $Pr(A|G) = 1 - p^n$ is bigger than that for $\hat{k} \neq n$, and it approaches one when $n \to \infty$.*

[1] Take p to be the 'true' underlying precision of the signal. When $k \neq n$, $Pr(C|I) = \sum_{j=k}^{n} \binom{n}{j} (1-p)^j p^{n-j}$, and, $Pr(A|G) = \sum_{j=0}^{k-1} \binom{n}{j} (1-p)^{n-j} p^j$.

1.4.3 The Probability That A Convicted Defendant Is Innocent under Unanimity Voting Rule

In this part, we check the probability that a convicted defendant is indeed innocent under strategic voting for the unanimity voting rule. Under informative voting, given the 'true' precision of the signal is equal to p, $Pr(I|C) = 1 - Pr(G|C) = 1 - \frac{p^n}{p^n+(1-p)^n}$, which converges to zero as the size of the jury grows to infinity (as $n \to \infty$). However, strategic voting under unanimity imposes a lower bound to the probability that the convicted defendant is innocent. There are two cases we can consider when voters vote strategically.

In the first case, the defendant is always convicted independently of signal s, such that $1 - \underline{p} > q$ when $m = 1$ and $1 - \bar{p} > q$ when $m < 1$. Therefore, since the prior is that half of the time a defendant happens to be innocent, convicting all defendants translates into making an error with probability 1/2. This says that if voters always vote to convict, then $Pr(I|C) = 1/2$.

In the second case, there is some juror who votes to acquit with positive probability, such that the probability that the defendant is guilty, for either type of voter, conditional on being pivotal and receiving either signal i or signal g, is less than or equal to the reasonable doubt q, $Pr(G|piv_j, s) \leq q$.

Proposition 1.2. *Under strategic voting and given the two-point non-common prior $\mathscr{P} = \{\underline{p}, \bar{p}\}$, consider any Nash equilibrium in which the defendant is convicted with strictly positive probability under the voting rule $\hat{k} = n$. If there exist heterogeneous beliefs with respect to the 'true' precision of the signal p across voters, $Pr(I|C)$ is bounded below by*

$$\min\{1/2, \frac{(1-\bar{p})(1-\underline{p})(1-q)}{(1-\underline{p})(1-q)+\bar{p}(\underline{p}+q-1)}\}.$$

The proof of Proposition 1.2 is provided in Appendix A.1.

If the true $p = \bar{p}$, and there exist heterogeneous beliefs regarding the p, we have

$$Pr(G|C) \leq \frac{q\bar{p}\bar{p}}{1-2\bar{p}-q+\bar{p}^2+2q\bar{p}},$$

and

$$Pr(I|C) = 1 - Pr(G|C) \geq \frac{(1-\bar{p})^2(1-q)}{1-2\bar{p}+\bar{p}^2-q+2q\bar{p}} \geq \frac{1-q}{1+q\frac{2\bar{p}-1}{1-\bar{p}^2}}.$$

If all voters' beliefs are homogeneous and correspond to the true $p = \bar{p}$, we have

$$Pr(I|C) \geq \frac{(1-\bar{p})^2(1-q)}{1-2\bar{p}+\bar{p}^2-q+2q\bar{p}} \geq \frac{1-q}{1+q\frac{2\bar{p}-1}{1-\bar{p}^2}}.$$

If all voters' beliefs are homogeneous, however, they correspond to the wrong p, we have

$$Pr(I|C) \geq \frac{(1-\underline{p})^2(1-q)}{1-2\underline{p}+\underline{p}^2-q+2q\underline{p}} \geq \frac{1-q}{1+q\frac{2\underline{p}-1}{1-\underline{p}^2}}.$$

Corollary 1.2. *Under unanimous voting with two-point ambiguous information structure, if the 'true' $p = \bar{p}$, the lower bound for the probability of a convicted defendant to be innocent is*

1. *the largest when all voters' beliefs correspond to the wrong p (everyone believes $\underline{p}$);*
2. *otherwise, the lower bound equals the case of a commonly known and certain p (as assumed by Feddersen and Pesendorfer (1998)).*

If the 'true' $p = \underline{p}$, and there exist heterogeneous beliefs regarding the p, we have

$$Pr(G|C) \leq \frac{1}{\frac{2\underline{p}-1}{\underline{p}} + \frac{1-\underline{p}}{\underline{p}}\frac{(1-q)(1-\bar{p})+q\bar{p}}{q\bar{p}}} = \frac{\bar{p}\underline{p}q}{(1-\underline{p})(1-q)+\bar{p}(\underline{p}+q-1)},$$

and

$$Pr(I|C) = 1 - Pr(G|C) \geq \frac{(1-\bar{p})(1-\underline{p})(1-q)}{(1-\underline{p})(1-q)+\bar{p}(\underline{p}+q-1)}.$$

If all voters' beliefs are homogeneous and correspond to the true $p = \underline{p}$, we have

$$Pr(I|C) \geq \frac{(1-\underline{p})^2(1-q)}{1-2\underline{p}+\underline{p}^2-q+2q\underline{p}} \geq \frac{1-q}{1+q\frac{2\underline{p}-1}{1-\underline{p}^2}}.$$

If all voters' beliefs are homogeneous, however, they correspond to the wrong p, we have

$$Pr(I|C) \geq \frac{(1-\bar{p})^2(1-q)}{1-2\bar{p}+\bar{p}^2-q+2q\bar{p}} \geq \frac{1-q}{1+q\frac{2\bar{p}-1}{1-\bar{p}^2}}.$$

Corollary 1.3. *Under unanimous voting with two-point non-common prior, if the 'true' $p = \underline{p}$, the lower bound for the probability of a convicted defendant to be innocent*

1. *is no larger than under a commonly known and certain p (as assumed by Feddersen and Pesendorfer (1998)); and,*

2. *has the smallest lower bound when there exist heterogeneous beliefs regarding the 'true' level of p.*

1.4.4 Symmetric Responsive Nash Equilibrium under Unanimity Voting Rule

In this section, we explicitly examine a specific strategic voting equilibrium under the unanimity rule, the Symmetric Responsive Nash equilibrium.

Assumption 1.1. ***Symmetric***
(i) $\sigma(\underline{i})_j = \sigma(\underline{i})_{-j}$, $\sigma(\underline{g})_j = \sigma(\underline{g})_{-j}$;
(ii) $\sigma(\bar{i})_j = \sigma(\bar{i})_{-j}$, $\sigma(\bar{g})_j = \sigma(\bar{g})_{-j}$;
(iii) if $\sigma(s)$ *is a pure strategy, then we require* $\sigma(\underline{s}) = \sigma(\bar{s})$.

Therefore, a strategy profile is symmetric if all jurors who receive the same signal take the same action; and if the voting strategy is a pure strategy, we assume the pure strategy is symmetric across different voters' types for simplicity.

For a strategy profile to be responsive all voters need to change their vote as a function of their private information with some positive probability. This definition for the responsive equilibrium follows that of Feddersen and Pesendorfer (1998). We first denote

$$\gamma_{\underline{G}} = \underline{p}\sigma(\underline{g}) + (1-\underline{p})\sigma(\underline{i})$$

as the probability that a sceptical juror votes to convict if the defendant is indeed guilty and

$$\gamma_{\underline{I}} = \underline{p}\sigma(\underline{i}) + (1-\underline{p})\sigma(\underline{g})$$

as the probability that a sceptical juror votes to convict if the defendant is indeed innocent. Analogously,

$$\gamma_{\bar{G}} = \bar{p}\sigma(\bar{g}) + (1-\bar{p})\sigma(\bar{i})$$

and

$$\gamma_{\bar{I}} = \bar{p}\sigma(\bar{i}) + (1-\bar{p})\sigma(\bar{g})$$

represent the probability that a trusting juror votes to convict if the defendant is guilty and if the defendant is innocent, respectively. If heterogeneity exists regarding the type of voters, being responsive means $\gamma_{\underline{G}} \neq \gamma_{\underline{I}}$ and $\gamma_{\bar{G}} \neq \gamma_{\bar{I}}$.

Assumption 1.2. ***Responsive***
(i) $\gamma_{\underline{G}} \neq \gamma_{\underline{I}}$*, such that* $\sigma(\underline{i}) \neq \sigma(\underline{g})$*;*
(ii) $\gamma_{\bar{G}} \neq \gamma_{\bar{I}}$*, such that* $\sigma(\bar{i}) \neq \sigma(\bar{g})$*.*

According to the above assumptions, we can remove the subscript j for all strategic profiles and other notations. Next, we concentrate on purely strategic and Symmetric Responsive Nash Equilibria. In addition to those, there are two strategic Symmetric Nash equilibria, $\{(\sigma(\underline{i}) = 0, \sigma(\underline{g}) = 0), (\sigma(\bar{i}) = 0, \sigma(\bar{g}) = 0)\}$ and $\{(\sigma(\underline{i}) = 1, \sigma(\underline{g}) = 1), (\sigma(\bar{i}) = 1, \sigma(\bar{g}) = 1)\}$, for which all voters ignore their private signals and symmetrically vote for either acquittal or conviction. According to Assumption 1.2, these two strategic equilibria are not responsive. Also, there is the case for which all voters vote informatively, $\{(\sigma(\underline{i}) = 0, \sigma(\underline{g}) = 1), (\sigma(\bar{i}) = 0, \sigma(\bar{g}) = 1)\}$.

Proposition 1.3. *Given* $\hat{k} = n$ *and* $0 < m < 1$*, there is one unique Symmetric Responsive Nash Equilibrium under two-point non-common prior, such that* $((0 < \sigma(\underline{i}) < 1, \sigma(\underline{g}) = 1), (0 < \sigma(\bar{i}) < 1, \sigma(\bar{g}) = 1))$ *as long as* $q > 1 - \underline{p}$*.*

The proof for this result can be found in Appendix A.2.

When calculating both types of errors, we define $\hat{\gamma}_G$ and $\hat{\gamma}_I$ as a function of the 'true' p, to represent the probabilities that the defendant gets convicted when the true state of world is either guilty or innocent, respectively. Although a juror's strategy is a direct function of each juror's idiosyncratic beliefs (prior and posterior beliefs, determining the individual probability of voting to convict), the effective ex-ante probability of receiving either of the signals is decided by the 'true' level of the signal accuracy, hence the probability that a collective decision, under a symmetric equilibrium, leads to errors of either type, can be derived as a function of the probability of voting to convict, which depends on both those subjective beliefs (whether right or wrong) and on the 'true' underlying precision with which signals come about.

Therefore, we have

$$\hat{\gamma}_{\underline{G}} = p + (1-p)\sigma(\underline{i}),$$

$$\hat{\gamma}_{\underline{I}} = p\sigma(\underline{i}) + (1-p),$$

and

$$\hat{\gamma}_{\bar{G}} = p + (1-p)\sigma(\bar{i}),$$

$$\hat{\gamma}_{\bar{I}} = p\sigma(\bar{i}) + (1-p).$$

Given the symmetric responsive strategy profile, the probability that a sceptic, and, a trusting, juror votes to convict if the defendant is indeed innocent is, respectively,

$$\hat{\gamma}_{\underline{I}} = \frac{[\frac{(1-q)(1-\underline{p})}{q\underline{p}}]^{\frac{1}{(n-1)}}(\underline{p}+p-1)+\underline{p}-p}{\underline{p}-(1-\underline{p})[\frac{(1-q)(1-\underline{p})}{q\underline{p}}]^{\frac{1}{(n-1)}}},$$

and

$$\hat{\gamma}_{\bar{I}} = \frac{[\frac{(1-q)(1-\bar{p})}{q\bar{p}}]^{\frac{1}{(n-1)}}(\bar{p}+p-1)+\bar{p}-p}{\bar{p}-(1-\bar{p})[\frac{(1-q)(1-\bar{p})}{q\bar{p}}]^{\frac{1}{(n-1)}}}.$$

Whereas, the probability that a sceptic, and, a trusting, juror votes to convict if the defendant is indeed guilty is, respectively,

$$\hat{\gamma}_{\underline{G}} = \frac{[\frac{(1-q)(1-\underline{p})}{q\underline{p}}]^{\frac{1}{(n-1)}}(\underline{p}-p)+(\underline{p}+p-1)}{\underline{p}-(1-\underline{p})[\frac{(1-q)(1-\underline{p})}{q\underline{p}}]^{\frac{1}{(n-1)}}},$$

and

$$\hat{\gamma}_{\bar{G}} = \frac{[\frac{(1-q)(1-\bar{p})}{q\bar{p}}]^{\frac{1}{(n-1)}}(\bar{p}-p)+(\bar{p}+p-1)}{\bar{p}-(1-\bar{p})[\frac{(1-q)(1-\bar{p})}{q\bar{p}}]^{\frac{1}{(n-1)}}}.$$

Therefore, we have type I error

$$Pr(C|I) = (\hat{\gamma}_I)^n = (\hat{\gamma}_{\underline{I}}^m \hat{\gamma}_{\bar{I}}^{1-m})^n,$$

and type II error

$$Pr(A|G) = 1-(\hat{\gamma}_G)^n = (\hat{\gamma}_{\underline{G}}^m \hat{\gamma}_{\bar{G}}^{1-m})^n.$$

When $n \to \infty$, the limit results of both types of errors are

$$\lim_{n\to\infty} Pr(C|I) = \lim_{n\to\infty}(\hat{\gamma}_I)^n = (\frac{(1-q)(1-\underline{p})}{q\underline{p}})^{\frac{pm}{2p-1}}(\frac{(1-q)(1-\bar{p})}{q\bar{p}})^{\frac{p(1-m)}{2p-1}},$$

and

$$\lim_{n\to\infty} Pr(A|G) = \lim_{n\to\infty} 1-(\hat{\gamma}_G)^n = 1-(\frac{(1-q)(1-\underline{p})}{q\underline{p}})^{\frac{(1-p)m}{2p-1}}(\frac{(1-q)(1-\bar{p})}{q\bar{p}})^{\frac{(1-p)(1-m)}{2p-1}}.$$

Also, the limit result of the lower bound of the probability of a convicted defendant to be innocent equals

$$\lim_{n\to\infty} Pr(I|C) = \lim_{n\to\infty}\frac{1}{1+\frac{Pr(C|G)}{Pr(C|I)}} = \frac{1}{1+\frac{\lim_{n\to\infty}(1-Pr(A|G))}{\lim_{n\to\infty}Pr(C|I)}} = \frac{1}{1+(\frac{q\underline{p}}{(1-q)(1-\underline{p})})^m(\frac{q\bar{p}}{(1-q)(1-\bar{p})})^{1-m}}.$$

Proposition 1.4. *Under the two-point non-common prior $\mathscr{P} = \{\underline{p}, \bar{p}\}$ the Symmetric Responsive Nash Equilibrium,*

1. *for $m = 0$, leads to the smallest probability of convicting an innocent independently of the 'true' level of p;*
2. *for $m = 1$, leads to the smallest probability of acquitting a guilty independently of the 'true' level of p.*

Thus, when the information structure allows for voters to adopt multiple priors, in order to reach the lowest probability of convicting the innocent, all voters should be trusting and believe in $\bar{p}$. However, if the aim is to lower the probability of acquitting the guilty, then all voters should be sceptical and believe in $\underline{p}$ instead. In other words, if all voters are trusting, the incidence of type I errors is the smallest, though type II errors become more frequent.

Corollary 1.4. *If the equilibrium strategic voting profile is symmetric and responsive,*

1. *And the 'true'* $p = \underline{p}$*,*

   (i) $Pr(C|I)$ *under two-point non-common prior is no larger than that under a given* p*, which is unique and commonly known to all voters;*

   (ii) $Pr(A|G)$ *under two-point non-common prior is no less than that under a given* p*, which is unique and commonly known to all voters.*

2. *However, if the 'true'* $p = \bar{p}$*,*

   (i) $Pr(C|I)$ *under two-point non-common prior is no less than that under a given* p*, which is unique and commonly known to all voters;*

   (ii) $Pr(A|G)$ *under two-point non-common prior is no larger than that under a given* p*, which is unique and commonly known to all voters.*

Remark 1.1. *The lower bound of the probability that a convicted defendant is innocent,* $Pr(I|C)$*, is the smallest(highest) whenever* $m = 0$ *(*$m = 1$*).*

1.5 Two-point Prior Model under Non-unanimous Voting Rule

We define the non-unanimous voting rule as $\hat{k} = \alpha n$, with $0 < \alpha < 1$. We find that the symmetric responsive Nash equilibrium exists for either type of voters under two-point non-common prior. Moreover, when $n \to \infty$, both types of errors approach to zero.

The probability the convicted is guilty is

$$Pr(G|C) = \frac{1}{1 + (\frac{(\gamma_{\underline{I}})^{m\alpha}(\gamma_{\bar{I}})^{(1-m)\alpha}(1-\gamma_{\underline{I}})^{m(1-\alpha)}(1-\gamma_{\bar{I}})^{(1-m)(1-\alpha)}}{(\gamma_{\underline{G}})^{m\alpha}(\gamma_{\bar{G}})^{(1-m)\alpha}(1-\gamma_{\underline{G}})^{m(1-\alpha)}(1-\gamma_{\bar{G}})^{(1-m)(1-\alpha)}})^n}.$$

And we know that when $n \to \infty$, the actual share of guilty votes converges to the expected share of guilty votes in each state of the world. Then, for larger n, the defendant is guilty with probability one if the above equation approaches one; the defendant is innocent with probability one if the above equation approaches zero.

We consider the case when $1/2 \leq \alpha < 1$ to make sure that $0 \leq \sigma(i) < 1$ and $\sigma(g) = 1$ and measure the probability of making either type of error for voting rules such as simple majority and two-thirds majority. The proofs of the derivation of this equilibrium are confined to Appendix A.3.

Then, in the limit case, we have

$$\hat{\gamma}_{\underline{G}} = p + (1-p)\frac{\underline{p}(1+(\frac{1-\underline{p}}{\underline{p}})^{(1-\alpha)/\alpha}) - 1}{\underline{p} - (\frac{1-\underline{p}}{\underline{p}})^{(1-\alpha)/\alpha}(1-\underline{p})}$$

$$\hat{\gamma}_{\underline{I}} = p\frac{\underline{p}(1+(\frac{1-\underline{p}}{\underline{p}})^{(1-\alpha)/\alpha}) - 1}{\underline{p} - (\frac{1-\underline{p}}{\underline{p}})^{(1-\alpha)/\alpha}(1-\underline{p})} + (1-p)$$

$$\hat{\gamma}_{\bar{G}} = p + (1-p)\frac{\bar{p}(1+(\frac{1-\bar{p}}{\bar{p}})^{(1-\alpha)/\alpha}) - 1}{\bar{p} - (\frac{1-\bar{p}}{\bar{p}})^{(1-\alpha)/\alpha}(1-\bar{p})}$$

$$\hat{\gamma}_{\bar{I}} = p\frac{\bar{p}(1+(\frac{1-\bar{p}}{\bar{p}})^{(1-\alpha)/\alpha})-1}{\bar{p}-(\frac{1-\bar{p}}{\bar{p}})^{(1-\alpha)/\alpha}(1-\bar{p})} + (1-p)$$

And we can see that $\hat{\gamma}_{\underline{G}} > \hat{\gamma}_{\bar{G}} > \hat{\gamma}_{\underline{I}} > \hat{\gamma}_{\bar{I}}$.

Whenever $m = 0$, i.e., all voters believe in $\bar{p}$, in order for the limit of the probability of either type of errors to approach zero as the jury size gets larger (tends to infinity) the following condition needs to hold: $\hat{\gamma}_{\bar{G}} > \alpha > \hat{\gamma}_{\bar{I}}$.

However, whenever $m = 1$, i.e., all voters believe in $\underline{p}$, in order for the limit of the probability of either type of errors to approach zero as the jury size gets larger (tends to infinity) the following condition needs to hold: $\hat{\gamma}_{\underline{G}} > \alpha > \hat{\gamma}_{\underline{I}}$.

If there exist heterogeneous beliefs regarding p, we need $(1-m)\gamma_{\bar{G}} + m\gamma_{\underline{G}} > \alpha$ and $(1-m)\gamma_{\bar{I}} + m\gamma_{\underline{I}} < \alpha$ for the probability that the convicted defendant is indeed guilty, $Pr(G|C)$, to tend to one as $n \to \infty$, i.e.,

$$\lim_{n\to\infty} Pr(G|C) = \lim_{n\to\infty} \frac{1}{1 + (\frac{\gamma_{\bar{I}}}{\gamma_{\bar{G}}})^{(1-m)\alpha n}(\frac{\gamma_{\underline{I}}}{\gamma_{\underline{G}}})^{m\alpha n}(\frac{1-\gamma_{\bar{I}}}{1-\gamma_{\bar{G}}})^{(1-m)(1-\alpha)n}(\frac{1-\gamma_{\underline{I}}}{1-\gamma_{\underline{G}}})^{m(1-\alpha)n}} = 1.$$

This result is in sharp contrast to those obtained under the unanimous voting case, for which both types of errors are bounded away from zero.

For both types of errors to be zero as n grows sufficiently large, the conditions above can also be rewritten as $\hat{\gamma}_{\underline{G}} > \hat{\gamma}_{\bar{G}} > \alpha > \hat{\gamma}_{\underline{I}} > \hat{\gamma}_{\bar{I}}$.[2]

Proposition 1.5 and Corollary 1.5 summarise these results.[3]

Proposition 1.5. *Under two-point non-common prior and* $1/2 \leq \alpha < 1$

1. *there is a Symmetric Responsive Equilibrium for* $n \to \infty$*, such that* $\{(0 \leq \sigma(\underline{i}) < 1, \sigma(\underline{g}) = 1), (0 \leq \sigma(\bar{i}) < 1, \sigma(\bar{g}) = 1)\}$*; and,*
2. *both types of errors approach zero as long as* $\gamma_{\underline{G}} > \gamma_{\bar{G}} > \alpha > \gamma_{\underline{I}} > \gamma_{\bar{I}}$.

Corollary 1.5. *Under two-point non-common prior, it is always possible to select an* α^* *– for the non-unanimous voting role with* $1/2 \leq \alpha^* < 1$*, such that type I and type II errors tend to zero as* $n \to \infty$ *(*$\hat{\gamma}_{\underline{G}} > \hat{\gamma}_{\bar{G}} > \alpha^* > \hat{\gamma}_{\underline{I}} > \hat{\gamma}_{\bar{I}}$*) for any given combinations of* p*,* $\underline{p}$ *and* $\bar{p}$.

Remark 1.2. *This suggests that the virtues of eliminating type I and type II errors can also be found in other voting rules but the simple majority, whenever there is scope for multiple priors to exist in regard to the accuracy of the information provided to voters casting their votes.*

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[2] Under this symmetric responsive Nash equilibrium, as long as $\hat{\gamma}_{\underline{G}} > \hat{\gamma}_{\bar{G}} > \alpha > \hat{\gamma}_{\underline{I}} > \hat{\gamma}_{\bar{I}}$, we can demonstrate that if the second term of the denominator for the expression of the probability of convicting a guilty defendant approaches zero as n tends to infinity, the entire probability of convicting a guilty defendant approaches one. That is so, because that second term can be rewritten as follows:

$$\lim_{n\to\infty}\left(\frac{\hat{\gamma}_{\bar{I}}}{\hat{\gamma}_{\bar{G}}}\right)^{(1-m)\alpha n}\left(\frac{\hat{\gamma}_{\underline{I}}}{\hat{\gamma}_{\underline{G}}}\right)^{m\alpha n}\left(\frac{1-\hat{\gamma}_{\bar{I}}}{1-\hat{\gamma}_{\bar{G}}}\right)^{(1-m)(1-\alpha)n}\left(\frac{1-\hat{\gamma}_{\underline{I}}}{1-\hat{\gamma}_{\underline{G}}}\right)^{m(1-\alpha)n} = 0.$$

Hence, this is equivalent to imposing the following restriction:

$$\left(\frac{(\hat{\gamma}_{\underline{I}})^{m\alpha}(\hat{\gamma}_{\bar{I}})^{(1-m)\alpha}(1-\hat{\gamma}_{\underline{I}})^{m(1-\alpha)}(1-\hat{\gamma}_{\bar{I}})^{(1-m)(1-\alpha)}}{(\hat{\gamma}_{\underline{G}})^{m\alpha}(\hat{\gamma}_{\bar{G}})^{(1-m)\alpha}(1-\hat{\gamma}_{\underline{G}})^{m(1-\alpha)}(1-\hat{\gamma}_{\bar{G}})^{(1-m)(1-\alpha)}}\right) < 1,$$

which translates in the condition $\hat{\gamma}_{\underline{G}} > \hat{\gamma}_{\bar{G}} > \alpha > \hat{\gamma}_{\underline{I}} > \hat{\gamma}_{\bar{I}}$.

[3] Note that, yet another symmetric responsive Nash equilibrium exists for the less general case of $0 \leq \alpha < 1/2$, for which the derivation can also be found in Appendix A.3. This case corresponds to decisions dictated by minorities, which is less plausible than that for which at least simple majority of votes in favour of one alternative is required for those votes to be decisive in that direction.

1.6 Simulations: Finite Jury Size

In real life, most of collective decisions are made by small groups with a finite size of members. Therefore, in this section, we use a series of parameters to perform simulations for voting with imprecise probabilities by a jury/committee of finite size. Our aim is to obtain results with respect to the performance of different voting rules, in regard to type-I error in particular. For that, we simulate results for those errors along various dimensions, from simple majority to unanimity rule, and both for the canonical jury model and our two-point non-common prior model, using a finite jury/committee size.

We start the simulation with comparing different levels of the information precision, p, and the reasonable doubt, q, with a given jury size[4], $n = 5$. Given the chosen group size, simple majority rule requires $k = 3$; and, unanimity rule, $k = 5$.

Also, we assume that the 'true' level of the signal accuracy, p, is either 0.6 or 0.9, with an unknown distribution. In the canonical model, jurors' priors equal the true p. In the two-point non-common prior model, jurors might not form identical priors with respect to the accuracy of the private signals they receive. Some of the jurors, the sceptics, believe the accuracy to be equal to 0.6; whereas, the trusting juror, believes that accuracy to be equal to 0.9.

Moreover, we assume all voters either adopt $q = 0.6$ or $q = 0.9$ as their reasonable doubt, or they select $q = 0.6$ when they are sceptical and $q = 0.9$ when they are trusting. The composition of different type of jurors, m, as before, is let free to vary, so that $m \in [0,1]$.

Table 1.1 contains the individual strategies with the resulted errors under simple majority and the unanimity voting rules for the parameter settings: $n = 5$, true $p = 0.6$ with the reasonable doubt being either $q = 0.6$ or $q = 0.9$ for all jurors or $q = \{0.6, 0.9\}$, which depends on

[4]Any finite jury size could have been chosen, but we restrict our simulations to the special case of size of 5 to compare those to the experimental data generated with collective decision-making by groups of size 5.

the juror's type. Then, we set the true p to be 0.9. The performance of different voting rules for both the canonical and the two-point non-common prior model can be found in Table 1.2. Before discussing the results, note that, as a benchmark simulation for the canonical model, with $n = 5$, we do not reproduce the 'Jury Paradox'.[5]

Nevertheless, the results of the simulations are startling for several reasons. First, cases where we observe that unanimity rule is inferior to majority rule in term of resulting in higher type I error are: (1) in both model setups, if the true $p = 0.9$ and $q = 0.6$; (2) in the two-point non-common prior setup, if $p = 0.9$, $q = \{0.6, 0.9\}$ and $m = 1$, that is, all voters are trusting and believe in the right p. Otherwise, both under the canonical setup and the two-point non-common prior setup, unanimity rule always outperforms majority rule for all levels of q as it leads to lower type I errors. Second, both types of errors remain the same: type I error, $Pr(C|I) = 0.3174$ and type II error, $Pr(A|G) = 0.3174$, if the true $p = 0.6$; and type I error, $Pr(C|I) = 0.0086$ and type II error, $Pr(A|G) = 0.0086$, if the true $p = 0.9$. Third, when majority rule is adopted, jurors vote informatively both in the canonical and two-point non-common prior model. However, jurors vote symmetrically and responsively when unanimous voting rule is used, except if a high level of reasonable doubt, $q = 0.9$, is applied to the sceptics. Fourth, with a finite jury size we find that type I error decreases with a higher proportion of sceptical voters, if the same level of reasonable doubt is exogenously given to all jurors.[6] This result is opposite to the limit results of the two-point non-common prior model, which instead predicts that with more jurors being trusting, type I error becomes smaller. This is an important difference, since with finite size, type I error gets smaller only when being sceptical. Finally, in the two-point non-common prior model when the type I error is at its lowest, the type II error is at its highest: decreasing the type I error comes at the cost of increasing type II error, more in general. However, since the focus of the existing literature is on the debate around the shortcomings of the unanimity rule, in that it delivers

[5] 'Jury Paradox' can be found with other parameter setups when jury size are bigger, for example, $n = 12$, $p = 0.8$ and $q = 0.9$ as illustrated in Feddersen and Pesendorfer (1998).

[6] When jurors have different levels of reasonable doubt depending on their types, simulation results are consistent with the limit results of the two-point prior model.

Table 1.1 Performance of Simple Majority and Unanimous Voting rules under The Canonical Model Setup and The Two-point Prior Model, given 'True' $p = 0.6$

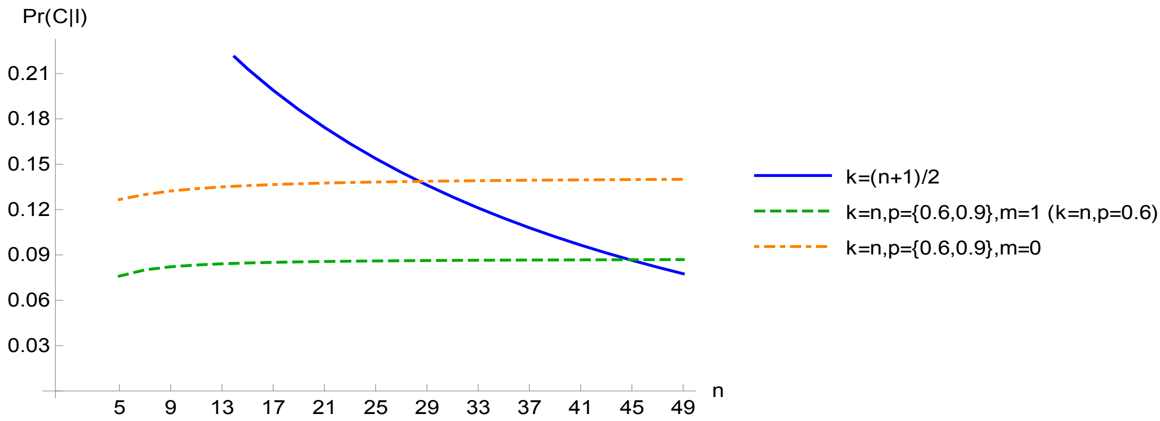
	$q = 0.6$		$q = 0.9$		$q = \{0.6, 0.9\}$	
$n = 5$	$k = 3$	$k = 5$	$k = 3$	$k = 5$	$k = 3$	$k = 5$
Canonical Model						
$\sigma(g)$	1	1	1	1	1	1
$\sigma(i)$	0	0.3288	0	0	0	0.3288
$Pr(C\|I)$	0.3174	0.0760	0.3174	0.0102	0.3174	0.0760
$Pr(A\|G)$	0.3174	0.7905	0.3174	0.9222	0.3174	0.7905
Two-point Prior Model						
$\sigma(\underline{g})$	1	1	1	1	1	1
$\sigma(\underline{i})$	0	0.3288	0	0	0	0.3288
$\sigma(\bar{g})$	1	1	1	1	1	1
$\sigma(\bar{i})$	0	0.4358	0	0.2308	0	0.2308
$Pr(C\|I)$						
$m = 1$	0.3174	0.0760	0.3174	0.0102	0.3174	0.0760
$m = 0.8$	0.3174	0.0842	0.3174	0.0138	0.3174	0.0685
$m = 0.6$	0.3174	0.0932	0.3174	0.0186	0.3174	0.0618
$m = 0.4$	0.3174	0.1033	0.3174	0.0250	0.3174	0.0557
$m = 0.2$	0.3174	0.1144	0.3174	0.0336	0.3174	0.0502
$m = 0$	0.3174	0.1267	0.3174	0.0453	0.3174	0.0453
$Pr(A\|G)$						
$m = 1$	0.3174	0.7905	0.3174	0.9222	0.3174	0.7905
$m = 0.8$	0.3174	0.7783	0.3174	0.9103	0.3174	0.8018
$m = 0.6$	0.3174	0.7653	0.3174	0.8965	0.3174	0.8124
$m = 0.4$	0.3174	0.7515	0.3174	0.8805	0.3174	0.8224
$m = 0.2$	0.3174	0.7370	0.3174	0.8622	0.3174	0.8320
$m = 0$	0.3174	0.7216	0.3174	0.8410	0.3174	0.8410

Table 1.2 Performance of Simple Majority and Unanimous Voting rules under The Canonical Model Setup and The Two-point Prior Model, given 'True' $p = 0.9$

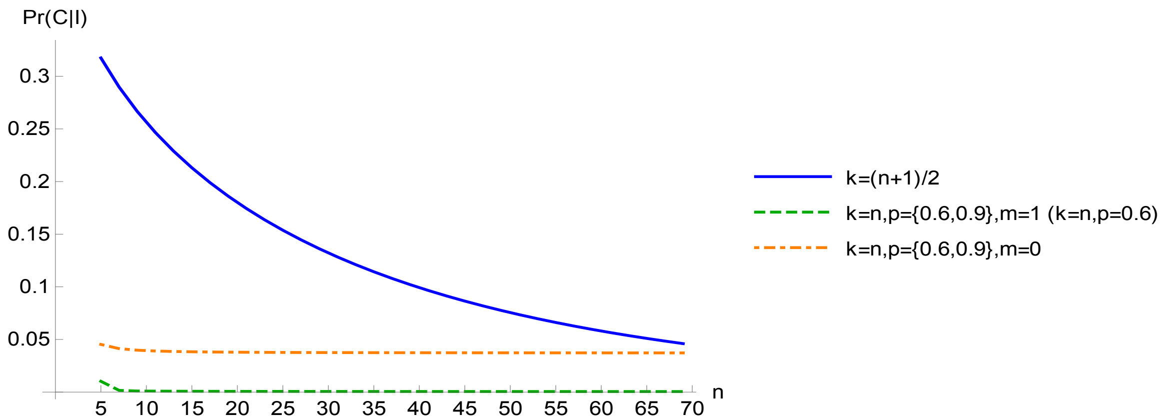
	$q = 0.6$		$q = 0.9$		$q = \{0.6, 0.9\}$	
$n = 5$	$k = 3$	$k = 5$	$k = 3$	$k = 5$	$k = 3$	$k = 5$
Canonical Model						
$\sigma(g)$	1	1	1	1	1	1
$\sigma(i)$	0	0.4358	0	0.2308	0	0.2308
$Pr(C\|I)$	0.0086	0.0289	0.0086	0.0028	0.0086	0.0028
$Pr(A\|G)$	0.0086	0.2520	0.0086	0.3298	0.0086	0.3298
Two-point Prior Model						
$\sigma(\underline{g})$	1	1	1	1	1	1
$\sigma(\underline{i})$	0	0.3288	0	0	0	0.3288
$\sigma(\bar{g})$	1	1	1	1	1	1
$\sigma(\bar{i})$	0	0.4358	0	0.2308	0	0.2308
$Pr(C\|I)$						
$m = 1$	0.0086	0.0097	0.0086	0.0000	0.0086	0.0097
$m = 0.8$	0.0086	0.0121	0.0086	0.0000	0.0086	0.0076
$m = 0.6$	0.0086	0.0150	0.0086	0.0001	0.0086	0.0059
$m = 0.4$	0.0086	0.0187	0.0086	0.0003	0.0086	0.0046
$m = 0.2$	0.0086	0.0232	0.0086	0.0009	0.0086	0.0035
$m = 0$	0.0086	0.0289	0.0086	0.0028	0.0086	0.0028
$Pr(A\|G)$						
$m = 1$	0.0086	0.2935	0.0086	0.4095	0.0086	0.2935
$m = 0.8$	0.0086	0.2854	0.0086	0.3944	0.0086	0.3009
$m = 0.6$	0.0086	0.2772	0.0086	0.3788	0.0086	0.3082
$m = 0.4$	0.0086	0.2689	0.0086	0.3629	0.0086	0.3155
$m = 0.2$	0.0086	0.2605	0.0086	0.3466	0.0086	0.3227
$m = 0$	0.0086	0.2520	0.0086	0.3298	0.0086	0.3298

the highest type I errors, it is worth stressing how the probability of convicting the innocent varies, rather than how the probability of acquitting the guilty varies, when allowing for non-common priors.

Next, we conduct the simulations by varying the jury size n each time with a given true level of p and an identical q for all jurors.



(a) 'True' $p = 0.6$, $q = 0.6$



(b) 'True' $p = 0.6$, $q = 0.9$

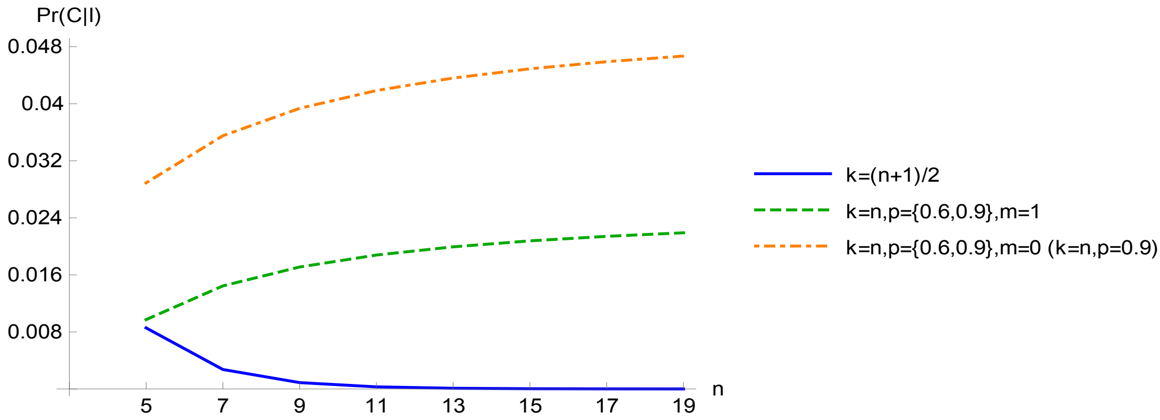
Fig. 1.1 Type I Errors of Different Voting Rules under The Canonical Model and The Two-point Prior Model, given 'True' $p = 0.6$

Figure 1.1 and Figure 1.2 illustrate the probability of convicting an innocent for both the canonical model and the two-point non-common prior model, given the true level of the signal precision equals $p = 0.6$ and $p = 0.9$, respectively. From the previous simulations, we know that type I errors under the canonical model are the same as those under the two-point non-common prior model; and they are represented by the solid blue lines in the figures, which are downward sloping.

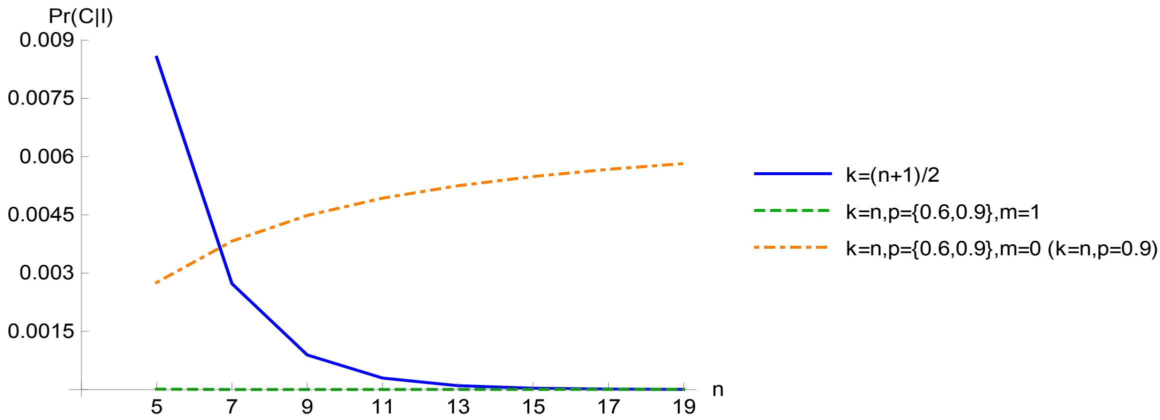
In Figure 1.1, the type I errors under the unanimous voting rule are the same across the canonical model and the two-point non-common prior model, if all jurors believes $p = 0.6$, that is, $m = 1$. This results in the lowest probability of convicting an innocent; and it is illustrated by the green dashed line. In Figure 1.1a, we can see that unanimous voting rule is preferred to majority voting rule as long as the jury size is no larger than 45. In Figure 1.1b, with $q = 0.9$, unanimous voting rule is preferred even when the jury size approaches 70.[7] If $m = 0$, with every juror believing $p = 0.9$, we have the highest type I error instead. The highest level of type I errors exceeds the one under the majority rule if $n > 31$ in Figure 1.1a and if the jury size reaches 73 approximately in Figure 1.1b. This suggests that if the information about the precision of the signal allows for potential multiple priors (it is not uniquely determined), unanimous voting rule can deliver higher type I errors if jurors adopt differing priors, which is the case when $m \neq 1$.

However, in Figure 1.2, we can see that the existence of the two-point non-common prior provides an improvement for the unanimous rule in terms of lowering the type I error, as opposed to that of the canonical model, in which the precision of the information is unique and commonly known to all jurors. In Figure 1.2a, although the majority rule outperforms the unanimity rule for all n, the type I error can be reduced if more jurors become sceptical. The most interesting case is illustrated in Figure 1.2b. We can see that the unanimity voting rule is preferred to the majority rule if the jury size is smaller than 7 and all jurors' beliefs coincide with the true p, $p = 0.9$. However, as long as any juror deviates from this belief, which means some of juror believes p to be equal to 0.6 instead of 0.9, the type I error decreases under the two-point non-common prior model. Moreover, the lowest type I error can be obtained if $m = 1$, and all voters are sceptics, with a preferable jury size for using the unanimity rule

[7] When everyone believes in the right p, that is, $p = 0.6$, the maximum jury size, for which unanimous rule should be adopted is 271, given $q = 0.9$. Please see Figure A.1 in Appendix A.4.



(a) 'True' $p = 0.9$, $q = 0.6$



(b) 'True' $p = 0.9$, $q = 0.9$

Fig. 1.2 Type I Errors of Different Voting Rules under The Canonical Model and The Two-point Prior Model, given 'True' $p = 0.9$

which can be extended to up to 17. This suggests that if the private information is not precise, the unanimity voting rule is superior to the simple majority rule for a not too large jury size and given certain parameter setups, which are very plausible to observe in the real world; and also, it is still possible for the unanimity rule to outperform the majority rule even for a larger jury size as long as not all jurors are trusting.

Details of the simulation of various sizes of the jury can be found in Appendix A.4.

1.7 Conclusion

In this chapter, we study collective decision-making processes using theoretical modelling and simulations for the case in which the common prior assumption is relaxed. To that end, we introduce a two-point non-common prior model, and run simulations, to capture the instances in which voters may possess differing priors (no common priors) beyond private information about the case at hand, when casting their votes. We do so, to account for the objective probability measure of the precision of the information possessed by each voter to be unmeasurable, as often is for many decision-making situations members of various committees are confronted with.

For the purpose of comparability with existing results in the literature for collective decision-making under the common prior assumption, we develop the study taking the leading example of the jury trial, and embedding unmeasurable probabilities to it. We take the case of jurors who form extreme beliefs/priors in regard to the accuracy of the information provided to them, say, in the course of a trial. Using this model, we find that the non-unanimous voting rule continues to lead to a zero probability of committing both types of errors – convicting the innocent defendant and acquitting the guilty defendant – when the size of jury gets sufficiently large. However, the unanimous voting rule does not appear to be as bad as under models using the common prior assumption, as studied extensively by Feddersen and Pesendorfer (1998). This is so, despite under strategic voting, the probability of making both types of errors is strictly bounded away from zero, even when the size of the jury approaches infinity. There are two ways to improve unanimous voting in terms of lowering the type I errors when the jury size tends to infinity: either (i) being genuinely trusting, which means strongly believing in a higher quality/credibility of the information; or (ii) framing the information towards the higher level of quality/credibility, such that not all voters are sceptics. In either of those alternatives, the results are improved as compared to the case in which the quality of the information is commonly known and uniquely given to all voters.

Furthermore, our simulation results, which focus on finite jury sizes, suggest that unanimity can improve upon non-unanimous voting rule, when the opposite is true, that is (i) being genuinely sceptical, or (ii) framing the information towards the lower level of quality/credibility, such that not all voters are trusting, leads to lower probabilities of committing the type I errors. Therefore, in the real-life judicial trial, if ambiguity matters, having a strong prosecutor against a weak defence lawyer creates the worst case scenario for implementing justice because a strong prosecutor is likely to frame jurors to form beliefs toward the higher level of guiltiness. Under such circumstances, only an even stronger defence lawyer outweighing the persuasion skills of the prosecutor would help rebalance beliefs in the right direction, hence restoring the chance for justice to take its course, thereby reducing the type I errors to the lowest level, for any given/chosen threshold of conviction.

Our simulation results deliver interesting testable predictions under two-point non-common prior model. First, we obtain useful comparative results regarding the model where the information is purely risky (the signal is correct with a commonly known and unique probability measure/level), providing predictions that for a small group size, that the unanimity voting rule not only does not always generate higher type I errors than majority voting rule, but also that the errors predicted by the canonical model for unanimity under a small jury size can be improved upon, when abandoning the common prior environment. These results provide the confirmation/validation of our theoretical results for a finite jury size, by which being sceptical is beneficial in lowering the occurrence of type I errors. These predictions form the basis for testing them experimentally. This is the focus of chapter 2, where we emulate jury-trials consistent with either model of the common prior or the two-point non-common prior assumption, by means of human-subject laboratory experiments.

Our research suggests a novel way to study collective deliberation outcomes when abandoning the common prior assumption with respect to the quality of information possessed by committee members. By doing so, we are able to highlight differences in conceived results in the existing literature. We do so, by assuming in particular that the quality of the private information each members possesses is unmeasurable (for example, because it is ambiguous in nature), so that voters' subjective beliefs are let free to differ, leading to possibly heterogeneous priors. By developing a theoretical model that incorporates the scope for heterogeneous priors to matter, we obtain insights about the way to improve the performance of the unanimity voting rule. Our results imply that one could manipulate the trusting/sceptical attitude of voters in a collective deliberation setting by framing the information so as to improve the performance of the unanimity voting rule. Our two-point non-common prior model represents but one way of relaxing the common prior assumption. An alternative way could be to allow for an interval within which those priors can be formed. This is explored in chapter 3 of this thesis, where ambiguity aversion is taken explicitly into account by voters, when casting their votes.

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Appendix A

Proofs for Chapter 1

A.1

Proof of The Lower Bound of $Pr(I|C)$ under Strategic Voting

When there is some juror who votes for acquittal with positive probability, that is because his/her posterior probability of the defendant being guilty, conditional on being pivotal and receiving either of the two signals – an innocent signal, i or an guilty signal, g – is not overwhelming the reasonable doubt, such that $Pr(G|piv_j, s) \leq q$.

$$Pr(G|piv_j, s) = \frac{Pr(prv_j|G)Pr(s|G)Pr(G)}{Pr(prv_j|G)Pr(s|G)Pr(G) + Pr(prv_j|I)Pr(s|I)Pr(I)}.$$

The prior about the 'true' state of the world is $Pr(G) = Pr(I) = 0.5$, and

$$\because Pr(piv_j|G) = \frac{Pr(piv_j|G)}{Pr(piv_j|G) + Pr(piv_j|I)},$$

$$\therefore Pr(G|piv_j, s) = \frac{\frac{Pr(prv_j|G)}{Pr(prv_j|G)+Pr(prv_j|I)}Pr(s|G)}{\frac{Pr(prv_j|G)}{Pr(prv_j|G)+Pr(prv_j|I)}Pr(s|G) + \frac{Pr(prv_j|I)}{Pr(prv_j|G)+Pr(prv_j|I)}Pr(s|I)};$$

and according to Bayes' Rule,

$$Pr(G|piv_j, s) = \frac{Pr(G|piv_j)Pr(s|G)}{Pr(G|piv_j)Pr(s|G) + (1 - Pr(G|piv_j))Pr(s|I)}.$$

We know that if voter j believes in $\underline{p}$, we have

$$Pr(s|G) = \begin{cases} Pr(i|G) = 1 - \underline{p} \\ Pr(g|G) = \underline{p}; \end{cases}$$

And

$$Pr(s|I) = \begin{cases} Pr(i|I) = \underline{p} \\ Pr(g|I) = 1 - \underline{p}. \end{cases}$$

We know that if voter j believes in $\bar{p}$,

$$Pr(s|G) = \begin{cases} Pr(i|G) = 1 - \bar{p} \\ Pr(g|G) = \bar{p}; \end{cases}$$

And

$$Pr(s|I) = \begin{cases} Pr(i|I) = \bar{p} \\ Pr(g|I) = 1 - \bar{p}. \end{cases}$$

Therefore,

$$Pr(G|piv_j, \underline{g}) = \frac{Pr(G|piv_j)Pr(\underline{g}|G)}{Pr(G|piv_j)Pr(\underline{g}|G) + (1 - Pr(G|piv_j))Pr(\underline{g}|I)}$$

$$= \frac{Pr(G|piv_j)\underline{p}}{Pr(G|piv_j)\underline{p} + (1 - Pr(G|piv_j))(1 - \underline{p})};$$

$$\because q \geq Pr(G|piv_j, \underline{g}),$$

$$\therefore Pr(G|piv_j) \leq \frac{q\underline{p} - q}{2q\underline{p} - q - \underline{p}}.$$

And

$$Pr(G|piv_j, \underline{i}) = \frac{Pr(G|piv_j)Pr(\underline{i}|G)}{Pr(G|piv_j)Pr(\underline{i}|G) + (1 - Pr(G|piv_j))Pr(\underline{i}|I)}$$

$$= \frac{Pr(G|piv_j)(1 - \underline{p})}{Pr(G|piv_j)(1 - \underline{p}) + (1 - Pr(G|piv_j))\underline{p}};$$

$$\because q \geq Pr(G|piv_j, \underline{i}),$$

$$\therefore Pr(G|piv_j) \leq \frac{q\underline{p}}{(1-q)(1-\underline{p})+q\underline{p}}.$$

Referring to the sceptical voters, for either of the signal to satisfy the condition that the posterior probability is not overwhelming the reasonable doubt, we obtain that $Pr(G|piv_j) \leq \frac{q\underline{p}}{(1-q)(1-\underline{p})+q\underline{p}}$.

Regarding the trusting voters, a similar approach can be applied, then,

$$\begin{aligned} Pr(G|piv_j,\bar{g}) &= \frac{Pr(G|piv_j)Pr(\bar{g}|G)}{Pr(G|piv_j)Pr(\bar{g}|G)+(1-Pr(G|piv_j))Pr(\bar{g}|I)} \\ &= \frac{Pr(G|piv_j)\bar{p}}{Pr(G|piv_j)\bar{p}+(1-Pr(G|piv_j))(1-\bar{p})}; \\ &\because q \geq Pr(G|piv_j,\bar{g}), \\ &\therefore Pr(G|piv_j) \leq \frac{q\bar{p}-q}{2q\bar{p}-q-\bar{p}}. \end{aligned}$$

$$\begin{aligned} Pr(G|piv_j,\bar{i}) &= \frac{Pr(G|piv_j)Pr(\bar{i}|G)}{Pr(G|piv_j)Pr(\bar{i}|G)+(1-Pr(G|piv_j))Pr(\bar{i}|I)} \\ &= \frac{Pr(G|piv_j)(1-\bar{p})}{Pr(G|piv_j)(1-\bar{p})+(1-Pr(G|piv_j))\bar{p}}; \\ &\because q \geq Pr(G|piv_j,\bar{i}), \\ &\therefore Pr(G|piv_j) \leq \frac{q\bar{p}}{(1-q)(1-\bar{p})+q\bar{p}}. \end{aligned}$$

With respect to the trusting voter, for either of the signal to satisfy the condition, $Pr(G|piv_j,\bar{s}) \leq q$, we obtain that $Pr(G|piv_j) \leq \frac{q\bar{p}}{(1-q)(1-\bar{p})+q\bar{p}}$.

Because we do not require every voter to vote to acquit with positive probability, as long as $Pr(G|piv_j) \leq \frac{q\bar{p}}{(1-q)(1-\bar{p})+q\bar{p}}$, at least those who are trusting will vote accordingly.

Define γ_{Gj} as the probability that the guilty defendant is convicted. γ_{Ij} is the probability the innocent defendant is convicted. Neither γ_{Gj} nor γ_{Ij} is the function of voter j's own perception of the signal accuracy, but the function of their voting strategies and the true quality of private signals, which is labelled as p.

Thus, for those who believe in $\bar{p}$,

$\gamma_{\bar{G}j} = p\sigma_j(\bar{g}) + (1-p)\sigma_j(\bar{i})$,

and
$\gamma_{\bar{I}j} = p\sigma_j(\bar{i}) + (1-p)\sigma_j(\bar{g})$.

We assume that at least one of $\sigma(\bar{i})$ and $\sigma(\bar{g})$ lies in $(0,1]$.

Thus, we first check whether $\frac{\gamma_{\bar{I}j}}{\gamma_{\bar{G}j}} \geq \frac{1-p}{p}$ when $\sigma_j(\bar{g}) \in (0,1]$.
We have

$$\frac{\gamma_{\bar{I}j}}{\gamma_{\bar{G}j}} = \frac{p\sigma_j(\bar{i}) + (1-p)\sigma_j(\bar{g})}{p\sigma_j(\bar{g}) + (1-p)\sigma_j(\bar{i})} = \frac{p\frac{\sigma(\bar{i})}{\sigma(\bar{g})} + (1-p)}{\bar{p} + (1-p)\frac{\sigma(\bar{i})}{\sigma(\bar{g})}};$$

$$\because \frac{\sigma(\bar{i})}{\sigma(\bar{g})} \in [0,+\infty),$$

And, also,

$$p > 1-p,$$

Thus, $p\frac{\sigma(\bar{i})}{\sigma(\bar{g})} \geq (1-p)\frac{\sigma(\bar{i})}{\sigma(\bar{g})}$; and the minimum of $\frac{\gamma_{\bar{I}j}}{\gamma_{\bar{G}j}}$ is $\frac{1-p}{p}$.

Therefore, we proved that when $\sigma_j(\bar{g}) \in (0,1]$; and thus, we have

$$\frac{\gamma_{\bar{I}j}}{\gamma_{\bar{G}j}} \geq \frac{1-p}{p}.$$

Now, we check whether $\frac{\gamma_{\bar{I}j}}{\gamma_{\bar{G}j}} \geq \frac{1-p}{p}$ when $\sigma_j(\bar{i}) \in (0,1]$.

We have

$$\frac{\gamma_{\bar{I}j}}{\gamma_{\bar{G}j}} = \frac{p\frac{\sigma(\bar{i})}{\sigma(\bar{g})} + (1-p)}{p + (1-p)\frac{\sigma(\bar{i})}{\sigma(\bar{g})}};$$

because

$$\frac{\sigma(\bar{i})}{\sigma(\bar{g})} \in (0,+\infty),$$

thus,

$$\frac{p\frac{\sigma(\bar{i})}{\sigma(\bar{g})}+(1-p)}{p+(1-p)\frac{\sigma(\bar{i})}{\sigma(\bar{g})}} > \frac{1-p}{p}.$$

Thus, we have proved that $\frac{\gamma_{\bar{I}j}}{\gamma_{\bar{G}j}} \geq \frac{1-p}{p}$, when at least one of $\sigma(\bar{i})$ and $\sigma(\bar{g})$ lies in $(0,1]$.

Therefore,

$$Pr(G|C) = \frac{Pr(G|piv_j)\gamma_{\bar{G}j}}{Pr(G|piv_j)\gamma_{\bar{G}j}+(1-Pr(G|piv_j))\gamma_{\bar{I}j}} = \frac{Pr(G|piv_j)}{Pr(G|piv_j)+(1-Pr(G|piv_j))\frac{\gamma_{\bar{I}j}}{\gamma_{\bar{G}j}}}.$$

$$\because \frac{\gamma_{\bar{I}j}}{\gamma_{\bar{G}j}} \geq \frac{1-p}{p},$$

$$\begin{aligned}\therefore Pr(G|C) &\leq \frac{Pr(G|piv_j)}{Pr(G|piv_j)+(1-Pr(G|piv_j))(\frac{1-p}{p})}\\ &\leq \frac{1}{\frac{2p-1}{p}+\frac{1-p}{\frac{q\bar{p}p}{(1-q)(1-\bar{p})+q\bar{p}}}} = \frac{(1-\bar{p})(1-\underline{p})(1-q)}{(1-\underline{p})(1-q)+\bar{p}(\underline{p}+q-1)}.\end{aligned}$$

A.2

Proof of The Existence of Symmetric Responsive Nash Equilibrium under Unanimous Voting

Define the probability the guilty defendant is convicted as γ_G and the probability the innocent defendant is convicted as γ_I, we then have

$$\gamma_{\underline{G}} = \underline{p}\sigma(\underline{g}) + (1-\underline{p})\sigma(\underline{i}),$$

$$\gamma_{\underline{I}} = \underline{p}\sigma(\underline{i}) + (1-\underline{p})\sigma(\underline{g}),$$

$$\gamma_{\bar{G}} = \bar{p}\sigma(\bar{g}) + (1-\bar{p})\sigma(\bar{i}),$$

$$\gamma_{\bar{I}} = \bar{p}\sigma(\bar{i}) + (1-\bar{p})\sigma(\bar{g}).$$

Here, γ is a function of the private signals voters receive and voters beliefs about the quality of the signals. The reason is because when a voter considers being pivotal, he/she assumes that all others vote for conviction, and the pivotal voter will apply the belief he/she adopted to all other voters.

Define all possible combinations of the share of sceptics and trusting voters, given the true state of world G, for these $n-1$ voters as $\mathscr{A}$; and all possible combinations of the share of sceptics and trusting voters, given the true state of world I, for these $n-1$ voters as $\mathscr{B}$. Therefore:

$$\mathscr{A} = \sum_{j=0}^{n-1} \begin{pmatrix} n-1 \\ j \end{pmatrix} (1-\mu)^j \mu^{n-1-j} = 1,$$

and

$$\mathscr{B} = \sum_{j=0}^{n-1} \begin{pmatrix} n-1 \\ j \end{pmatrix} \mu^j (1-\mu)^{n-1-j} = 1.$$

First, we will prove that for those who believe in $\underline{p}$, there exists a symmetric responsive equilibrium $(\sigma(\underline{g}) = 1, 0 < \sigma(\underline{i}) < 1)$.

Assume there exists a symmetric equilibrium $(\sigma(\underline{g}) = 1, 0 < \sigma(\underline{i}) \leq 1)$.

If $0 < \sigma(\underline{i}) \leq 1$, it indicates $\beta(n-1, n, 1-\underline{p}) \geq q$, then,

$$\frac{1}{1+(\frac{\gamma_{\underline{I}}}{\gamma_{\underline{G}}})^{n-1}\frac{\underline{p}}{1-\underline{p}}} \geq q.$$

This gives us

$$(\frac{\gamma_{\underline{I}}}{\gamma_{\underline{G}}})^{n-1} \leq \frac{1-q}{q}\frac{1-\underline{p}}{\underline{p}}.$$

In the case when the pivotal voter receives g, it must be true that

$$\frac{1}{1+(\frac{\gamma_{\underline{I}}}{\gamma_{\underline{G}}})^{n-1}\frac{1-\underline{p}}{\underline{p}}} \geq \frac{1}{1+\frac{1-q}{q}\frac{1-\underline{p}}{\underline{p}}\frac{1-\underline{p}}{\underline{p}}} > q.$$

Thus, the posterior belief about the defendant being guilty is $\beta(n, n, \underline{p}) > q$, which indicates that those who receive signal g and believe in $\underline{p}$ will vote guilty for sure, $\sigma(\underline{g}) = 1$.

As the strategic profile needs to be responsive, $\sigma(\underline{i})$ has to be smaller than 1, thus, $(\sigma(\underline{g}) = 1, 0 < \sigma(\underline{i}) < 1)$.

However, if $0 \leq \sigma(\underline{g}) < 1$, then $\beta(n, n, \underline{p}) \leq q$. We have

$$\frac{1}{1+(\frac{\gamma_{\underline{I}}}{\gamma_{\underline{G}}})^{n-1}\frac{1-\underline{p}}{\underline{p}}} \leq q;$$

and then,

$$(\frac{\gamma_{\underline{I}}}{\gamma_{\underline{G}}})^{n-1} \geq \frac{1-q}{q}\frac{\underline{p}}{1-\underline{p}}.$$

We obtain

$$\frac{1}{1+\frac{1-q}{q}\frac{\underline{p}}{1-\underline{p}}\frac{\underline{p}}{1-\underline{p}}} < q,$$

which says $\beta(n-1, n, 1-\underline{p}) < q$. This shows that $\sigma(\underline{i}) = 0$, if jurors receive signal i and believe in $\underline{p}$, they will never vote for conviction. Noticeably, being pivotal means there are $n-1$ guilty votes. Since those who gets i do not vote for guilty, it has to be the case that those who gets g vote for conviction. However, this contradicts our assumption of $\sigma(\underline{g}) < 1$. Therefore, $\sigma(\underline{g}) < 1$ cannot be an equilibrium.

Hence, $\sigma(\underline{g}) = 1$ and $0 < \sigma(\underline{i}) < 1$ is the unique symmetric responsive strategy profile for sceptical voters, who believes in $\underline{p}$.

We then have
$\gamma_{\underline{G}} = p + (1-p)\sigma(\underline{i})$,

and
$\gamma_{\underline{I}} = p\sigma(\underline{i}) + (1-p)$.

When $0 < \sigma(\underline{i}) < 1$, it means that

$$\frac{1}{1+(\frac{\underline{p}}{1-\underline{p}})(\frac{\gamma_{\underline{I}}}{\gamma_{\underline{G}}})^{n-1}} = \frac{1}{1+(\frac{\underline{p}}{1-\underline{p}})(\frac{\underline{p}\sigma(\underline{i})+(1-\underline{p})}{\underline{p}+(1-\underline{p})\sigma(\underline{i})})^{n-1}} = q.$$

$$\sigma(\underline{i}) = \frac{[\frac{(1-q)(1-\underline{p})}{q\underline{p}}]^{\frac{1}{(n-1)}}\underline{p} - (1-\underline{p})}{\underline{p} - [\frac{(1-q)(1-\underline{p})}{q\underline{p}}]^{\frac{1}{(n-1)}}(1-\underline{p})}$$

Examining the above equation, we can see hat $\sigma(\underline{i}) < 1$ as long as $q > 1-\underline{p}$.

Using a similar proof, we can get that $\sigma(\bar{g}) = 1$ and $0 < \sigma(\bar{i}) < 1$ is the unique symmetric responsive strategy profile for trusting voters, who believe in $\bar{p}$. Also,

$$\sigma(\bar{i}) = \frac{[\frac{(1-q)(1-\bar{p})}{q\bar{p}}]^{\frac{1}{(n-1)}}\bar{p} - (1-\bar{p})}{\bar{p} - [\frac{(1-q)(1-\bar{p})}{q\bar{p}}]^{\frac{1}{(n-1)}}(1-\bar{p})} < 1,$$

as long as $q > 1-\bar{p}$.

Thus, we have one unique symmetric responsive Nash equilibrium, $((0 < \sigma(\underline{i}) < 1, \sigma(\underline{g}) = 1), (0 < \sigma(\bar{i}) < 1, \sigma(\bar{g}) = 1))$, when there exists heterogeneity in voters' types caused by the two-point non-common prior model.

A.3

Proof of The Existence of Symmetric Responsive Nash Equilibrium under Non-unanimous Voting

Define the non-unanimous voting rule as $\hat{k} = \alpha n$, $0 < \alpha < 1$; all possible combinations of the share of sceptics and trusting voters, given the true state of world G, for these $\alpha n - 1$ voters as $\mathscr{A}$; and all possible combinations of the share of sceptics and trusting voters, given the true state of world I, for these $\alpha n - 1$ voters as $\mathscr{B}$. Therefore,

$$\mathscr{A} = \sum_{j=0}^{\alpha n-1} \begin{pmatrix} \alpha n - 1 \\ j \end{pmatrix} (1-\mu)^j \mu^{\alpha n - 1 - j} = 1,$$

and

$$\mathscr{B} = \sum_{j=0}^{\alpha n-1} \begin{pmatrix} \alpha n - 1 \\ j \end{pmatrix} \mu^j (1-\mu)^{\alpha n - 1 - j} = 1.$$

If $\sigma(\underline{i}) < 1$, we must have

$$\frac{1}{1 + \frac{\underline{p}(\gamma_{\underline{I}})^{\hat{k}-1}(1-\gamma_{\underline{I}})^{n-\hat{k}}}{(1-\underline{p})(\gamma_{\underline{G}})^{\hat{k}-1}(1-\gamma_{\underline{G}})^{n-\hat{k}}}} \leq q,$$

with equality holding if $1 > \sigma(\underline{i}) > 0$.

Similarly, when $\sigma(\underline{g}) > 0$, it must be true that

$$\frac{1}{1 + \frac{(1-\underline{p})(\gamma_{\underline{I}})^{\hat{k}-1}(1-\gamma_{\underline{I}})^{n-\hat{k}}}{\underline{p}(\gamma_{\underline{G}})^{\hat{k}-1}(1-\gamma_{\underline{G}})^{n-\hat{k}}}} \geq q,$$

with equality holding if $1 > \sigma(\underline{g}) > 0$.

We first prove, in any responsive equilibrium, we must have either $\sigma(\underline{i}) = 0$ and $1 \geq \sigma(\underline{g}) > 0$ or $0 \leq \sigma(\underline{i}) < 1$ and $\sigma(\underline{g}) = 1$.

Suppose $1 \geq \sigma(\underline{g}) > 0$, we have

$$\frac{1}{1+\frac{(1-\underline{p})(\gamma_{\underline{I}})^{\hat{k}-1}(1-\gamma_{\underline{I}})^{n-\hat{k}}}{\underline{p}(\gamma_{\underline{G}})^{\hat{k}-1}(1-\gamma_{\underline{G}})^{n-\hat{k}}}} \geq q,$$

then,

$$\frac{(1-q)\underline{p}}{q(1-\underline{p})} \geq \frac{(\gamma_{\underline{I}})^{\hat{k}-1}(1-\gamma_{\underline{I}})^{n-\hat{k}}}{(\gamma_{\underline{G}})^{\hat{k}-1}(1-\gamma_{\underline{G}})^{n-\hat{k}}}.$$

Thus, we have

$$\frac{1}{1+\frac{1-q}{q}(\frac{\underline{p}}{1-\underline{p}})^2} < q,$$

which implies $\sigma(\underline{i}) = 0$.

Suppose $1 > \sigma(\underline{i}) \geq 0$, we have

$$\frac{1}{1+\frac{\underline{p}(\gamma_{\underline{I}})^{\hat{k}-1}(1-\gamma_{\underline{I}})^{n-\hat{k}}}{(1-\underline{p})(\gamma_{\underline{G}})^{\hat{k}-1}(1-\gamma_{\underline{G}})^{n-\hat{k}}}} \leq q,$$

then,

$$\frac{(1-q)(1-\underline{p})}{q\underline{p}} \leq \frac{(\gamma_{\underline{I}})^{\hat{k}-1}(1-\gamma_{\underline{I}})^{n-\hat{k}}}{(\gamma_{\underline{G}})^{\hat{k}-1}(1-\gamma_{\underline{G}})^{n-\hat{k}}}.$$

Thus, we have

$$\frac{1}{1+\frac{1-q}{q}(\frac{(1-\underline{p})}{\underline{p}})^2} > q,$$

which implies $\sigma(\underline{g}) = 1$.

Thus, in any responsive equilibrium we must have either $\sigma(\underline{i}) = 0$ and $0 < \sigma(\underline{g}) \leq 1$ or $0 \leq \sigma(\underline{i}) < 1$ and $\sigma(\underline{g}) = 1$.

Then, we need to check whether $\sigma(\underline{i}) = 0$ and $\sigma(\underline{g}) = 1$ is a responsive voting equilibrium.

Suppose it is indeed a responsive voting equilibrium, we then have

$$\gamma_{\underline{I}} = \underline{p},$$

and

$$\gamma_{\underline{G}} = 1 - \underline{p}.$$

$\sigma(\underline{i}) = 0$ indicates that

$$\frac{1}{1 + \frac{\underline{p}(1-\underline{p})^{\hat{k}-1}(\underline{p})^{n-\hat{k}}}{(1-\underline{p})(\underline{p})^{\hat{k}-1}(1-\underline{p})^{n-\hat{k}}}} < q;$$

and we obtain

$$\frac{(1-q)(1-\underline{p})}{q\underline{p}} < \frac{(1-\underline{p})^{\hat{k}-1}(\underline{p})^{n-\hat{k}}}{(\underline{p})^{\hat{k}-1}(1-\underline{p})^{n-\hat{k}}},$$

then,

$$\frac{1}{1 + \frac{(1-\underline{p})(1-\underline{p})^{\hat{k}-1}(\underline{p})^{n-\hat{k}}}{\underline{p}(\underline{p})^{\hat{k}-1}(1-\underline{p})^{n-\hat{k}}}} > q,$$

we proved that $\sigma(\underline{i}) = 0$ and $\sigma(\underline{g}) = 1$ is a responsive equilibrium.

If $\sigma(\underline{i}) = 0$, $\sigma(\underline{g}) = 1$ is not a responsive equilibrium, we then have two cases to consider.

If $\sigma(\underline{i}) = 0$ and $0 < \sigma(\underline{g}) < 1$, then,

$\gamma_{\underline{I}} = (1-\underline{p})\sigma(\underline{g})$,

and

$\gamma_{\underline{G}} = \underline{p}\sigma(\underline{g})$.

$$\frac{1}{1 + \frac{(1-\underline{p})(\gamma_{\underline{I}})^{\hat{k}-1}(1-\gamma_{\underline{I}})^{n-\hat{k}}}{\underline{p}(\gamma_{\underline{G}})^{\hat{k}-1}(1-\gamma_{\underline{G}})^{n-\hat{k}}}} = q,$$

$$\frac{(1-q)\underline{p}}{q(1-\underline{p})} = \frac{(\gamma_{\underline{I}})^{\hat{k}-1}(1-\gamma_{\underline{I}})^{n-\hat{k}}}{(\gamma_{\underline{G}})^{\hat{k}-1}(1-\gamma_{\underline{G}})^{n-\hat{k}}},$$

$$= \frac{((1-\underline{p})\sigma(\underline{g}))^{\hat{k}-1}(1-(1-\underline{p})\sigma(\underline{g}))^{n-\hat{k}}}{(\underline{p}\sigma(\underline{g}))^{\hat{k}-1}(1-\underline{p}\sigma(\underline{g})))^{n-\hat{k}}},$$

$$= (\frac{1-\underline{p}}{\underline{p}})^{\hat{k}-1}(\frac{1-(1-\underline{p})\sigma(\underline{g})}{1-\underline{p}\sigma(\underline{g})})^{n-\hat{k}}.$$

Therefore, we get

$$\frac{(1-q)}{q}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}} = (\frac{1-(1-\underline{p})\sigma(\underline{g})}{1-\underline{p}\sigma(\underline{g})})^{n-\hat{k}},$$

$$\frac{(1-\sigma(\underline{g})+\underline{p}\sigma(\underline{g}))}{(1-\underline{p}\sigma(\underline{g}))} = (\frac{(1-q)}{q}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}})^{1/(n-\hat{k})},$$

$$(\frac{1-q}{q}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}})^{\frac{1}{n-\hat{k}}}(1-\underline{p}\sigma(\underline{g})) = 1-\sigma(\underline{g})(1-\underline{p}),$$

$$(\frac{1-q}{q}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}})^{\frac{1}{n-\hat{k}}} - (\frac{1-q}{q}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}})^{\frac{1}{n-\hat{k}}}\underline{p}\sigma(\underline{g}) + \sigma(\underline{g}) - \underline{p}\sigma(\underline{g}) = 1,$$

$$\sigma(\underline{g})((1-\underline{p}) - \underline{p}(\frac{1-q}{q}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}})^{\frac{1}{n-\hat{k}}}) = 1-(\frac{1-q}{q}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}})^{\frac{1}{n-\hat{k}}},$$

$$\therefore \sigma(\underline{g}) = \frac{(\frac{1-q}{q}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}})^{\frac{1}{n-\hat{k}}}-1}{\underline{p}(\frac{1-q}{q}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}})^{\frac{1}{n-\hat{k}}}-(1-\underline{p})} = \frac{(\frac{1-q}{q}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}})^{\frac{1}{n-\hat{k}}}-1}{\underline{p}(\frac{1-q}{q}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}})^{\frac{1}{n-\hat{k}}}-1}.$$

This yields

$$\sigma(\underline{g}) = \frac{h-1}{\underline{p}(h+1)-1},$$

where

$$h = (\frac{(1-q)}{q}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}})^{1/(n-\hat{k})}.$$

If $\sigma(\underline{g}) = 1$ and $0 < \sigma(\underline{i}) < 1$, then,

$\gamma_{\underline{G}} = \underline{p} + (1-\underline{p})\sigma(\underline{i})$

and

$\gamma_{\underline{I}} = 1-\underline{p}+\underline{p}\sigma(\underline{i})$

$$\frac{(1-\underline{p})(\gamma_{\underline{G}})^{\hat{k}-1}(1-\gamma_{\underline{G}})^{n-\hat{k}}}{\underline{p}(\gamma_{\underline{I}})^{\hat{k}-1}(1-\gamma_{\underline{I}})^{n-\hat{k}}+(1-\underline{p})(\gamma_{\underline{G}})^{\hat{k}-1}(1-\gamma_{\underline{G}})^{n-\hat{k}}} < q,$$

$$\because \frac{(1-q)(1-\underline{p})}{q\underline{p}} = (\frac{\gamma_{\underline{I}}}{\gamma_{\underline{G}}})^{\hat{k}-1}(\frac{1-\gamma_{\underline{I}}}{1-\gamma_{\underline{G}}})^{n-\hat{k}},$$

$$\therefore \frac{(1-q)(1-\underline{p})}{q\underline{p}} = (\frac{1-\underline{p}(1-\sigma(\underline{i}))}{1-(1-\underline{p})(1-\sigma(\underline{i}))})^{\hat{k}-1}(\frac{\underline{p}(1-\sigma(\underline{i}))}{(1-\underline{p})(1-\sigma(\underline{i}))})^{n-\hat{k}},$$

$$\frac{(1-q)(1-\underline{p})}{q\underline{p}}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}-n} = (\frac{1-\underline{p}+\underline{p}\sigma(\underline{i})}{\underline{p}+(1-\underline{p})\sigma(\underline{i})})^{\hat{k}-1},$$

$$(\frac{(1-q)(1-\underline{p})}{q\underline{p}}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}-n})^{\frac{1}{\hat{k}-1}} = \frac{1-\underline{p}+\underline{p}\sigma(\underline{i})}{\underline{p}+(1-\underline{p})\sigma(\underline{i})},$$

$$((\frac{(1-q)(1-\underline{p})}{q\underline{p}}\frac{\underline{p}}{1-\underline{p}}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}-n-1}))^{\frac{1}{\hat{k}-1}} = \frac{1-\underline{p}+\underline{p}\sigma(\underline{i})}{\underline{p}+(1-\underline{p})\sigma(\underline{i})},$$

$$((\frac{1-q}{q})(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}-n-1})^{\frac{1}{\hat{k}-1}}(\underline{p}+(1-\underline{p})\sigma(\underline{i})) = 1-\underline{p}+\underline{p}\sigma(\underline{i}),$$

$$\underline{p}((\frac{1-q}{q})(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}-n-1})^{\frac{1}{\hat{k}-1}} + (1-\underline{p})\sigma(\underline{i})((\frac{1-q}{q})(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}-n-1})^{\frac{1}{\hat{k}-1}} - \underline{p}\sigma(\underline{i}) = 1-\underline{p},$$

$$\sigma(\underline{i})((1-\underline{p})((\frac{1-q}{q})(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}-n-1})^{\frac{1}{\hat{k}-1}} - \underline{p}) = 1-\underline{p}-\underline{p}((\frac{1-q}{q})(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}-n-1})^{\frac{1}{\hat{k}-1}},$$

$$\therefore \sigma(\underline{i}) = \frac{1-\underline{p}-\underline{p}((\frac{1-q}{q})(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}-n-1})^{\frac{1}{\hat{k}-1}}}{(1-\underline{p})((\frac{1-q}{q})(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}-n-1})^{\frac{1}{\hat{k}-1}} - \underline{p}} = \frac{1-\underline{p}(1+((\frac{1-q}{q})(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}-n-1})^{\frac{1}{\hat{k}-1}})}{(1-\underline{p})((\frac{1-q}{q})(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}-n-1})^{\frac{1}{\hat{k}-1}} - \underline{p}}.$$

Thus, an interior solution in this case, is

$$\sigma(\underline{i}) = \frac{\underline{p}(1+f)-1}{\underline{p}-f(1-\underline{p})},$$

where

$$f = (\frac{(1-q)}{q}(\frac{(1-\underline{p})}{\underline{p}})^{n-\hat{k}+1})^{1/(\hat{k}-1)}.$$

We could also prove that symmetric responsive Nash equilibrium is either $(0 \leq \sigma(\bar{i}) < 1, \sigma(\bar{g}) = 1)$ or $(\sigma(\bar{i}) = 0, 0 < \sigma(\bar{g}) \leq 1)$ for those who believe in $\bar{p}$ following a similar proof as we have used for those who believe in $\underline{p}$.

Next, we prove the existence of the symmetric responsive limit equilibrium.

When $n \to \infty$,

$$\lim_{n\to\infty} f = \lim_{n\to\infty} (\frac{(1-q)}{q}(\frac{(1-\underline{p})}{\underline{p}})^{n-\hat{k}+1})^{1/(\hat{k}-1)} = (\frac{1-\underline{p}}{\underline{p}})^{\frac{1-\alpha}{\alpha}};$$

and, therefore, we have

$$\sigma(\underline{i}) = \frac{\underline{p}(1+f)-1}{\underline{p}-f(1-\underline{p})} = \frac{\underline{p}(1+(\frac{1-\underline{p}}{\underline{p}})^{(1-\alpha)/\alpha})-1}{\underline{p}-(\frac{1-\underline{p}}{\underline{p}})^{(1-\alpha)/\alpha}(1-\underline{p})}.$$

We can see that when $1 > \alpha \geq 1/2$, $\sigma(\underline{i}) \to 1$ as $\alpha \to 1$.

Similarly,

$$\lim_{n\to\infty} h = \lim_{n\to\infty} (\frac{(1-q)}{q}(\frac{\underline{p}}{1-\underline{p}})^{\hat{k}})^{1/(n-\hat{k})} = (\frac{\underline{p}}{1-\underline{p}})^{\frac{\alpha}{1-\alpha}},$$

$$\sigma(\underline{g}) = \frac{h-1}{\underline{p}(h+1)-1} = \frac{(\frac{\underline{p}}{1-\underline{p}})^{\alpha/(1-\alpha)}-1}{\underline{p}((\frac{\underline{p}}{1-\underline{p}})^{\alpha/(1-\alpha)}+1)-1}.$$

When $0 < \alpha \leq 1/2$, $\sigma(\underline{g}) \to 0$ as $\alpha \to 0$.

This proves that for any $\alpha \in (0,1)$ there is a responsive limit equilibrium.

The limit strategy for those who believe in $\bar{p}$ can be obtained in a similar way, such that

$$\sigma(\bar{i}) = \frac{\bar{p}(1+f)-1}{\underline{p}-f(1-\bar{p})} = \frac{\bar{p}(1+(\frac{1-\bar{p}}{\bar{p}})^{(1-\alpha)/\alpha})-1}{\bar{p}-(\frac{1-\bar{p}}{\bar{p}})^{(1-\alpha)/\alpha}(1-\bar{p})},$$

and

$$\sigma(\bar{g}) = \frac{h-1}{\bar{p}(h+1)-1} = \frac{(\frac{\bar{p}}{1-\bar{p}})^{\alpha/(1-\alpha)}-1}{\bar{p}((\frac{\bar{p}}{1-\bar{p}})^{\alpha/(1-\alpha)}+1)-1}.$$

Thus, we consider the case when $1 > \alpha \geq 1/2$ to make sure that $0 \leq \sigma(i) < 1$ and $\sigma(g) = 1$ and measure the probability of making either type of error for voting rules such as simple majority and two-thirds majority.

A.4

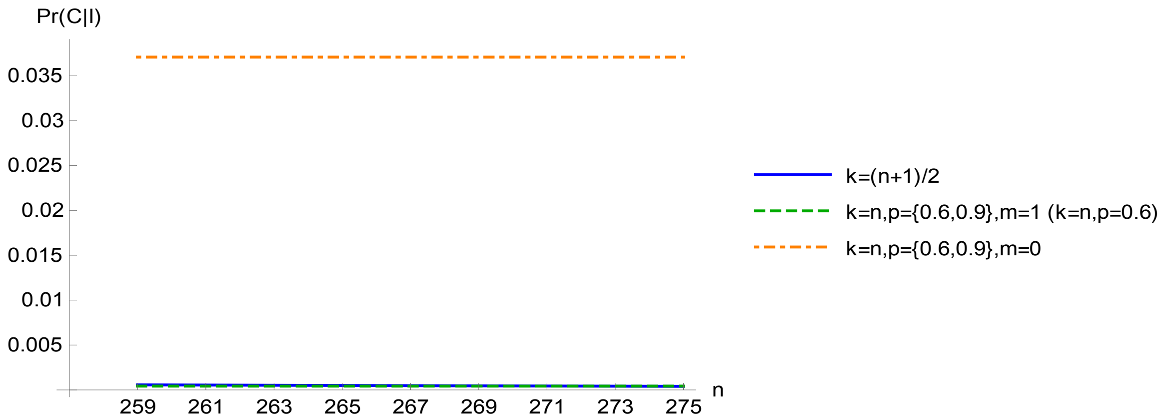
Simulation Results of Varying The Jury Size n

Table A.1 Type I Error of Different Voting Rules under The Canonical Model and The Two-point Prior Model, Given 'True' $p = 0.6$ and $q = 0.6$

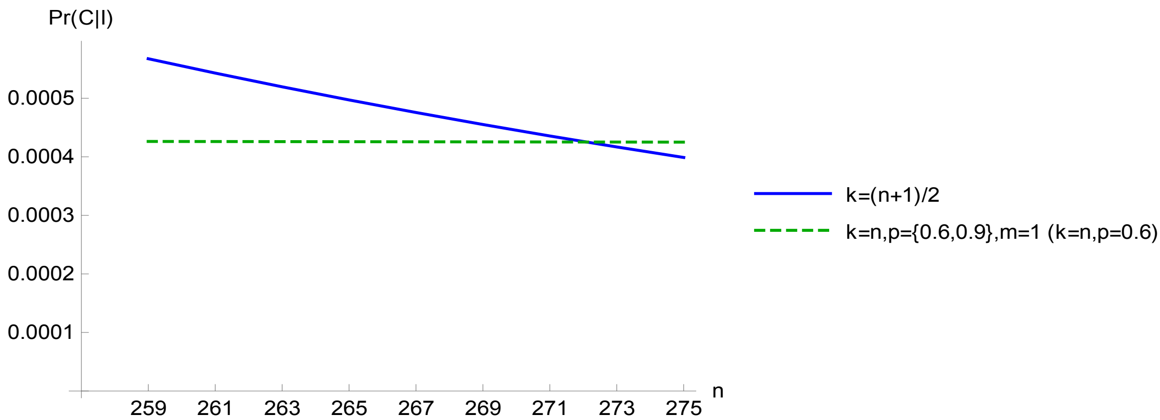
Type I Error, 'True' $p = 0.6$, $q = 0.6$										
n	5	7	...	27	29	31	...	43	45	47
Canonical Model										
$k = \frac{n+1}{2}$	0.32	0.29	...	0.14	0.14	0.13	...	0.09	0.09	0.08
$k = n$	0.08	0.08	...	0.09	0.09	0.09	...	0.09	0.09	0.09
Two-point Prior Model										
$k = \frac{n+1}{2}$	0.32	0.29	...	0.14	0.14	0.13	...	0.09	0.09	0.08
$k = n, m = 1$	0.08	0.08	...	0.09	0.09	0.09	...	0.09	0.09	0.09
$k = n, m = 0$	0.13	0.13	...	0.14	0.14	0.14	...	0.14	0.14	0.14

Table A.2 Type I Error of Different Voting Rules under The Canonical Model and The Two-point Prior Model, Given 'True' $p = 0.6$ and $q = 0.9$

Type I Error, 'True' $p = 0.6$, $q = 0.9$										
n	5	7	...	65	67	69	...	269	271	273
Canonical Model										
$k = \frac{n+1}{2}$	0.32	0.29	...	0.05	0.05	0.05	...	0.0005	0.0004	0.0004
$k = n$	0.01	0.02	...	0.0005	0.0005	0.0005	...	0.0004	0.0004	0.0004
Two-point Prior Model										
$k = \frac{n+1}{2}$	0.32	0.29	...	0.05	0.05	0.05	...	0.0005	0.0004	0.0004
$k = n, m = 1$	0.01	0.02	...	0.0005	0.0005	0.0005	...	0.0004	0.0004	0.0004
$k = n, m = 0$	0.05	0.04	...	0.04	0.04	0.04	...	0.04	0.04	0.04



(a) ‘True’ $p = 0.6$, $q = 0.9$



(b) ‘True’ $p = 0.6$, $q = 0.9$

Fig. A.1 Type I Errors of Different Voting Rules under The Canonical Model and The Two-point Prior Model, given ‘True’ $p = 0.9$, $q = 0.9$ for larger n

Table A.3 Type I Error of Different Voting Rules under The Canonical Model and The Two-point Prior Model, Given 'True' $p = 0.9$ and $q = 0.6$

Type I Error, 'True' $p = 0.9$, $q = 0.6$								
n	5	7	9	11	13	15	17	19
Canonical Model								
$k = \frac{n+1}{2}$	0.009	0.003	0.001	0.00	0.00	0.00	0.00	0.00
$k = n$	0.029	0.036	0.039	0.042	0.044	0.045	0.046	0.047
Two-point Prior Model								
$k = \frac{n+1}{2}$	0.009	0.003	0.001	0.00	0.00	0.00	0.00	0.00
$k = n$, $m = 1$	0.001	0.014	0.017	0.019	0.020	0.021	0.021	0.022
$k = n$, $m = 0$	0.029	0.036	0.039	0.042	0.044	0.045	0.046	0.047

Table A.4 Type I Error of Different Voting Rules under The Canonical Model and The Two-point Prior Model, Given 'True' $p = 0.6$ and $q = 0.9$

Type I Error, 'True' $p = 0.9$, $q = 0.9$								
n	5	7	9	11	13	15	17	19
Canonical Model								
$k = \frac{n+1}{2}$	0.0086	0.0027	0.0009	0.0003	0.0001	0.00	0.00	0.00
$k = n$	0.0028	0.0038	0.0045	0.0049	0.0052	0.0055	0.0057	0.0058
Two-point Prior Model								
$k = \frac{n+1}{2}$	0.0086	0.0027	0.0009	0.0003	0.0001	0.00	0.00	0.00
$k = n$, $m = 1$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$k = n$, $m = 0$	0.0028	0.0038	0.0045	0.0049	0.0052	0.0055	0.0057	0.0058

Chapter 2

Collective Deliberation Under Non-common Prior: An Experiment

2.1 Introduction

In this chapter, we introduce our laboratory experiments aimed at evaluating the effects of different institutions on the voting outcomes when there exist potentially multiple priors with respect to the quality with which private information can be obtained. Experiments also help understand the plausible behaviour of jurors in such setting.

Our plan is to draw an experimental parallel to the theoretical analysis conducted in the first chapter of the dissertation: the two-point prior model, with a focus on applications to the jury trial case. Furthermore, we want to examine the impact of free-form communication on collective decision-making outcomes.

For that, we report observations from the first lab experiments aimed at examining the results of collective voting under different institutions, when (1) the accuracy of private information is not univocally determined (scope for multiple priors); (2) the decision makers' types, that is, the degree of their trusting nature vis–à–vis the quality of the information provided to them, can vary; and (3) the communication among voters is allowed for, or not. We employed a $2 \times 2 \times 2$ design. We varied (i) the information structures for the accuracy of private information received (probabilistic or two-point non-common prior), (ii) two different institutional designs for determining the group decision (simple majority or unanimity), and (iii) the availability of free-form communication prior to participants making their choices.

It is important to stress that our experimental setup captures the core elements of not only the judicial trials, but also many other different small group deliberations, such as medical consulting teams, recruitment and tenure decisions by university committees, government legislatures, and more.

From the experiments, we found that the jury paradox is not always reproduced, so that the realised type I errors under unanimity voting are smaller than those under majority voting, regardless of the information structures. In particular, when multiple priors are possible, as long as not everyone is fully trusting the quality of the information to be the highest among two possible values provided, both type I and type II errors are reduced. Furthermore, free-form communication helps to reduce both type I errors and type II errors in this environment. In particular, when decision makers are free to talk to the group members, their voting strategies are remarkably close to the predictions of the two-point non-common prior model. Additionally, free-form communication helps to eliminate the differences across different voting rules. We found that if the information structure is probabilistic, both type I errors and type II errors are not significantly different across voting rules when free-form communication is available; however, if the information structure allows for multiple priors, type I errors are not significantly different across voting rules when free-form communication is available, however their levels are much reduced.

The remainder of this chapter is organised as follows. Section 2.2 contains a review of the recent experimental work on voting. Section 2.3 explains the experimental design and protocol we adopted for our own experiments. In section 2.4, we present the detailed experiment data and our preliminary experimental evidence and results. Section 2.5 concludes. Appendices B.1-B.6 contain the supplementary materials for the experimentations.

2.2 Related Literature

The classic Condorcet Jury Theorem demonstrates that, under the simple majority voting rule, increasing the number of jurors who receive a private, imperfect, yet informative and independent signal, about the true state of the world (innocence or guiltiness of the defendant) enhances the probability of reaching a correct verdict by the jury. In the limit, as the size of the jury reaches infinity, the probability of convicting an innocent tends to zero (eliminating type I errors). This is thanks to the information aggregation process generated by each juror sticking to the basic behaviour of voting according to the signal if the accuracy of the signal is above 1/2, and to vote against it, otherwise. Even though following such behaviour would help the group reach the 'correct' decision in this environment, committing to this behaviour needs not be individually rational. A fundamental weakness of this classic result is that it heavily relies on the assumption that jurors do not follow any strategic behaviour when casting their votes: each juror is assumed to vote sincerely, regardless of whether following this strategy is compatible with a Nash equilibrium under any circumstances.

In line with the predictions of the Condorcet jury theorem, Blinder and Morgan (2000) conducted a statistical urn problem to test the group decision quality as opposed to the individual decision quality. They consider two urns, each of which contains 40 marbles but comes with different compositions of the marbles: one of the urns contains 50% white marbles and 50% black marbles; another urn contains either 70% white marbles and 30% black ones, or 70% black marbles and 30% white ones. The urn with the composition '50-50' are presented to all subjects in the beginning of the experiment. And they are told that no later than the 10th round, the initial urn will be replaced with another urn, which could have the composition either '70-30' or '30-70'. Each subject could randomly draw a marble from the urn with replacement in each round up to the 40th round. And subjects need to decide in which round did this switch happen. The authors found that not only the collective decision is better than the individual decision-making in terms of the quality, but also that the collective decision does not take longer time to be made than decisions made by individuals.

However, these results were challenged on the ground that jurors need not stick to the strategy of voting for the alternative which has the higher probability of realisation. Rather, they will consider being pivotal and vote strategically, that is, each voter behaves according to whether his/her own vote will make a difference on the final decision, especially if that involves the verdict of the defendant being guilty or innocent.

In this spirit, Ladha et al. (1996) conducted an experiment to test whether the sincere voting equilibrium is observed in practice. And if not, whether voters' votes can truthfully reflect their private information; and then, whether majoritarian collective decisions are better than individual decision outcomes. In the experiment, subjects form groups of three and are asked to guess the colour of a marble. Two urns are provided: one of the urns contains 60 white marbles and 40 black marbles and another urn contains 100 black marbles. Group decisions are determined by different institutional designs. In the first setting subjects are told that the marble will be drawn from the urn containing both white and black marbles. According to Condorcet jury theorem, all subjects would vote for white and the group decisions would be correct 60% of the time. However, for that to be so, all subjects should vote sincerely, a behaviour which is not observed in the experiment. In the second setting, subjects are given private information, which is a clue about the colour of the hidden urn type: if the hidden urn is of the first type, subjects can observe the colour of a marble, which is randomly drawn from the urn containing both white and black marbles with replacement; if the hidden urn is of the second type, the marble will be randomly selected from the urn containing only black marbles with replacement. In either case, the colour of the randomly drawn marble is privately revealed to each subject. The results of the second experiment indicate the fact that subjects vote while considering themselves being pivotal. The equilibrium characterised by all subjects voting informatively is not observed in the experiment. In other words, these results show that subjects vote strategically rather than informatively.

There is another important assumption, which is missing in the Condorcet jury theorem, that is the availability of communication/deliberation. Emulating jury trials, Guarnaschelli et al. (2000) presents a statistical urn experiment with two urns, each of which contains 100 marbles. However, these two urns are different in the composition of the colour of the marbles. One of the urns contains 30 red marbles and 70 black ones. Another urn contains 30 black marbles and 70 red ones. Each subject observes the colour of a marble, which is randomly drawn from the selected urn. Then, subjects vote in the pre-vote stage before entering the final voting stage, at which point the collective decision will be determined according to the given voting rule. The difference of this experiment from the one conducted by Blinder and Morgan (2000) is that it allows the non-binding pre-vote deliberation, which is the straw poll voting before the official voting. When pre-vote deliberation is not available, results provide evidence of subjects voting strategically, in that a large fraction of subjects vote against their private signals. Also, the bigger the jury size, the larger the fraction of subjects voting strategically. Results of the experiments demonstrate that when subjects are given the chance to deliberate before casting their final votes, most of them reveal their signals truthfully by voting informatively. Although, when the straw vote is available, the probability that subjects vote strategically gets lower under unanimous voting, the opposite happens when the adopted voting rule is simple majority.

Goeree and Yariv (2011) provide a more comprehensive experimental study of collective deliberation. They focus on the effect of free-form communication under different institutional settings. The differences between this study and the previous ones are: (1) subjects could have heterogeneous preferences; (2) besides simple majority and unanimity voting rule, the intermediate two-thirds majority rule is also considered; (3) the pre-vote deliberation is designed as free-form communication. The statistical urn experiments conducted are similar to Guarnaschelli et al. (2000). Through a series of experiments, they found that without free-form communication, subjects vote strategically. Also, simple majority rule delivers more efficient outcomes of group decisions than unanimous voting rule. However, the availability of free-form communication eliminates the institutional difference in terms of increasing the collective decision quality. The experimental results indicate that most of the subjects reveal their private information truthfully although they vote strategically. Moreover, the communication protocol consists of two phases: information sharing and opinion aggregation. Thus, subjects first reveal their private information truthfully and publicly, then give opinion on which alternative they should vote for; and, ultimately, vote for that alternative. These results suggest that the availability of free-form deliberation and the communication protocol are vital in the collective decision-making process.

Different from other common value experiments, Suzuki and Li (2016) set up a detail-free environment to test the robustness of the strategic voting and the efficiency of different voting rules. They did not use the standard urn game. Rather, they test individuals' voting strategies and the collective decision quality by asking them to identify the correctness of a math question and a logic question. In their experiments, there is no explicit signal structure. Therefore, voters' beliefs are based on their own computational skills and reasoning abilities. Thus, voters' beliefs could be wrong due to the computational and reasoning errors. Moreover, voters might not realise that they form the wrong beliefs and they are uncertain whether their group members form the correct beliefs. In the experiments, Suzuki and Li (2016) found that subjects' behaviour is consistent with the pivotal reasoning regardless of the detail-free information structure. They also found that there are more approval votes if unanimity rule is adopted than the case when majority rule is selected. However, the focus of Suzuki and Li (2016) is the individual voting behaviour. They did not compare the group level decision outcomes under majority and unanimity voting rules.

In our experiment, we provide subjects with another form of 'detail-free environment', in which the measure of the objective probabilities, representing the level of the accuracy of the information available to them, is not univocally determined and has unknown distribution. In this environment, it is plausible to conceive that subjects will form their own beliefs as to which accuracy level to adopt/adhere to, which allows us to categorise subjects into two groups, or types, according to their adopted/subjective beliefs regarding the true state of the world, which are based on their beliefs in (weighting of) the private information they receive. This framework gives us a chance to not only analyse the individual voting behaviour, but also evaluate collective decision-making quality under different institutional designs.

2.3 Experimental Design and Protocol/Execution

Our experiments employ a $2 \times 2 \times 2$ design. Namely, our 8 experimental treatments involve (i) two different imperfect informational structures for the accuracy of signals received (probabilistic or two-point ambiguous structure), (ii) two different institution settings or voting rules for the group decision to be made (simple majority or unanimity), and (iii) two communication protocols, that is, whether or not free-form communication is available to subjects prior to them making their choices.

We used those 8 treatments to conduct an array of experiments to emulate a jury decision-making process. To be specific, preceding each session, subjects were asked to participate in a trial experiment to familiarise with the process. Sessions were designed so that in each experimental session, groups of five subjects were confronted with making a collective decision between one of two alternatives, representing neutral metaphors for either an acquittal or a conviction. Before casting their votes, subjects received private signals regarding the true state of the world. The private signals so received played an homologue role to the evidence and testimonies produced in a trial. Individual decisions (votes) translated into the group decision in favour of a given state of the world as a function of the voting rule applied. Subjects received the corresponding rewards reflecting the quality of the group decision(s) they were part of, that is, whether their group decision(s) was correct, or wrong, with respect to the underlying 'true' state of the world.

The experimental sessions were all computerised using the z-Tree software (Fischbacher, 2007). For each of the treatments envisaged, after all subjects arrived, instructions were provided by the experimenters and time was given to subjects to go through those instructions and to answer open questions, if any, before the experiment started. Each experimental session involved 20 decision tasks, one for each separate/independent rounds/periods.

For each of those sessions, we presented jars of potentially different composition to the subjects. Jars could either be red or blue. Each jar contained 10 balls with either majority of red or blue coloured balls. The 'red' jar was referred to the jar, which contained more red balls than blue ones; and the 'blue' jar was referred to as the jar, which contained more blue balls than red ones. The red (blue) jar was a metaphor for the underlying true state of the world, namely the state of the world in which the defendant happened to be guilty (innocent).

At the start of each of those 20 rounds, one of the jars was randomly chosen, and subjects were randomly and anonymously matched to a group of five then labelled from 1-5 by the computer. Then, each subject received a hint about the true colour of the selected jar by picking a ball from the chosen jar independently with replacement. The colour of the ball was revealed only to the subject himself/herself, which represented a token of the private information in our setting. Ultimately, subjects needed to cast their individual votes, which then contributed to the group-decision about what the true colour of the chosen jar was (believed to be). No feedback regarding the quality of the individual and each group's decision was provided to subjects after each rounds. Instead, each subject was paid according to whether the resulting collective decision of one of those round picked at random was correct or wrong. Subjects received a $10 show up fee and at the end of the experiment they also received a bonus of $15 if the randomly selected round for the group decision they were involved in was correct, and $5 otherwise. Any period of the experiment could have counted toward such payment, and, thus, subjects needed to pay attention throughout the experiment to maximise their expected payment for the collective deliberations they participated in.

As indicated above, there are three important components of this experimental design: the private information structure each subject faces, the voting rule in place, and the availability of free-form communication.

Private Information Structure: At the beginning of each period, subjects are informed that either the red jar or the blue jar was selected at random with equal probability. And then, each member within the group received an independent draw (with replacement) from the jar which was selected beforehand. In the baseline case of a probabilistic setting, subjects were given only two jars, each of them contained 10 balls with colour either red or blue. The red jar contained 6 red balls and 4 blue balls, and the blue jar contained 6 blue balls and 4 red ones. Thus, the subjects were made aware that the colour of the drawn ball matched the jar's colour with probability $p = 0.6$ (or 60%), which is referred to as the accuracy of the private signal. In the two-point non-common prior setting, four compositions of red and blue balls were feasible for those jars instead: there were two alternative ways in which a 'red' jar could come about, either when it contained 6 red balls, and 4 blue ones; or when it contained 9 red balls and 1 blue ball. The 'blue' jar could either be one which contained 6 blue balls and 4 red balls, or 9 blue balls and 1 red ball. The subjects were shown possible variations of the relative probability with which either the 'red' or the 'blue' type jars could come about, which did not leave any hints as to how such distribution happens with any regularity. To avoid participants to form intermediate priors, for example 0.75, rather than 0.6 or 0.9, we adopted the method Stecher et al. (2011) suggested in their paper to create true ambiguity. A graph including 50 histograms for the occurrence of either of the compositions, each consisting of 1000 times of such occurrence is provided to participants who are in the 'ambiguous' treatment. The purpose of presenting such histogram graph was to show the participants that there was no known prior, no known ex-ante distribution for either of the two possible realisations, either red or blue. All that was known was that the composition of an urn could either be 60-40 or 90-10 and nothing more. In addition, from the histograms, participants were not able to generate any patterns or regularity of the objective distribution of the possible compositions, which is indeed what unmeasurable probabilities (or ambiguity specifically) indicates, so that in this setting the objective probability was genuinely unmeasurable.[1] Thus, if the chosen jar was 'red' ('blue') in the sense just described, we could have had p equal to either 0.6 or 0.9. Once a jar was chosen, subjects could see the colour of a ball drawn at random from that jar, which served as their private signal before the casting of their votes. Their individual votes then determined the collective choice within the group they belong

[1]To understand this further, think of the parallel example of two possibly unfair coins, which could be selected: one of them is tilted to show one side 60% of the time, when flipping it many times; whereas the other unfair coin shows that same side 90% of the time, when flipped many times. And, one does not know which of the two coins is selected, prior to flipping it many times, making it impossible to predict the exact frequency of the possible realisations of heads and tails, a priori.

to, also depending on the selected voting rule (simple majority or unanimity) for the session they participated in.

Voting Rules: The voting rule was explained to the subjects at the outset of the experiment, depending on the threshold k given to reach the group decision/consensus, where the red jar was the group choice if and only if k of n subjects within the group voted for red. The colour red of our experiments can be taken as a metaphor for 'guilty', so that the colour 'blue' represents 'innocent', when interpreting results as emulating jury-trials. In the experiments, there were two types of treatments, corresponding to different voting rules: $k = 3$ (simple majority), and $k = 5$ (unanimity). Under unanimity, the pre-set default colour was blue, whenever subjects failed to reach an unanimous consensus for red.

Communication: In the 'communication' treatment, subjects first observed the private signal, then, they were free to communicate with their group members through the shared chat box that appeared on each of their screens. Messages could have taken any (written) form and sufficient time was given to the communication process before subjects were asked to submit their individual votes, which then contributed to the group decision. In the 'non-communication' treatment, subjects casted their votes directly after observing their private signals.

Appendix B.1 provides an illustrative sample of the experimental instructions we used for the sessions: risky treatment under majority voting rule without communication and ambiguity treatment under unanimity voting rule with communication.

2.4 Data and Preliminary Results

Based on our experimental design and protocol, and after receiving all relevant ethical clearances from the Human Ethics Committees of both the University of Auckland and Massey University[2], an array of experiments were conducted on 2–9 October 2015 at DECIDE (Laboratory for Business Decision Making), an experimental lab based at the University of Auckland. We recruited overall 165 subjects to participate in the experiments through the ORSEE system (Greiner, 2004) among the students of the University of Auckland[3]. In the course of these experimental sessions, we collected 3300 individual decision-making results and 660 group decision-making outcomes.

Using our experimental data, we start by comparing the realised individual strategies with the theoretical predictions of the two-point non-common prior model. Table 2.1 and Table 2.2 summarise participants' strategies and the realised errors for all treatments. The theoretical predictions of the two-point non-common prior model are presented in the round parentheses. As we can see, when free-form communication is not available before voters cast their votes, the experimental evidence both for the risky treatment and the ambiguity treatment is in line with what the theory predicts for each of those settings. First of all, in the absence of communication, reaching an unanimous decision is not easy. Thus, having a default set to failing to convict whenever not reaching unanimity, in order to emulate real-life situations, leads to observing a very small occurrence of type I errors under this voting rule for a small number of jurors, as expected. In other words, for a small jury size as predicted by the theory – and in line with previous experimental evidence, at least for the risky treatment equivalent to our experiments – the jury paradox is not reproduced, so that the realised type I errors under unanimity voting are smaller than those under majority voting, both in the

[2]Ethical approvals to conduct this research with human subjects were obtained from both the Massey University Human Ethics Committee and the University of Auckland Human Participants Ethics Committee on the 5th of March 2015 and on the 18th of June 2015 (with reference number 014565), respectively. Both ethical clearances are valid for a duration of three years. And the details of the Ethics clearance can be found in Appendix B.2.

[3]A copy of the invitation letter used for recruiting participants through ORSEE is provided in B.3. Also, extra material, which includes the participant information sheet, participant consent form and the experimental payment slip/receipt, is also provided in Appendix B.4, Appendix B.5 and B.6

risky and the ambiguous scenarios. Also, under the risky treatment, the probability of voting for red when receiving a blue signal with majority rule is significantly different from the probability of voting for red when receiving a blue signal with unanimous rule at the 5% level. However, when free-form communication is available, these two probabilities are not significantly different from each other. Furthermore, in the risky treatment without communication, both types of errors under different voting rules are significantly different at the 1% level. Nonetheless, the difference from different institutions are eliminated by the free-form communication. Similar results are observed under the ambiguity treatment. Differences in the probabilities of voting for red when receiving a blue signal and the difference in the type I errors under different voting rules are also eliminated when communication is viable.

In the treatment that allows for communication, we assume individuals could follow a specific strategy in equilibrium, that is to vote for the commonly preferred alternative within the group based on the signals they acknowledged through the communication process. The errors that would have resulted in the equilibrium in which voters were to believe what was reported in the communication stage are reported in the square brackets. If the information structure is purely risky – our baseline case – and individuals follow the commonly preferred alternative, they will commit higher type I errors; and, indeed, the higher type I errors are found in the experiment. However, when the information structure is ambiguous, free-form communication is very helpful in terms of lowering both types of errors, against the prediction of this specific strategy being followed in equilibrium as evidenced in other studies not allowing for multiple priors as we do; and, instead, more in line with our theoretical predictions, stemming from the two-point non-common prior model. And communication helps to eliminate the difference in the resulted type I errors from both the majority voting and the unanimity voting, although type II errors are still significantly different for any conventional levels of confidence.

In real life, if the quality of the information provided to jurors were genuinely ambiguous, neither would we be in a position to assess, nor even meaningfully discuss, what the ‘true’ level of accuracy really ought to be. We would not necessarily be able to pinpoint an exact probability measure for the quality of the information, limiting our ability to distinguish the real consequences of facing an ambiguous scenario. Instead, in an experiment we can design scenarios with and without ambiguity, starting with a predefined/specific level of accuracy for the risky treatment – our baseline treatment –, to compare results obtained in that environment against those obtained when altering the information, to allow the accuracy not to be measurable anymore. By identifying different levels for the 'true' p, we can derive the theoretical predictions for the two-point non-common prior model based on those, one at a time, to compare them against the evidence provided by the experimental data, as derived in a laboratory/controlled environment. By doing that, we find that the experimental data conforms very well with the predictions of the theoretical models considered, implying again that the availability of free-form communication results in lower errors. If we compare across the communication and the non-communication treatment, when the information structure is ambiguous, we can see that the level of the realised errors when communication is available, is closer to the two-point non-common prior model predictions, as illustrated in Table 2.3. This result suggests that individuals tend to vote symmetrically and responsively when their private signals are ambiguous especially when they are allowed to communicate with their group members before casting their votes. Similar results can be found when the true p is identified in different treatments.

To summarise, there are several insights we gain by comparing our experimental data with the model predictions.

(1) When information is purely risky – baseline treatment –, (1a) in the absence of free-form communication, individuals behave strategically, which is consistent with the predictions of the canonical model; (1b) when free-form communication is available, individuals tend to vote for the commonly preferred alternative, and, thus, collective decisions result in higher probability of committing the type I errors, and (1c) the difference between both type I and type II errors under different institutions is eliminated due to the communication,

although both types of errors are increased as opposed to the case when communication is not viable.

(2) When information is ambiguous (scope for multiple priors), (2a) individual's strategies are in the line with the predictions of the two-point non-common prior model if communication is not allowed during the collective decision-making process; (2b) however, if deliberations occur before individuals cast their votes, then, not only the difference between type I errors under different voting rules are eliminated, but individuals also vote symmetrically and responsively, much in the spirit of the two-point non-common prior model, matching the theoretical predictions very closely, and, consequently, resulting in smaller type I errors under the unanimity voting rule than under the majority voting rule.

Table 2.1 Experimental Realisations and Theory Predictions under Risky Treatment

	Without Communication		With Communication	
$n = 5$	$k = 3$	$k = 5$	$k = 3$	$k = 5$
Risky Treatment				
Number of individual decisions	400	500	300	300
Number of group decisions	80	100	60	60
Red votes with red signals	82% (100%)†	76% (100%)	85%/56%‡	100%/72%
Red votes with blue signals	17% (0%)	33% (60%)	39%/11%	100%/28%
Wrong jury outcomes	36% (32%)	35% (42%)	47% [47%]§	30% [31%]
True jar blue (Type I error)	37% (32%)	2% (25%)	45% [45%]	31% [34%]
True jar red (Type II error)	36% (32%)	81% (59%)	47% [47%]	29% [29%]

† Numbers in the round parentheses are the theory predictions without free-form communication.

‡ A pair of percentages x%/y% indicates the probability to vote to convict by a pivotal voter based on the realised signals, that is, when the optimal decision is red/blue.

§ Numbers in the square brackets are the resulted errors if voters vote for the commonly preferred alternative within the group based on the signals they acknowledged through the free-from communication process.

Table 2.2 Experimental Realisations and Theory Predictions under Ambiguity Treatment

	Without Communication		With Communication	
$n = 5$	$k = 3$	$k = 5$	$k = 3$	$k = 5$
Ambiguity Treatment				
Number of individual decisions	300	500	500	500
Number of group decisions	60	100	100	100
Sceptical types				
Red votes with red signals	63% (100%)	74% (100%)	100%/60%	93%/58%
Red votes with blue signals	26% (0%)	49% (60%)	53%/0%	80%/8%
Trusting types				
Red votes with red signals	93% (100%)	89% (100%)	98%/52%	90%/61%
Red votes with blue signals	3% (0%)	21% (50%)	28%/8%	91%/8%
Wrong jury outcomes	28% (16%)	38% (30%)	14% [10%]	27% [20%]
True jar blue (Type I error)	20% (15%)	5% (13%)	14% [9%]	8% [13%]
True jar red (Type II error)	37% (16%)	80% (50%)	14% [12%]	47% [28%]

Table 2.3 Experimental Realisations and Theory Predictions under Ambiguity Treatment with Specified 'True' p

	Without Communication				With Communication			
$n=5$	$k=3$		$k=5$		$k=3$		$k=5$	
	$p=0.6$	$p=0.9$	$p=0.6$	$p=0.9$	$p=0.6$	$p=0.9$	$p=0.6$	$p=0.9$
Type I error	36% (32%)	6% (1%)	8% (20%)	3% (7%)	31% [19%]	0% [0%]	15% [24%]	0% [0%]
Type II error	67% (32%)	7% (1%)	90% (65%)	60% (22%)	27% [22%]	0% [0%]	61% [46%]	29% [4%]
	$p=0.6$	$p=0.9$	$p=0.6$	$p=0.9$	$p=0.6$	$p=0.9$	$p=0.6$	$p=0.9$
Type I error								
$m<0.5$	42% (32%)	7% (1%)	11% (19%)	0% (6%)	30% [17%]	0% [0%]	19% [31%]	0% [0%]
$m>0.5$	0% (32%)	0% (1%)	0% (22%)	14% (8%)	33% [33%]	0% [0%]	0% [0%]	0% [0%]
Type II error								
$m<0.5$	64% (32%)	8% (1%)	88% (65%)	62% (22%)	30% [24%]	0% [0%]	63% [42%]	27% [6%]
$m>0.5$	75% (32%)	0% (1%)	100% (62%)	50% (19%)	0% [0%]	0% [0%]	56% [56%]	33% [0%]

2.5 Conclusion

In this chapter, we report results from an experiment designed to capture the collective voting behaviour under two-point non-common prior model and to contrast them against results of canonical collective voting behaviour models, such as those already studied, both theoretically and experimentally, in the existing literature. Our aim is to investigate the collective decision-making outcomes under different voting rules when the quality of the private information given to voters when casting their votes is somewhat unmeasurable, triggering voters to adopt potentially differing beliefs about it.

The experimental results proved that (1) the voters tend to vote strategically, consistent with the existing literature; (2) the 'Jury paradox' was not manifesting itself, given the relatively small size of the group voting collectively and consistent with the existing literature; (3) when group members form heterogeneous priors, both types I and II errors are reduced, this is a novel result in the literature; especially, when free-form communication is allowed before voters casting their votes; (4) if the information is purely risky – baseline treatment –, voters vote according to the majority of the signals they received within a group, consistent with the existing literature; (5) however, when information is ambiguous, voters vote symmetrically and responsively, this is a novel result in the literature.

The results of the experiments replicate the theoretical predictions of voting under two-point non-common prior model. This suggests the importance of the information structure in the collective deliberation outcomes. Our theoretical and experimental results call into question preconceived results about the performance of different institutional designs. When the objective probability of information is imprecisely measured, that is when the common-prior assumption is relaxed, novel results arise which deserve further exploration, challenging our views about the virtues of adopting, say, majority voting, as opposed to unanimity voting, to avoid the bad outcome of exacerbating the odds of convicting an innocent defendant (jury paradox). Our results help establish when, in the finite case, unanimity voting rule can outperform majority voting rule if voters adopts two-point non-common priors.

Although in our experiments we have tested only one specific type of imprecisely measured information structure, two simple voting rules and one particular communication protocol, the free-form communication, we believe that the qualitative conclusions hold more generally. In this respect, our experiments can be extended so as to capture more realistic set-ups with respect to collective deliberation processes. The simple way of doing so is to vary the jury size and/or let the information precision vary within an interval, to next check how voters' behaviour and the performance of alternative institutional voting rules could change to secure the lowest occurrence of instances of miscarriage of justice. Next chapter, chapter 3 is devoted to explore this question further. In particular, in chapter 3 we aim at addressing the question of whether when voters are confronted with information that comes with varying degrees of precision (within an interval), and could form subjective beliefs, say, according to the Maxmin approach, the performance of alternative voting rules replicates some of the flavour of the results obtained under the specific case of the two-point non-common prior; and, hence, whether there is any scope for resurrecting unanimity against majority, when facing an ambiguous world.

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Appendix B

Supplementary Materials for Experimentation

B.1 Samples of Experimental Instructions

B.1.1 Experimental Instructions for Risky Treatment without Communication under Majority Voting Rule

Welcome to the Lab

- Welcome and thank you for registering for today's experimental session!

- Shortly, we will be giving you some preliminary information and, if you choose to stay and participate in today's experiment, you will next receive more detailed instructions on it.

- From now on, please do not communicate with other participants.

***__Remember__**: Similar rules of behavior as during examinations apply to today's experimental session.*

Outline of the experiment

- You will participate in 20 rounds of decision tasks
  – At the beginning of each round, you will be randomly assigned to 4 other participants to form a group tasked with making a collective decision.
  – The decision reached within your group will determine your overall payment in this experiment.
- Remuneration
  – You will receive <u>$10 as a show-up fee</u>.
  – You will receive an <u>additional payment of either $5 or $15 depending on your group performance</u>.
- Ethics considerations
  – Participation is entirely voluntary and anonymous.

What's next…

- Earlier on:
  – When we invited you, we sent you a link to the **PARTICIPANT INFORMATION SHEET** with details of the experiment (instructions will come again, in greater details in a few seconds).
- Today:
  – If you agree to participate, <u>please sign one copy of</u> **PARTICIPANT CONSENT FORM** for the experimenter to collect (<u>one copy is for you to keep</u>).
  – If not, please raise your hand so that the experimenter can assist you with collecting your show-up fee of $10 and with leaving the experimental lab.

Before we get started:
Some 'House-Keeping' Rules

- Please use computers only as instructed.
  - Do not start or end any programs, unless told to do so.
  - Do not change any settings.

- Please only use the material provided.
- We will now give you instructions step by step.

***__Please note__**: If you have any questions either now, or during the experiment, raise your hand, and an experimenter will come and assist you privately.*

Experimental Instructions

- Today, you will participate in an experiment in group decision-making.
  - You will be involved in making decisions for 20 consecutive rounds.
  - In each round, you will be randomly matched to 4 other participants, with whom you will form a group, tasked with making a decision, for that round.
  - The entire experiment will take place through computer terminals.
  - All interaction between you and your group members will take place via the computer.

Respect this rule*: It is important that you do not talk to any other participants during the experiment.*

The Decision Task

- In each round,
  - ✓ You are randomized into a group of 5 members.
  - ✓ One of two colored jars is randomly chosen by the computer, with equal probability. Either
    - ➢ **Red** jar containing 6 red balls and 4 blue balls; or
    - ➢ **Blue** jar containing 6 blue balls and 4 red balls.
  - ✓ You get to see the color of a randomly selected ball for you, with replacement, from the chosen jar for that round.
    - ➢ ***Note:*** *The color you see and the one your group members see may not be the same.*
  - ✓ You and your group members each submit a decision about which is the chosen jar for that round, determining the group decision for that round.

Majority Voting

- Your group decision is based on majority voting.

  ➢ "Red Jar" is your group decision in a given round, if 3, 4 or 5 of the members you are matched with in that round choose "Red Jar" [e.g., even if you choose "Blue Jar" but 3 or more of your group members choose "Red Jar", the group decision is "Red Jar"].

  ➢ "Blue Jar" is your group decision in a given round, if 3, 4 or 5 of the members you are matched with in that round choose "Blue Jar" [e.g., even if you choose "Red Jar" but 3 or more of your group members choose "Blue Jar", the group decision is "Blue Jar"].

Your Payment in the Experiment

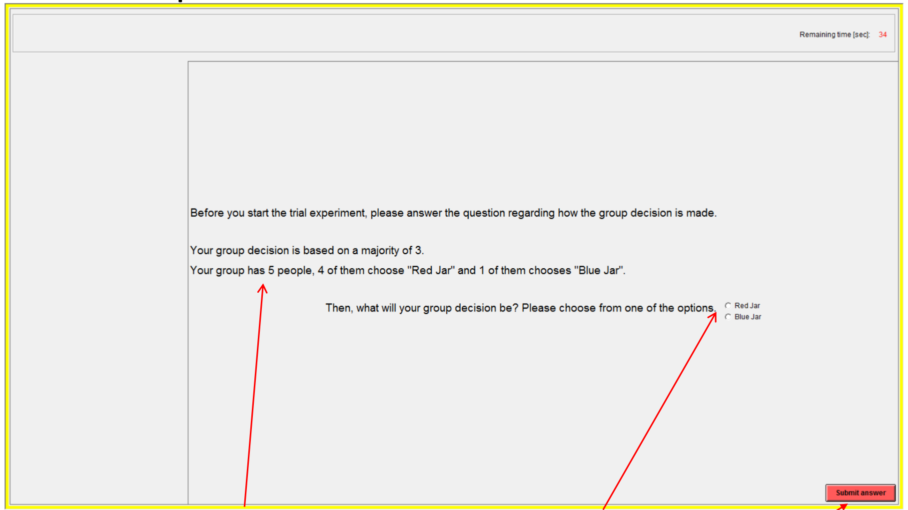
- You will be paid in cash
  - ✓ At the end of the experiment, the computer will randomly choose one of 20 rounds, and you will be paid according to whether the group decision for that round was correct.
    - ➢ **If your group decision is correct, you will get $25** as reward ($10 "show up" fee + $15).
    - ➢ **If your group decision is wrong, you will get $15** as reward ($10 "show up" fee + $5).
  - ✓ We will call you by your computer terminal number (see sticker on your desk) at the end of the experiment and pay you privately.

Note*: Different subjects may earn different amounts. You need not tell any other participant how much you earned.*

Some Warming Up Exercise!

- Before we get into the real experiment, let's do some practice (go through some trials).
- The next slides are meant to familiarise you
  - ✓ With the **voting rule** in place for the group decision.
  - ✓ With the **computer interface** you will use for this experiment.

Experimental Interface-Decision Rule

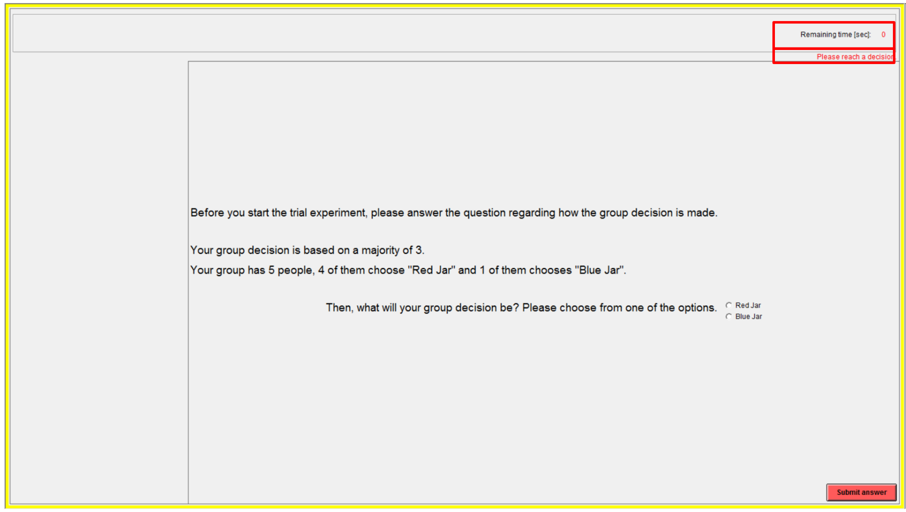


Before the real experiment starts, you will be asked about how a group decision is formed, for a specific fictitious scenario, to make sure the voting rule in place is well understood.

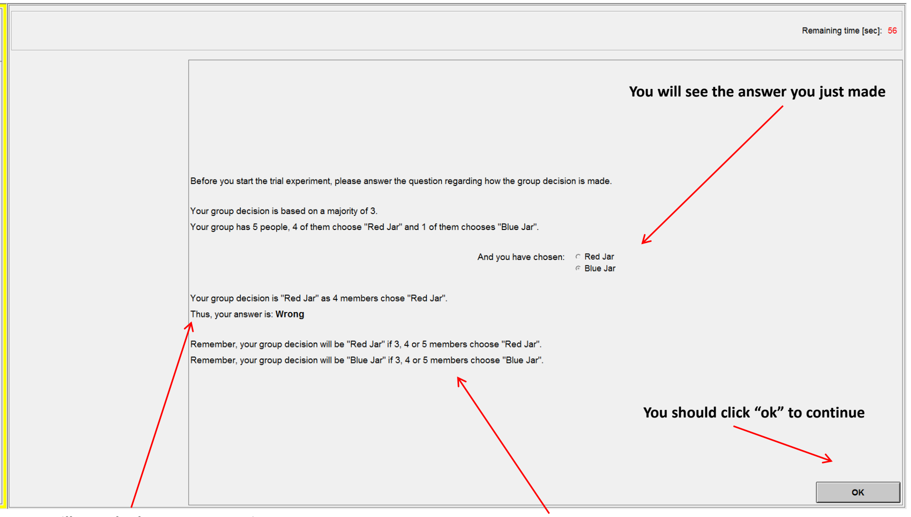
You need to choose either of two options

Then, click "Submit answer" to continue

Experimental Interface-Decision Rule



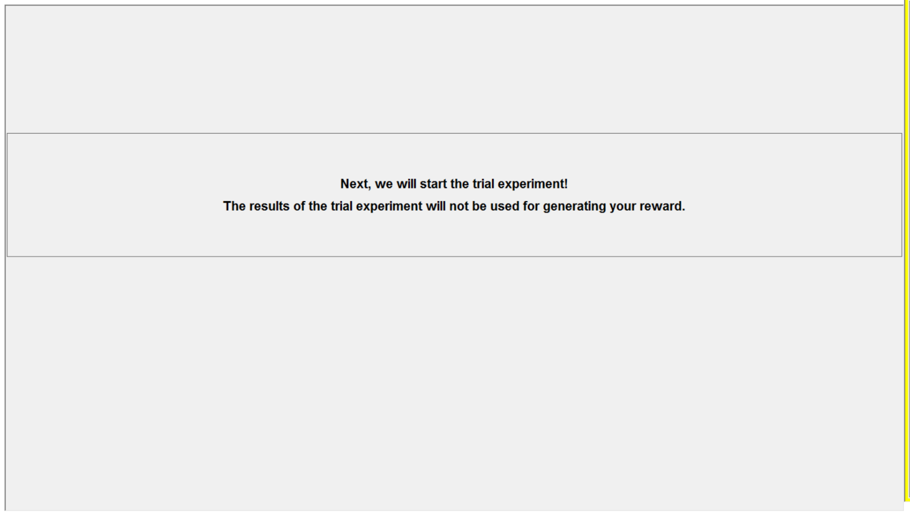
Experimental Interface-Decision Rule



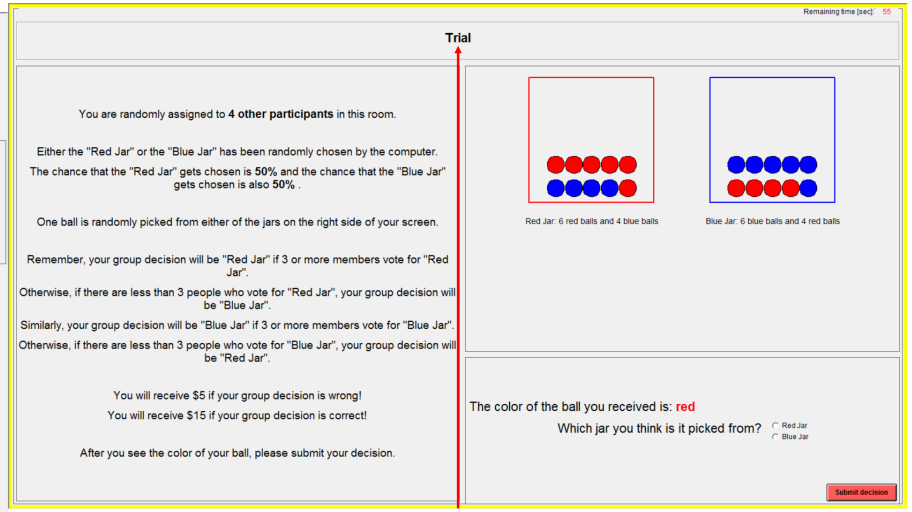
You will see whether your answer is correct

The computer reminds you again how the group decisions are made

Experimental Interface-Trial

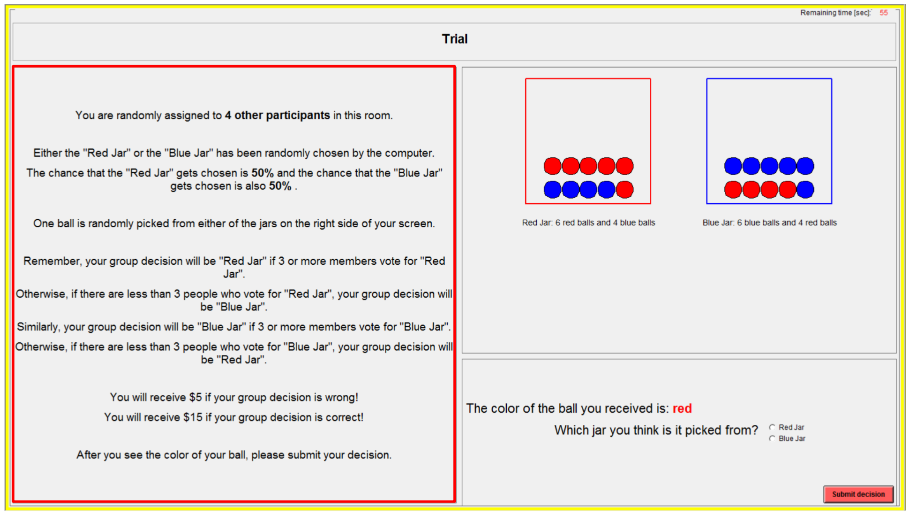


Experimental-Trial



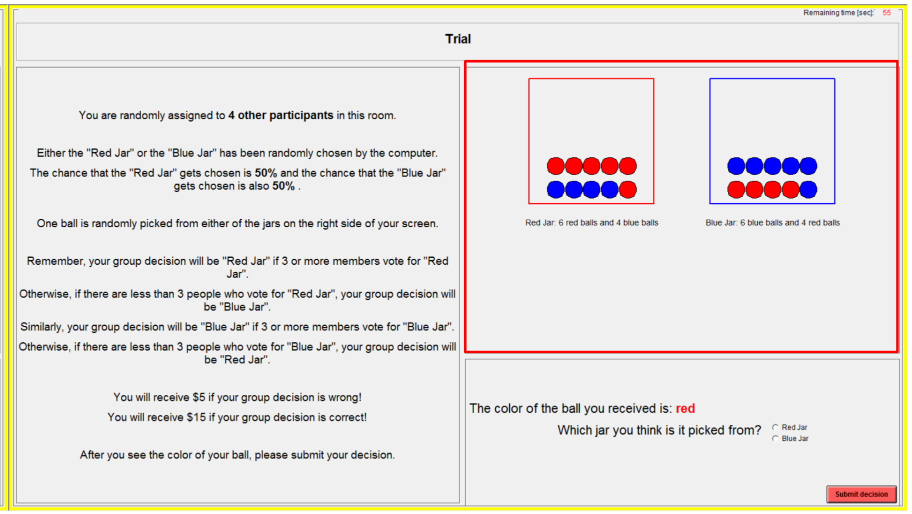
The top centre part of the screen indicates which round you are in. If you are in the trial experiment, it shows "Trial". If you are in round 1 of the experiment, it shows "Round 1 out of 20"

Experimental-Trial



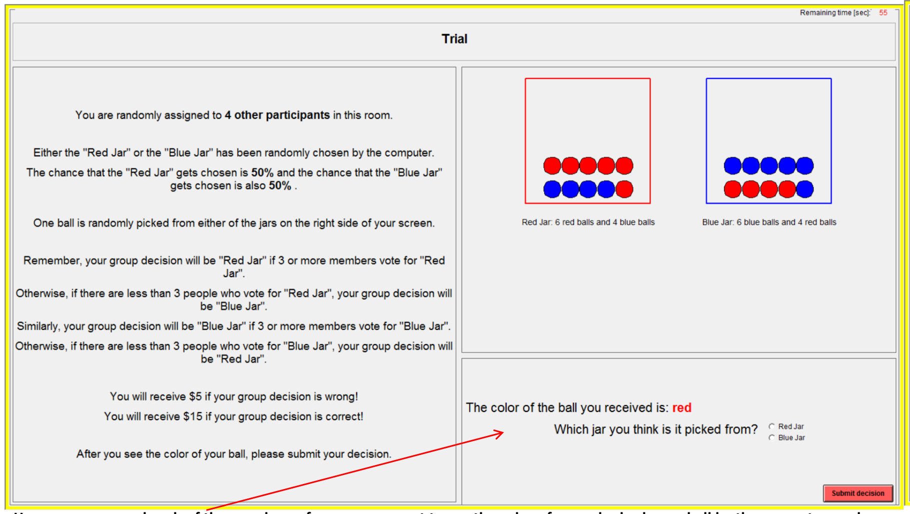
In the left side of the screen, important rules of this experiment are stated (the same as in the written instructions just covered)

Experimental-Trial



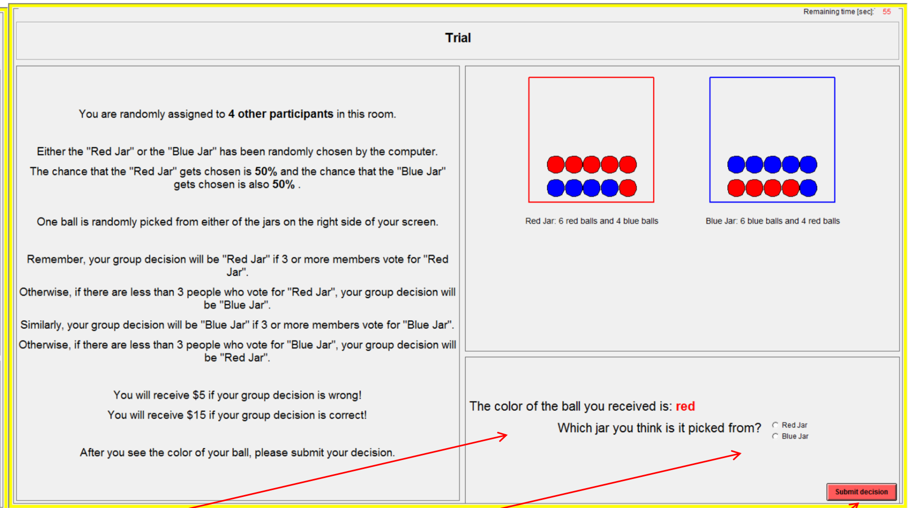
There are two possible jars: a "Red Jar" and a "Blue Jar" illustrated in the upper right portion of your screen. The red jar contains 6 red balls and 4 blue balls. The blue jar contains 6 blue balls and 4 red balls. One of the two jars will be chosen randomly by the computer, but you don't know which one

Experimental-Trial



However, you – and each of the members of your group – get to see the color of a ramdonly chosen ball by the computer, each with replacement, from the randomly selected jar. The color of the ball you get to see is displayed, before you make your choice which, together with your other group members' choices, contributes to the group decision

Experimental-Trial

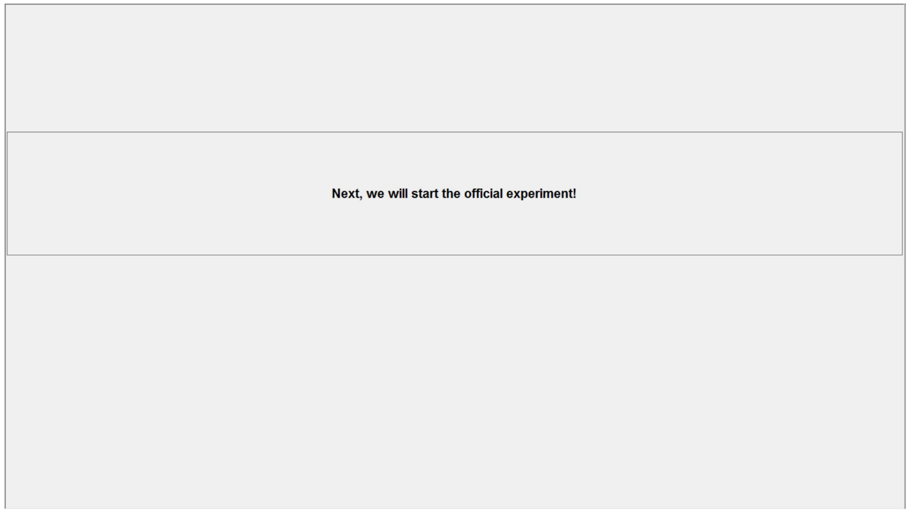


The color of the ball the computer randomly selected for you is displayed on your screen

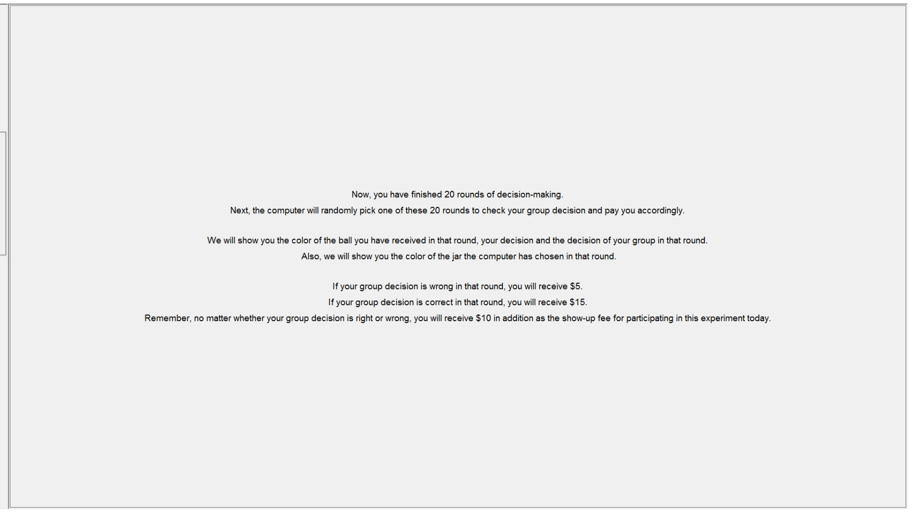
You need to make your decision on which jar you think the ball is picked from by choosing either of the options

Click "Submit decision" to continue

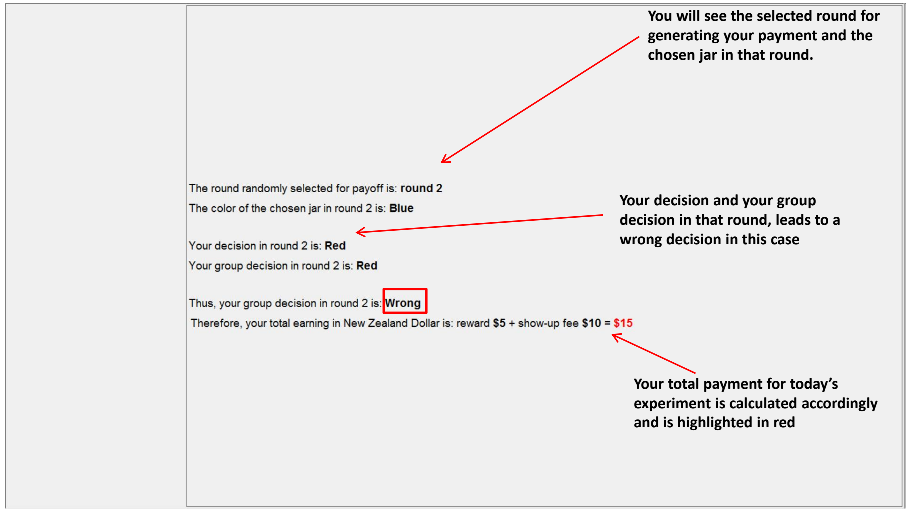
Experimental Interface-Official Experiment



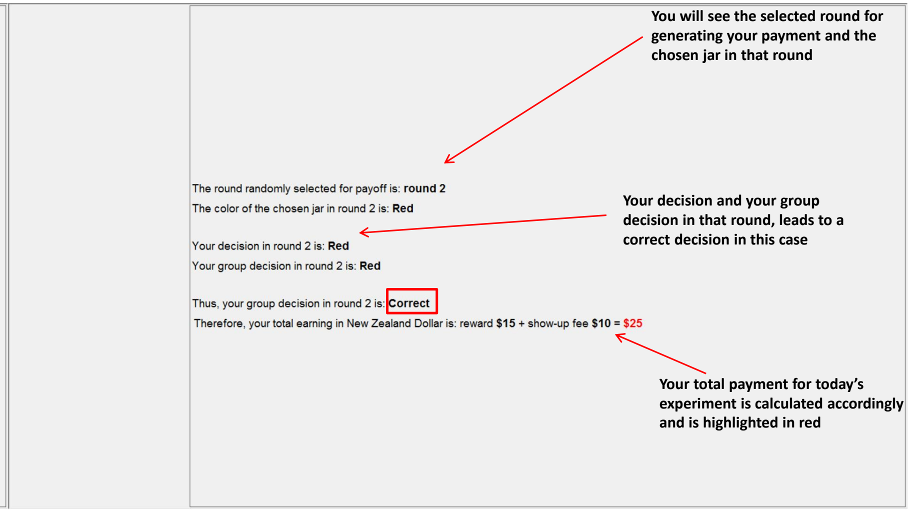
Experimental Interface-Payment



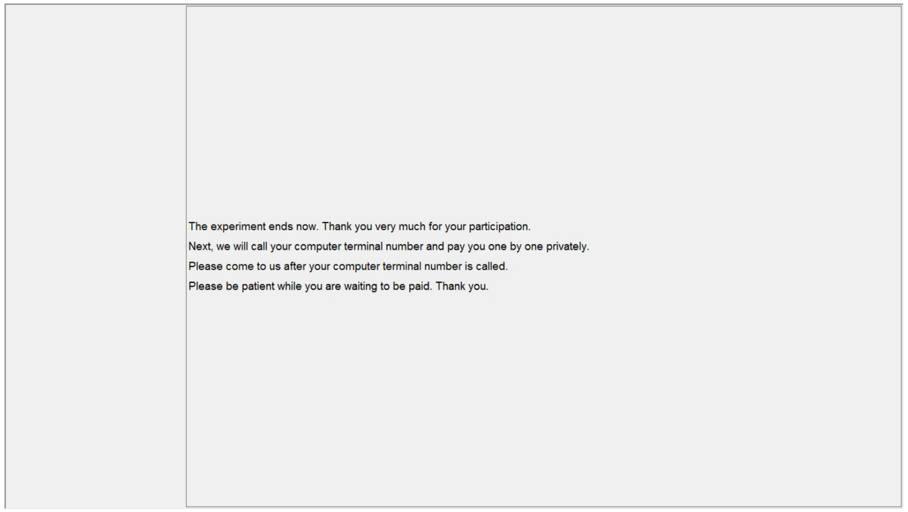
Experimental Interface-Payment



Experimental Interface-Payment



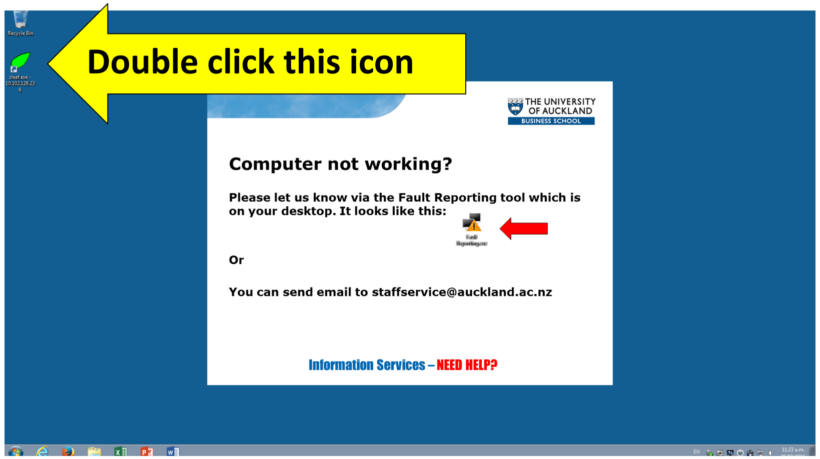
Experimental Interface-Payment



Let the Experiment Begin!

- If there are no questions, we will now begin the experiment.

To get started



B.1.2 Experimental Instructions for Ambiguity Treatment with Communication under Unanimity Voting Rule

Welcome to the Lab

- Welcome and thank you for registering for today's experimental session!

- Shortly, we will be giving you some preliminary information and, if you choose to stay and participate in today's experiment, you will next receive more detailed instructions on it.

- From now on, please do not communicate with other participants.

Remember: *Similar rules of behavior as during examinations apply to today's experimental session.*

Outline of the experiment

- You will participate in 20 rounds of decision tasks
  - At the beginning of each round, you will be randomly assigned to 4 other participants to form a group tasked with making a collective decision.
  - The decision reached within your group will determine your overall payment in this experiment.
- Remuneration
  - You will receive $10 as a show-up fee.
  - You will receive an additional payment of either $5 or $15 depending on your group performance.
- Ethics considerations
  - Participation is entirely voluntary and anonymous.

What's next…

- Earlier on:
  - When we invited you, we sent you a link to the **PARTICIPANT INFORMATION SHEET** with details of the experiment (instructions will come again, in greater details in a few seconds).
- Today:
  - If you agree to participate, please sign one copy of **PARTICIPANT CONSENT FORM** for the experimenter to collect (one copy is for you to keep).
  - If not, please raise your hand so that the experimenter can assist you with collecting your show-up fee of $10 and with leaving the experimental lab.

Before we get started: Some 'House-Keeping' Rules

- Please use computers only as instructed.
  - Do not start or end any programs, unless told to do so.
  - Do not change any settings.

- Please only use the material provided.
- We will now give you instructions step by step.

Please note: *If you have any questions either now, or during the experiment, raise your hand, and an experimenter will come and assist you privately.*

Experimental Instructions

- Today, you will participate in an experiment in <u>group decision-making</u>.
  - You will be involved in making decisions for 20 consecutive rounds.
  - In each round, you will be randomly matched to 4 other participants, with whom you will form a group, tasked with making a decision, for that round.
  - The entire experiment will take place through computer terminals.
  - All interaction between you and your group members will take place via the computer.

<u>Respect this rule</u>*: It is important that you do not talk to any other participants during the experiment.*

The Decision Task

- In each round,
  - ✓ You are randomized into a group of 5 members.
  - ✓ One of two possible colored jars is randomly chosen by the computer, with equal probability: a "Red Jar" or a "Blue Jar".
  - ✓ There are two types, with unknown distributions, in which a "Red Jar" and a "Blue Jar" can come about.
    - ➢ **Red-type** jar can either contain:
    - ✧ 6 red balls and 4 blue balls; or,
    - ✧ 9 red balls and 1 blue ball.
    - ➢ **Blue-type** jar can either contain:
    - ✧ 6 blue balls and 4 red balls; or,
    - ✧ 9 blue balls and 1 red ball.

The Decision Task (Continued)

- ✓ The computer randomly selects the composition of the red or blue jar which was previously chosen at random.
- ✓ <u>You do not get to see which jar color and which composition is chosen by the computer</u>, but what you get to see is the color of a randomly selected ball for you, with replacement, from the chosen jar (one ball for each decision round).
  - ➢ ***<u>Note</u>:*** *The color you see and the one your group members see may not be the same.*

Three-Step Decision Task

① You each first <u>choose which is the likely composition of the selected jar</u>.

② Next, you <u>observe the color of a randomly picked ball from the selected jar</u>, and have an opportunity to <u>chat with members of your group, via a chat-box</u>.

③ Finally, you are asked to <u>choose whether the selected jar is red or blue</u>, determining the group decision for that round.

Unanimity Voting

- Your group decision is based on unanimity voting.
  - "Red Jar" is your group decision in a given round, if all members in your group choose "Red Jar".
  - "Blue Jar" is your group decision in a given round, if at least one member in your group chooses "Blue Jar" .

Your Payment in the Experiment

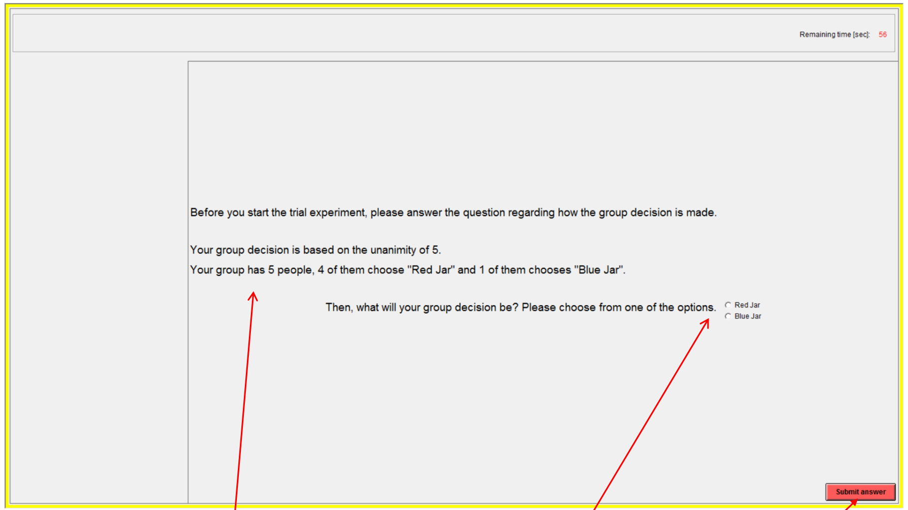
- You will be paid in cash
  - ✓ At the end of the experiment, the computer will randomly choose one of 20 rounds, and you will be paid according to whether the group decision for that round was correct.
    - **If your group decision is correct, you will get $25** as reward ($10 "show up" fee + $15).
    - **If your group decision is wrong, you will get $15** as reward ($10 "show up" fee + $5).
  - ✓ We will call you by your computer terminal number (see sticker on your desk) at the end of the experiment and pay you privately.

Note*: Different subjects may earn different amounts. You need not tell any other participant how much you earned.*

Some Warming Up Exercise!

- Before we get into the real experiment, let's do some practice (go through some trials).
- The next slides are meant to familiarise you
  - ✓ With the **voting rule** in place for the group decision.
  - ✓ With the **computer interface** you will use for this experiment.

Experimental Interface-Decision Rule

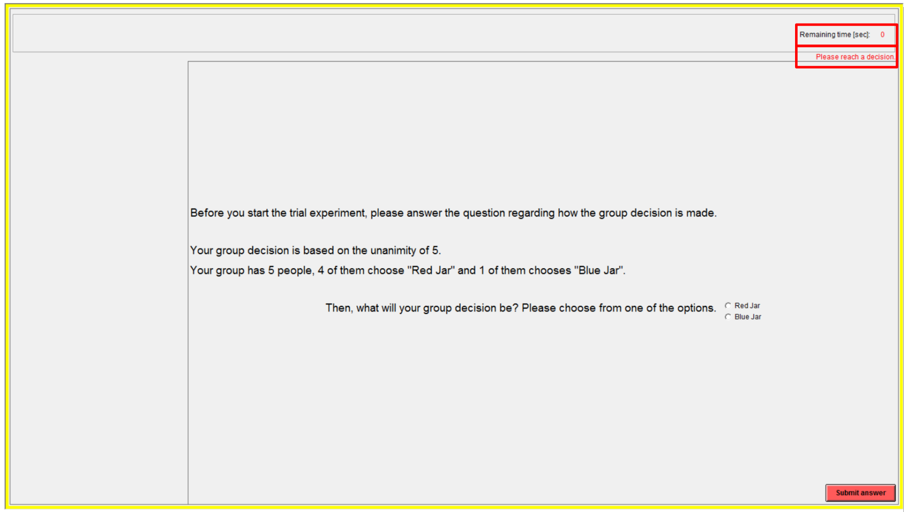


Before the real experiment starts, you will be asked about how a group decision is formed, for a specific fictitious scenario, to make sure the voting rule in place is well understood

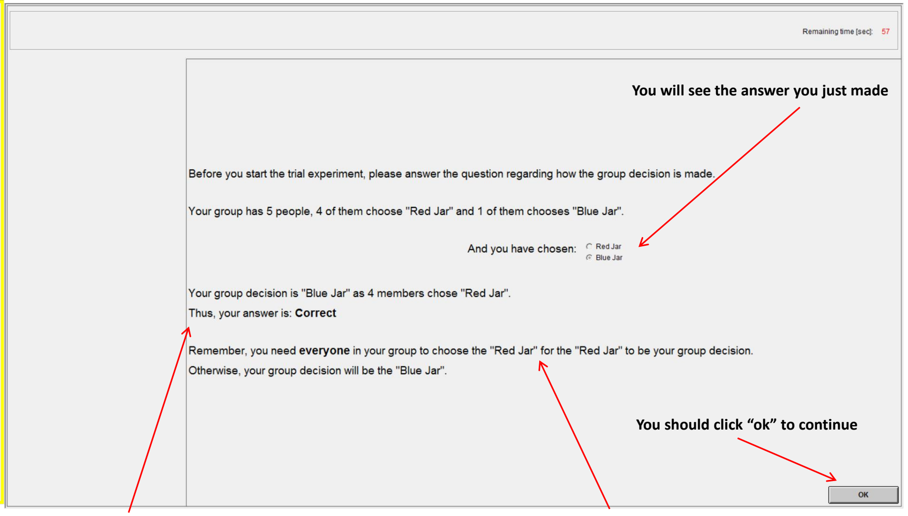
You need to choose either of two options

Then, click "Submit answer" to continue

Experimental Interface-Decision Rule

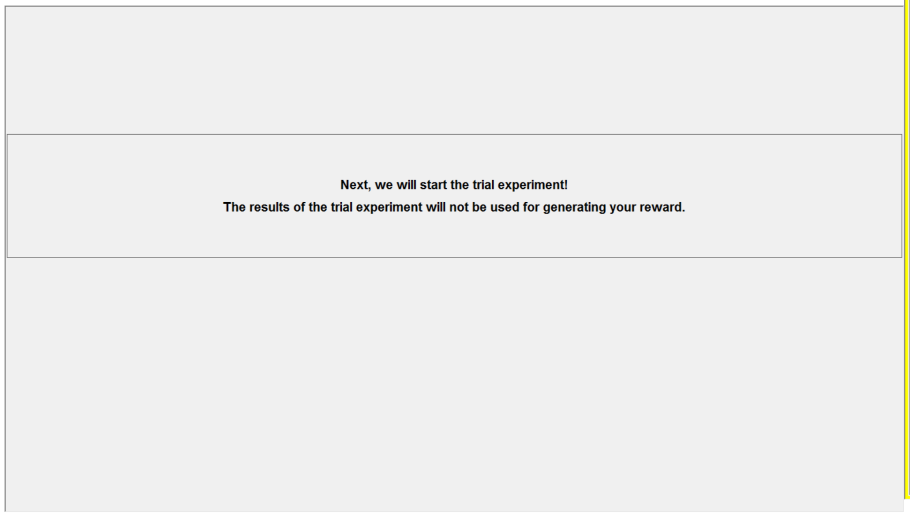


Experimental Interface-Decision Rule

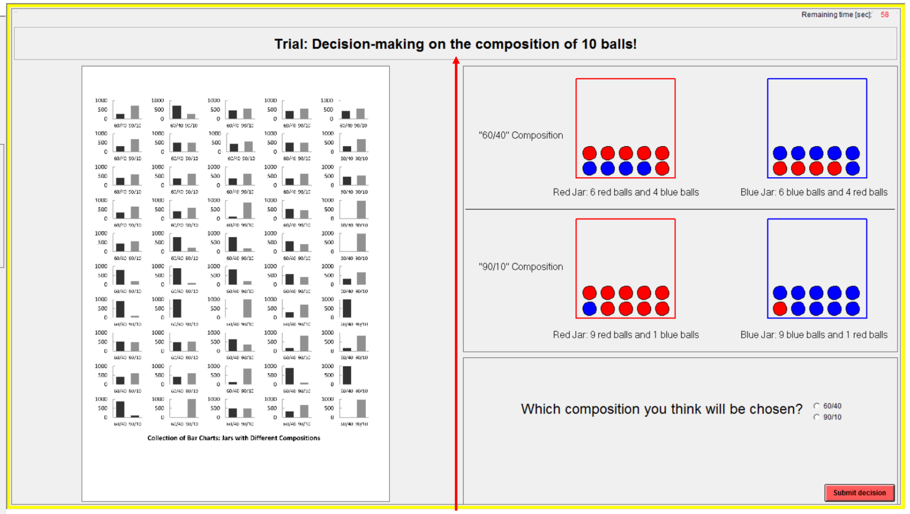


You will see whether your answer is correct **The computer reminds you again how the group decision will be made**

Experimental Interface-Trial

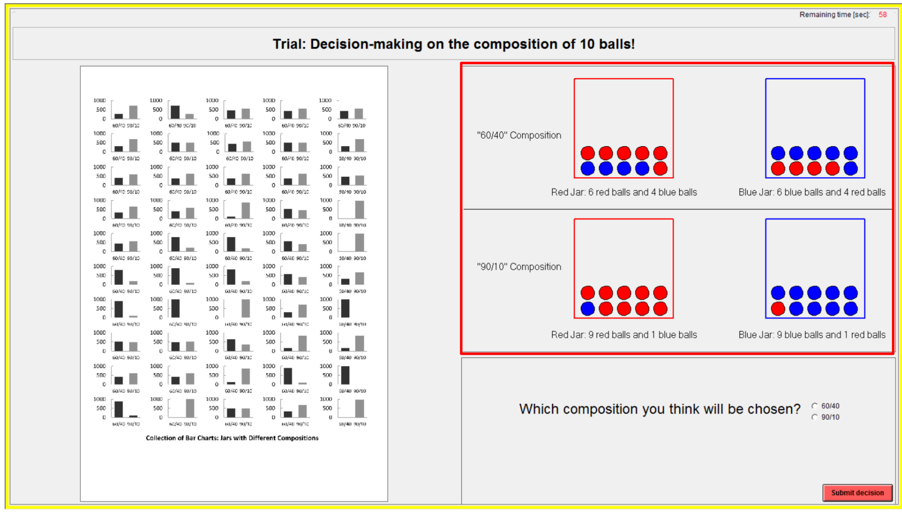


Experimental Interface-Trial



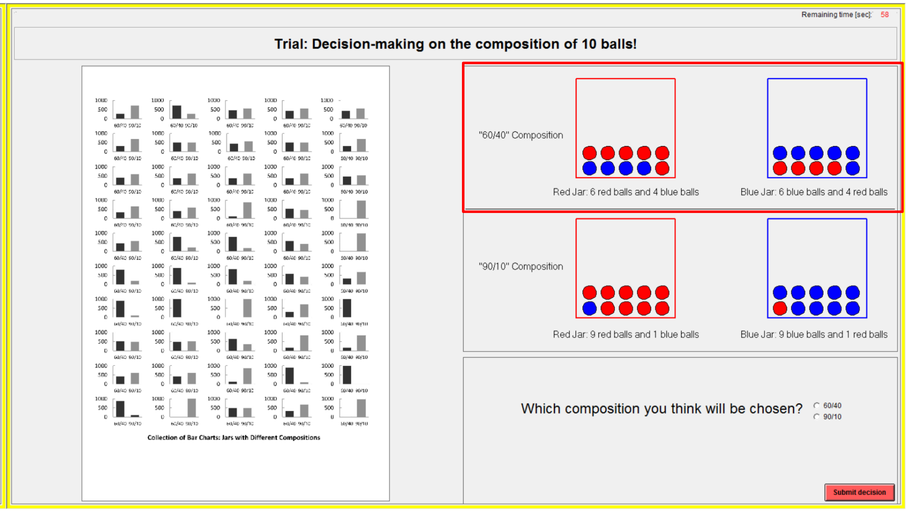
The top centre part of the screen indicates which round you are in. If you are in the trial experiment, it shows "Trial". If you are in round 1 of the experiment, it shows "Round 1 out of 20"

Experimental Interface-Trial



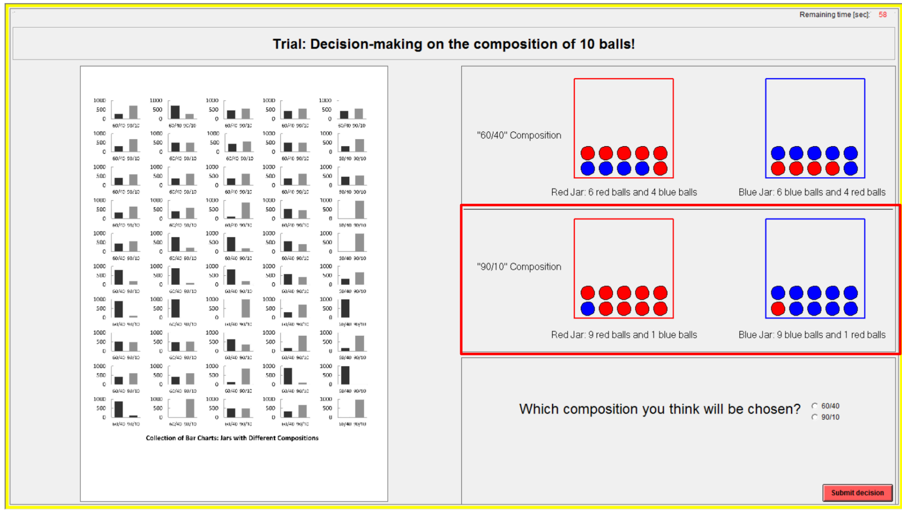
In the upper right portion of the screen, you can see 4 jars with different composition of red and blue balls

Experimental Interface-Trial



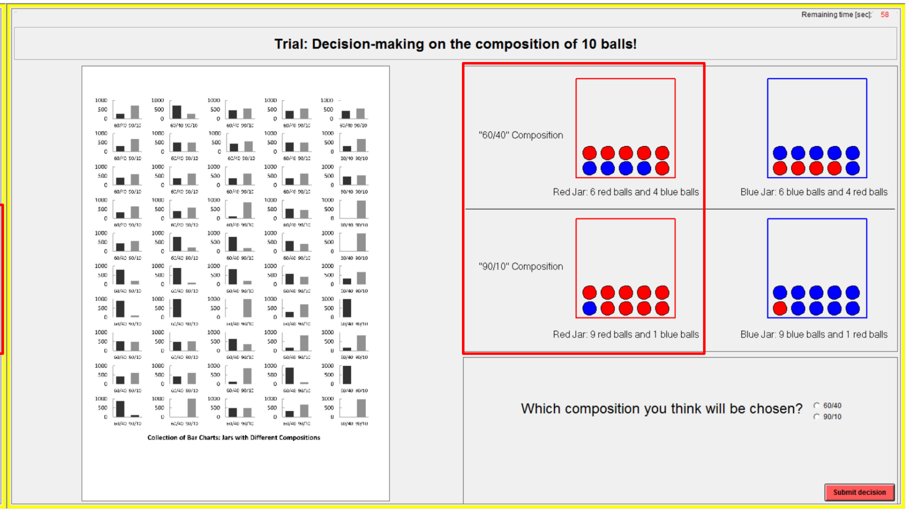
The 2 jars on the top level have the "60/40" and "40/60" composition. This means that those jars either have 6 red balls and 4 blue balls or 6 blue balls and 4 red balls, respectively

Experimental Interface-Trial



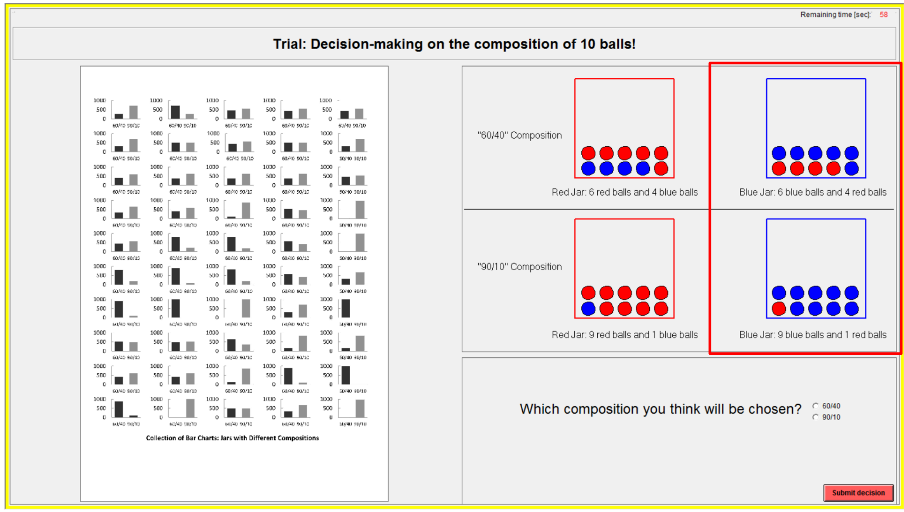
The 2 jars at the bottom level have the "90/10" and "10/90" composition. This means that those jars either have 9 red balls and 1 blue ball or 9 blue balls and 1 blue ball, respectively

Experimental Interface-Trial



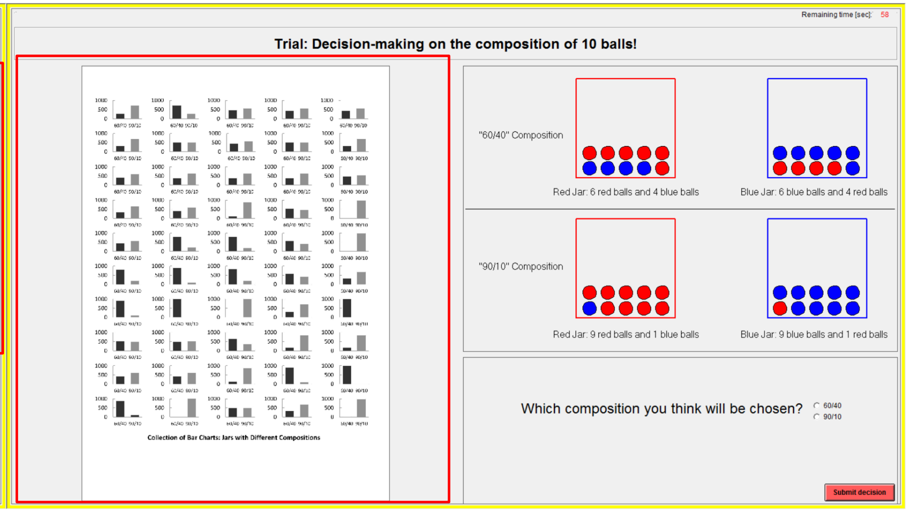
We refer to a "Red Jar" as the jar containing more red balls than blue balls, whether that red jar contains 6 or 9 red balls

Experimental Interface-Trial



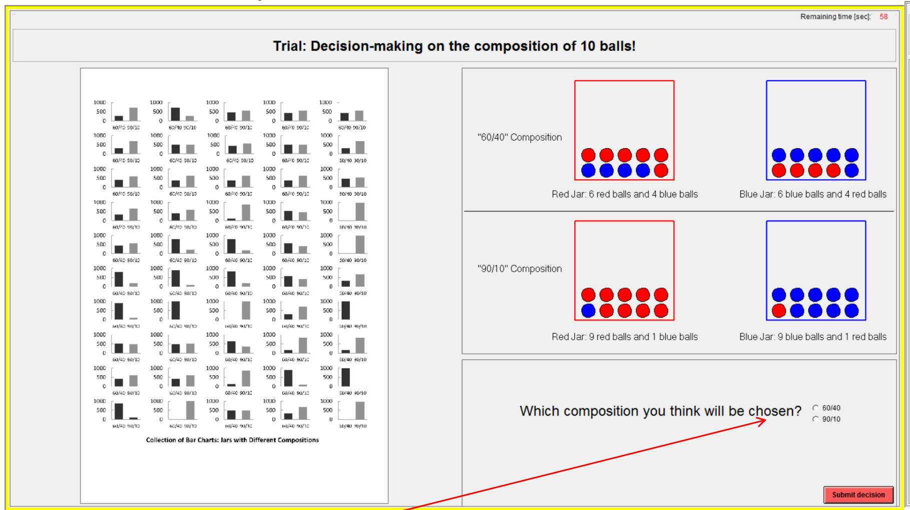
We refer to a **"Blue Jar"** as the jar containing more blue balls than red balls, whether that blue jar contains **6 or 9 blue balls**

Experimental Interface-Trial



In the left side of the screen, a graph is displayed, which contains an illustration of the possible relative frequency with which a specific composition can materialise, when repeating random draws 1,000 times, for each of those graphs (50,000 draws in total). The graphs illustrates that there is no known precise distribution for those compositions

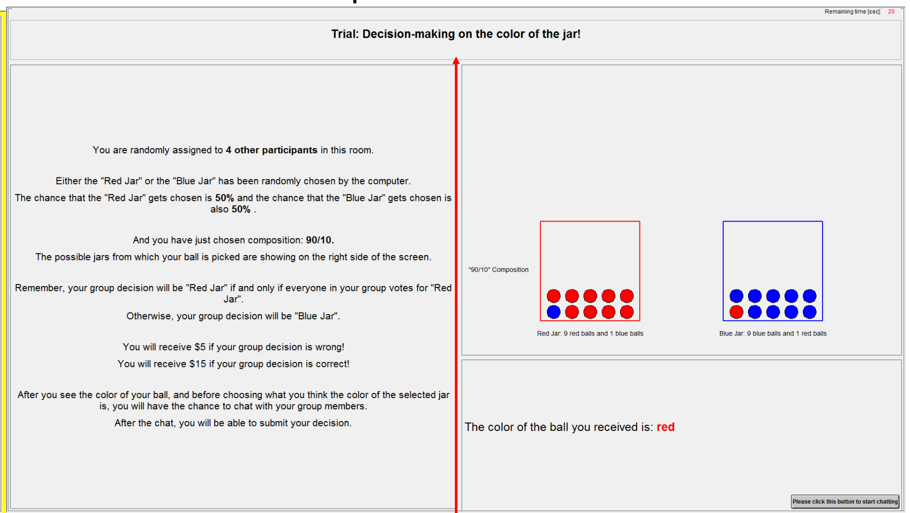
Experimental Interface-Trial



In this environment, you are asked to first make a choice with respect to what composition you think will be picked at random by the computer

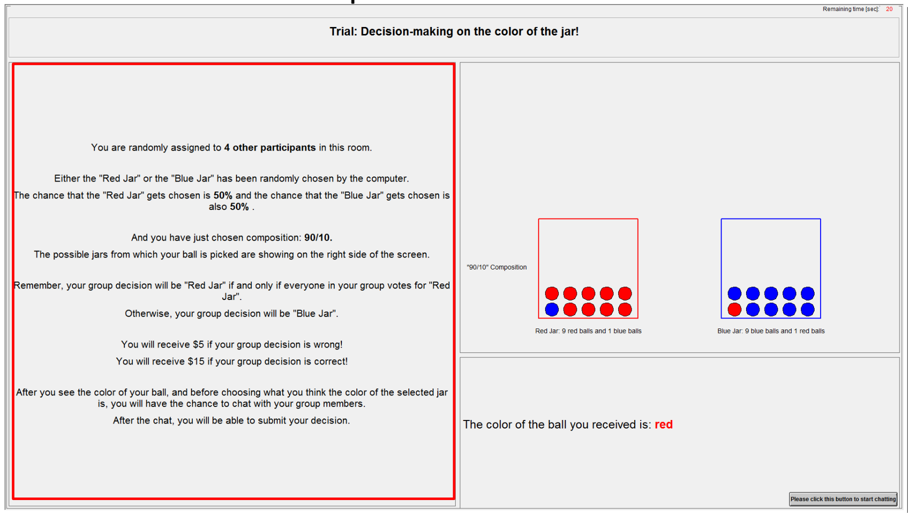
Then, you should click "Submit decision" to continue

Experimental-Trial



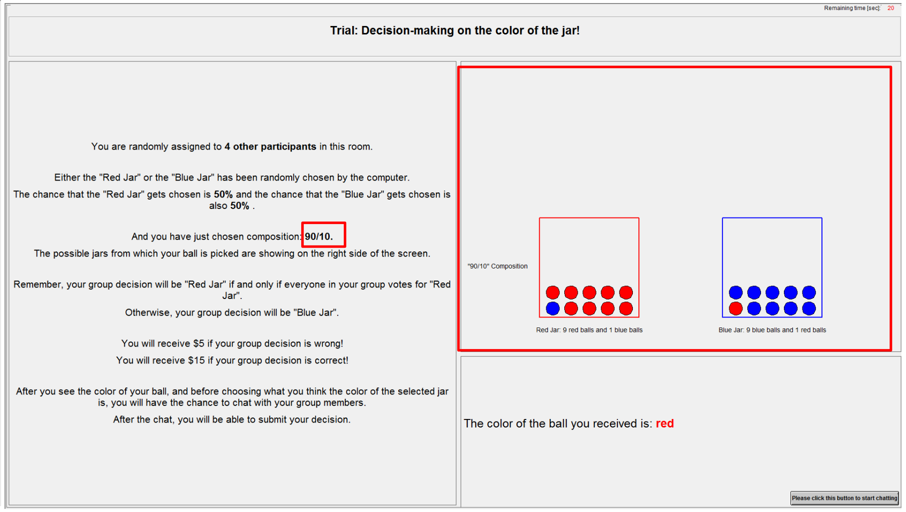
After observing the color of a randomly picked ball from the selected jar and before making your choice with respect to what is the color of the randomly selected jar, you have an opportunity to chat with the members of your group, via a chat box appearing on your screen

Experimental-Trial



To help you in that choice, in the left side of the screen, important rules of this experiment are stated (the same as in the written instructions just covered)

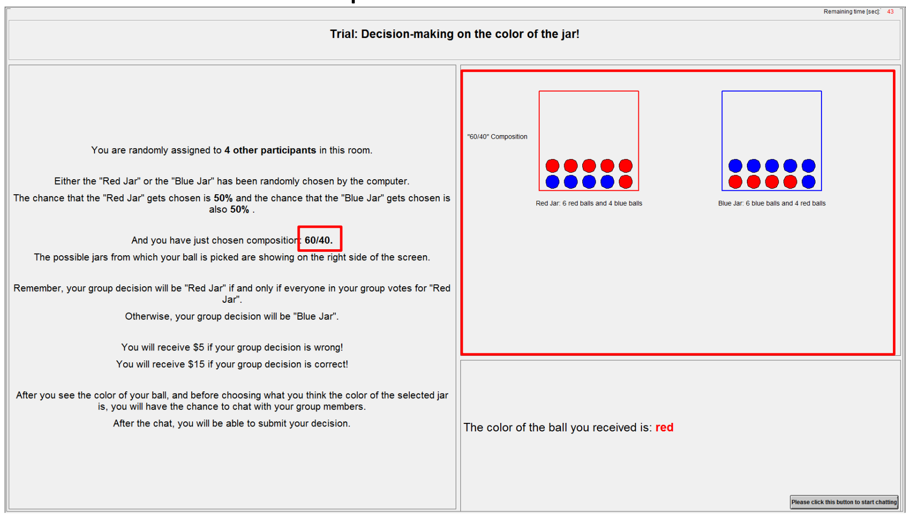
Experimental-Trial



While your decision on the composition you have just made is displayed in the left side of the screen…

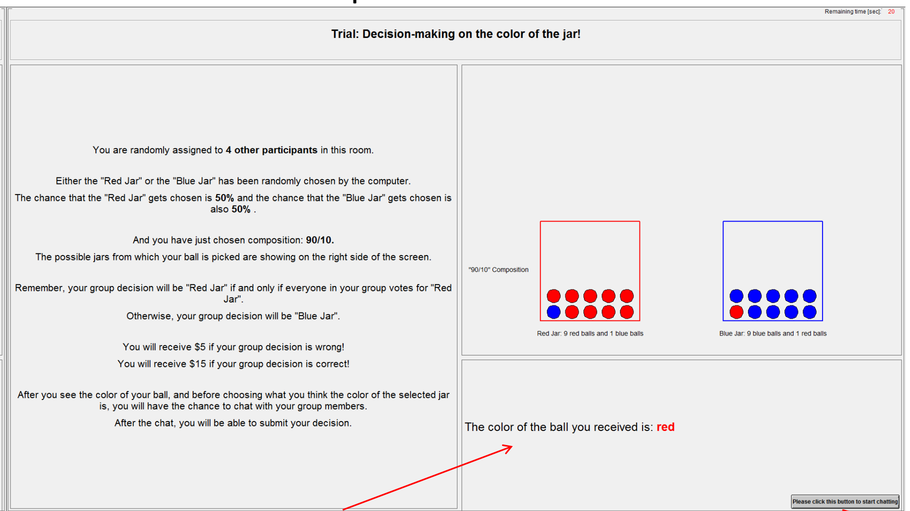
…the two jars, a red jar and a blue jar, whose composition correspond to the decision you have just made (in this example it was "90/10") are displayed in the upper right portion of your screen

Experimental-Trial



Instead, if your decision on the composition of the balls was "60/40", the two jars, a red jar and a blue jar, with the "60/40" composition are displayed in the upper right portion of your screen

Experimental-Trial



You will also get to see the color of the ball the computer randomly selected from the chosen jar

You should then click this button to start chatting with your group members

Experimental-Trial

Remaining time [sec]: 37

Group member #2: I have a red ball
Group member #1: I got a red one too
Group member #5: Well, I got blue

Help
To chat with your group members, type something in the light blue field and press 'Enter' to submit.

You will see your group members' messages in this dialogue box

Experimental-Trial

Remaining time [sec]: 58

Trial: Decision-making on the color of the jar!

You are randomly assigned to **4 other participants** in this room.

Either the "Red Jar" or the "Blue Jar" has been randomly chosen by the computer.
The chance that the "Red Jar" gets chosen is **50%** and the chance that the "Blue Jar" gets chosen is also **50%** .

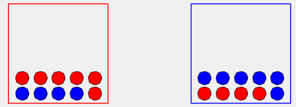
And you have just chosen composition: **60/40.**
The possible jars from which your ball is picked are showing on the right side of the screen.

Remember, your group decision will be "Red Jar" if and only if everyone in your group votes for "Red Jar".
Otherwise, your group decision will be "Blue Jar".

You will receive $5 if your group decision is wrong!
You will receive $15 if your group decision is correct!

After you see the color of your ball, and before choosing what you think the color of the selected jar is, you will have the chance to chat with your group members.
After the chat, you will be able to submit your decision.

"60/40" Composition



Red Jar: 6 red balls and 4 blue balls

Blue Jar: 6 blue balls and 4 red balls

The color of the ball you received is: **red**

Which jar you think is it picked from? Red Jar / Blue Jar

Submit decision

After chatting with your group members, you need to make your decision on which jar you think the ball is picked from by choosing either of the two options

You should then click "Submit decision" to continue

Experimental Interface-Official Experiment

Next, we will start the official experiment!

Experimental Interface-Payment

Now, you have finished 20 rounds of decision-making.
Next, the computer will randomly pick one of these 20 rounds to check your group decision and pay you accordingly.

We will show you the color of the ball you have received in that round, your decision and the decision of your group in that round.
Also, we will show you the color of the jar the computer has chosen in that round.

If your group decision is wrong in that round, you will receive $5.
If your group decision is correct in that round, you will receive $15.
Remember, no matter whether your group decision is right or wrong, you will receive $10 in addition as the show-up fee for participating in this experiment today.

Experimental Interface-Payment

You will see the selected round for generating your payment and the chosen jar in that round.

The round randomly selected for payoff is: **round 2**
The color of the chosen jar in round 2 is: **Blue**

Your decision and your group decision in that round, leads to a wrong decision in this case

Your decision in round 2 is: **Red**
Your group decision in round 2 is: **Red**

Thus, your group decision in round 2 is: **Wrong**
Therefore, your total earning in New Zealand Dollar is: reward **$5** + show-up fee **$10** = **$15**

Your total payment for today's experiment is calculated accordingly and is highlighted in red

Experimental Interface-Payment

You will see the selected round for generating your payment and the chosen jar in that round

The round randomly selected for payoff is: **round 2**
The color of the chosen jar in round 2 is: **Red**

Your decision and your group decision in that round, leads to a correct decision in this case

Your decision in round 2 is: **Red**
Your group decision in round 2 is: **Red**

Thus, your group decision in round 2 is: **Correct**
Therefore, your total earning in New Zealand Dollar is: reward **$15** + show-up fee **$10** = **$25**

Your total payment for today's experiment is calculated accordingly and is highlighted in red

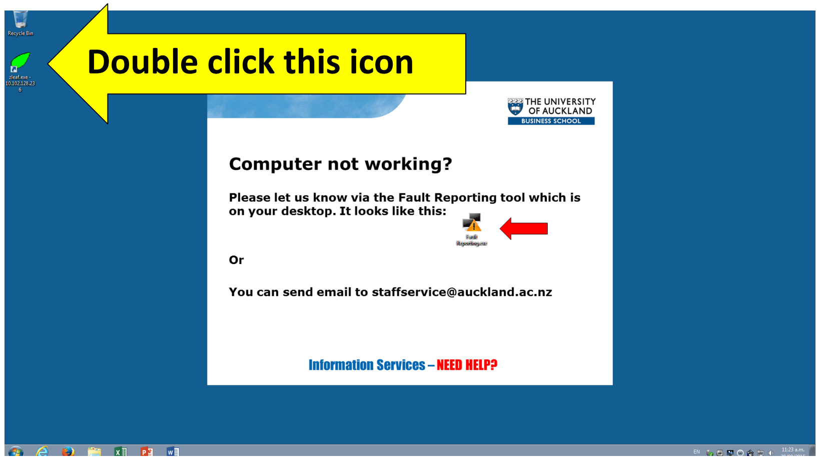
Experimental Interface-Payment

The experiment ends now. Thank you very much for your participation.
Next, we will call your computer terminal number and pay you one by one privately.
Please come to us after your computer terminal number is called.
Please be patient while you are waiting to be paid. Thank you.

Let the Experiment Begin!

- If there are no questions, we will now begin the experiment.

To get started



B.2

Ethical Clearance

MASSEY UNIVERSITY
ALBANY

05 March 2015

Simona Fabrizi
School of Economics and Finance
Albany Campus

Dear Simona

Re: Collective deliberation under ambiguity: theory and experimentation

Thank you for your Low Risk Notification which was received on 05 March 2015.

Your project has been recorded on the Low Risk Database which is reported in the Annual Report of the Massey University Human Ethics Committees.

You are reminded that staff researchers and supervisors are fully responsible for ensuring that the information in the low risk notification has met the requirements and guidelines for submission of a low risk notification.

The low risk notification for this project is valid for a maximum of three years.

Please notify me if situations subsequently occur which cause you to reconsider your initial ethical analysis that it is safe to proceed without approval by one of the University's Human Ethics Committees.

Please note that travel undertaken by students must be approved by the supervisor and the relevant Pro Vice-Chancellor and be in accordance with the Policy and Procedures for Course-Related Student Travel Overseas. In addition, the supervisor must advise the University's Insurance Officer.

A reminder to include the following statement on all public documents:

> *"This project has been evaluated by peer review and judged to be low risk. Consequently, it has not been reviewed by one of the University's Human Ethics Committees. The researcher(s) named above are responsible for the ethical conduct of this research.*
>
> *If you have any concerns about the conduct of this research that you wish to raise with someone other than the researcher(s), please contact Dr Brian Finch, Director (Research Ethics), telephone 06 356 9099, extn 84459, e-mail humanethics@massey.ac.nz".*

Please note that if a sponsoring organisation, funding authority or a journal in which you wish to publish requires evidence of committee approval (with an approval number), you will have to provide a full application to one of the University's Human Ethics Committees. You should also note that such an approval can only be provided prior to the commencement of the research.

Yours sincerely

Brian T Finch (Dr)
Chair, Human Ethics Chairs' Committee and
Director (Research Ethics)

cc Professor Martin Young
School of Economics & Finance
Palmerston North

Office of the Vice-Chancellor
Finance, Ethics and Compliance

The University of Auckland
Private Bag 92019
Auckland, New Zealand

Level 10, 49 Symonds Street
Telephone: 64 9 373 7599
Extension: 87830 / 83761
Facsimile: 64 9 373 7432

UNIVERSITY OF AUCKLAND HUMAN PARTICIPANTS ETHICS COMMITTEE (UAHPEC)

18-Jun-2015

MEMORANDUM TO:

Dr Valery Pavlov
Info Systems & Operations Mgmt

Re: Application for Ethics Approval (Our Ref. 014565): Approved

The Committee considered your application for ethics approval for your project entitled **Collective deliberation under ambiguity: theory and experimentation**.

We are pleased to inform you that ethics approval is granted for a period of three years.

The expiry date for this approval is 18-Jun-2018.

If the project changes significantly, you are required to submit a new application to UAHPEC for further consideration.

If you have obtained funding other than from UniServices, send a copy of this approval letter to the Research Office, at ro-awards@auckland.ac.nz. For UniServices contracts, send a copy of the approval letter to the Contract Manager, UniServices.

In order that an up-to-date record can be maintained, you are requested to notify UAHPEC once your project is completed.

The Chair and the members of UAHPEC would be happy to discuss general matters relating to ethics approvals. If you wish to do so, please contact the UAHPEC Ethics Administrators at ro-ethics@auckland.ac.nz in the first instance.

Please quote reference number: **014565** on all communication with the UAHPEC regarding this application.

(This is a computer generated letter. No signature required.)

UAHPEC Administrators
University of Auckland Human Participants Ethics Committee

c.c. Head of Department / School, Info Systems & Operations Mgmt
Steffen Lippert

Additional information:

1. Do not forget to fill in the 'approval wording' on the Participant Information Sheets and Consent Forms, giving the dates of approval and the reference number, before you send them out to your participants.

2. Should you need to make any changes to the project,please complete the online proposed changes and include any revised documentation.

3. At the end of three years, or if the project is completed before the expiry, please advise UAHPEC of its completion.

4. Should you require an extension, please complete the online Amendment Request form associated with this approval number giving full details along with revised documentation. An extension can be granted for up to three years, after which a new application must be submitted.

5. Please note that UAHPEC may from time to time conduct audits of approved projects to ensure that the research has been carried out according to the approval that was given.

B.3 Invitation Letter Sent Through ORSEE System for Recruiting Participants

Invitation email

Subject: Invitation to participate in a decision-making experiment

Dear #FirstName# #LastName#,

we invite you to participate in a laboratory study of collective decision-making.

1. This study has been approved by the University of Auckland Human Participants Ethics Committee. The data collected during the experiment is fully anonymous. Your participation is entirely voluntary and you will be able to withdraw from participation at any time and leave the room without providing any explanations. For more information, please refer to the Participant Information Sheet.

2. For your participation you will receive a show-up fee of $10 and some amount that depends on the decisions you will have made in the experiment. We expect that on average participants will be paid about $20 as a result of participation in a 90-min session. The individual payoffs, however, can be different among the participants, ranging from $15 (including the show-up fee) to possibly $25.

3. The sessions are scheduled for the following times:

   #sessionlist#

   If you want to participate, you can register by clicking on the following link: #registration link# (If you cannot click on the link, copy and paste it into the address line in your browser.)

Kind regards,

You experimenters.

This E-Mail was sent to you by the experiment participant recruitment system. If you want to change or to delete your data, please follow the link: [Link to the subscriber page]

The University of Auckland DECIDE Laboratory for Behavioural Decision-Making

APPROVED BY THE UNIVERSITY OF AUCKLAND HUMAN PARTICIPANTS ETHICS COMMITTEE ON 18-06-2015 for (3) years, Reference Number _ 014565__

B.4

Participant Information Sheet

DEPARTMENT OF INFORMATION SYSTEMS
AND OPERATIONS MANAGEMENT

PROJECT TITLE:

COLLECTIVE DELIBERATION UNDER AMBIGUITY: THEORY AND EXPERIMENTATION

Researchers: Dr Valery Pavlov (Co-Director of the Laboratory for Business Decision Making (DECIDE)); Dr Simona Fabrizi, Prof Thomas Pfeiffer and Miss Addison (Siwen) Pan (all Massey University), A/Prof Matthew Ryan (AUT) and Dr Steffen Lippert (Department of Economics, University of Auckland).

THE UNIVERSITY OF AUCKLAND
BUSINESS SCHOOL

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www.isom.auckland.ac.nz

The University of Auckland
Private Bag 92019
Auckland 1142
New Zealand

PARTICIPANT INFORMATION SHEET

Project description

You are invited to participate in an experiment studying how individual information and individual's choices determine the quality of collective decision-making. Think of a hiring committee which must vote "Yes" or "No" on an applicant for a position; or a team of surgeons who must vote "Yes" or "No" on a patient receiving an organ for a transplant; or jurors voting to "Acquit" or "Convict" a defendant. In all these situations, deliberation and voting procedures over these binary choices amongst a few, each bringing their own views to the decision task, determine whether to approve one option or another, thereby potentially affecting the quality of those decisions.

The goal of this research is to study the impact of a variety of information protocols and voting procedures on the quality of collective decision-making. To that end, we seek to investigate experimentally decision-making scenarios over binary options in as neutral environments as possible, to gain insights, which could be applied to various small group decision-making more in general, be it, e.g., for any selection decision made by a hiring committee, or for reaching a verdict in a jury trial.

Project procedures

The data will be collected in a laboratory experiment simulating collective deliberations over binary choices. A typical collective deliberation could be described by the information available to each decision-maker, the size of the decision-group, as well as the voting rule, that is, the minimum number of votes to be collected in favor of one option, for that option to be selected. The simulation of collective deliberation will be facilitated via computer-generated scenarios to include the information transmitted to the participants in the experiment regarding the case at hand. In an experiment setting, such information is disclosed to participants via sharing with them what is known about, say, the composition of the colored balls within two urns, either urn A or B, representing two possible states of nature, or, equivalently, the set of binary choices individuals in a small group decision-making context need to make.

The computer then randomly draws one of the two urns and which urn was picked is going to be revealed to all participants at the end of the experiment.

First, the information about the possible composition of each of the two urns is disseminated to all participants, then one of the two urns is randomly selected by the computer, after which, participants receive signals about the selected urn, by observing the color of computer-generated randomly drawn balls from the

B.5

Participant Consent Form

DEPARTMENT OF INFORMATION SYSTEMS
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PROJECT TITLE:

COLLECTIVE DELIBERATION UNDER AMBIGUITY: THEORY AND EXPERIMENTATION

Researchers: Dr Valery Pavlov (Co-Director of the Laboratory for Business Decision Making (DECIDE)); Dr Simona Fabrizi, Prof Thomas Pfeiffer and Miss Addison (Siwen) Pan (all Massey University), A/Prof Matthew Ryan (AUT) and Dr Steffen Lippert (Department of Economics, University of Auckland).

PARTICIPANT CONSENT FORM

THIS FORM WILL BE HELD FOR A PERIOD OF 6 YEARS

I have read the Participant Information Sheet, have understood the nature of the research and why I have been selected. I have had the opportunity to ask questions and have them answered to my satisfaction.

I agree to take part in this research.

I understand that this research is funded by the University of Auckland and Massey University.

I understand that I have been assured that the fact of my participation or non-participation in this study will have no effect on my grades or my relationship with the university and that I may contact the Head of Department should I feel this assurance is not met.

I understand that I am free to withdraw participation at any time, and to withdraw any data traceable to me up to 01-Dec-2015.

I understand that the experimental session may take up to 90 minutes.

I understand that if I participate in the experiment then at the end of the experimental session I will be paid $10 for participation even if I withdraw from participation during the session, and also an amount of money in the range from $5.00 to $15.00 such that the average total payoff across all the participants will be about $20.

I understand that the data collected for research purposes will not contain any information that would allow anyone to identify the decisions made by me in this experiment, and that this data will be kept indefinitely long and, possibly, shared with other research institutions world-wide.

I understand that in order to get paid I need to provide identification information and that this data will be used by the appropriate University finance division, separately from the data collected for research purposes. Confidentiality of this data is protected by the University policies.

I wish / do not wish to receive the summary of findings (should you wish us to send you a Summary, please add your email address here___________________________________)

Name ___________________________ Signature ___________________________ Date _________________

APPROVED BY THE UNIVERSITY OF AUCKLAND HUMAN PARTICIPANTS ETHICS COMMITTEE ON 18-06-2015 for (3) years, Reference Number _ 014565__

B.6

Experiments Payment Slip/Receipt Form

THE UNIVERSITY OF AUCKLAND
Te Whare Wānanga o Tāmaki Makaurau
NEW ZEALAND

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DEPARTMENT OF ECONOMICS

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Payment Slip/Receipt for Experimental Session on "Collective Deliberation"

Principal Investigators:
Drs Simona Fabrizi (Massey University) and Steffen Lippert (University of Auckland)

Date _________________ Time ________________

I affirm that I have received from Drs Simona Fabrizi and Steffen Lippert the amount shown against my name below, in cash. I further affirm that I was provided adequate information at the beginning of the session and I agreed to take part in this research project willingly. My signature below signifies my acceptance of the financial remuneration for participating in the session, as well as a reaffirmation of my informed consent towards participation in the experiment.

Terminal #	**Last Name**	**First Name**	**Unique Personal Identifier (UPI)**	**NZ$ Paid**	**Signature (*)**

(*) **Signature also counts as participant consent form.**

Chapter 3

A Generalisation of Feddersen and Pesendorfer (1998): Voting Under Ambiguity

3.1 Introduction

Consider once more a standard voting game in which voters simultaneously cast votes to contribute to the collective decision-making outcome. Assume again that, before casting her vote, each voter receives a private signal, which is positively related to the true state of the world. In chapter 1 we analyse the case in which the private information that each voter receives is imprecisely measured and the precision of the signal can take two possible levels, hence, we derive results for the two-point non-common prior model. There, we assume that individual's prior is exogenously determined. The approach in this chapter is very different from it.

First, the private signal each voter gets is ambiguous within a continuum, rather than being only ambiguous with respect two points. Second, we assume voters are ambiguity averse. Therefore, we use Maxmin Expected Utility Theorem (MMEU) of Gilboa and Schmeidler (1989) to capture the voter's attitude towards ambiguity to analyse how this impacts the collective voting outcomes under both the majority rule and the unanimity rule. According to MMEU, voters assess each of their available actions (to acquit or to convict) by the minimum expected utility associated with each of those actions.

Our aim is to address the following key questions.

1. In a general ambiguity setting, what are the possible priors that voters would adopt?

2. Does voting strategically survive as a plausible equilibrium in the presence of ambiguity and ambiguity averse voters?

3. Does the jury paradox hold when voters are not ambiguous neutral?

To do so, we generalise the jury voting model of Feddersen and Pesendorfer (1998) by embedding ambiguity into the private signal structure and considering voters who, being ambiguity averse, adopt a Maxmin approach to form subjective beliefs. Within this framework we prove the existence of an informative voting equilibrium and of strategic voting equilibria. Moreover, we find that if ambiguity exists in the precision of the private information, it is easier to sustain informative voting as an equilibrium strategy, that is, there exists a larger set of reasonable doubt levels for the unanimity voting rule to prevail as an equilibrium of the voting game. This is an important result as voting informatively, especially under unanimity helps maintain the efficiency of information aggregation.

The rest of the chapter proceeds as follows. Section 3.2 offers a review of the related literature on ambiguity, and ambiguity aversion in particular. Section 3.3 describes the collective voting game with the interval ambiguous information structure and ambiguity averse voters. In this section we also prove the existence of both an informative voting equilibrium and of the strategic voting equilibrium for both the unanimous voting rule and the majority rule. Some numerical examples and comparative statics results are provided in section 3.4. Section 3.5 concludes.

3.2 Related Literature

The Expected Utility (EU) theory of Neumann and Morgenstern (1947) assumes that the outcomes of the events under examination have objectively known probabilities. They define the preferences over acts by a real-valued utility function of the choices weighted by the objective probabilities of the outcomes of the states.

However, cases when the probability measure of the events are known to all decision makers hardly exist in real life. Decision makers are not able to form purely objective beliefs regarding the states unless they are confronted with a fair coin, a perfect die, or a well-made roulette wheel. Knight (1921) is the first person to distinguish ‘risk’ from ‘uncertainty’ by referring to the existence/absence of objective probabilities. ‘Risk’ is defined by events the objective probability measure of which could either (i) be theoretically deduced, which means that individuals are able to form priori probabilities; or (ii) be determined by empirical frequencies, which means individuals can generate statistical probabilities. Knight uses the notion of ‘uncertainty,’ when referring to events that do not fall within these two categories, that is, if either of the previous methods are not available for measuring the objective probabilities of such events. He also suggests that even in the uncertain cases, individuals can form estimates, which represent the concept of subjective probabilities, when making decisions based on them.

Savage (1954) suggested that probabilities are not necessarily something objectively known. Instead, decision makers have their subjective beliefs regarding the probability measure of the states. For example, unlike the roulette lottery, the horse lottery does not associate a known chance with each observation of the lottery. In other words, the decision maker cannot assign a specific probability to the outcomes of a horse lottery.

Thus, in Subjective Expected Utility (SEU) theory, preference relations over acts are represented by some real-valued utility function on the set of the consequences weighted by the subjective probabilities of the states; whereas the individual’s choice behaviour in situations of risk is predictable under certain postulates, such as complete ordering and the sure-thing principle.

Anscombe and Aumann (1963) established the theory of State-Dependent Expected Utility by combining EU and SEU. They started by redefining the word 'probability'. They separated 'probability' into two very different concepts. When it is interpreted within the 'logical' sense, it means the plausibility of some events or reasonableness of some expectations, whereas if it is interpreted within the sense of 'physics', it is roughly identical to the word 'chances', which refers to the proportion of successes in some events in the statistical way. This allows to transform a choice under uncertainty into a two-stage lottery-act framework.

Although SEU gives a rather accurate prediction of a decision maker's gambling choice and his/her reflective choice behaviour, Ellsberg (1961) points out that Savage's normative rules are not applicable whenever there is an unmeasurable uncertainty in the relative likelihood of the events. In his paper, ambiguity exists whenever there is inadequate information regarding the relative likelihood of the events. For example, ambiguity could be caused purely by lack of information. It could also be due to the fact that the decision maker receives contradicting information or/and the source of information is not credible. He provided a famous thought experiment and proved that there is a non-negligible minority of decision makers who violate Savage's axioms, who are not able to reduce the unmeasurable uncertainty to risk, or to apply the von Neumann–Morgenstern Expected Utility Theory.

In the Ellsberg two-colour urn experiment, decision makers are faced with two urns containing 100 balls each of either red or black colour, from which one ball will be randomly drawn. Let us suppose that in Urn A, the composition of red and black balls is not known to the decision maker. However, in Urn B, there are 50 red balls and 50 black ones. Decision makers are asked which one they prefer, (1) to bet on Red_A or to bet on $Black_A$? (2) to bet on Red_B or $Black_B$? (3) to bet on Red_A or Red_B? (4) to bet on $Black_A$ or $Black_B$? To 'bet on Red_A' means that the decision maker chooses to draw a ball from Urn A; and that he/she will receive a prize a if the drawn ball is red, which means Red_A occurs. If the drawn ball is black, then the decision maker receives the prize b, which means not–Red_A occurs; and the amount of prize a is bigger than b. Also, Red_A, $Black_A$, Red_B and $Black_B$ are mutually exclusive.

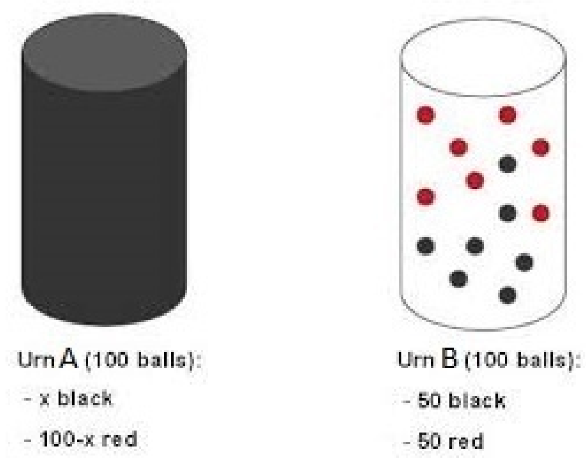

Fig. 3.1 Ellsberg Two-colour Urn Experiment

According to Savage's theorem, the individuals should be indifferent to either of the options for these four questions. This means that individuals should be indifferent with respect to the colour they bet on. Moreover, they should also be indifferent with respect to the urn they choose to bet on. A number of people, including Savage himself, although being indifferent between the options of questions (1) and (2), and those of questions (3) and (4), nevertheless prefer betting on Red_B to Red_A, and $Black_B$ to $Black_A$. This preference obviously violates the Savage Axioms. Thus, the preferences elicited from the Ellsberg Urn game cannot be explained by the Savage Axioms. This contradiction between ambiguity and SEU theory becomes a major challenge to game theory and rational choice theory.

As stated in Table 3.1 below, let a and b be the payoffs, $a > b$, such that, for example, in gamble I, if 'R_A' occurs, the payoff of betting on 'R_A' is a. According to Savage's theorem, individual should be indifferent between gamble I and gamble II. Also, they should be indifferent between gamble III and gamble IV. Following Savage's postulates, complete ordering and the sure-thing principle, individuals are indifferent between gamble V and gamble VI, which means that decision makers are not only indifferent to bet on either of the colour from each urn, but also are indifferent to bet on either of the urns.

Table 3.1 Ellsberg Two-Colour Urn Game

	R_A	B_A	R_B	B_B
I	a	b	b	b
II	b	a	b	b
III	b	b	a	b
IV	b	b	b	a
V	a	a	b	b
VI	b	b	a	a

Then, starting with the assumption that the individual prefers gamble III to gamble I, we could make certain transformations toward gamble I and gamble III on the basis of complete ordering and the sure-thing principle and keep the preference unchanged, that is individuals always prefer the second gamble in the five pairs listed in Table 3.2. If the payoff of betting on $Black_B$ changes from b to a, we have the payoffs as gamble I' and III' in Table 3.2. According to the sure-thing principle, preference regarding a pair of gambles will not change by the payoff values of events, for which both gambles have the same payoffs. Thus, gamble III' is preferred to gamble I'. As gamble III' is equivalent to gamble VI in Table 3.1 and gamble VI is indifferent to gamble V, we can transform III' to III". Gamble III"' is preferred to gamble I"' after we apply the sure-thing principle by changing the value of the payoffs of the event Red_A from a to b under both gambles. Then, we get that gamble III"" is preferred to gamble I"" as gamble III"' is equivalent to gamble II in Table 3.1 and individuals are indifferent between II and I. However, gamble III"" (equivalent to I) is preferred to gamble I"" (equivalent to III), which contradicts the assumption that gamble III is preferred to I. Thus, the Savage Axioms cannot explain these preference relations, opening up the door for alternative explanations.

Table 3.2 Transformed Ellsberg Two-Colour Urn Game

	R_A	B_A	R_B	B_B
I	a	b	b	b
III	b	b	a	b
I’	a	b	b	a
III’	b	b	a	a
I”	a	b	b	a
III”	a	a	b	b
I”’	b	b	b	a
III”’	b	a	b	b
I””	b	b	a	b
III””	a	b	b	b

In Ellsberg’s three-colour urn game, the participants exhibit the same pattern as they do in the previous two-colour experiment. The participants are given an urn containing 90 balls, of which 30 are red and the remaining 60 are either black or yellow. In this alternative experiment, participants prefer betting on events for which they know more about the probability measure over the states—the event that a red (black or yellow) ball will be picked out of this urn to betting on black (red) one. As shown in Table 3.3, betting on red has a winning probability of 1/3 and betting on either black or yellow has a winning probability of 2/3; thus, the decision maker prefers X to Y and Y' to X'. Since these two pairs of acts are identical without taking yellow into consideration, if $X \succ Y$, then $X' \succ Y'$. However, such a pattern also violates the sure-thing principle. To be consistent with the sure-thing principle, X is preferred to Y and X' is preferred to Y', since the sure-thing principle requires decision makers to ignore the states in which the act leads to the same payoff. This means that the state yellow will not influence the individuals’ choice when comparing acts of X and Y; and the same applies to X' and Y'.

In addition, the first order stochastic dominance axiom is violated by Ellsberg-type preferences. In the two-colour urn game, the probability of winning by betting on Red_A is

Table 3.3 Ellsberg Three-Colour Urn Game

Number of balls			
	30	60	
Act	Red	Black	Yellow
X	W	0	0
Y	0	W	0
X’	W	0	W
Y’	0	W	W

higher than from betting on Red_B if the composition of red and black in Urn A is $(60,40)$, for instance. However, $Red_B \succ Red_A$ to the Ellsberg type. The explanation of the three-colour urn game is fairly similar to that of the two-colour case.

Moreover, not only Savage’s theorem but also other subjective utility theories with additive probabilities are proved to be implausible, as they fail to infer the probabilities from the decision maker’s choice for Ellsberg’s ambiguous urn game. In the two-colour case, the individual prefers to bet on Red_B rather than Red_A. This means the same as that the decision maker believes that $P(Red_B) > P(Red_A)$, which indicates that $1 - P(Red_B) < 1 - P(Red_A)$. However, this contradicts the preference of the individual, $P(Black_B) > P(Black_A)$. Analogously, in the three-colour urn game, denote the subjective probabilities of drawing a red, black and yellow by $P(Red)$, $P(Black)$ and $P(Yellow)$, respectively. $Y' \succ X'$ indicates $P(Black \cup Yellow) > P(Red \cup Yellow)$. Thus, when probabilities are additive, $P(Black \cup Yellow) = P(Black) + P(Yellow)$ and $P(Red \cup Yellow) = P(Red) + P(Yellow)$. However, given the preference $Y' \succ X'$, $P(Red) < P(Black)$. Thus, this contradicts the preference $X \succ Y$.

A series of empirical studies have been conducted following Ellsberg’s thought experiment so as to test the existence of ambiguity and ambiguity aversion. The aversion to ambiguous choices have been well demonstrated in the replications of the Ellsberg urn experiments in Becker and Brownson (1964), Larson (1980), Hogarth and Einhorn (1990), Bernasconi and Loomes (1992), Seidenfeld and Wasserman (1993), Keren and Gerritsen (1999), Ivanov (2011), among others. In these experiments, as in Ellsberg's Urn experiment, objective probabilities exist; nevertheless, individuals can only partially access such measurements. If individuals were allowed to access the whole objective probabilities measurement, for example, by looking into Ellsberg's Urn A, and by seeing every ball in it, then, they would know the exact measurement of the objective probabilities of each ball to be drawn from that urn. Thus, Urn A would be no longer ambiguous, rather a risky urn and each individual would be able to settle on the same explicit probability measure of the event of a particular ball being drawn from it and, hence, form an identical prior/belief from such well-defined probability measure. However, there is another type of ambiguity, where the underlying objective probabilities measure is intrinsically unknown/unmeasurable. Unlike Ellsberg's design of the game, some experiments have taken natural events, such as betting on future stock prices, or GNP, that is, events for which there exists conflicting advice regarding their probability distributions. Those are instances of events with ambiguous probabilities, as objective probability measures can neither be deduced theoretically nor generated by obtaining sufficiently empirical frequencies for them, to test decision makers' attitudes towards them, as in MacCrimmon (1968), Goldsmith and Sahlin (1983), and Einhorn and Hogarth (1985).

Other experiments have found that individuals exhibit an ambiguity seeking attitude when the probability for gain is low and when the probability for loss is high (see Kahn and Sarin (1988), Curley et al. (1989)).

Besides decision analysis, the concepts of ambiguity and ambiguity aversion also prevail in other realistic applications. Kellner (2010) argues that a tournament contract is preferred to an independent contract in an ambiguous situation, where the relationship between effort and output is opaque. Tournament contracts will not be favourable when agents are risk averse but ambiguity neutral. However, as long as ambiguity aversion occurs, rank-dependent tournaments will most often be attractive for agents over effort-dependent contracts, although they may not be optimal. When the relation between effort and output is opaque, agents prefer a tournament contract, where the wage is based on the ranks of the agents' contributions, to an independent contract, where the wage is solely based on the agent's own performance, because a tournament contract removes this ambiguity from the unknown distribution of output in the principal–agent problem (Kellner and Riener (2011)).

Dickhaut et al. (2011) study investment behaviour both under uncertainty and ambiguous probabilities. They find that only one-third of investors act consistently with SEU in a first-price sealed bid auction when deciding on their investment in a financial asset. This shows that bidders tend to bid higher than the expected return of the assets, given the range of the expected return when not informed of the specific probabilities of each asset. The experimental work of Bossaerts et al. (2010) also displays substantial heterogeneity in attitudes towards ambiguity when choosing a portfolio with ambiguous Arrow securities. This generates different financial market equilibria than with the traditional approach, which assumes that agents are ambiguity neutral.

Also, in the auction market, the effect of ambiguity is amplified by its mechanism (Malmendier and Szeidl, 2008). It is claimed that bidders with the most market experience overbid more frequently than inexperienced ones. Bidders face ambiguous information as to all alternative auction goods and their winning bid. Bidders may over-value the good, because of the anchoring effect. In addition, unlike commodity-type goods, bidders are constrained by differentiated private values for goods like antiques, paintings, and collectibles, as the private values of the other competing bidders are unknown.

Similarly, when compared to individuals, firms in the market face an even more perplexing strategic environment. Armstrong and Huck (2010) found that entrepreneurs are over-optimistic, caring more about satisfactory results than optimisation, and resorting to rules of thumb when making strategic decisions for firms. In comparing the present or future outcomes with the historical figures of the firm itself or with the previous outcomes of their peers, entrepreneurs feel happy as long as the outcome remains as high as previous outcomes, when the probability of reaching the target set by the optimal strategy is rather ambiguous. Thus, entrepreneurs do not spend time on calculating the optimal equilibrium strategies even when there is no search cost. They adopt rules of thumb, resorting to imitating the successful strategies of their rivals or peers, against other strategies which would have been optimal instead.

Given the abundant evidence for ambiguity averse attitudes in well controlled laboratory experiments and in real life, other theories than SEU have been explored in order to solve/overcome the Ellsberg paradox, which incorporate ambiguity aversion. Inspired by this idea, Gilboa and Schmeidler (1989) assume that the decision makers formulate a set of possible additive probabilities when faced with ambiguity. They redefine the independence axiom for a non-unique prior. Moreover, they define uncertainty aversion. Thus, in the ambiguous urn game, the uncertainty averse decision maker takes the minimal expected utility over the prior set as his/her utility, for all priors in this set. In other words, preferences are represented by the minimum expected utilities over the set of possible probability measures. In Ellsberg's two-colour urn game, if the decision maker forms a prior set with all possible probability distributions of red and black balls in Urn A, the minimal expected utility of betting on Urn A will be zero for a utility maximising decision maker. With the probability of a red (black) ball picked from the risky Urn B as 1/2, the decision maker will prefer betting on the risky urn as long as the payoff of betting on Urn B is bigger than zero. Similarly, in the three-colour problem, the probability measure of $P(Black)$ could be $[0, 2/3]$. We assume the reward of winning the bet is W. Then, the expected utility is $(1/3)W$ and the minimum expected utility is 0 if the individual bets on the ambiguous urn. However, both the expected utility and the minimum expected utility is $(1/3)W$ if the individual bets on the unambiguous urn with an equal number of red, black and yellow balls in it. This explains why most individuals prefer the unambiguous urn to the ambiguous one.

Schmeidler (1989) restates Ellsberg's point that the probability assigned to an uncertain event is not only based on the information the decision maker receives when forming such a probability; the missing information reflects a heuristic part, which the decision maker takes into consideration to assess the uncertainty probability component. He axiomatises SEU in an Anscombe and Aumann framework. Schmeidler replaces the classical independence axiom with a weaker condition: co-monotonic independence; this allows SEU to be generalised to allow non-additive probability measures. A non-additive probability describes the probabilities of two equally likely events as being equal but not necessarily 1/2, unless the information set for assigning the probabilities is rich enough. In the Ellsberg experiment, the probability measure on the set of states need not be additive, due to the fact that the decision maker receives little information. For example, in the two-colour urn, $P(Black_A) = P(Red_A)$. However, the sum of $P(Black_A)$ and $P(Red_A)$ need not be 1. $1 - P(Black_A) - P(Red_A)$ measures the decision maker's confidence in the probability. Thus, the capacity of a red (black) ball to be picked from Urn B is 0.5 and the capacity of getting a red (black) ball picked from Urn A is smaller than 1/2. We might assign 0 capacities to the events of Red_A and $Black_A$, so that betting on either of the colours from urn A gives the decision maker a zero utility. Thus, SEU with non-additive probability also gives a conceivable explanation of the observed preferences from the ambiguous urn game.

Although ambiguity aversion has been observed through well controlled laboratory experiments and in real life, studies only focus on comparing decision-makings under different scenarios, without and with ambiguity. This means individuals are given both the risky environment and the ambiguous one, and they are asked to make decisions as if they were confronted with the Ellsberg Urn game. In reality, decision makers do not always have both risky and ambiguous scenarios to choose from. The risky world and the ambiguous world are mutually exclusive, which means individuals could start with being in the ambiguous world, with the risky world never becoming available. In the remainder of this chapter, we study voters confronted with ambiguity: voters cannot choose to switch to a non-ambiguous world.

3.3 The Collective Voting Game Under Ambiguity: The MMEU Approach

As in Feddersen and Pesendorfer (1998), and as in chapter 1, a group of n jurors, $j = 1, \cdots, n$, have to reach a verdict on a defendant, who could be either "guilty"-G, or "innocent"-I with ex-ante equal probability, i.e., $Pr(G) = Pr(I) = 1/2$. Each juror is expected to cast a vote $\{C, A\}$ either to 'C=convict' or to 'A=acquit' the defendant based on the evidence received, with signal's precision p, where $p = Pr(g|G) = Pr(i|I)$. The individual votes then contribute towards the collective verdict. As before, we assume that all jurors have the same preferences with respect to the outcome of the verdict, that is, they all want to reach the correct judgment. Their preferences are defined as follows:

$$u(A, I) = u(C, G) = 0,$$

$$u(C, I) = -q,$$

$$u(A, G) = -(1-q),$$

with $q \in (0, 1)$ representing once again the threshold of reasonable doubt for conviction.

However, before jurors cast their votes, each voter j receives an independent randomly drawn private and imperfect signal $s_j \in \{g, i\}$ as the evidence, with random and ambiguous precision p with $p \in \mathscr{P} = [\underline{p}, \bar{p}]$ and $1/2 < \underline{p} < \bar{p} < 1$. No further probabilistic information about the signal precision is provided. This implies that, differing once more from the existing jury voting literature, the quality of the private signals is imprecisely measured. In particular, we allow such precision to fall within two levels, in the sense that the domain of the information quality is a closed interval, rather than a set including only two points as in chapter 1, or a singleton (Feddersen and Pesendorfer, 1998). The interval ambiguity with respect to the signal precision could be understood as the case in which a piece of evidence, say, hinting toward the defendant being guilty, tells us that the probability that the defendant is guilty is at least 60%, but at most 90%. However, except for this, there is no extra information provided regarding the probability measure of the underling true accuracy of the information (evidence) each voter receives.

Define the set of all possible priors as Π, $\Pi = [\underline{p}, \bar{p}]$. The ambiguity averse voter chooses the action that maximises the minimum utility across all possible signal precisions.

With this in mind, we maintain the assumption that after observing the private signals, each juror casts her vote simultaneously, according to the strategy $\sigma_j(s_j, \pi_j)$, which is the probability that voter j votes for conviction conditional on her private signal s_j and her subjectively formed prior π_j. As before, the collective decision is determined by the voting rule $\hat{k}$, $\hat{k} \leq n$. The given voting rule is the simple majority rule when $\hat{k} = (n+1)/2$; and it is the unanimity rule when $\hat{k} = n$. And, as always, the verdict is either acquittal or conviction, depending on whether the threshold of necessary votes to convict is either not reached, or reached.

3.3.1 Informative Voting

A voter j's expected utility of voting for acquittal, conditional on being pivotal and receiving an innocent signal is

$$E[u_j(A,\cdot) \mid piv, s_j = i] = u_j(A|I)Pr(I|piv, s_j = i) + u_j(A|G)Pr(G|piv, s_j = i).$$

Because $u_j(A|I) = 0$ and $u_j(A|G) = -(1-q)$, we then have

$$E[u_j(A,\cdot) \mid piv, s_j = i] = -(1-q)Pr(G|piv, s_j = i).$$

Denote the posterior belief that the defendant is guilty conditional on the voter being pivotal and having received signal i, when all other voters vote informatively, as $\beta_G^i(\pi_j, \sigma(\cdot))$. Hence,

$$Pr(G|piv, s_j = i) = \beta_G^i(\pi_j, \sigma(\cdot)) = \frac{1}{1 + (\frac{\pi_j}{1-\pi_j})(\frac{1-\pi_j}{\pi_j})^{n-1}}.$$

For an ambiguity averse voter j, we can determine what is the selected belief or prior π_j, that corresponds to the action leading to the highest among the minimum expected utilities from choosing, say, to acquit a defendant.

To do so, we first determine the prior, among those one can hold, which leads to the lowest utility of acquitting a guilty when receiving the innocent signal:

$$min_{\pi_j \in \Pi} E[u_j(A,\cdot) \mid piv, s_j = i] = -(1-q) max_{\pi_j \in \Pi} \beta_G^i(\pi_j, \sigma(\cdot)).$$

Denote $max_{\pi_j \in \Pi} \beta_G^i(\pi_j, \sigma(\cdot))$ as $\bar{\beta}_G^i(\pi, \sigma(\cdot))$, so that

$$min_{\pi_j \in \Pi} E[u_j(A,\cdot) \mid piv, s_j = i] = -(1-q)\bar{\beta}_G^i(\pi_j, \sigma(\cdot)).$$

We can repeat the exercise for a voter j's expected utility of voting for conviction, conditional on being pivotal and receiving an innocent signal. This leads to:

$$E[u_j(C,\cdot) \mid piv, s_j = i] = u_j(C|I)Pr(I|piv, s_j = i) + u_j(C|G)Pr(G|piv, s_j = i),$$

which is equivalent to

$$E[u_j(C,\cdot) \mid piv, s_j = i] = -qPr(I|piv, s_j = i).$$

An ambiguity averse voter assesses her action to vote to convict by the minimum expected utility of this action, conditional on being pivotal and receiving an innocent signal. That is,

$$min_{\pi_j \in \Pi} E[u_j(C,\cdot) \mid piv, s_j = i] = -q max_{\pi_j \in \Pi} \beta_I^i(\pi_j, \sigma(\cdot)).$$

We know that $\beta_I^i(\pi_j) = 1 - \beta_G^i(\pi_j)$. Hence, $max_{\pi_j \in \Pi} \beta_I^i(\pi_j, \sigma(\cdot)) = 1 - min_{\pi_j \in \Pi} \beta_G^i(\pi_j, \sigma(\cdot))$. Define $min_{\pi_j \in \Pi} \beta_G^i(\pi_j, \sigma(\cdot))$ as $\underline{\beta}_G^i(\pi_j, \sigma(\cdot))$, we then have

$$min_{\pi_j \in \Pi} E[u_j(C,\cdot) \mid piv, s_j = i] = -q(1 - \underline{\beta}_G^i(\pi_j, \sigma(\cdot))).$$

Thus, the ambiguity averse voter will vote for acquittal informatively if and only if her minimum expected utility of voting for acquittal is bigger than the minimum utility of voting for conviction, given her private signal is i, conditional on being pivotal and all other voters voting informatively, i.e.,

$$min_{\pi_j \in \Pi} E[u_j(A,\cdot) \mid s_j = i] > min_{\pi_j \in \Pi} E[u_j(C,\cdot) \mid s_j = i].$$

As the utility of convicting the guilty and acquitting the innocent is zero, we have

$$min_{\pi_j \in \Pi} E[u_j(A,G) \mid s_j = i] > min_{\pi_j \in \Pi} E[u_j(C,I) \mid s_j = i], \tag{3.1}$$

which is equivalent to:

$$-(1-q)max_{\pi_j\in\Pi}Pr(G|s_j=i) > -qmax_{\pi_j\in\Pi}Pr(I|s_j=i).$$

Therefore, requiring condition (3.1) to be satisfied is equivalent to verifying that the following condition holds:

$$-(1-q)\bar{\beta}^i_G(\pi_j,\sigma(\cdot)) > -q\bar{\beta}^i_I(\pi_j,\sigma(\cdot)).$$

Because $\bar{\beta}^i_I(\pi_j,\sigma(\cdot)) = 1-\underline{\beta}^i_G(\pi_j,\sigma(\cdot))$, we then have

$$\frac{\bar{\beta}^i_G(\pi_j,\sigma(\cdot))}{1-\underline{\beta}^i_G(\pi_j,\sigma(\cdot))} < \frac{q}{1-q}.$$

Analogously, the ambiguity averse j voter will vote for conviction informatively if and only if

$$min_{\pi_j\in\Pi}E[u_j(C,\cdot) \mid piv, s_j=g] > min_{\pi_j\in\Pi}E[u_j(A,\cdot) \mid piv, s_j=g].$$

As the utility of convicting the guilty and acquitting the innocent is zero, we have

$$min_{\pi_j\in\Pi}E[u_j(C,I) \mid s_j=i] > min_{\pi_j\in\Pi}E[u_j(A,G) \mid s_j=i], \tag{3.2}$$

which is equivalent to:

$$-q(1-\underline{\beta}^g_G(\pi_j,\sigma(\cdot))) > -(1-q)\bar{\beta}^g_G(\pi_j,\sigma(\cdot)),$$

that is

$$\frac{\bar{\beta}^g_G(\pi_j,\sigma(\cdot))}{1-\underline{\beta}^g_G(\pi_j,\sigma(\cdot))} > \frac{q}{1-q}.$$

Given that in an informative equilibrium all jurors behave the same and that $\sigma(i)=0$ and $\sigma(g)=1$, in the remainder of this chapter, we omit the index j, when referring to the equilibrium beliefs and strategies of a specific juror, and, for the rest of this subsection, we also omit to specify the equilibrium strategy when describing the belief function.

Thus, the condition for informative voting being a Nash equilibrium, can simply be written as $\frac{\bar{\beta}^i_G(\pi)}{1-\underline{\beta}^i_G(\pi)} < q < \frac{\bar{\beta}^g_G(\pi)}{1-\underline{\beta}^g_G(\pi)}$.

Notice that

$$\beta^i_G(\pi) = \frac{(1-\pi)\pi^{\hat{k}-1}(1-\pi)^{n-\hat{k}}}{(1-\pi)\pi^{\hat{k}-1}(1-\pi)^{n-\hat{k}} + \pi(1-\pi)^{\hat{k}-1}\pi^{n-\hat{k}}} = \frac{1}{1+(\frac{1-\pi}{\pi})^{2\hat{k}-n-2}}$$

is strictly increasing with π when $k \geq \frac{n+2}{2}$, for example, when $\hat{k} = n$, it reaches its maximum when $\pi = \bar{p}$; whereas, if $k < \frac{n+2}{2}$, for example, when $\hat{k} = \frac{n+1}{2}$, β^i_G reaches its maximum when $\pi = \underline{p}$; and,

$$\beta^g_G(\pi) = \frac{\pi\pi^{\hat{k}-1}(1-\pi)^{n-\hat{k}}}{\pi\pi^{\hat{k}-1}(1-\pi)^{n-\hat{k}} + (1-\pi)(1-\pi)^{\hat{k}-1}\pi^{n-\hat{k}}} = \frac{1}{1+(\frac{1-\pi}{\pi})^{2\hat{k}-n}}$$

is strictly increasing with p when $k \geq \frac{n}{2}$, when $\pi = \underline{p}$, it reaches its minimum.

Proposition 3.1. *Under the Maxmin approach and ambiguous information p, with* $p \in [\underline{p}, \bar{p}]$, *informative voting is an equilibrium for ambiguity averse voters if and only if* $\frac{\bar{\beta}^i_G(\pi)}{1-\underline{\beta}^i_G(\pi)} < \frac{q}{1-q} < \frac{\bar{\beta}^g_G(\pi)}{1-\underline{\beta}^g_G(\pi)}$.

3.3.2 Strategic Voting Under Unanimity

In this section, we study the Symmetric Responsive Nash Equilibrium under unanimous voting rule, when informative voting is not an equilibrium, that is, equation (3.1) and equation (3.2) are not satisfied at the same time. Under the Maxmin approach, voter strategic behaviour is still captured by considering the minimum level of utility of either votes one can cast, and each voter chooses the action, which gives the highest utility between the two.

A voter j's expected utility of voting for acquittal, conditional on being pivotal and receiving an innocent signal is

$$E[u_j(A,\cdot) \mid piv, s_j = i] = -(1-q)Pr(G|piv, s_j = i).$$

We denote $Pr(G|piv, s_j = i)$ as $\beta_G^i(\pi, \sigma(\cdot))$, which is the posterior belief that the defendant is guilty conditional on the voter being pivotal and receive signal i, that is

$$\beta_G^i(\pi, \sigma(\cdot)) = \frac{1}{1+(\frac{\pi}{1-\pi})(\frac{\gamma_I}{\gamma_G})^{n-1}},$$

where

$$\gamma_I(\pi, \sigma(\cdot)) = \pi\sigma(i) + (1-\pi)\sigma(g);$$

and

$$\gamma_G(\pi, \sigma(\cdot)) = \pi\sigma(g) + (1-\pi)\sigma(i).$$

An ambiguity averse voter assesses her action to acquit by its minimum expected utility among all possible priors, that is,

$$min_{\pi\in\Pi}E[u_j(A,\cdot) \mid piv, s_j = i] = -(1-q)max_{\pi\in\Pi}\beta_G^i(\pi, \sigma(\cdot)).$$

Define $max_{\pi\in\Pi}\beta_G^i(\pi, \sigma(\cdot))$ as $\bar{\beta}_G^i(\pi, \sigma(\cdot))$, we have

$$min_{\pi\in\Pi}E[u_j(A,\cdot) \mid piv, s_j = i] = -(1-q)\bar{\beta}_G^i(\pi, \sigma(\cdot)). \tag{3.3}$$

An ambiguity averse voter accesses her action to convict by its minimum expected utility, conditional on being pivotal and receiving an innocent signal. That is,

$$min_{\pi\in\Pi}E[u_j(C,\cdot) \mid piv, s_j = i] = -qmax_{\pi\in\Pi}\beta_I^i(\pi,\sigma(\cdot)).$$

We know $\beta_I^i(\pi,\sigma(\cdot)) = 1-\beta_G^i(\pi,\sigma(\cdot))$. Hence, $max_{\pi\in\Pi}\beta_I^i(\pi,\sigma(\cdot)) = 1-min_{\pi\in\Pi}\beta_G^i(\pi,\sigma(\cdot))$. Define $min_{\pi\in\Pi}\beta_G^i(\pi,\sigma(\cdot))$ as $\underline{\beta}^i_G(\pi,\sigma(\cdot))$, we then have

$$min_{\pi\in\Pi}E[u_j(C,\cdot) \mid piv, s_j = i] = -q(1-\underline{\beta}^i_G(\pi,\sigma(\cdot))). \tag{3.4}$$

Similarly, an ambiguity averse voter's minimum expected utility of voting to acquit, conditional on being pivotal and receiving a guilty signal is

$$min_{\pi\in\Pi}E[u_j(A,\cdot) \mid piv, s_j = g] = -(1-q)\bar{\beta}_G^g(\pi,\sigma(\cdot)); \tag{3.5}$$

and her minimum expected utility of voting to convict, conditional on being pivotal and receiving a guilty signal is

$$min_{\pi\in\Pi}E[u_j(C,\cdot) \mid piv, s_j = g] = -q(1-\underline{\beta}^g_G(\pi,\sigma(\cdot))), \tag{3.6}$$

where

$$\beta_G^g(\pi,\sigma(\cdot)) = \frac{1}{1+(\frac{1-\pi}{\pi})(\frac{\gamma_I}{\gamma_G})^{n-1}},$$

where

$$\gamma_I(\pi,\sigma(\cdot)) = \pi\sigma(i)+(1-\pi)\sigma(g);$$

and

$$\gamma_G(\pi,\sigma(\cdot)) = \pi\sigma(g)+(1-\pi)\sigma(i).$$

Hence, $\bar{\beta}_G^g(\pi,\sigma(\cdot))$ and $\underline{\beta}^g_G(\pi,\sigma(\cdot))$ are respectively the maximum level of the posterior belief and the minimum level of the posterior belief that the defendant is guilty conditional on the voter being pivotal and receiving signal g.

Notice that because receiving a guilty signal can never be information in favour of the innocence of the defendant more than receiving an innocent signal can ever be, we know that $\beta_G^g(\pi,\sigma(\cdot)) > \beta_G^i(\pi,\sigma(\cdot))$. Therefore, $\bar{\beta}_G^g(\pi,\sigma(\cdot)) > \bar{\beta}_G^i(\pi,\sigma(\cdot))$; and $1-\underline{\beta}_G^g(\pi,\sigma(\cdot)) < 1-\underline{\beta}_G^i(\pi,\sigma(\cdot))$. Thus:

$$\frac{\bar{\beta}_G^g(\pi,\sigma(\cdot))}{1-\underline{\beta}_G^g(\pi,\sigma(\cdot))} > \frac{\bar{\beta}_G^i(\pi,\sigma(\cdot))}{1-\underline{\beta}_G^i(\pi,\sigma(\cdot))}. \tag{3.7}$$

Lemma 3.1. *If* $\frac{\bar{\beta}_G^g(\pi,\sigma(\cdot))}{1-\underline{\beta}_G^g(\pi,\sigma(\cdot))} > \frac{\bar{\beta}_G^i(\pi,\sigma(\cdot))}{1-\underline{\beta}_G^i(\pi,\sigma(\cdot))}$, $(0<\sigma(i)<1, \sigma(g)=1)$ *is the Symmetric Responsive Nash Equilibrium.*

Proof. Assume equation (3.1) does not hold, that is voter's minimum expected utility of voting for acquittal is no larger than the minimum expected utility of voting for conviction, conditional on being pivotal and receiving an innocent signal, that is

$$-(1-q)\bar{\beta}_G^i(\pi,\sigma(\cdot)) \leq -q(1-\underline{\beta}_G^i(\pi,\sigma(\cdot))).$$

It is equivalent to

$$\frac{\bar{\beta}_G^i(\pi,\sigma(\cdot))}{1-\underline{\beta}_G^i(\pi,\sigma(\cdot))} \geq \frac{q}{1-q}.$$

If inequality (3.7) holds, it must be

$$\frac{\bar{\beta}_G^g(\pi,\sigma(\cdot))}{1-\underline{\beta}_G^g(\pi,\sigma(\cdot))} > \frac{q}{1-q}.$$

And this proves that if $0<\sigma(i)\leq 1$, then $\sigma(g)=1$. Because the strategic voting equilibrium has to be responsive, then we have the equilibrium, such that $(0<\sigma(i)<1, \sigma(g)=1)$. And $0<\sigma(i)<1$ simply means that voters randomise when receiving an innocent signal, which requires

$$\frac{\bar{\beta}_G^i(\pi,\sigma(i))}{1-\underline{\beta}_G^i(\pi,\sigma(i))} = \frac{q}{1-q}.$$

If equation (3.2) fails to hold, that is voter's minimum expected utility of voting for conviction is no larger than the minimum expected utility of voting for acquittal, conditional on being pivotal and receiving a guilty signal, that is

$$-q(1-\underline{\beta}^g_G(\pi,\sigma(\cdot))) \leq -(1-q)\bar{\beta}^g_G(\pi,\sigma(\cdot)),$$

we can also conclude that

$$\frac{\bar{\beta}^g_G(\pi,\sigma(\cdot))}{1-\underline{\beta}^g_G(\pi,\sigma(\cdot))} \leq \frac{q}{1-q}.$$

And because of equation (3.9), we have

$$\frac{\bar{\beta}^i_G(\pi,\sigma(\cdot))}{1-\underline{\beta}^i_G(\pi,\sigma(\cdot))} < \frac{q}{1-q},$$

leading to a contradiction, since it says that when $0 \leq \sigma(g) < 1$, $\sigma(i) = 0$). Due to the requirement of being responsive, $\sigma(g)$ cannot equal to 0, however, $(0 < \sigma(g) < 1, \sigma(i) = 0)$ would not satisfy being a symmetric responsive Nash equilibrium. When $0 < \sigma(g) < 1$, being pivotal means that the other $n-1$ voters all received signal g, because if they received signal i, they would vote to acquit with probability 1. But if this is the case, then a pivotal voter with a guilty signal would always vote to convict. Thus, this contradicts the assumption $0 < \sigma(g) < 1$.

□

Lemma 3.2. *The function* $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))}$ *is continuous at every* $\sigma(i) \in [0,1]$.

Proof. Let $\Pi = (0.5,1)$. Define $\phi : [0,1] \to 2^\Pi$ by $\phi(\sigma(i)) = [\underline{\pi},\bar{\pi}]$ for every $\sigma(i) \in [0,1]$. Note that ϕ is nonempty, continuous and compact-valued and that $\beta^i_G(\pi,\sigma(i))$ is continuous in both π and $\sigma(i)$. Thus by Berge Maximum Theorem (Aliprantis and Border, 2006), both $\bar{\beta}^i_G(\pi,\sigma(i))$ and $\underline{\beta}^i_G(\pi,\sigma(i))$ are continuous, and so is $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))}$. □

Therefore, Lemma 3.1 suggests that the symmetric responsive equilibrium exists when $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))} = \frac{q}{1-q}$. As shown in Figure 3.2,[1] if $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))} < \frac{q}{1-q}$, voters will vote informatively, which is the orange shaded area. If $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))} > \frac{q}{1-q}$, then voters vote for conviction

[1]Figure 3.2 has been obtained by a numerical simulation of the function $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))}$, when fixing $n = 5$, $p \in [0.70, 0.80]$ and letting the variable $\sigma(i)$ vary between 0 and 1. Other simulations for other values of n and the interval of p led to similar qualitative behaviours for this function. Since we do not have a formal

regardless of the signals, that is $(\sigma(i)=1,\sigma(g)=1)$, which is the green shaded area. In between is the are where voters can randomise their strategy when receiving an innocent signal. And there exists such strategy $0<\sigma^*(i)<1$ if and only if there is a horizontal line $\frac{q}{1-q}$ intercepting the continuous function $\frac{\bar{\beta}^i_G(\pi,\sigma^*(i))}{1-\underline{\beta}^i_G(\pi,\sigma^*(i))}$.

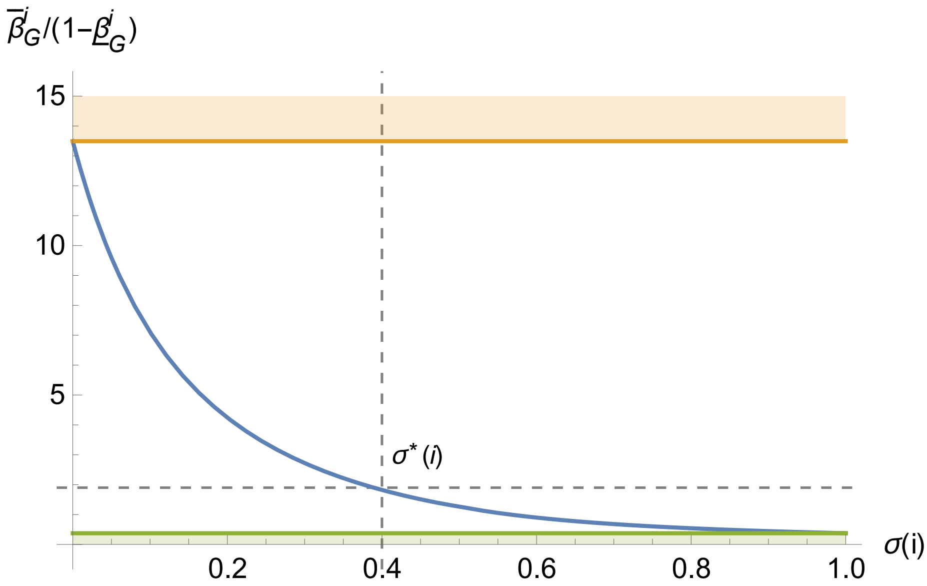

Fig. 3.2 Symmetric Responsive Equilibrium $(0<\sigma(i)<1,\sigma(g)=1)$

Proposition 3.2. *Under the Maxmin approach and ambiguous information p, with* $p\in[\underline{p},\bar{p}]$*, there exists a Symmetric Responsive Nash Equilibrium for the unanimity rule, when* $\frac{1-\underline{p}}{1-\underline{p}+\bar{p}}<q<\frac{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}}{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}(2+(\frac{1-\bar{p}}{\bar{p}})^{n-2})}$*, such that* $0<\sigma^*(i)<1$*, and such that* $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))}=\frac{q}{1-q}$.

Proof. Given $\sigma(g)=1$, then

$$\gamma_I(\pi,\sigma(i))=\pi\sigma(i)+(1-\pi);$$

and

$$\gamma_G(\pi,\sigma(i))=\pi+(1-\pi)\sigma(i),$$

proof that the function is strictly monotonic, we provide this figure for purely illustrative purposes, and only claim existence of a strategic equilibrium for intermediate levels of q, leaving the proof of uniqueness of such equilibrium to further research.

with $0 < \sigma(i) < 1$. Then,

$$\beta_G^i(\pi, \sigma(i)) = \frac{1}{1 + (\frac{\pi}{1-\pi})(\frac{\pi\sigma(i)+(1-\pi)}{\pi+(1-\pi)\sigma(i)})^{n-1}}.$$

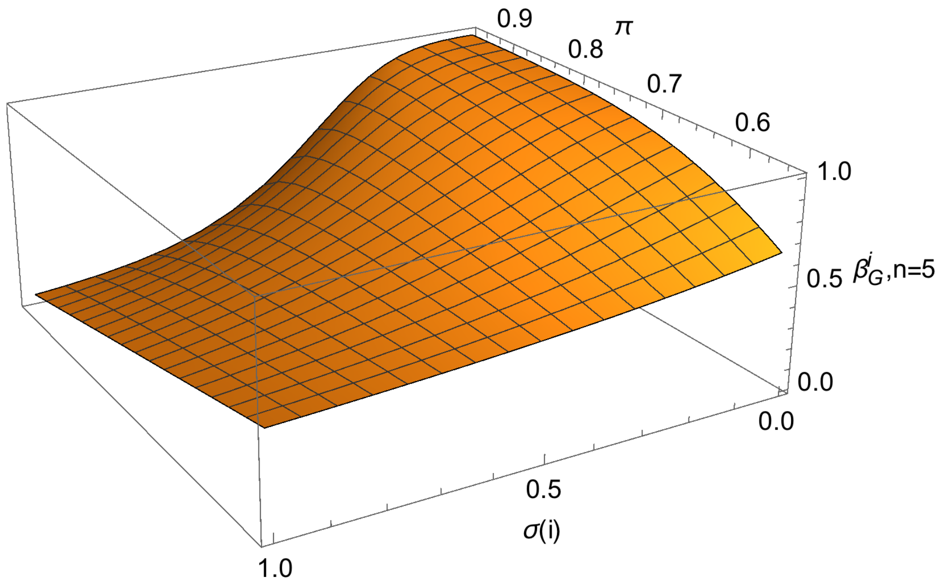

Fig. 3.3 3D Plot of $\beta_G^i(\pi, \sigma(i))$

Because $\beta_G^i(\pi, \sigma(i))$ is continuous at $\sigma(i) = 0$, as shown in Figure 3.3, we have

$$\lim_{\sigma(i)\to 0^+} \beta_G^i(\pi, \sigma(i)) = \frac{1}{1 + (\frac{1-\pi}{\pi})^{n-2}}.$$

Hence,

$$\lim_{\sigma(i)\to 0^+} \bar{\beta}_G^i(\pi^*, \sigma(i)) = \frac{1}{1 + (\frac{1-\pi^*}{\pi^*})^{n-2}},$$

with $\pi^* = \bar{p}$, and

$$\lim_{\sigma(i)\to 0^+} \underline{\beta}_G^i(\pi^*, \sigma(i)) = \frac{1}{1 + (\frac{1-\pi^*}{\pi^*})^{n-2}},$$

with $\pi^* = \underline{p}$. Therefore, by continuity

$$\lim_{\sigma(i)\to 0^+} \frac{\bar{\beta}_G^i(\pi,\sigma(i))}{1-\underline{\beta}_G^i(\pi,\sigma(i))} = \frac{\lim_{\sigma(i)\to 0^+} \bar{\beta}_G^i(\pi,\sigma(i))}{1-\lim_{\sigma(i)\to 0^+} \underline{\beta}_G^i(\pi,\sigma(i))} = \frac{\frac{1}{1+(\frac{1-\bar{p}}{\bar{p}})^{n-2}}}{1-\frac{1}{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}}}. \tag{3.8}$$

Similarly, $\beta_G^i(\pi,\sigma(i))$ is continues at $\sigma(i)=1$, and then,

$$\lim_{\sigma(i)\to 1^-} \beta_G^i(\pi,\sigma(i)) = \frac{1}{1+(\frac{\pi}{1-\pi})}.$$

Hence,

$$\lim_{\sigma(i)\to 1^-} \bar{\beta}_G^i(\pi^*,\sigma(i)) = \frac{1}{1+(\frac{\pi^*}{1-\pi^*})},$$

with $\pi^* = \underline{p}$, and

$$\lim_{\sigma(i)\to 1^-} \underline{\beta}_G^i(\pi^*,\sigma(i)) = \frac{1}{1+(\frac{\pi^*}{1-\pi^*})},$$

with $\pi^* = \bar{p}$. Therefore, by continuity

$$\lim_{\sigma(i)\to 1^-} \frac{\bar{\beta}_G^i(\pi,\sigma(i))}{1-\underline{\beta}_G^i(\pi,\sigma(i))} = \frac{\lim_{\sigma(i)\to 1^-} \bar{\beta}_G^i(\pi,\sigma(i))}{1-\lim_{\sigma(i)\to 1^-} \underline{\beta}_G^i(\pi,\sigma(i))} = \frac{\frac{1}{1+(\frac{\underline{p}}{1-\underline{p}})}}{1-\frac{1}{1+(\frac{\bar{p}}{1-\bar{p}})}} = \frac{1-\underline{p}}{\bar{p}}. \tag{3.9}$$

Therefore, whenever $\frac{q}{1-q}$ is strictly between the values identified in equations (3.8) and (3.9), that is $\frac{\bar{\beta}_G^i(\pi,\sigma(i))}{1-\underline{\beta}_G^i(\pi,\sigma(i))} = \frac{q}{1-q}$, there exists $0 < \sigma(i) < 1$ as an equilibrium.

Note that this condition can also be split into two components, as follows:

$$q < \frac{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}}{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}(2+(\frac{1-\bar{p}}{\bar{p}})^{n-2})}; \tag{3.10}$$

and

$$q > \frac{1-\underline{p}}{1-\underline{p}+\bar{p}}. \tag{3.11}$$

Therefore, we proved that as long as $\frac{1-\underline{p}}{1-\underline{p}+\bar{p}} < q < \frac{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}}{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}(2+(\frac{1-\bar{p}}{\bar{p}})^{n-2})}$, there exists $0 < \sigma^*(i) < 1$, which is the equilibrium strategy when voter receives signal i.

$\square$

In addition, if condition (3.11) is violated, that is $q < \frac{1-\underline{p}}{1-\underline{p}+\bar{p}}$, there exists a Symmetric Non-Responsive Strategic Nash Equilibrium, that is $(\sigma(i) = 1, \sigma(g) = 1)$, where voters vote for conviction regardless of their signals. This strategy leads to the highest type I error, $Pr(C|I) = 1$, and the lowest type II error, $Pr(A|G) = 0$.

Corollary 3.1. *Under the Maxmin approach and ambiguous information p, with* $p \in [\underline{p}, \bar{p}]$*, for unanimous voting, there exists Symmetric Non-Responsive Nash Equilibrium with* $(\sigma(i) = 1, \sigma(g) = 1)$ *if and only if* $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))} > \frac{q}{1-q}$*, that is as long as* $q < \frac{1-\underline{p}}{1-\underline{p}+\bar{p}}$.

3.3.3 Strategic Voting Under Non-Unanimity

For non-unanimous voting, $\hat{k} \neq n$, we have

$$\beta_G^i(\pi, \sigma(\cdot), \hat{k}) = \frac{1}{1 + (\frac{\pi}{1-\pi})(\frac{\gamma_I}{\gamma_G})^{\hat{k}-1}(\frac{1-\gamma_I}{1-\gamma_G})^{n-\hat{k}}}$$

and

$$\beta_G^g(\pi, \sigma(\cdot), \hat{k}) = \frac{1}{1 + (\frac{1-\pi}{\pi})(\frac{\gamma_I}{\gamma_G})^{\hat{k}-1}(\frac{1-\gamma_I}{1-\gamma_G})^{n-\hat{k}}},$$

where

$$\gamma_I(\pi, \sigma(\cdot)) = \pi\sigma(i) + (1-\pi)\sigma(g),$$

and

$$\gamma_G(\pi, \sigma(\cdot)) = \pi\sigma(g) + (1-\pi)\sigma(i).$$

Because $\beta_G^g(\pi, \sigma(\cdot), \hat{k}) > \beta_G^i(\pi, \sigma(\cdot), \hat{k})$, Lemma 3.1 also holds for the case where $\hat{k} \neq n$. Hence, we also have a Symmetric Responsive Nash Equilibrium for non-unanimous voting, $(0 < \sigma(i) < 1, \sigma(g) = 1)$. Using Lemma 3.2, we can also prove the existence of the symmetric responsive Nash equilibrium for non-unanimous voting rule. The formal proofs can be found in Appendix C.1, whereas the main results for this case are summarised below.

Proposition 3.3. *Under the Maxmin approach and ambiguous information p, with $p \in [\underline{p}, \bar{p}]$, for the non-unanimous voting rule, there exists Symmetric Responsive Nash Equilibria, $(0 < \sigma(i) < 1, \sigma(g) = 1)$, if and only if $\frac{\bar{\beta}_G^i(\pi, \sigma(i), \hat{k})}{1-\underline{\beta}_G^i(\pi, \sigma(i), \hat{k})} = \frac{q}{1-q}$, that is,*

1. *if $\hat{k} > \frac{n+2}{2}$ and $\frac{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})} < q < \frac{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}}{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}(2+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2})}$;*

2. *if $0 < \hat{k} \leq \frac{n+2}{2}$ and $\frac{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})} < q < \frac{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}}{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}(2+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2})}$.*

Corollary 3.2. *Under the Maxmin approach and ambiguous information p, with $p \in [\underline{p}, \bar{p}]$, for non-unanimous voting, if $q < \frac{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})}$, there exists Symmetric Non-Responsive Nash Equilibria, that is $(\sigma(i) = 1, \sigma(g) = 1)$, where voters vote for conviction* *regardless of their signals, which leads to the highest type I error,* $Pr(C|I) = 1$*, and the lowest type II error,* $Pr(A|G) = 0$*.*

3.4 Comparative Statics Results

Consider the 12-person jury example as in Feddersen and Pesendorfer (1998), that is when $n = 12$, $p = 0.8$ and $\hat{k} = 12$ or 7. We know that when signal precision is uniquely defined, voters behave symmetrically and responsibly if and only if $1 - p < q < \frac{1}{1+(\frac{1-p}{p})^{n-2}}$. Especially, from Figure 3.4, we can see the cut-off value of q for informative voting being an equilibrium is very high, which is almost 1. This says that, it is almost impossible for voters to vote informatively in this scenario. For any level of q, which is exogenously given and set equal to $q = 0.9$, which is below the cut-off value for informative voting, strategic equilibria arise. We can compute the voter's strategy for this case, which is exactly equal to $\sigma(i) = 0.575$, $\sigma(g) = 1$.

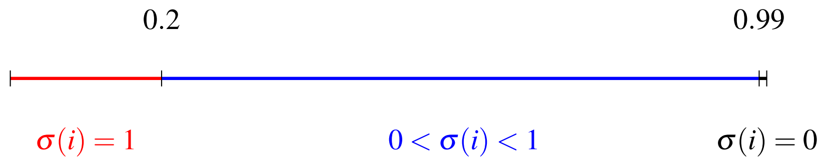

Fig. 3.4 Threshold values of q for different voting equilibria when $p = 0.8$

However, if we were in the presence of ambiguous information and voters were ambiguity averse and choosing their beliefs according to the Maxmin analysed in this chapter, we would be able to observe voters voting informatively under the unanimity rule, for a larger range of thresholds of reasonable doubts, below the level 0.9.

Take the signal precision to belong to $p \in [0.6, 0.8]$, from Figure 3.5, the cut-off value for informative voting being an equilibrium is 0.894, which is smaller than the given reasonable doubt level 0.9. Voters' strategy is $\sigma(i) = 0$, $\sigma(g) = 1$ in this case, since voters cast their votes according to the signals they receive.

On the other hand, although the cut-off value for the strategy $\sigma(i) = 1$, $\sigma(g) = 1$ being an equilibrium is increased, it will never exceed 0.5.

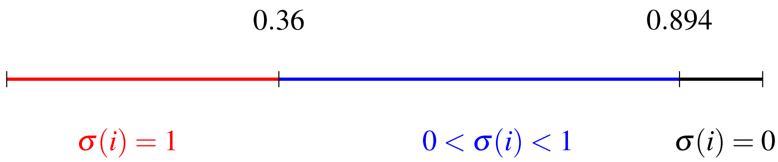

Fig. 3.5 Threshold values of q for different voting equilibria when $p = [0.55, 0.8]$

Under majority voting rule, if the signal precision is ambiguous, we observe similar results as we do under the unanimous voting rule, that is, the threshold for informative voting being an equilibrium is lower given $p \in [0.6, 0.8]$ than that when $p = 0.8$. The results are summarised in Table 3.4.

In Table 3.4, we looked at the three different voting rules, unanimity, simple majority and super majority. We found that (1) when information is downward ambiguous, the threshold level of q for voting informatively is lower. Thus, (2) informative voting is the only equilibrium for these three voting rules. The unanimity rule is the least preferred one when $p = 0.8$. However, (3) when $p = [0.55, 0.8]$, it outperforms other voting rules as it leads to the smallest type I error as opposed to other rules.

In chapter 2, we presented and discussed our experiments for the two-point non-common prior model. Based on the same parameters as the ones used in those experiments, $n = 5$, $q = 0.5$, $p = \{0.6, 0.9\}$, we first check how the threshold level of the reasonable doubt required for informative voting to be an equilibrium is affected by the introduction of imprecise probabilities belonging to an interval, rather, as opposed to the case when the precision of the signal is known and unique. We do so, by conducting a simple simulation so as to find out what is the voting strategy under unanimous voting rule when interval ambiguous information is provided instead of two-point ambiguous information, that is when signal precision is at least 0.6, but at most 0.9.

Table 3.5 shows that if the signal precision is amplified from its initial value 0.6, the threshold level of q for voting informatively under unanimity voting is increased. Whereas, if the signal precision is undermined from its initial value 0.9, the threshold level of q under the unanimity rule for voting informatively is decreased. Conversely for the majority voting rule. Because in the experiments we conducted we set the $q = 0.5$, which is very low, we do not observe the switch of voting strategy.

However, the ambiguous information not only affects the threshold level of q, it also affects the symmetric responsive voting strategy $\sigma(i)$. From Table 3.6, we do observe the dramatic decrease of type I error for unanimous voting rule when the information is amplified from 0.6, which is caused by the decrease in $\sigma(i)$. This suggests that if q is set fairly low, we should amplify the information precision from its initial level as it will lower the probability of voting against the received private signals.

Table 3.4 12-Person Jury Case under Different Information Structures, given $q = 0.9$

Signal Precision	Voting Rule	Threshold for $\sigma(i) = 1$	Threshold for $\sigma(i) = 0$	Informative Voting	$Pr(C\|I)$	$Pr(A\|G)$
$p = 0.8$	$\hat{k} = 12$	0.2	0.99	No	0.0069	0.6540
	$\hat{k} = 8$	0.00098	0.94	No	0.0011	0.0666
	$\hat{k} = 7$	0.00025	0.5	Yes	0.0039	0.0194
$p = [0.55, 0.8]$	$\hat{k} = 12$	0.36	0.894	Yes	0.0000	0.9313
	$\hat{k} = 8$	0.218	0.701	Yes	0.0006	0.0726
	$\hat{k} = 7$	0.188	0.5	Yes	0.0039	0.0194

Table 3.5 Group Decision under Different Information Structures, given $n = 5$, $q = 0.5$

Signal Precision	Voting Rule	Threshold for $\sigma(i) = 0$	Informative Voting
$p = 0.6$	$\hat{k} = 5$	0.7714	No
	$\hat{k} = 3$	0.4	Yes
$p = 0.9$	$\hat{k} = 5$	0.9986	No
	$\hat{k} = 3$	0.1	Yes
$p = [0.6, 0.9]$	$\hat{k} = 5$	0.8137	No
	$\hat{k} = 3$	0.3077	Yes

Table 3.6 Voting Strategies and Resulted Errors across Different Information Structures, given $n = 5$, $q = 0.5$

$n = 5$	$p = 0.6$	$p = 0.9$	$p = \{0.6, 0.9\}$		$p = (0.6, 0.9)$	
$\hat{k} = 5$			True $p = 0.6$	True $p = 0.9$	True $p = 0.6$	True $p = 0.9$
$\sigma(i)$	0.5959	0.4982	0.5618	0.5618	0.5248	0.5248
$Pr(C\|I)$	0.25	0.0496	0.2176	0.0815	0.1867	0.0614
$Pr(A\|G)$	0.59	0.2227	0.6185	0.2007	0.6515	0.2161
$\hat{k} = 3$			True $p = 0.6$	True $p = 0.9$	True $p = 0.6$	True $p = 0.9$
$\sigma(i)$	0	0	0	0	0	0
$Pr(C\|I)$	0.32	0.0086	0.32	0.0086	0.32	0.0086
$Pr(A\|G)$	0.32	0.0086	0.32	0.0086	0.32	0.0086

3.5 Conclusion

In this chapter we generalise the jury voting model of Feddersen and Pesendorfer (1998) by embedding ambiguity into the private signal structure and considering voters who, being ambiguity averse, adopt a Maxmin approach to assess each available action. We start our analysis by adopting the Maxmin approach (Gilboa and Schmeidler, 1989) to analyse the collective deliberation problem in the presence of an ambiguous precision of the private information voters/jurors possess. According to the Maxmin expected utility theorem, ambiguity averse voters assign the least favourable prior to evaluate each of their actions; and choose the action among the alternatives available to them, which gives the highest expected utility among the worst case scenarios.

Notice that, when using the standard Bayesian updating rule, $Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$, the implicit assumption requires both $A \cap B$ and B to be measurable sets. This suggests that under ambiguity, as no sufficient information is available for decision-makers to assign a probability for all relevant events, the conditional probability $Pr(A|B)$ is not well-defined. Thus, the standard Bayesian updating rule is not a proper updating rule to use under ambiguity. In this chapter, we use the Full Bayesian updating rule to update the conditional probability of the defendant being guilty conditional on the pivotal voter gets certain signal. The Full Bayesian updating rule requires updating all possible priors according to Baye's rule, without excluding any of them. Pires (2002) provided a decision-theoretic axiomatisation of this updating rule, which allows us to use Full Bayesian updating rule in the context of MMEU. Also, Eichberger et al. (2007) provides an axiomatic proof for updating non-additive capacities by using Full Bayesian update rule.

Following the Maxmin approach, we proved existence of both informative voting equilibrium and the strategic voting equilibria. However, there exist ways to improve upon the type I errors induced by the adoption of the unanimity rule, by selecting the appropriate width of the interval within which the ambiguity lies, for any adopted level of the threshold of reasonable doubt and size of the jury combinations. For example, we found that if the reasonable doubt level is set very high, at least, if we can undermine the information precision from its initial level, we can allow for a wider range of the reasonable doubt for informative voting being an equilibrium. And thus, unanimity outperforms non-unanimous voting. If the reasonable doubt level is fairly low, we might not see the switch of the equilibrium under the unanimity rule. If this is the case, then, amplifying the information precision from its initial level lowers the probability of voting against the private signal, and thus, lowers type I errors.

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Appendix C

Proofs of Results from Chapter 3

C.1 Proof of Proposition 3

Proof. Given $\sigma(g) = 1$, we have

$$\beta_G^i(\pi, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{\pi}{1-\pi})(\frac{\pi\sigma(i)+(1-\pi)}{\pi+(1-\pi)\sigma(i)})^{\hat{k}-1}(\frac{\pi}{1-\pi})^{n-\hat{k}}} = \frac{1}{1 + (\frac{\pi}{1-\pi})^{n-\hat{k}+1}(\frac{\pi\sigma(i)+(1-\pi)}{\pi+(1-\pi)\sigma(i)})^{\hat{k}-1}}.$$

Because $\beta_G^i(\pi, \sigma(i), \hat{k})$ is continuous at $\sigma(i) = 1$, and then,

$$\lim_{\sigma(i)\to 1^-} \beta_G^i(\pi, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{\pi}{1-\pi})^{n-\hat{k}+1}}.$$

Hence,

$$\lim_{\sigma(i)\to 1^-} \bar{\beta}_G^i(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{\pi^*}{1-\pi^*})^{n-\hat{k}+1}},$$

with $\pi^* = \underline{p}$, and

$$\lim_{\sigma(i)\to 1^-} \underline{\beta}_G^i(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{\pi^*}{1-\pi^*})^{n-\hat{k}+1}},$$

with $\pi^* = \bar{p}$. Therefore,

$$\lim_{\sigma(i)\to 1^-} \frac{\bar{\beta}_G^i(\pi, \sigma(i), \hat{k})}{1 - \underline{\beta}_G^i(\pi, \sigma(i), \hat{k})} = \frac{\lim_{\sigma(i)\to 1^-} \bar{\beta}_G^i(\pi, \sigma(i), \hat{k})}{1 - \lim_{\sigma(i)\to 1^-} \underline{\beta}_G^i(\pi, \sigma(i), \hat{k})} = \frac{\frac{1}{1+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1}}}{1 - \frac{1}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}}. \tag{C.1}$$

And $\beta_G^i(\pi,\sigma(i),\hat{k})$ is continuous at $\sigma(i)=0$, then, we have

$$\lim_{\sigma(i)\to 0^+}\beta_G^i(\pi,\sigma(i),\hat{k})=\frac{1}{1+(\frac{1-\pi}{\pi})^{2\hat{k}-n-2}}.$$

We can see that the monotonicity of $\lim_{\sigma(i)\to 0^+}\beta_G^i(\pi,\sigma(i),\hat{k})$ depends on $\hat{k}$. When $\hat{k}>\frac{n+2}{2}$, $\lim_{\sigma(i)\to 0^+}\beta_G^i(\pi,\sigma(i)v)$ is an increasing function of π. When $\frac{n+1}{2}<\hat{k}\leq\frac{n+2}{2}$, $\lim_{\sigma(i)\to 0^+}\beta_G^i(\pi,\sigma(i),\hat{k})$ is an increasing function of π.

If $\hat{k}>\frac{n+2}{2}$,

$$\lim_{\sigma(i)\to 0^+}\bar{\beta}_G^i(\pi^*,\sigma(i),\hat{k})=\frac{1}{1+(\frac{1-\pi^*}{\pi^*})^{2\hat{k}-n-2}},$$

with $\pi^*=\bar{p}$, and

$$\lim_{\sigma(i)\to 0^+}\underline{\beta}_G^i(\pi^*,\sigma(i),\hat{k})=\frac{1}{1+(\frac{1-\pi^*}{\pi^*})^{2\hat{k}-n-2}},$$

with $\pi^*=\underline{p}$. Therefore,

$$\lim_{\sigma(i)\to 0^+}\frac{\bar{\beta}_G^i(\pi,\sigma(i),\hat{k})}{1-\underline{\beta}_G^i(\pi,\sigma(i),\hat{k})}=\frac{\lim_{\sigma(i)\to 0^+}\bar{\beta}_G^i(\pi,\sigma(i),\hat{k})}{1-\lim_{\sigma(i)\to 0^+}\underline{\beta}_G^i(\pi,\sigma(i),\hat{k})}=\frac{\frac{1}{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}}}{1-\frac{1}{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}}}. \tag{C.2}$$

Therefore, whenever $\frac{q}{1-q}$ is strictly between the values identified in equations (C.1) and (C.2), there exists $\sigma^*(i)\in(0,1)$ such that $\frac{\bar{\beta}_G^i(\pi,\sigma^*(i),\hat{k})}{1-\underline{\beta}_G^i(\pi,\sigma^*(i),\hat{k})}=\frac{q}{1-q}$.

This is equivalent to requiring that conditions (C.3) and (C.4) below are satisfied at the same time:

$$q>\frac{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})}; \tag{C.3}$$

and

$$q<\frac{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}}{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}(2+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2})}. \tag{C.4}$$

Thus, if $\hat{k}>\frac{n+2}{2}$, as long as $\frac{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})}<q<\frac{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}}{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}(2+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2})}$, we have $0<\sigma(i)<1$ as the equilibrium strategy, which indicates that voters are indifferent to vote for convicting and acquitting when receiving signal i.

If $0 < \hat{k} < \frac{n+2}{2}$,

$$\lim_{\sigma(i)\to 0^+} \bar{\beta}^i_G(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1+(\frac{1-\pi^*}{\pi^*})^{2\hat{k}-n-2}},$$

with $\pi^* = \underline{p}$, and

$$\lim_{\sigma(i)\to 0^+} \underline{\beta}^i_G(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1+(\frac{1-\pi^*}{\pi^*})^{2\hat{k}-n-2}},$$

with $\pi^* = \bar{p}$. Therefore,

$$\lim_{\sigma(i)\to 0^+} \frac{\bar{\beta}^i_G(\pi, \sigma(i), \hat{k})}{1-\underline{\beta}^i_G(\pi, \sigma(i), \hat{k})} = \frac{\lim_{\sigma(i)\to 0^+} \bar{\beta}^i_G(\pi, \sigma(i), \hat{k})}{1-\lim_{\sigma(i)\to 0^+} \underline{\beta}^i_G(\pi, \sigma(i), \hat{k})} = \frac{\frac{1}{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}}}{1-\frac{1}{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}}}. \tag{C.5}$$

Therefore, whenever $\frac{q}{1-q}$ is strictly between the limit identified in equation (C.1) and the one identified in equation (C.5), there exists $\sigma^*(i) \in (0,1)$ such that $\frac{\bar{\beta}^i_G(\pi,\sigma^*(i),\hat{k})}{1-\underline{\beta}^i_G(\pi,\sigma^*(i),\hat{k})} = \frac{q}{1-q}$. Notice that this condition can be broken down into two conditions, as follows:

$$q > \frac{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})}; \tag{C.6}$$

and

$$q < \frac{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}}{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}(2+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2})}. \tag{C.7}$$

If $\hat{k} = \frac{n+2}{2}$, we have

$$\lim_{\sigma(i)\to 0^+} \bar{\beta}^i_G(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1+(\frac{1-\pi^*}{\pi^*})^{2\hat{k}-n-2}} = \frac{1}{2},$$

and

$$\lim_{\sigma(i)\to 0^+} \underline{\beta}^i_G(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1+(\frac{1-\pi^*}{\pi^*})^{2\hat{k}-n-2}} = \frac{1}{2}.$$

Therefore,

$$\lim_{\sigma(i)\to 0^+} \frac{\bar{\beta}^i_G(\pi, \sigma(i), \hat{k})}{1-\underline{\beta}^i_G(\pi, \sigma(i), \hat{k})} = \frac{\lim_{\sigma(i)\to 0^+} \bar{\beta}^i_G(\pi, \sigma(i), \hat{k})}{1-\lim_{\sigma(i)\to 0^+} \underline{\beta}^i_G(\pi, \sigma(i), \hat{k})} = 1. \tag{C.8}$$

And, we know that when $\hat{k} \leq \frac{n+2}{2}$, the limit value identified by condition (C.5) is larger than 1. Therefore, whenever $\frac{q}{1-q}$ is strictly between the limit identified in equation (C.1) and the one identified in equation (C.5), there exists $0 < \sigma(i) < 1$ as the symmetric responsive equilibrium strategy, that is, $\frac{n+1}{2} < \hat{k} \leq \frac{n+2}{2}$, as long as $\frac{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})} < q < \frac{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}}{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}(2+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2})}$.

$\square$

Chapter 4

Conclusion And Further Research

Much decision-making takes place in small groups. Members gather together to cast votes over alternatives proposed to them, be it for a bill to be passed in a Congress, a project to be selected for financing, a patient to be put on a transplant list, or a defendant's fate in a court of law.

The way in which decision-making occurs, its process and the voting rule adopted, as well as the information used by members of a committee to finalise a decision, are all important elements in the quality of that decision. Although, it is always possible to conjecture which decision(s) would be reached by a small decision-group under specified preferences and information structure, that is, whenever a common prior is shared by all parties involved, it is far from obvious to anticipate what the final decision could be in the presence of information which is inherently *ambiguous*.

Using the jury trial as our leading example, and to advance our understanding of how ambiguity can play a role in a jury trial setting, in this thesis we investigated how the embedding of identical, but – at least partially – ambiguous information into the canonical jury decision-making model of Feddersen and Pesendorfer (1998) changes its main findings. We did so by providing more realistically imprecise measures of information reliability and by studying how those impact on information aggregation for the group decision-making. Specifically, we were interested in studying how the introduction of ambiguity in the precision of the signals voters receive before casting their votes affects the probabilities of convicting the innocent (type I error) and acquitting the guilty (type II error) when compared to the canonical jury trial case.

To that end, in chapter 1 we began by exploring a model in which jurors may distrust the precision of the information given to them, leading to jurors adopting potentially differing priors and altering the formation of their posteriors, used when casting votes to convict or to acquit a defendant. Within this model, we obtained the following findings. As the size of the jury grows sufficiently large, when voters share the same 'trusting' level of belief, voting according to their private signals leads to a smaller probability of convicting an innocent defendant than in the canonical model. This suggest that if there were ways of framing all voters to believe that the quality of the private information is the highest among alternative ones provided to them, and that belief is wrong (jurors trust the precision to be higher than its 'true' underlying level), type I errors would be reduced, if not even eliminated. Therefore, asymptotically, being trusting of the information received or framing the information to induce more trust in it, makes the unanimity voting rule less unappealing. However, for a small jury size, distrusting the information provided would be best to reduce type I errors and to improve the performance of the unanimity rule.

In chapter 2, we reported results from an array of experiments designed to capture the collective voting behaviour under the two-point non-common prior model introduced in chapter 1 and to contrast them against results of canonical collective voting behaviour models. Our aim was to investigate the collective decision-making outcomes under different voting rules when the quality of the private information given to voters when casting their votes is unmeasurable, triggering voters to adopt potentially differing beliefs about it. The results of these experiments validate the theoretical predictions of voting under the two-point non-common prior model, suggesting the importance of the quality of the information structure in determining the collective deliberation outcomes. These results help establish when, in the finite case, the unanimity voting rule can outperform majority voting rule if voters adopts two-point non-common priors.

In chapter 3 we generalised the jury voting model of Feddersen and Pesendorfer (1998) by embedding ambiguity into the private signal structure and considering voters who, being ambiguity averse, adopt a Maxmin approach to form subjective beliefs. The Maxmin Expected Utility Theorem (MMEU) of Gilboa and Schmeidler (1989) helped capture the voter's attitude towards ambiguity to analyse how this impacts the collective voting outcomes under both the majority rule and the unanimity rule. According to MMEU, voters assign their priors in an act-contingent manner, that is, ambiguity averse voters assign the prior, which gives them the best among the worst expected utility levels when evaluating alternatives choices (in this context, voting choices, namely whether to vote to convict or to acquit). Within this framework we proved the existence of an informative voting equilibrium and of strategic voting equilibria. Moreover, we found that if ambiguity exists in the precision of the private information, it is easier to sustain informative voting as an equilibrium strategy, that is, there exists a larger set of reasonable doubt levels for the unanimity voting rule to prevail as an equilibrium of the voting game. This is an important result as voting informatively, especially under unanimity helps maintain the efficiency of information aggregation: there is scope for resurrecting unanimity against majority, when facing an ambiguous world.

Our theoretical and experimental results call into question preconceived results about the performance of different institutional designs and voting rules for collective deliberation under differing information structures. When the objective probability of the information is imprecisely measured, that is when the common-prior assumption is relaxed, novel results arise which deserve further exploration, challenging our views about the virtues of adopting, say, majority voting, as opposed to unanimity voting, to avoid the bad outcome of exacerbating the odds of convicting an innocent defendant (jury paradox).

Drawing from the lessons from our laboratory experiments conducted at the University of Auckland, and on theoretical predictions obtained in chapter 3, we plan to conduct additional laboratory experiments, emulating real life decision-making scenarios. Our experimental designs could be varied along the following dimensions: (i) The voting rule (unanimity versus majority vote); (ii) The size of the decision-making group (small, medium, or large); (iii) The communication channel amongst subjects to emulate various deliberation processes in real life (with and without free-form communication); (iv) The source of non-common priors (ambiguous information versus maximum likelihood belief revision); and (v) The spread and the positioning of the ambiguous information with respect to some 'true' level of precision of the signal to be taken as a benchmark. Additionally, we could also control for whether information is (vi) provided in a neutral fashion or with context; and (vii) acquired descriptively or experientially. With these experimental designs we will continue to effectively engage in decision-making engineering, with control and treatment groups, to disentangle what information structures and belief revision processes are most conducive to sound group decision-making.