APPLICATIONS OF MATHEMATICAL PROGRAMMING ON FOUR NEW ZEALAND HORTICULTURAL HOLDINGS

by

A.N. RAE

Thesis Presented in Partial Fulfilment
of the Requirements for the Degree
of
Master of Horticultural Science
in
Massey University

November 1968

ACKNOWLEDGEMENTS

The author wishes to thank his two supervisors, Professor W.V. Candler and his successor, Professor A.R. Frampton, for their advice and encouragement received during this study. Special thanks are due to Professor Candler who obtained a solution to the quadratic programming study of Chapter 5 at Purdue University, Indiana, U.S.A. But for a lack of time a solution to the quadratic programme of Chapter 6 would have been obtained from the same source.

The author also wishes to express his thanks to the four horticulturalists who willingly gave of their time to allow the various programming models to be constructed.

Thanks are due to Mesdames J. Bishop and M. Clifford for typing, and to the library staff for their assistance in obtaining references.

Finally, thanks are due to members of the Horticulture Department, Massey University, with whom the author had discussions from time to time.

CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION AND THESIS OUTLINE

1.1 Introduction

Although fifteen years have passed since the publication of Dorfman's article[1] describing linear programming in terms readily understood by the most non-mathematical agricultural economist, and fourteen years have lapsed since Heady published an article[2] demonstrating the obvious potential of linear programming in solving a large class of farm management problems, 'real life' applications of programming, particularly those concerned with horticultural management are surprisingly few.[3]

---

1. Dorfman, Robert, "Mathematical or 'Linear' Programming, a Non-Mathematical Exposition," American Economic Review, vol.43, p.797, 1953.
2. Heady, Earl O., "Simplified Presentation and Logical Aspects of Linear Programming Technique," Journal of Farm Economics, vol.36, p.1035, 1954.
3. For interesting applications of programming to horticultural or part-horticultural holdings, see:
   Simpson, I.G., Hales, A.W., and Fletcher, A., "Linear Programming and Uncertain Prices in Horticulture," Journal of Agricultural Economics, vol.15, p.617, 1963;
   Camm, B.M., "Risk in Vegetable Production on a Fen Farm," The Farm Economist, vol.10, p.89, 1962-65;
   Wesney, D., "A Study of the Financial Returns to Process Pea Growers in Hawkes Bay," unpublished M.Agr.Sc. thesis, Massey University Library, 1964; and
   Tyler, G.J., "An Application of Linear Programming," Journal of Agricultural Economics, vol.13, p.473, 1960.

Linear programming has been accepted in the U.S.A. as an extremely useful and versatile tool for both farm management research and advisory work but has not as yet been widely accepted in the United Kingdom, where simpler techniques such as Programme Planning[4/] are advocated. Official advisory services in New Zealand tend to be based on techniques used in the United Kingdom and hence linear programming has not been given adequate opportunity to demonstrate its usefulness.

1.2 Objectives of the Study

The objective of this study is to evaluate the usefulness of horticultural management plans formulated by linear and nonlinear programming. Such an evaluation will include the feasibility of the plan itself, the nature of additional information (such as the imputed value of resources) obtained from the programmed solution (which is not easily provided by the simpler budgeting methods), and the extent to which profits may be increased if the programmed solution was put into practice. Rather than comparing the programmed profits with those the horticulturalist received in the previous season, they should be compared with the expected profit, using the prices, yields and costs assumed in the programme, for the management plan the horticulturalist considered best before being presented with the programmed solution. This is considered desirable due to the wide fluctuations in horticultur-

---

4. Clarke, G.B. and Simpson, I.G., "A Theoretical Approach to the Profit Maximisation Problems in Farm Management," Journal of Agricultural Economics, vol.13, p.250, 1959. For a comparison of the merits of Programme Planning and Linear Programming see Candler, Wilfred and Warren Musgrave, "A Practical Approach to the Profit Maximisation Problems in Farm Management," Journal of Agricultural Economics, vol.14, p.208, 1960.

al incomes that may occur from one year to the next - it is possible that a linear programme may show a lower level of profit than that obtained the previous season, simply due to high prices the previous year.

It will be possible, then, to see just how near the horticulturalist's management plan is to the optimum and dependent on this, how useful mathematical programming may be in horticultural management advisory work.

1.3 The Approach to the Study

Two fresh vegetable growers, one process vegetable grower, and a nurseryman were chosen as sources of information to allow their respective production situations to be analysed by either linear or quadratic programming.

Although an orchardist was not included in the study, problems of orchard development are dealt with since the process vegetable grower intends to plant his property in orchard, gradually phasing out the vegetable crops grown for processing.

In general, two visits to each grower were necessary to collect the data required for the programme. The problem was then solved and taken back to the grower for discussion. This resulted, in some cases, in modification of the programme, so that a further visit to the grower was necessary before his comments on the final solution could be obtained.

1.4 Some Common Criticisms of Mathematical Programming for Horticulture

It is the belief of some that for various reasons, linear programming is unsuitable as a method of formulating horticultural management plans. Some of these reasons are now mentioned, along with the way in which this study attempted to overcome, or nullify, these criticisms.

1.4.1 Problems of price and yield variability

Great difficulty is experienced in estimating the next season's prices of some horticultural products, especially vegetables and flowers sold at auction, and this uncertainty regarding prices is no doubt the greatest problem encountered when programming fresh vegetable holdings in particular. Since weather tends to be an unpredictable factor of production, problems of yield variability also exist.

It is emphasised, however, that the uncertainty attached to prices and yields is not a weakness of linear programming techniques - prices and yields would still require estimation even if crop management planning was carried out using the most simple budgeting techniques.

Furthermore, a horticulturalist, in deciding what crops to grow in the following season, either implicitly or explicitly makes some estimation of expected prices and yields.

1.4.2 Methods of overcoming price and yield variability

Two methods are demonstrated in the thesis.

First, the linear programme solution includes the range of crop net returns over which the optimum plan will remain unchanged. If it can be shown that the range of net returns for each crop permits some considerable degree of price and/or yield variation without altering the optimum combination of crops, then the uncertainty attached to price and yield estimation will not hinder the practical application of the results as much as had been previously anticipated.

The second method of overcoming price and yield variability is through the use of quadratic programming to take specific account of risk aversion, where data is available on price and yield variability. Since risk (as opposed to uncertainty) refers to variability which is measurable in an empirical manner, this approach assumes that although prices and yields cannot be exactly predicted in any specified year, the <u>probability</u> of their outcomes can be established.

1.4.3 The problem of lack of farm records

Although well kept farm records can readily provide much of the information required to set up a linear (or nonlinear) programming model, the lack of such records does <u>not</u> mean that mathematical programming is impossible.

It was the author's experience that although all growers contacted were becoming increasingly aware of the benefits of keeping production and marketing records, the lack of a wide range of data did not prevent the various production situations from being programmed. By careful questioning and discussion with the grower, estimates of all the required information could be obtained.

It is possible that information carried 'in the grower's head' may be more subject to error than if such information was available from records. Again, this is no reason for rejecting mathematical programming - exactly the same information has been used as the grower himself would have available in his memory to allow him to plan for the future.

1.4.4 Problems peculiar to process vegetable production

At first sight, mathematical programming may appear to be of little use to process vegetable growers, since the acreages of crops grown is beyond the control of the grower, being set in the form of contracts by the processing company. This means that the process vegetable grower has 'room for manoeuvre' only in the production of those crops he may grow for the fresh market, or in the carrying of livestock. This, however, does not detract from the usefulness of a programming study - first, it is intended to show just what room for manoeuvre (production possibilities) the grower actually has, and secondly, by imputing values to the contracts, the programme will indicate what each contract is worth to the grower, and how he should attempt to alter them.

1.5 An Outline of the Thesis

In Chapter 2, both linear and quadratic programming have been described and concepts used in the thesis, such as the range of crop net returns mentioned in section 1.4.2, have been developed.

The next four chapters discuss the formulation of various programming models and their solutions. Where necessary, reference has been made to Chapter 2 to avoid duplication of the theoretical or mathematical background of programming.

Chapter 3 illustrates an application of linear programming to an Otaki fresh vegetable holding and Chapter 4 describes an intertemporal linear programming application to a Heretaunga Plains process vegetable holding.

Chapters 5 and 6 both illustrate quadratic programming as applied to horticultural management. Chapter 5 formulates the optimum management plan for a nurseryman who faces downward-sloping demand curves for some of his products, and Chapter 6 deals with risk aversion on a Wanganui fresh vegetable holding.

Chapter 7 concludes the thesis by drawing together the various ideas developed in the preceding chapters.

CHAPTER 2

A DESCRIPTION OF LINEAR AND QUADRATIC PROGRAMMING

2.1 Linear Programming

2.1.1 Introduction[1]

Linear programming problems are concerned with the efficient allocation of scarce resources to meet desired objectives. Many problems which have a definite objective, alternative methods of achieving the objective and restrictions (for example on the availability of resources) can be expressed as linear programming problems.

The objective may be the maximisation of a linear profit function, or the minimisation of a linear cost function. Specification of the problem includes the quantity of resources available, the possible outputs, the quantity of resources required per unit of output, and the profit (or cost) from a unit of output.

A solution to the linear programme that satisfies both the conditions of the problem and maximises (or minimises) the given objective, is termed an optimum feasible solution.

---

1. For an elementary exposition of linear programming, see: Heady, Earl O. and Wilfred Candler, "Linear Programming Methods", Iowa State University Press, 1958, Chapters 1 and 2; and Dorfman, R., Samuelson, P. and Solow, R., "Linear Programming and Economic Analysis", McGraw-Hill Book Company, New York, 1958, Chapter 2.

2.1.2 The assumptions

Linear programming makes use of several assumptions, all of which must apply to the problem under consideration if realistic solutions are to be obtained.[2/]

2.1.2.1. The linearity assumption

Linear programming requires all input-input, input-output and output-output relationships to be of a linear (straight-line) nature. That is, the curvilinear isoquants, production functions and transformation curves of production economics theory are replaced by segmented straight-line relationships.[3/]

This is analagous to a constant-returns-to-scale assumption, in that a given change in resource input levels must change output from the same resources in the same proportion.

2.1.2.2 The additivity assumption

This assumption states that the total output and amounts of resources used by several enterprises must be equal to the sums of those of the individual enterprises. That is, a complete accounting, by enterprises, can be made for each resource and product.[4/]

---

2. If some assumptions do not hold, linear programming may still provide a useful approximation to the optimum plan.
3. This does not mean that curvilinear relationships cannot be closely approximated. See Heady, op.cit., p.1038.
4. This does not mean that non-additive relationships cannot be handled. Thus a rotation activity for tomatoes, sweet corn and ryegrass may give a greater yield of all three crops than the sum of each crop's yield when the crops are grown continuously in the same land.

2.1.2.3 The non-negativity assumption

The solution must contain variables at non-negative levels only, since the production of a negative output is meaningless.[5]

2.1.2.4 Single-value expectations

Linear programming assumes that resource supplies, input-output coefficients and prices are known with certainty.[6] Although this assumption would appear unrealistic in horticultural production, it must be remembered that this same assumption applies to other management techniques such as budgeting.[7] Therefore linear programming can provide solutions as realistic as those obtained from other methods which employ the same assumptions.

2.1.2.5 The convexity assumption

The set of feasible solutions to a linear programming problem (if a feasible solution exists) is a closed convex set. Therefore if any two points x_1 and x_2 are in the convex set $\mathbf{X}$ of feasible plans then any point on the line segment joining x_1 and x_2 is also feasible (in the set $\mathbf{X}$).[8]

---

5. However an activity could be defined as buying, for example, hay, and another (with coefficients of opposite sign) as selling hay. This effectively results in the introduction of activities which can occur at negative levels.
6. If the variance of prices is known, a quadratic objective function may be constructed to minimise income variance for each level of expected income. See section 2.3 on risk programming.
7. For a comparison of linear programming and budgeting see Heady, op.cit., p.1035, and Heady and Candler, op.cit., pp.42-46. Briefly, budgeting is used to find which of a relatively few likely production situations yields the most profit, but there is no guarantee that this is the maximum possible profit. Linear programming has the advantage that the unique maximum profit plan is found by examining a large number of alternatives. It can also provide much extra information such as the value of resources, which is not easily obtained with budgeting.
8. Situations with increasing marginal products give rise to non-convex sets of feasible plans and such situations cannot be handled with linear programming. Heady and Candler, opcit., pp.220-224, suggest a way around this problem.

2.1.3 The objective function

The objective of the linear programme models of Chapters 3 and 4 is to maximise a linear profit function of the form [9/]

$$Z = \sum_{j=1}^{n} c_j x_j \tag{2-1}$$

where Z is the linear programme profit,

c_j are per unit profits and

x_j are the operation levels of the n cropping alternatives included in the model.

Given the c_j values, the linear programming problem is to find values of x_j such that Z is a maximum, subject to the appropriate restraints.

2.1.4 The activities

Two processes of production represent different activities if they

(i) use different resources;

(ii) produce different products;

(iii) require different proportions of the same resources to produce the same product; or

(iv) use the same resources in the same proportions, but produce products in different proportions.

For example, a comparison of various ways of producing a crop may be included in a linear programme. This would appear to be an important consideration in horticultural management since profits may

---

9. The notation used is similar to that found in Heady and Candler, op.cit., except that the input-output coefficients (r_{ij} in the above text) will be represented by a_{ij}).

just as likely be increased by improving technical efficiency as by changing the structure of output.

2.1.5 The restraints

For a linear programme solution to be feasible, the total amount of each resource used in producing the total output must be no greater than the supply of each resource available. That is,

$$\sum_{j=1}^{n} a_{ij}x_j \leqslant b_i \quad ; \quad i = 1 \ldots m \quad ; \qquad (2\text{-}2)$$

where a_{ij} is the amount of the i^{th} resource used in the production of one unit of the j^{th} activity; and

b_i is the total supply of the i^{th} resource.

2.1.6 The non-negativity requirement

The non-negativity assumption states that each activity must be operated at a zero or positive level. That is,

$$x_j \geqslant 0 \quad ; \quad j = 1 \ldots n \qquad (2\text{-}3)$$

2.1.7 A summary of the linear programme problem

The problem concerns the maximisation of a linear objective function (equation 2-1) subject to a set of linear inequalities (2-2) and the non-negativity condition (2-3).

2.1.8 Disposal activities

Even though the presence of inequalities in (2-2) gives rise to computational difficulties, it is desirable to allow the possibility of less than the total supply of a resource being used, since this may result in more profitable plans than if it was necessary to use exactly all resources.

Therefore the inequalities of (2-2) are converted to equalities by adding disposal activities, one for each restriction.[10/] Inequality (2-2) then becomes

$$\sum_{j=1}^{n} a_{ij}x_j + x_{n+i} = b_i \quad ; \quad i = 1 \ldots m \qquad (2\text{-}4)$$

where x_{n+i} is a disposal activity to allow the i^{th} resource to remain partly or wholly unused.

To distinguish between the n activities (the x_j's) and the m disposal activities, the former will be referred to as real activities.

2.1.9 The valuation of resources

Associated with every linear programming maximisation problem (such as described in section 2.1.7) is a closely related minimisation problem, with such pairs of problems termed dual linear programming problems.[11/]

Whereas the primal problem is to maximise (2-1) subject to the restraints (2-2) and (2-3), the dual problem may be represented as:

$$\text{minimise} \quad G = \sum_{i=1}^{m} w_i b_i \qquad (2\text{-}5)$$

$$\text{subject to} \quad \sum_{i=1}^{m} a_{ij}w_i \geq c_j \quad ; \quad j = 1 \ldots n \; ; \qquad (2\text{-}6)$$

$$\text{and} \quad w_i \geq 0 \quad ; \quad i = 1 \ldots m \qquad (2\text{-}7)$$

---

10. Strictly, disposal activities are required only for those inequalities which specify that the amount of a resource used must be 'less than or equal to' the supply. For treatment of 'greater than or equal to' and equality restraints, see Hadley, G., "Linear Programming," Addison-Wesley Publishing Company Inc., U.S.A., 1962, pp.72-76.
11. Dual linear programming problems and their interpretation are discussed in Heady and Candler, op.cit., pp.90-107; Hadley, op.cit., Chapter 8 and pp.483-486; and Baumol, William J., "Economic Theory and Operations Analysis," Prentice-Hall, Inc., New Jersey, 1961, Chapter 6 (second edition only).

where w_i is a value imputed to the i^{th} resource.[12/]

The dual problem therefore determines the w_i values so that the total value of resources is minimised and the value of resources used in the production of one unit of activity j is at least as great as the profit received on the sale of one unit of the j^{th} activity.

For either the primal or dual problem to be optimised it is necessary that resources be valued so that total profits are completely imputed to the resources, and total profits equal the total value of resources. Hence for an optimum solution,

$$Z = \sum_{j=1}^{n} c_j x_j = G = \sum_{i=1}^{m} w_i b_i \qquad (2\text{-}8)$$

The imputed values are often referred to as shadow prices, and measure the rate of change of the objective function with respect to changes in the resource supply. Thus valuation of resources by means of the shadow prices is an opportunity cost valuation and has <u>no</u> relation to the <u>actual</u> costs of the resources.

2.1.10 Linear programme solutions and their interpretation

Any set of x_j which satisfies the conditions (2-3) is a <u>solution</u> to the linear programme problem; a solution which also satisfies the conditions (2-2) is a feasible solution; and any feasible solution which optimises the objective function (2-1) is an <u>optimum</u> feasible solution.

The simplex algorithm[13/] may be employed to obtain the optimum

---

12. Linear programming problems may be solved as either the primal or dual as both give the same information. Since the activities of a primal problem form the restraints of the dual, (primal) problems with many restraints but few activities may be solved with less effort by expressing the problem in its dual form.
13. For a lucid description of the simplex method, see Heady, op.cit., pp.1039-1048. A comprehensive treatment of the simplex algorithm is to be found in Hadley, op.cit., Chapters 3 to 5.

solution, with the final simplex tableau providing much useful data.[14]

2.1.11 Sensitivity analysis

2.1.11.1 Stability of the optimum plan to price changes

In Chapter 1, section 1.4.2 it was mentioned that the range of crop net returns over which the optimum solution would remain unaltered, can be calculated directly from the final simplex tableau. A solution which permits the net returns of all included activities to fluctuate over a wide range can be considered stable, and hence problems of price variation may be less serious than is commonly thought.

The maximum increase in the net return from an activity included in the solution (that is, a basic activity) which would leave the optimum solution unchanged is given by:

$$\Delta c_i = \min_j \left[\frac{-(z_j - c_j)}{a_{ij}} \quad ; \quad a_{ij} < 0 \right] \qquad (2\text{-}9)$$

where $z_j = \sum_{i=1}^{m} c_i a_{ij}$ and

$\min_j$ refers to the limiting variable which would enter the basis should the net return of the i^{th} basic activity increase by more than Δc_i.[15] The upper price (net return) limit will be equal to $(c_i + \Delta c_i)$, where c_i is the original price of the i^{th} basic activity.

Similarly, the lower price (net return) limit is $(c_i + \Delta c_i)$,

---

14. Puterbough, H.L., Kehrberg, E.W., and Dunbar, J.O., "Analysing the Solution Tableau of a Simplex Linear Programming Problem in Farm Organisation," Journal of Farm Economics, vol.39, p.478, 1957.
15. Should the net return of the i^{th} basic activity become equal to either the upper or lower limit, the j^{th} non-basic activity can be included in the optimum plan with no effect on total profits.

where Δc_i, the maximum decrease in the price of the i^{th} basic activity which would leave the optimum plan unchanged, is given by

$$\Delta c_i = \min_j \left[\frac{-(z_j - c_j)}{a_{ij}} \quad ; \quad a_{ij} > 0 \right] \qquad (2\text{-}10)$$

It can be seen, then, that so long as the realised net return from a crop included in the optimum plan lies somewhere between the upper and lower limits, the optimum plan initially recommended to the horticulturist will still remain the optimum.[16/]

Assuming that the net returns of all basic activities remain unchanged, the stability of the solution will be affected by the likelihood of non-basic activities entering the basis. The $(z_j - c_j)$ value of the j^{th} non-basic activity represents the extent to which profits would fall should the activity be forced into the basis, and such a value may be interpreted as the marginal opportunity cost of including the non-basic activity in the solution. Therefore the extent to which the net returns of non-basic activities would have to increase to make their marginal opportunity costs equal to zero are easily found. The likelihood of these higher net returns being realised will determine which non-basic activities form stable elements, and which form non-stable elements of the optimum solution.[17/]

2.1.11.2 The limits to which non-basic activities may enter the basis

Should a real non-basic activity be forced into the basis profits will fall by an amount which depends upon the marginal opportunity cost

---

16. This will be true, as long as only one net return has altered - all other net returns must remain unchanged.
17. Again, this assumes that all other net returns remain constant.

of that activity. However, profits will increase should the supply of a limiting (scarce) resource be added to, with the increase in profits depending upon the shadow price (or marginal value product) of that resource.

It is possible to compute the limits over which a given marginal cost or marginal value product will hold. For example, as the supply of a scarce resource is increased, its shadow price will remain constant for a range, and then, at some discrete point, fall to a lower level.

The upper limit to the amount of a non-basic (real or disposal) activity which can be added to the plan without changing the shadow price is given by:

$$\min_{i} \left[\frac{b_i}{a_{ij}} \; ; \quad a_{ij} > 0 \right] \qquad (2\text{-}11)$$

where $\min_{i}$ gives the limiting basic activity and

$\frac{b_i}{a_{ij}}$ gives the upper limit at which the j^{th} non-basic activity may enter the plan to replace the i^{th} basic activity, without the imputed value being changed.

Similarly, the lower limit (or amount of the activity which could be "removed from the plan" without changing the marginal value product) is given by:

$$\min_{i} \left[\frac{b_i}{a_{ij}} \; ; \quad a_{ij} < 0 \right] \qquad (2\text{-}12)$$

Since the lower limits are negative, they have no significance as far as real activities are concerned since a real activity cannot exist at a negative level. The lower limits are most important for disposal activities, however, since they indicate the limit to a decrease in the resource disposal level (that is, an increase in resource supply), which leaves the shadow price of the resource unchanged.

2.2 Quadratic Programming

2.2.1 Introduction[18]

This section discusses the slightly more general problem of optimising a quadratic function[19] subject to linear inequalities.[20]

Section 2.1 dealt with linear programming in which it was assumed that product (and factor) prices remain constant no matter what output is produced. This restricts linear programming analyses to firms in perfect competition, and excludes study of imperfectly competitive market forms such as monopoly and oligopoly.

Quadratic programming can handle the latter market forms. For example, if a firm must lower product prices in order to sell extra output, it may double all inputs and find that profits increase proportionately less. The firm's optimum production plan can be found

---

18. For a practical application of quadratic programming see: Louwes, S.L., Boot, J.C.G., and Wage, S., "A Quadratic Programming approach to the Problem of the Optimal Use of Milk in the Netherlands", Journal of Farm Economics, vol.49, p.309, 1963.
19. A quadratic function is of the form:

$$Z = ax_1^2 + bx_2^2 + cx_1x_2 + dx_1 + ex_2$$

which contains no powers or products of the variables of higher degree than the second.

20. For a general coverage of non-linear (quadratic) programming, see: Hadley, G., "Nonlinear and Dynamic Programming", Addison-Wesley Publishing Company, Inc., 1964; Dorfman, Samuelson and Solow, op.cit., Chapter 8; and Baumol, op.cit., Chapter 7.

by quadratic programming since the problem is one in which the rate of change of the objective function with respect to changes in activity levels diminishes.

2.2.2 Statement of the quadratic programming problem[21]

The problem is to find a 1 x n vector $\underline{x}$ such that

$$Z = \underline{cx}' + \underline{x}B\underline{x}' \text{ is maximised} \quad (2\text{-}13)$$

subject to $A\underline{x}' \leqslant \underline{b}'$ (2-14)

and $\underline{x} \geqslant \underline{0}$ (2-15)

where $\underline{c}$ and $\underline{x}$ are 1 x n vectors of activity net returns and operation levels respectively;

$\underline{b}$ is 1 x m vector of resource supplies;

A is an m x n matrix of input-output coefficients; and

B is an n x n negative definite[22] matrix.

It will be noticed that the restraints (2-14) and (2-15) are similar to the linear programming restraints (2-2) amd (2-3), except that the former are written in matrix notation.

2.2.3 The related lagrangian problem

By the addition of disposal activities, inequality (2-14) may be converted to an equality:

$$\underline{b}' - A\underline{x}' - \underline{\pi}' = \underline{0}' \quad (2\text{-}16)$$

where $\underline{\pi}$ is a 1 x m vector of disposal activities, and therefore (2-15) is augmented to:

$$\underline{x},\ \underline{\pi} \geqslant 0 \quad (2\text{-}17)$$

---

21. Candler, Wilfred, and Evans, D.A., "Classical Quadratic Maximisation and Parametric Quadratic Programming", Technical Discussion Paper No. 2, Department of Agricultural Economics and Farm Management, Massey University, October 1963.
22. B is negative definite when $\underline{x}B\underline{x}' < 0$, for all values of $\underline{x}$.

Equations (2-13) and (2-16) may now be re-written in lagrangian form. Since (2-16) is equal to zero, it can be added to equation (2-13) without causing any change in the value of the objective function. The lagrangian form is:

$$Z^* = \underline{cx}' + \underline{x}B\underline{x}' + \underline{\lambda}(\underline{b}' - A\underline{x}' - \underline{\pi}') \text{ a maximum,} \qquad (2\text{-}18)$$

subject to (2-15), where $\underline{\lambda}$ is a 1 x m vector of lagrangian multipliers.

If the i^{th} restraint in (2-18) is effective, π_i will equal zero and λ_i (the shadow price of the restraint) will be either positive or zero.[23] Should the i^{th} restraint be non-effective, however (so that the inequality holds), π_i will be positive (the i^{th} resource will be in disposal) and λ_i will be zero.

Therefore a further condition for finding the vector $\underline{x}$ which maximises the lagrangian expression (2-18) and the vector $\underline{\lambda}$ which minimises the lagrangian expression is:

$$\underline{\lambda\pi'} = \underline{0}' \qquad (2\text{-}19)$$

$$\text{and} \quad \underline{\lambda}, \underline{\pi} \geq \underline{0} \qquad (2\text{-}20)$$

2.2.4 The partial derivatives

To maximise (2-18) with respect to an element of $\underline{x}$ (x_j), the partial derivative with respect to the element x_j is taken, to obtain:

$$\frac{\partial Z^*}{\partial x_j} = c_j + 2\underline{\beta}_j\underline{x}' - \underline{\lambda\alpha}'_j = 0 \; ; \; x_j > 0 \; ; \qquad (2\text{-}21)$$

$$\leq 0 \; ; \; x_j = 0 \; ; \qquad (2\text{-}21)'$$

where $\underline{\beta}_j$ is the row of B conforming to x_j, and

$\underline{\alpha}'_j$ is the column of A conforming to x_j.

---

23. Both $\pi_i = 0$ and $\lambda_i = 0$ represent the degenerate case where the optimum plan uses up exactly the available amount of resource i, but any further units of the resource would be allocated to disposal.

Writing $\underline{\nabla x}$ as the vector of partial derivatives of Z* with respect to the elements of $\underline{x}$, gives:

$$\underline{\nabla x}' \quad = \quad \underline{c}' \; + \; 2B\underline{x}' \; - \; A'\underline{\lambda}' \qquad (2\text{-}22)$$

To express the possibilities of (2-21) and (2-21)', equation (2-22) may be modified to give the conditions:

$$\underline{c}' \; + \; 2B\underline{x}' \; - \; A'\underline{\lambda}' \; - \; \underline{y}' \quad = \quad \underline{0}' \qquad (2\text{-}23)$$

$$\underline{xy}' \quad = \quad \underline{0}' \qquad (2\text{-}24)$$

$$\underline{x},\underline{y} \quad \geqslant \quad \underline{0} \qquad (2\text{-}25)$$

where $\underline{y}$ is a 1 x n non-negative vector.

The maximisation of the lagrangian expression also requires that the partial derivatives of that expression with respect to the lagrangian multipliers ($\underline{\lambda}$) be obtained. Setting these partial derivatives equal to zero gives:

$$\underline{\nabla\lambda}' \quad = \quad \underline{b}' \; - \; A\underline{x}' \; - \; \underline{\pi}' \quad = \quad \underline{0}' \qquad (2\text{-}26)$$

where $\underline{\nabla\lambda}$ is a 1 x m vector of partial derivatives of Z* with respect to the elements of $\underline{\lambda}$.

The conditions for maximising the lagrangian expression may now be summarised:

$$\underline{c}' \; + \; 2B\underline{x}' \; - \; A'\underline{\lambda}' \; + \; \underline{y}' \quad = \quad \underline{0}' \qquad (2\text{-}27)$$

$$-\,\underline{b}' \; + \; A\underline{x}' \; + \; \underline{\pi}' \quad = \quad \underline{0}' \qquad (2\text{-}28)$$

$$\underline{x},\underline{y} \quad \geqslant \quad \underline{0}' \qquad (2\text{-}29)$$

$$\underline{\lambda},\underline{\pi} \quad \geqslant \quad \underline{0}' \qquad (2\text{-}30)$$

$$\underline{xy}' \quad = \quad \underline{0}' \qquad (2\text{-}31)$$

$$\underline{\lambda\pi}' \quad = \quad \underline{0}' \qquad (2\text{-}32)$$

where (2-27) is the condition that $\underline{\nabla x'} = \underline{0}'$,

(2-28) is the condition that $\underline{\nabla \lambda'} = \underline{0}'$,

(2-29) and (2-31) are (2-25) and (2-24), and (2-30) and (2-32) are (2-20) and (2-19).

Equations (2-31) and (2-32) are often referred to as the orthogonality conditions.

2.2.5 The related linear programming problem

Equations (2-27) to (2-32) represent an (n+m) x (n+m) linear programme, without an objective function, but with the addition of the orthogonality conditions $\underline{xy}' = \underline{\lambda \pi}' = 0$. The problem, therefore, is to find an initial basis of equations (2-27) and (2-28) subject to the conditions (2-29) to (2-32). Equation (2-31) ensures that only n of the 2n elements x_j, y_j may be positive, and equation (2-32) ensures that only m of the 2m elements λ_i, π_i may be positive.

This means that the basis will consist of a maximum of (n+m) activities at positive levels, and equations (2-31) and (2-32) imply that each activity can be replaced in the basis by only one other. That is, x_j can only be replaced by y_j, and λ_i can only be replaced by π_i.

2.2.6 An algorithm for solving quadratic programming problems[24]

The disposal activities y_j and π_i are used to form an initial simplex tableau as in table 2.1. The algorithm explained here involves simplex-type calculations and can be divided into two phases. After each iteration the columns are rearranged to maintain the format of

24. A good summary of alternative algorithms is to be found in Hadley, G., "Nonlinear and Dynamic Programming", op.cit, Chapter 7.

table 2.1.

Table 2.1 Initial Simplex Tableau for Quadratic Programming

Disposal Activity	B		$x_1 \ldots x_n$	$\lambda_1 \ldots \lambda_m$	$y_1 \ldots y_n$	$\pi_1 \ldots \pi_m$
y_1	$-c_1$	=			1	
.	.				.	
.	.		$[2B]$	$-[A']$	.	
.	.				.	
y_n	$-c_n$	=			1	
π_1	b_1	=				1
.	.					.
.	.					.
.	.					.
π_m	b_m	=	$[A]$	$[0]$		1

Phase I: (a) Find the most negative of the first n elements in the B column and call this row p. (If $a_{po} \geqslant 0$, go to Phase II).

(b) The R ratios are calculated:

$$r_p = \frac{a_{po}}{a_{pp}} \quad ; \quad a_{pp} < 0 \quad ;$$

$$r_p = \infty \quad ; \quad a_{pp} \geqslant 0 \quad ;$$

$$r_i = \frac{a_{io}}{a_{ip}} \quad ; \quad a_{io} \geqslant 0 \quad ; \quad a_{ip} > 0 \; ; \quad i \neq p \; ; \; i = 1 \ldots n.$$

(c) The smallest r_i is found in order to locate the pivot. If the smallest r_i is r_p,[25] the pivot is a_{pp} and step (a) is repeated.

If the smallest r_i is less than r_p, the row in which r_i occurs is called row q, and the pivot is a_{qq}. The next step is (b).

Phase II: (d) The most negative element in the B column for $i = n+1 \ldots n+m$ is found, and this row is row p. If $a_{po} \geqslant 0$, the problem is solved.

(e) The R ratios are calculated:

$$r_p = \frac{a_{po}}{a_{pp}} \; ; \quad a_{pp} < 0 \; ;$$

$$r_p = \infty \; ; \quad a_{pp} \geqslant 0 \; ;$$

$$r_i = \frac{a_{io}}{a_{ip}} \; ; \quad a_{io} \geqslant 0 \; ; \quad a_{ip} > 0 \; ; \; i \neq p \; ; \; i=1 \ldots n+m.$$

(f) The smallest r_i is located. If it is equal to ∞, the problem is infeasible.

If the smallest r_i is r_p, a_{pp} is the pivot, and the algorithm returns to step (d).

If the smallest r_i is less than r_p, the row in which it occurs is row q, and the pivot is a_{qq}. The next step is (e).

---

25. If the smallest pivot is $r_p = \infty$, then the matrix B is not negative definite (contrary to the initial assumption).

2.2.7 Interpretation of the solution

Basic and non-basic variables have the same economic interpretation as their linear programming counterparts.

2.2.7.1 Real activities in the basis

If $x_j > 0$, the j^{th} activity is in the optimum solution, and should be operated at a level x_j.

2.2.7.2 Real activities not in the basis

If $y_j > 0$, the marginal revenue product of the j^{th} activity is negative, and equal to $- y_j$. Should this activity be forced into the solution, profit will be reduced by an amount equal to y_j.

2.2.7.3 Imputed values of scarce resources

If $\lambda_i > 0$, the i^{th} restraint is effective, and its shadow price is equal to λ_i.

2.2.7.4 Resources in disposal

If $\pi_i > 0$, the i^{th} resource is in disposal by an amount equal to π_i. That is, the i^{th} restraint is ineffective.

2.3 Risk Programming

2.3.1 Introduction

One of the linear programming assumptions (section 2.1.2.4) is that all data, (that is resource supplies, input-output coefficients, and prices, costs, and yields) must be known with certainty. In other words, it is assumed that none of the coefficients have associated error terms.[26/]

In practice the error terms associated with some types of horticultural production data (especially that for fresh vegetables) merit special consideration, due to the degree of risk associated with this type of farming.

An example is afforded by fresh vegetable auction prices. These vary from season to season, mainly due to 'cobweb'[27/] effects, that is a large crop in one season will be marketed at relatively low prices, therefore persuading growers to reduce the acreage planted in the following year and so giving rise to a higher level of prices.

The grower will usually consider a range of price expectations, which he may formulate in either an ordinal or cardinal sense.[28/] If formulation is ordinal only, the grower will consider a range of prices, some of which are more likely to occur than others. If, on the other hand, future price expectations are formulated in a cardinal sense, the grower would attach an exact but subjective probability to the likely occurrence of any possible price. The grower, rather than saying price A was more likely than price B, would say that price A was, for example, exactly twice as likely as price B.

---

26. Any coefficient, a, can be written in terms of two parts: part A, which corresponds exactly to the 'real world' value of a; and part u, which corresponds to a deviation or measurement error between the real world value and the estimated value used in the computations. That is,

$$a = A + u.$$

In linear programming the assumption is that $u = 0$.

27. Allen, G.R., "Agricultural Marketing Policies", Blackwell, 1959, pp.39-42.
28. Heady, Earl O., "Economics of Agricultural Production and Resource Use", Prentice-Hall, Inc., New Jersey, 1952, pp.445-446.

2.3.2 Some examples of stochastic errors in horticultural production data

The variability inherent in horticultural production data is of a stochastic nature due to random or "chance" variations in the real world. Stochastic errors will arise for many reasons, and a few of the more important are given below:

(i) the price of produce sold on the auction floor may vary, both from day to day, and from one period of the year to the corresponding period of the following year;

(ii) yields will vary from season to season due to both climatic conditions and the incidence of disease (the latter may be partly influenced by the weather conditions);

(iii) the time of planting may need to be altered due to bad weather at the scheduled planting time, or an unexpected frost may damage the young seedlings so that a second and consequently later planting is required;

(iv) the labour input for many operations such as land preparation, hand-weeding, and spraying will vary, due mainly to climatic conditions; and

(v) the labour supply may be reduced through sickness, or difficulties may be experienced in obtaining labour when required.

2.3.3 An outline of risk programming [29] [30]

Markowitz [31] has developed a quadratic programming technique which allows the selection of efficient combinations of securities which minimises risk for specified levels of expected income.

The problem of choosing optimal combinations of horticultural enterprises when data contains stochastic error terms is similar to Markowitz's portfolio selection problem, and a quadratic programming algorithm similar to that of Markowitz could be used to obtain a solution. [32]

In risk programming, it is assumed that the horticulturalist has an E-V indifference system, [33] as in figure 2.1. This means that he will be 'better off', the higher his expected income and the lower his income variability.

In figure 2.1, I_1, I_2, and I_3 represent successively higher indifference curves. The horticulturalist is assumed to be indifferent between all points on any one indifference curve - each indifference curve gives all possible combinations of expected income and income variability which result in equal satisfaction to the horticulturalist.

---

29. McFarquhar, A.M.M., "Rational Decision Making and Risk in Farm Planning - An Application of Quadratic Programming in British Arable Farming", Journal of Agricultural Economics, vol.14, p.552, 1961.
30. For applications of risk programming see:
Camm, B.M., op.cit., and
Heady and Candler, op.cit., Chapter 17.
31. Markowitz, Harry M., "Portfolio Selection - Efficient Diversification of Investments", John Wiley and Sons, Inc., 1959.
32. Markowitz, op.cit., pp.170-187.
33. Heady and Candler, op.cit., pp.557-558.

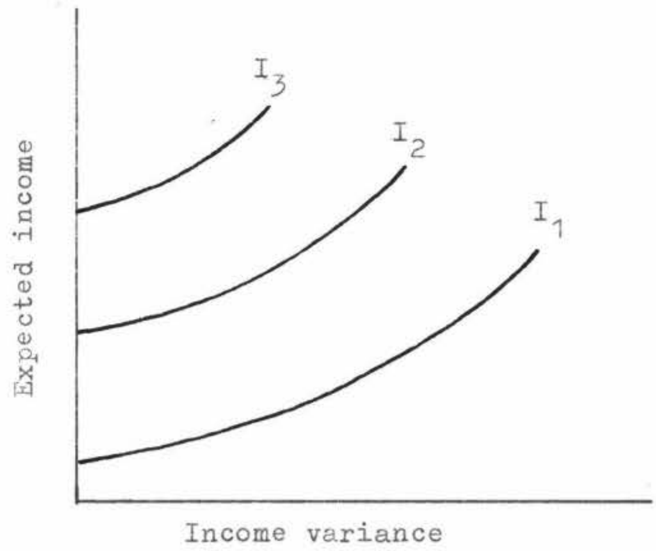

Figure 2.1 An E-V Indifference System

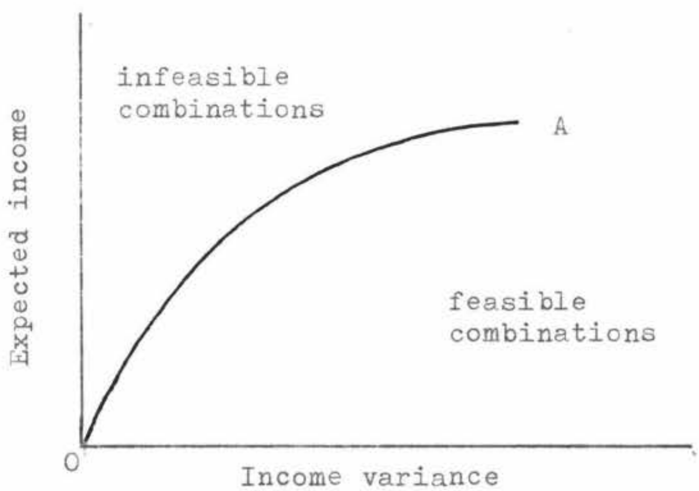

Figure 2.2 Expected Income as a Function of Income Variance

For any given production situation there will be a minimum level of income variance associated with any given expected income, or the same thing, a maximum level of expected income associated with any given income variance.

In figure 2.2, OA separates infeasible income-variance combinations from those combinations which are feasible. Therefore the risk programming problem consists of finding the set of farm plans which maximises expected income for each level of income variance, which is the same as finding the farm plans associated with each point on OA.

Any point to the right of OA represents a feasible income-variance combination. All points on OA are preferred to all points horizontally to the right since such points on OA represent plans with smaller income variance or greater expected income than any other feasible plan with the same income or variance.

2.3.4 A measure of risk

In the quadratic programming approach to risk aversion, the variance of incomes is taken as an index of the degree of risk attached to a particular plan. This leads to a programme which gives minimum income variance for each level of expected income.

Consider a farm plan which includes three activities, P1, P2 and P3, at levels x_1, x_2 and x_3 respectively. The variance of the resulting net revenue may be measured by:

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underline{x}B\underline{x}' \qquad (2\text{-}33)$$

where σ_{11}, σ_{22} and σ_{33} are the net revenue variances of P1, P2 and P3 respectively, and

$\sigma_{12} = \sigma_{21}$,

$\sigma_{13} = \sigma_{31}$, and

$\sigma_{23} = \sigma_{32}$, are the net revenue covariances between activities P1 and P2, P1 and P3, and P2 and P3 respectively.

An unbiassed estimate of the population variance computed from a random sample is given by: 34/

$$\sigma_i^2 = \frac{\sum_k (c_i - \mu_i)^2}{n-1} , \qquad (2\text{-}34)$$

where c_i is the observed net revenue from the i^{th} activity in each of the k years,

μ_i is the mean net revenue of the i^{th} activity, and

n is the number of observations of net revenue for the i^{th} activity.

An unbiassed estimate of covariance is given by:

$$\sigma_{ij} = \frac{\sum_k (c_i - \mu_i)(c_j - \mu_j)}{n-1} , \qquad (2\text{-}35)$$

where c_i and c_j are the observed net revenues from the i^{th} and j^{th} activities respectively in each of the k years,

μ_i and μ_j are the mean net revenues of the i^{th} and j^{th} activities respectively, and

n is the number of observations.

A measure of net revenue covariance between each pair of activities which may be included in the plan is necessary since, for example,

---

34. $\sigma_i^2 = \frac{\sum (c_i - \mu_i)^2}{n}$ is not an unbiassed estimate of variance. If repeated samples of size n are taken and the resulting sample variances averaged, the average estimated variance will be smaller than the true variance by the factor $\frac{n-1}{n}$. For small samples, this factor becomes important. See: Hoel, Paul G., "Introduction to Mathematical Statistics", John Wiley and Sons, Inc., 1947, p.198.

crops A and B may both be subject to high net revenue variance and therefore high risk, while combinations of A and B may provide a net revenue subject to much less variance if the net returns from both crops are shown to be inversely correlated - that is, a low return from A in one year will be offset to some degree by a high return from B.[35]

2.3.5 Mathematical statement of the risk programming problem

It is assumed that the net revenue from a unit level of each activity carried out under risk conditions is in the form of a random variate which follows some probability distribution.[36] This distribution may be defined as representing a measure of the probability of certain outcomes (net revenues) occuring. Since observed (historic) data are used in the programme, it is assumed that the probabilities of particular outcomes in the future is the same as observed in the past.

Consider the distribution of net revenue due to some cropping programme $\underline{x}$, where $\underline{x}$ is the vector of activity levels in the programme. If the net revenue from each activity is assumed to have a mean μ_i and variance σ_i^2, and the covariance between the net revenue of two activities is defined as σ_{ij}, then the net revenue of programme $\underline{x}$, will have

---

35. If the correlation coefficient is -1.0, the two enterprises will serve optimally as a precaution against risk. Should the correlation coefficient be +1.0, combination of two enterprises need not reduce income variability, although for anything less than perfect correlation between outcomes a combination of crops will result in some offsetting effect. Therefore there will be a tendency for the combined variance to be less than the variance if all resources were devoted to either crop alone. See:
Heady, "Economics of Agricultural Production and Resource Use", op.cit., pp.510-524.
36. Freund, R.J., "The Introduction of Risk into a Programming Model", Econometrica, vol.24, p.253, 1956.

mean $\underline{\mu}\underline{x}'$ and variance $\underline{x}B\underline{x}'$, where B is the matrix of net revenue variances and covariances of the i activities.

Thus the risk programming problem may be defined as:

$$\text{maximise } Z = \phi\underline{\mu}\underline{x}' + \underline{x}B^*\underline{x}' , \qquad (2\text{-}36)$$

$$\text{subject to} \quad A\underline{x}' \leq \underline{b}', \qquad (2\text{-}37)$$

$$\text{and} \quad \underline{x} \geq \underline{0} , \qquad (2\text{-}38)$$

where $\underline{\mu}$ is the vector of mean (expected) net revenues μ_i,

B^* is the negative of the net revenue variance-covariance matrix (that is, $B^* = -B$),

A is a matrix of input-output coefficients,

$\underline{b}$ is a vector of resource supplies, and

ϕ is a risk-aversion parameter (that is, a weight applied to the linear terms of the objective function).

To obtain the efficient set of plans (the boundary OA in figure 2.2) the risk aversion parameter is varied between zero and infinity.[37/]

---

37. Wolfe has developed a quadratic programming algorithm to handle such a parametric objective function which gives a solution for all $\phi \geq 0$. See:
Wolfe, P., "The Simplex Method for Quadratic Programming", Econometrica, vol.27, pp.382-398, 1959, and
Hadley, G., "Nonlinear and Dynamic Programming", op.cit., Chapter 7, sections 7-5 and 7-6.

CHAPTER 3

LINEAR PROGRAMMING AND PROFIT MAXIMISATION ON AN OTAKI HORTICULTURAL HOLDING

3.1 Introduction

This chapter will illustrate the use of linear programming to formulate a profit-maximising production plan for an Otaki fresh vegetable producer. After a brief description of the holding, the restraints on production and the activities will be discussed. Price, cost, and yield expectations are made, and the input-output coefficients derived. The solution to the linear programme will then be examined in some detail and compared with the grower's proposed plan.

The production season for which the linear programme solution was planned was that beginning in September 1967.

3.2 A Description of the Holding

3.2.1 Location and size

The holding is a four acre property situated near the Otaki Railway township.

3.2.2 Leased land

The grower has arranged to lease nine acres of land from a nearby farmer for the coming season. Although the leasing of land is a permanent arrangement, the acreage leased may vary from year to year depending on the size of the paddocks which become available.

The land is ploughed out of pasture and is excellent for the production of tomatoes since rotation problems will not exist.

The lease is for a period of 14 months, from September through until October of the following year.

3.2.3 Glasshouse area

The grower owns a 3,000 sq.ft. unheated glasshouse which is used for the production of cucumbers.

3.2.4 Cropping practice

A fairly typical range of crops for the district is grown, the main crop being outdoor tomatoes. Other crops grown during the past season were cucumbers (both outdoor and hothouse), pumpkin, cauliflower, lettuce, rhubarb and Soleil d'Ors (a yellow tazetta daffodil). The latter two crops are perennials, with rhubarb being replanted every five years, and Soleil d'Ors every seven years. Last season, the rhubarb planting was half an acre, and the Soleil d'Ors planting two-thirds of an acre.

3.2.5 Soil types

The freehold land is a river silt loam, varying in depth from one to eight feet, with a gravel subsoil. Drainage is excellent which makes this land suitable for winter crops, and since the soil is slightly alkaline it is best suited for crops such as cabbage and cauliflower.

The leased land consists of river silt loam varying in depth from one to six feet, under which is a clay subsoil. Because of poor drainage during excessively wet periods this land is not suited to the production of winter crops. Since the soil retains moisture well during the summer, good pumpkin, cucumber and cauliflower crops may be obtained at this time of the year. The leased land is slightly more acidic than the freehold land and is therefore better suited to pumpkin, cucumber, and tomato crops.

3.2.6 Rainfall and irrigation

Since rainfall is regular and averages about 3.5 inches per month, irrigation is not usually necessary. However, should either January or February be months of low rainfall, fortnightly irrigation may then be required.

3.2.7 Disease problems

Crops prone to disease, such as tomatoes and lettuce, are sprayed regularly to prevent any serious outbreak. Pests and diseases which may cause a problem from time to time are late blight and Botrytis on tomato, mildew and aphids on cucumbers, aphids on lettuce, and slugs and snails. Late blight is controlled with maneb sprays, and zineb is used for the control of Botrytis. Mildew control is obtained with cuprox, and malathion gives control over aphids. Slugs and snails are controlled with metaldehyde pellets.

An insect problem which may be on the increase in the freehold land is due to nematodes and soil sterilisation may become necessary.

Deficiencies of some trace elements are common in soils around Otaki and the grower finds that cauliflower crops occasionally show signs of molybdenum and boron deficiencies. Molybdenum deficiency is overcome by either spraying the crop with ammonium molybdate, or by applying sodium molybdate with the base dressing of fertiliser. Boron deficiency is overcome by the application of borax with the base fertiliser application.

3.3 The Restraints

3.3.1 Problems of timing of production

Figure 3.1 indicates the months over which various crops require land. All except two of these crops occupy land for a period less than 12 months; for example the cauliflower activity (P1)[1] will require leased land from the beginning of September until the end of December, and cauliflower (P13) will require freehold land from the beginning of May until the end of December. This meant that the total land supply in each month was considered as a possible restraint because land is used intensively and production needs to be timed accurately.

Since the leased land is acquired in September, this was the most convenient month to use as the beginning of the season. It is apparent from figure 3.1 though, that production is a continuous process and one season's cropping programme will often carry over into that of the following season.

---

1. A full description of the activities will follow in section 3.4. A capital letter P is used to denote an activity : since there are 14 activities in all, these will be number P1 - P14. The restraints will be numbered from R1 onwards.

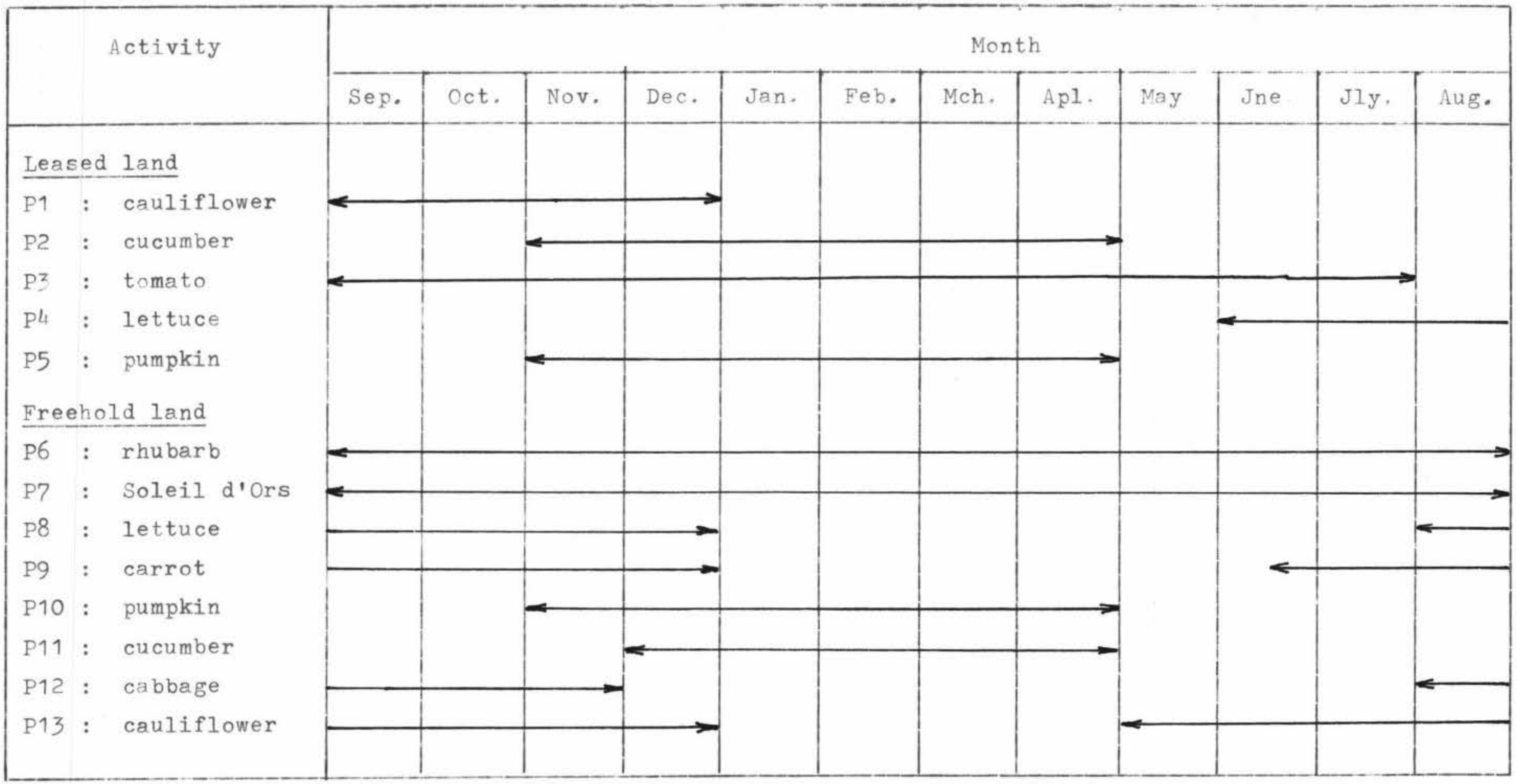

Figure 3.1 Activity Land Requirements

Notes:

1. Activity P14 is a glasshouse crop and does not require cropland.
2. Leased land cropping is shown for 12 months only - the lease continues for a further two months though, with lettuce (P4) being the only crop which may occupy the land over these months.

3.3.2 Freehold land (restraints R1 - R7)

A minimum of seven time periods may be identified (beginning at September), a combination of which will define the production period for any of the activities P6 to P13 which can be grown only on freehold land. Thus the annual supply of freehold land is divided into the following seven restraints:

R1 : September - October,

R2 : November,

R3 : December,

R4 : January - April,

R5 : May - $June_1$ [2],

R6 : $June_2$ - July,

R7 : August,

where R1, for example, represents the restraint which the supply of freehold land imposes on production of certain crops during the months of September and October. The supply of each of the above restraints is four acres, this being the area of freehold land.

3.3.3 Leased land (restraints R8 - R15)

A minimum of eight time periods is required to define the production periods of the five activities which may be cropped on the leased land.

The leased land resource supply is nine acres, and the restraints (which have a similar interpretation to the freehold land restraints)

---

2. Where months require division into half-month periods, the subscripts 1 or 2 are used to denote the first or second half of the month respectively.

are:

R8 : September - October,

R9 : November,

R10 : $December_1$,

R11 : $December_2$,

R12 : January - April,

R13 : May,

R14 : June - July,

R15 : August .

The thirteenth and fourteenth months (September and October) after the beginning of the lease need not enter the restraints, since the cropped acreage in these months will be the sum of the September plantings of cauliflower (P1) and tomato (P3) on the newly-leased land, plus the acreage of lettuce (P4) in the previous season's leased land. Therefore providing the September - October (R8) and August (R15) restraints are not violated, the production level in the thirteenth and fourteenth months must be feasible.

3.3.4 Labour availability (restraints R16 - R39)

The total labour supply consists of the owner, one man and two women (who comprise the permanent staff) and three men and three women who are employed for part of the year only.

The owner supplies an average 50 hours per week, the permanent man 42 hours per week, while the two permanent women work for 42 hours per week from the second half of January until the end of May, and 30 hours per week from June until the third week of December.

The three casual female workers and one of the male casual workers are employed only in the tomato harvesting season, the former working 25 hours per week each from February until May[1] and the latter working for 41 hours per week from March until the end of May.

Of the two other male casual workers, one works for 45 hours per week from October until the end of June and the other, 30 hours per week from November until the end of May.

The labour restraints and the corresponding resource supplies are set out in table 3.1.

Table 3.1 Labour Restraints and Supply

Month	Labour Supply in first half of month (hours)	Labour Supply in second half of month (hours)
January	R16 : 362	R17 : 544
February	R18 : 706	R19 : 706
March	R20 : 795	R21 : 795
April	R22 : 795	R23 : 795
May	R24 : 795	R25 : 633
June	R26 : 427	R27 : 427
July	R28 : 329	R29 : 329
August	R30 : 329	R31 : 329
September	R32 : 329	R33 : 329
October	R34 : 427	R35 : 427
November	R36 : 492	R37 : 492
December	R38 : 492	R39 : 422

Note: One half-month period is taken as $\frac{52}{24}$ = (2-1/6) weeks.

Time spent travelling between the freehold and leased land has been deducted from the labour supply, and the owner and permanent staff are assumed to take their holidays in slack months.

All work that cannot be allocated to the various activities is assumed to be carried out during months when labour is in disposal. (No provision is made in the model to ensure that sufficient labour is available for this 'overhead' work. However, should the grower think that insufficient labour is in disposal, the model can easily be modified by estimating the labour requirements of such work and over what months it is likely to be carried out).

3.3.5 Cropping limits (restraints R40 - R52)

The grower placed restrictions on the acreage of all crops included in the programme.

This reflects a desire for diversification due to the possibility of widely fluctuating prices (and to a lesser extent yields) from season to season, as well as for other reasons such as crop health and soil fertility.

Outdoor cucumbers may be grown on either freehold or leased land, but since the former land is less suited to this crop yields are somewhat lower than for cucumber crops grown on the leased land. The grower has also found that cucumbers grown on his own land are more prone to attack by aphids. For these reasons the grower insisted that a maximum of half an acre of cucumber be grown on the freehold land.

Of the two perennial crops (rhubarb and Soleil d'Ors) the grower was prepared to increase the planting of rhubarb only, from half an acre to a maximum of one acre.

The cropping limit restraints are set out in table 3.2.

Table 3.2 Cropping Limit Restraints

Crop Restraint			Maximum Acreage
R40	:	cauliflower (P1)	3.00
R41	:	cucumber	2.50
R42	:	cucumber (P11)	0.50
R43	:	tomato	5.00
R44	:	lettuce (P4)	1.50
R45	:	pumpkin	4.00
R46	:	rhubarb	1.00
R47	:	Soleil d'Ors	0.67
R48	:	lettuce (P8)	1.50
R49	:	carrot	0.50
R50	:	cabbage	2.00
R51	:	cauliflower (P13)	1.00
R52	:	glasshouse cucumbers	825 plants

Note: The size of the glasshouse limits the production of glasshouse cucumbers to 825 plants so restraint R52 is a glasshouse restraint, rather than a cropping limit.

3.4 The Activities

This section will describe the production activities which the grower wished to have considered for inclusion in the optimum plan. Unless otherwise stated, all produce is sold by auction.

3.4.1 Production on leased and freehold land

Five crops (cauliflower, tomato, cucumber, pumpkin, and lettuce) may be grown on the leased land block. Tomato, the grower's main crop in past seasons, requires new land each year to help avoid disease problems, so all tomatoes are grown on the leased land which is ploughed out of pasture. The grower finds that a spring cauliflower crop can be grown and harvested before the land is required by the later tomato plantings. Once the tomatoes have been harvested, the grower considered spring lettuce as the only possible crop to follow the tomatoes.

Of the crops grown on freehold land, lettuce, cabbage, carrot and cauliflower cannot be grown on the leased land simply because the timing of production of these crops does not coincide with the lease.

The permanent beds of rhubarb and Soleil d'Ors are planted in the freehold land.

3.4.2 Cauliflower (P1)

An acre of this activity consists of two half-acre crops. The plants are raised on the property in seedbeds prepared and sown during July, and then transplanted into the open ground (the first crop during September_1 and the second during September_2). Each planting is hand-weeded and mechanically cultivated during October and a side-dressing of fertiliser is applied during the first half of November. The crop is harvested from November_2 until the end of December.

3.4.3 Cucumber (P2)

The single crop is sown direct into the paddock during November_1. The plants are hand and mechanically cultivated for weed control three times, in December_1, January_1, and January_2. Harvesting is spread from January_2 through until the end of April.

3.4.4 Tomato (P3)

An acre of the tomato activity consists of five one-fifth-acre plantings. A local nurseryman raises the plants from seed collected by the grower from the previous season's crop. The five plantings are made during $October_1$, $October_2$, $December_1$, $December_2$, and $January_1$ at the rate of 5,000 plants per acre. Two to four weeks after planting, stakes are driven in along the rows and the first of three double wires attached which are then clipped together, one wire on each side of the plants. At the same time the plants will be pruned and both hand and mechanically cultivated. Each planting is pruned a total of five times, at intervals of a fortnight. A month after their first cultivation the plants will be mechanically cultivated for a second time and a side-dressing of fertiliser applied. The second wires are attached about a month after the first wiring, and after another month the third wires are set out and clipped together. The complete crop is sprayed every fortnight (from planting until harvesting is completed) with a fungicide-insecticide mixture.

Harvesting begins in the early part of January and continues until June. Most fruit is sold through the auction markets, but all rough-grade fruit unsuitable for table tomatoes is sold for processing. (Last season the grower contracted to supply 29 tons of rough-grade fruit for processing for a price of $40.00 per ton).

The first of the five crops is cleared from the land during $June_1$, and the remaining four crops are removed from $June_2$ until the end of July.

3.4.5 Lettuce (P4)

The lettuce plants are raised in a seedbed during $April_2$ and the single planting is made during $June_1$. The crop is wheel-hoed and sprayed three times, during $July_1$, $August_1$ and $September_1$. Harvesting is from $September_2$ until the end of October.

3.4.6 Pumpkin (P5 and P10)

Seed is drilled directly into the ground during the first half of November. The crop is both hand and mechanically cultivated three times, in $December_1$, $December_2$, and $January_1$. All pumpkins are harvested in $April_2$ and treated with a fungicide to prevent them from deteriorating in storage. The pumpkins are turned during July and August, and sold from $September_1$ until October.

3.4.7 Rhubarb (P6)

Rhubarb is a perennial crop, being replanted every five years. Planting is carried out in November and irrigation may be necessary until the plants are established. The crop is mechanically cultivated five times during a year, in May_1, $June_1$, $August_1$, $October_1$ and $December_1$, and fertiliser topdressings are applied during $April_1$, $August_1$ and $October_1$.

Although some rhubarb is harvested in the first year, the yield will increase each year. Harvesting is carried out from May_2 until $November_2$ with the monthly yield tending to be greatest during October and November.

3.4.8 Soleil d'Ors (P7)

This yellow tazetta daffodil is a perennial crop, being replanted every seven years. The crop is sprayed with a weedicide in $April_1$ and mechanically cultivated during May_2. The flowers are picked from $June_2$ until the end of August and the number of flowers picked increases each year as the bulbs multiply. At the end of the seventh year when the bulbs are ploughed out, only sufficient for replanting are kept, the remainder being sold.

3.4.9 Lettuce (P8)

This activity consists of one planting only. The seedbed is prepared and sown during $July_2$, with the seedlings transplanted into the open ground during $September_1$, (this land being prepared for planting during August). The crop is wheel-hoed and sprayed three times in all, in $September_2$, $October_2$, and November. The entire crop is harvested during December.

3.4.10 Carrot (P9)

The land to be sown in carrots is ploughed in $June_2$, disced, rotary hoed and levelled during $July_1$, and the seed is drilled during $July_2$. The crop is sprayed with a weedicide as well as wheel-hoed during $August_2$. Thinning is carried out in $September_1$, and the crop is wheel-hoed twice more, in $September_2$ and $October_2$. The carrots are harvested from the beginning of November until the end of December.

3.4.11 Cucumber (P11)

Following ploughing, discing and fertilising during $December_1$, the paddock is levelled and the seed drilled in $December_2$. Crop husbandry then consists of scarifying and hand weeding. Each operation is carried out three times, during $January_1$, $January_2$ and February.

The cucumbers are harvested in March and April, and most of the fruit from both outdoor cucumber crops (P2 and P11) is sold under contract to processing firms in Wanganui and Wellington.

Cucumber grown on the freehold land has appeared in the past to be more susceptible to aphid and mildew infestation and regular spraying throughout the growing season is considered necessary. This, coupled with the fact that the leased land is better suited to cucumber crops, (it is somewhat more acidic and retains moisture better during the summer months) means that the yield from cucumber on freehold land is lower than that from the other cucumber activity.

3.4.12 Cabbage (P12)

Cabbage plants are raised in a seedbed during the second half of July. The land to be planted is ploughed during $August_1$ and disced, levelled and fertilised in $September_1$ prior to transplantation. The crop is both hand and mechanically cultivated during $September_2$ and $October_1$. Harvesting takes place from $October_2$ until the end of November.

3.4.13 Cauliflower (P13)

An acre of this activity includes three one-third-acre plantings. The seedbed is prepared and sown in $April_2$ and three crops are transplanted into the open ground, during $July_1$, $July_2$ and $August_1$. The plants are hand-weeded twice during $July_2$ and $August_2$, and side-dressed with fertiliser in $September_1$. The first crop to be planted is ready for harvesting in $October_2$ and harvesting then proceeds continuously until the end of December.

3.4.14 Glasshouse cucumber (P14)

Soil preparation commences in $July_1$ with a mecnanical cultivation, immediately after which the soil is sterilised. During $August_1$ the soil is mechanically cultivated twice more with fertiliser applied between cultivations. The cucumber plants are purchased from a nurseryman and planted during August$_2$. In September$_1$ the crop is hand-hoed and sprayed, and string is hung from crosswires to the base of each plant to provide support for the vines. The plants are sprayed and trained up the strings every fortnight and irrigated regularly. The cucumbers are harvested and sprayed every fortnight from November$_2$ until the end of January. Once harvesting has been completed the glasshouse is cleared, and a greencrop may be sown.

3.5 The Objective Function

3.5.1 Introduction

To allow the objective function[3/] to be maximised it is necessary to calculate the net return per unit of each activity. The net return per acre of an activity may be estimated as the gross margin per acre of the activity,[4/] which is defined as the activity's gross return less the variable costs incurred in producing that activity. The gross return is the product of the expected price and expected yield, and variable costs are those directly attributable to the activity.

Variable costs may be divided into two parts - 'observable' variable costs (which includes such items as seed, fertiliser, and spray materials), and 'imputed' variable costs (an example of which is tractor running costs which need to be calculated from the hours of tractor usage per activity as stated by the grower).

---

3. See Chapter 2, section 2.1.3.
4. Wesney, op.cit., pp.68-118 and p.141, and Frampton, A.R., "The Economics of Growing Sugar Beet on Farms in South Otago", unpublished M.Agr.Sc. thesis, Massey University Library, pp.50-53.

By maximising the revenue based on gross margins (subject to the restraints) the returns to the fixed resources are maximised. Net profit is obtained by deducting the fixed (overhead) costs from the profits shown by the linear programme solution.

3.5.2 Calculation of gross margins[5]

3.5.2.1 Expected prices and yields

A suitable estimate of prices and yields could be obtained by averaging prices and yields over a period of, say, the past three or four years.[6] The grower in this study, however, had been recording price and yield data over the immediate past season only. For each crop the grower gave an indication as to whether the past season's prices and yields were above, below, or "about the average". These prices and yields were then adjusted so that in the grower's opinion they were the best possible estimate of the next season's prices and yields. The activity prices and yields used in the linear programme are set out in table 3.3, and multiplication of yield by price gives the gross return per unit of activity.

The cauliflower activity (P1) was expected to return a slightly higher price than the other cauliflower activity (P13), since the former crop is harvested from November_2 until December_2. (The average price over this period is expected to be higher than the average for the October_2 - December_2 period which is the harvesting time of activity P13).

---

5. The present section discusses gross margins of annual crops only. Returns and costs for the two perennial crops are discussed in a subsequent section.
6. Heady, "Economics of Agricultural Production and Resource Use", op.cit., pp.475-496, discusses several expectation models of which the average prices and yields is but one.

Table 3.3 Price and Yield Assumptions

Activity		Unit	Yield	Price ($)	Gross Return ($ per unit)
P1	: cauliflower	1 acre	564 cases	1.20/case	676.80
P2	: cucumber	"	13.2 tons	109.10/ton	1440.12
P3	: tomato	"	2880 cases	1.39/case	4003.20
P4	: lettuce	"	600 cases	1.06/case	636.00
P5 & P10	: pumpkin	"	80 sacks	6.40/sack	512.00
P8	: lettuce	"	600 cases	0.94/case	564.00
P9	: carrot	"	504 cases	1.58/case	796.32
P11	: cucumber	"	11.9 tons	109.10/ton	1298.29
P12	: cabbage	"	600 cases	0.80/case	480.00
P13	: cauliflower	"	564 cases	1.14/case	642.96
P14	: glasshouse cucumber	825 plants	7 cucumbers/ plant	0.79/plant	651.75

Notes:
1. Cauliflower, lettuce, carrot, and cabbage are marketed in banana cases, while tomatoes are packed into smaller, 20 lb. cases. A pumpkin sack, when full, weighs 140 lbs.
2. The expected price for glasshouse cucumbers is $0.79 per seven fruit (i.e., $0.79 per plant).

The lettuce activity (P4) is harvested from September$_2$ until October$_2$ when the grower expects prices to be somewhat higher than those prevailing during December, when the lettuce activity (P8) is harvested.

It was the grower's opinion that yields would be identical for cauliflower grown on either land block and that yields and costs would be identical for the two lettuce and pumpkin activities, whether grown on leased or freehold land. However, the grower estimated the yield from cucumber grown on freehold land to be 10 percent less than from cucumber grown on the leased land (for reasons given in section 3.4.11).

3.5.2.2 Variable costs

As costs do not change from year to year as markedly as do prices and yields, more certainty can be attached to cost estimates than to estimates of gross revenue. Costs were based on those of the past season with the exception of container costs, which were directly related to the yield estimates.

The activity variable costs are given in table 3.4.

3.5.2.3 Gross Margins

The gross margins are calculated by subtracting the variable costs total of table 3.4 from the gross revenue of the same activity given in table 3.3.

Table 3.5 gives the gross revenue, variable costs, and gross margins for all activities with the exception of rhubarb (P6) and Soleil d'Ors (P7).

Table 3.4 Variable Costs ($)

Activity			Unit	Fertiliser	Seed/ Plants	Spray	Tractor	Container & Sundry	Total
P1	:	cauliflower	1 acre	56.00	12.00	-	3.80	28.20	100.00
P2	:	cucumber	"	52.00	20.00	-	6.60	-	78.60
P3	:	tomato	"	110.00	72.40	32.00	5.20	212.80	432.40
P4 & P8	:	lettuce	"	91.00	12.00	13.40	4.00	30.00	150.40
P5 & P10	:	pumpkin	"	24.00	12.00	3.00	3.40	4.00	46.40
P9	:	carrot	"	-	12.40	12.20	3.20	25.20	53.00
P11	:	cucumber	"	52.00	20.00	9.50	6.60	-	88.10
P12	:	cabbage	"	23.00	2.40	-	6.40	30.00	61.80
P13	:	cauliflower	"	56.00	12.00	-	3.20	28.20	99.40
P14	:	glasshouse cucumbers	825 plants	7.60	82.40	4.40	0.80	35.00	130.20

Notes:

1. Calculation of tractor running costs is similar to that used by Frampton. See Frampton op.cit., p.51.
2. Banana cases cost $0.15 each to buy, but $0.10 is refunded by the auction firm once the produce is sold, so that the net cost to the grower is $0.05. The net cost of sacks and the smaller cases used for glasshouse tomatoes is also $0.05 each.

Table 3.5 Gross Margins ($)

Activity			Unit	Gross Return	Variable Costs	Gross Margin
P1	:	cauliflower	1 acre	676.80	100.00	576.80
P2	:	cucumber	"	1440.12	78.60	1361.52
P3	:	tomato	"	4003.20	432.40	3570.80
P4	:	lettuce	"	636.00	150.40	485.60
P5 & P10	:	pumpkin	"	512.00	46.40	465.60
P8	:	lettuce	"	564.00	150.40	413.60
P9	:	carrot	"	796.32	53.00	743.32
P11	:	cucumber	"	1298.29	88.10	1210.19
P12	:	cabbage	"	480.00	61.80	418.20
P13	:	cauliflower	"	642.96	99.40	543.56
P14	:	glasshouse cucumber	825 plants	651.75	130.20	521.55

3.5.3 Perennial crops and discounting

Since returns and yields for the perennial crops increase each year, it was necessary to derive a gross margin for these crops which would indicate their profitability when compared with the gross margins (net returns) of the annual crops. This is achieved by first discounting the annual future net returns of the perennial crops back to the present.

3.5.3.1 Present value of rhubarb (P6)

During the past season the yield from this crop was 722 cases per acre, which was sold at an average price of $1.96 per case. The grower thought that the same price would be a reasonable estimate for the following season. He also stated that the rhubarb yield in the first year after planting would be 300 cases per acre, increasing to 1200 cases per acre in the fifth and final year.

Annual variable costs of rhubarb are given in table 3.6.

Table 3.6 Variable Costs ($ per acre) - Rhubarb (P6)

Year	Fertiliser	Cases	Tractor	Total
1	216.00	15.00	2.00	233.00
2	108.00	26.30	-	134.30
3	108.00	37.50	-	145.50
4	108.00	48.80	-	156.80
5	108.00	60.00	-	168.00

Given the price, yield and variable costs in each of the five years, the net returns in each year can be calculated and the "stream" of net returns discounted back to the present to give the present value of the future net returns of the activity.

The present value is calculated from:[7/]

$$PV = \frac{a_1}{(1+r)} + \frac{a_2}{(1+r)^2} + \frac{a_3}{(1+r)^3} + \frac{a_4}{(1+r)^4} + \frac{a_5}{(1+r)^5} \qquad (3\text{-}1)$$

where PV is the present value of the activity,

a_1 to a_5 are the net returns in years 1 to 5, and

r is the market interest rate expressed as a decimal (taken to be 0.06).

7. Equation (3-1) assumes that all costs and returns occur at the end of each year.

Table 3.7 contains the present value estimation.

Table 3.7 Present Value of Rhubarb (P6)

Year	Yield (cases/ acre	Price ($/case)	Gross Return	Variable Costs	Net Return	Present Value
			($/acre)			
1	300	1.96	588.00	233.00	355.00	334.91
2	525	1.96	1028.40	134.30	894.10	795.46
3	750	1.96	1470.00	145.50	1324.50	1112.09
4	975	1.96	1911.00	156.80	1754.20	1388.92
5	1200	1.96	2352.00	168.00	2184.00	1631.07
					Σ PV =	5262.45

Therefore the present value of future net returns from rhubarb amounts to $5262.45 per acre.

3.5.3.2 Present value of Soleil d'Ors (P7)

Last season this activity gave a gross return of $354.0 from 140 x 12 dozen flowers, being the yield from two-thirds of an acre. As this was thought to be an average return a price of $0.21 per dozen flowers was estimated for the coming season.

Of the two-thirds of an acre of Soleil d'Ors, one-third of an acre is one year old, while the other one-third acre is in its seventh and final year. Last season, 259 dozen flowers were taken from the 10,000 bulbs planted in the one-year-old block (the planting rate is 30,000 bulbs per acre), giving one flower for about every three bulbs planted. In the same season, 1,421 dozen flowers were picked from the seven-year-old block and by assuming one flower was still produced for every three bulbs, this meant that by the end of the seventh year the 10,000 bulbs planted had multiplied to about 50,000.

The number of bulbs per acre for years 2 to 6 were calculated on the assumption that bulb multiplication would conform to a geometric progression,[8/] given that the number of bulbs in the first year was 30,000 and in the seventh, 150,000. Following this, the yield of flowers in each year was calculated on the assumption that one flower is obtained for every three bulbs. The gross return in each year could then be obtained.

The variable costs in each year are set out in table 3.8.

Table 3.8 Variable Costs ($ per acre) - Soleil d'Ors (P7)

Year	Bulbs	Spray	Tractor	Cases	Total
1	510.00	13.00	5.80	0.45	529.25
2	-	13.00	-	0.60	13.60
3	-	13.00	-	0.80	13.80
4	-	13.00	-	1.05	14.05
5	-	13.00	-	1.35	14.35
6	-	13.00	-	1.75	14.75
7	-	13.00	1.80	2.30	17.10

8. That is, one bulb multiplies to two the next year, four the year after, and so on. However, due to pests and diseases the actual number of bulbs will be less than the theoretical number.

The net returns for each year were calculated from the price, yield and cost estimates, and the "stream" of net returns was discounted to a present value using:

$$PV = \sum_{n=1}^{7} \frac{a_n}{(1+r)^n} \quad ; \qquad (3\text{-}2)$$

where a_n is the net return in each of the seven years.

The present value calculations are given in table 3.9.

Table 3.9 Present Value of Soleil d'Ors (P7)

Year	Bulbs (per acre)	Flowers (doz/ acre)	Gross Return	Variable Costs	Net Return	Present Value
			←	($ per	acre)	→
1	30,000	833	174.93	529.25	-354.32	-334.26
2	39,000	1083	227.43	13.60	213.83	190.24
3	51,000	1417	297.57	13.80	283.77	238.26
4	66,000	1833	384.93	14.05	370.88	293.65
5	87,000	2417	507.57	14.35	493.22	368.35
6	114,000	3167	665.07	14.75	650.32	458.29
7	150,000	4167	2915.07	17.10	2897.97	1928.12
					Σ PV =	3142.65

Note: Gross revenue in the seventh year includes the returns from the sale of all bulbs over and above the planting requirement of 30,000.

The present value of future net returns from Soleil d'Ors is therefore \$3142.65 per acre.

3.5.4 Calculation of annuities

Once the present value of future net returns from the perennial crops had been established, an annuity could be calculated,[9] using equation (3-3).[10]

$$A = PV\left[\frac{r\ (1+r)^n}{(1+r)^n-1}\right] , \qquad (3\text{-}3)$$

where A is the annuity,

PV is present value (computed in section 3.5.3),

r is the market rate of interest, expressed as a decimal, and

n is the life span of the activity.

The annuities are included in the linear programme as the annual gross margins of the perennial activities.

3.5.4.1 Rhubarb (P6) annuity

The present value of the rhubarb activity was found to be \$5262.45 per acre. Given that n = 5 years and r = 0.06, the annuity may be calculated:

$$A = 5262.45\left[\frac{0.06\ (1.06)^5}{(1.06)^5 - 1}\right]$$

$$= \underline{\$1247.15 \text{ per acre}}$$

---

9. An annuity is that constant value whose present value is the same as the present value of the stream of uneven net returns, over the same period of time.
10. Faris, J. Edwin, "Analytical Techniques used in Determining the Optimum Replacement Pattern", Journal of Farm Economics, vol.42, p.759, 1960.

3.5.4.2 Soleil d'Ors (P7) annuity

The present value of Soleil d'Ors is \$3142.65 per acre, n = 7 years, and the annuity is:

$$A = 3142.65 \left[\frac{0.06\ (1.06)^7}{(1.06)^7 - 1} \right]$$

$$= \underline{\$563.45 \text{ per acre}}$$

3.5.5 Overhead costs

The objective function has been constructed so as to maximise returns to the fixed resources of the holding. Once the linear programme has been solved (that is, the objective function maximised) overhead costs are deducted from the programmed profit to obtain the net profits of the farm plan.

The overhead costs of the holding are shown in table 3.10 and, with the exception of wages, were taken from the grower's 1966/67 balance sheet. Wages of all employees[11/] were determined from the hours of labour available, costed at the appropriate wage rate.

Table 3.10 Overhead Costs (\$)

Rent and rates	849.70
Power and light	25.90
Plant and vehicle repairs and maintenance	1,129.70
Plant and vehicle depreciation	599.00
Buildings repairs and maintenance	114.00
Buildings depreciation	8.00
Employees' wages	10,840.00
Motor expenses	2,112.50
Administration expenses	260.80
Refreshments	150.00
Total overhead cost	\$16,089.60

11. These do not include the owner's wages, and hence the grower's "true" profits are those remaining once his own salary has been withdrawn from net farm profits.

3.6 Input-Output Coefficients[12/]

3.6.1 Freehold land restraints

Should production of any of those activities which require freehold land increase by one unit (one acre) the unused supply of this land would be reduced by one acre and therefore the input-output coefficients are equal to unity. As an example, restraint R4 may be written as an inequality, the supply of freehold land being equal to four acres. Therefore,

$$1.0\ x_6 + 1.0\ x_7 + 1.0\ x_{10} + 1.0\ x_{11} \leq 4 \qquad (3\text{-}4)$$

which expresses the condition that the total acreage of freehold land in production during the January - April period, must be less than or equal to the supply of four acres (where x_6, for example, is the production level of activity P6).

The inequalities for the other freehold land restraints are derived in a similar manner.

3.6.2 Leased land restraints

The cauliflower (P1) and tomato (P3) activities consist of two and five plantings respectively and the land required by each planting need not be prepared or cleared all at the same time. For example, the land for both cauliflower plantings is ploughed in September_1, but by December_2 the first crop has been harvested and the land is available for another crop so that for every one acre ploughed in September_1 only half an acre is required in the second half of December. Therefore

---

12. The input-output coefficients are the a_{ij} values already mentioned in Chapter 2, section 2.1.5.

the cauliflower activity will have a coefficient of unity for the September - October leased land restraint, but a coefficient of 0.5 for the December_2 leased land restraint.

Turning now to the tomato activity, land for the first two plantings is prepared in September, that for the third and fourth plantings is prepared in December_1 and December_2 respectively, while land for the fifth planting is prepared in January_1. The grower estimated that in June_1 he would clear the first planting and about half of the second from the paddock since harvesting of them would be complete. The remainder of the tomato crop would remain in the ground until the end of July.

Thus the leased land coefficients for one acre of the tomato activity would include 0.4 for the September - October and November restraints, 0.6 for the December_1 restraint, 0.8 for the December_2 restraint, 1.0 for the January - April and May restraints and 0.7 for the June - July restraint.

To provide an example of a leased land inequality, reference to figure 3.1 shows that during November four activities (cauliflower (P1), cucumber (P2), tomato (P3) and pumpkin (P5)) require leased land. The supply of leased land is nine acres, and the November leased land restraint (R9) is defined by:

$$1.0\ x_1 + 1.0\ x_2 + 0.4\ x_3 + 1.0\ x_5 \leq 9. \qquad (3\text{-}5)$$

Inequality (3-5) states that the linear programme must not use more than nine acres of leased land in the production of activities P1, P2, P3 and P5, during November. (Since the lettuce activity (P4) does not require leased land during November, it has a zero input-output coefficient and has been omitted from (3-5)).

3.6.3 Labour restraints

To enable the labour input-output coefficients to be determined and therefore the labour restraints to be numerically specified, it was necessary to obtain from the grower an estimate of the amount of time spent in each half-month period on the various operations involved in producing one unit of each activity.

The grower outlined all operations performed on each activity, at what time they occurred and the tractor and labour hours involved. He stressed that these coefficients, although considered reasonable for a normal season, could be subject to variation brought about mainly by the prevailing weather conditions. For example, ploughing would take longer in wet than in dry, loose soil, or more or less time may need to be allocated to weeding or spraying than estimated. (The weather may also affect the timing of operations although some flexibility does exist in the linear programme in that jobs may be carried out any time within a period of half a month).

Table 3.11 shows the tractor and labour inputs per one acre output of the lettuce activity (P8) as an example of the information collected. Ploughing (for example) is carried out during $August_1$, only one stroke is necessary (that is the ground is ploughed once only), the tractor is running for 2.5 hours and 3.5 hours of labour are required.

The labour hours per acre are summed for each half-month period to provide the labour input-output coefficients. For example, the total labour requirement of the lettuce activity (P8) during $July_2$ is 1.7 hours per acre - should production of this activity be increased by one acre, 1.7 hours of labour will be required from the $July_2$ labour supply.

Table 3.11 Labour Input - Lettuce Activity (P8)

	Operation	Date	Strokes	Total Machine (hours/ acre	Total Labour (hours/ acre
In seedbed:	Seedbed preparation and sowing	$July_2$	-	-	1.4
	Cover with glass	$July_2$	-	-	0.3
	Spray seedbed	$Aug._1$	-	-	0.2
In open ground:	Plough	$Aug._1$	1	2.5	3.5
	Disc	$Aug._2$	3	4.0	4.0
	Level	$Aug._2$	1	1.0	1.0
	Fertiliser application	$Aug._2$	1	1.0	1.5
	Plant	$Sept._1$		-	160.0
	Wheel hoe	$Sept._2$	1	-	34.0
	Spray	$Sept._2$	1	2.0	2.5
	Wheel hoe	$Oct._2$	1	-	34.0
	Spray	$Oct._2$	1	2.0	2.5
	Wheel hoe	$Nov._1$	1	-	34.0
	Spray	$Nov._2$	1	2.0	2.5
	Harvesting	$Dec._1$	-	-	50.0
	Harvesting	$Dec._2$	-	-	100.0

The 'total machine hours per acre' column was summed for each activity, and used to calculate the tractor running cost per acre (see table 3.4).

Inequality (3-6) specifies the $July_2$ labour restraint (R29) where the supply of labour during this period is 329 hours:

$$0.3x_1 + 21.6x_3 + 6.0x_5 + 20.0x_6 + 60.0x_7 + 1.7x_8 + 2.5x_9 + 6.0x_{10} + 2.0x_{12} + 10.6x_{13} \leq 329 \quad . \qquad (3\text{-}6)$$

Difficulty was experienced in the calculation of accurate labour coefficients for the two perennial crops, rhubarb and Soleil d'Ors. First, should the acreage of these crops included in the linear programme solution be greater than (less than) those of the past season extra labour will be required for planting (digging out). Secondly, the labour requirements for harvesting will increase each year until the crop is due for replacement, since yields increase each year. The labour input coefficients for the perennial activities, then, do not include any allowance for new plantings and conform to the grower's estimate of the labour requirements for weeding, spraying, and harvesting in the average year.

3.6.4 Cropping limit restraints

The maximum acreage the grower would consider planting of each activity has been specified and each of these restraints is an inequality. For example, the acreage planted in outdoor cucumbers (activities P2 and P11) must not exceed 2.5 acres (restraint R41), and (3-7) ensures that this restraint is not violated:

$$1.0x_2 + 1.0x_{11} \leq 2.5 \qquad (3\text{-}7)$$

If less than 2.5 acres is planted in cucumbers, then the difference between 2.5 acres and the acreage planted will be in disposal.

3.6.4.1 The glasshouse restraint (R52)

The area of the glasshouse is 3,000 square feet and can be planted with up to 825 cucumber plants. Since one glasshouse unit is the space required by 825 plants (3,000 sq.ft.) and one unit of the glasshouse cucumber (P14) activity has been defined as 825 plants, the condition that the capacity of the glasshouse must not be exceeded is ensured by:

$$1.0x_{14} \leq 1.0 \qquad (3\text{-}8)$$

3.6.5 Dominated restraints[13]

A dominated restraint in a linear programme can never limit production since there will always exist at least one other restraint which will effectively limit production before the dominated restraint. Since dominated restraints are superfluous, they may be omitted from the linear programming problem without affecting the solution, but with a consequent reduction in the size of the simplex tableau and hence computational effort. It was found that the September - October, January - April, May - $June_1$, $June_2$ - July and August freehold land restraints were dominated, as were the September - October, November, May and August leased land restraints, and the $January_2$ and $June_2$ labour restraints.

3.7 The Basic Matrix

The basic matrix is presented as table 3.12. The supplies of each resource (restraint) form the first, or B, column of the basic matrix, while each of the fourteen activities P1 to P14 is represented by the remaining columns. Each restraint forms a row of the basic matrix. Construction of the matrix is easily understood by first following the derivation of the restraints (such as inequalities 3-4, 3-5, 3-6, 3-7 and 3-8) and then examining their inclusion in table 3.12. The resource supply is entered in the B column, and the coefficients are placed in the appropriate activity columns. The row at the top of the

---

13. Heady and Candler, op.cit., pp.151-154.

Table 3.12 The Basic Matrix

Restraints	unit	B	Relationship	P1 Cauliflower	P2 Cucumber	P3 Tomato	P4 Lettuce	P5 Pumpkin	P6 Rhubarb	P7 Soleil d'Ors	P8 Lettuce	P9 Carrot	P10 Pumpkin	P11 Cucumber	P12 Cabbage	P13 Cauliflower	P14 Glasshouse cucumber
Gross margin ($)				576.80	1361.52	3570.80	485.60	465.60	1247.15	563.45	413.60	743.32	465.60	1210.19	418.20	543.56	521.55
	unit→			1 acre	1 acre	1 acre	1 acre	1 acre	1 acre	1 acre	1 acre	1 acre	1 acre	1 acre	1 acre	1 acre	825 plants
Freehold land:																	
November	1 acre	4	>						1.0	1.0	1.0	1.0	1.0		1.0	1.0	
December	"	4	>						1.0	1.0	1.0	1.0	1.0	1.0		1.0	
Leased land:																	
December 1	"	9	>	1.0	1.0	0.6		1.0									
December 2	"	9	>	0.5	1.0	0.8		1.0									
January-April	"	9	>		1.0	1.0		1.0									
June-July	"	9	>			0.7	1.0										
Labour:																	
January 1	1 hour	362	>		7.5	43.3		6.0					6.0	7.5			21.0
January 2	"	544	>		10.5	57.2								7.5			25.0
February 1	"	706	>		29.7	98.2								0.5			
February 2	"	706	>		47.5	93.6								7.0			
March 1	"	795	>		47.5	89.0								48.0			
March 2	"	795	>		47.5	81.2								62.0			
April 1	"	795	>		29.7	80.4			1.5	4.0				62.0			
April 2	"	795	>		14.8	80.4	2.5	24.0	60.0				24.0	48.0		0.7	
May 1	"	795	>			63.0	0.3	60.0	4.5				60.0			3.5	
May 2	"	633	>			92.8			36.0	20.0						0.1	
June 1	"	427	>			21.0	169.8		40.5							0.1	
June 2	"	427	>			4.0			20.0	4.5		3.0				4.5	
July 1	"	329	>	2.0		27.6	36.5	6.0	20.0	60.0		7.5	6.0			3.3	5.0
July 2	"	329	>	0.3		21.6		6.0	20.0	60.0	1.7	2.5	6.0		2.0	10.6	
August 1	"	329	>				36.5	6.0	26.0	60.0	3.7		6.0		3.0	3.3	2.5
August 2	"	329	>	0.3				6.0	36.0	12.0	6.5	14.0	6.0			33.0	3.5
September 1	"	329	>	24.0	3.5	2.5	36.5	12.0	36.0		160.0	180.0	12.0		47.5	2.0	9.5
September 2	"	329	>	15.0			32.0	12.0	36.0		36.5	12.5	12.0		48.0		2.0
October 1	"	427	>	33.0		5.6	64.0	12.0	54.0				12.0		48.0		2.0
October 2	"	427	>	33.0		5.2	64.0		48.0		36.5	12.5			20.0	15.2	2.5
November 1	"	492	>	2.0	10.0	13.3		14.0	48.0		34.0	36.0	14.0		40.0	30.5	2.5
November 2	"	492	>	49.5		7.9			48.0		2.5	36.0			20.0	61.0	6.5
December 1	"	492	>	99.0	7.5	22.8		7.8	4.5		50.0	36.0	7.8	18.0		61.0	22.5
December 2	"	422	>	49.5		28.3		8.0			100.0	36.0	8.0	4.0		30.5	21.0
Cropping limits:																	
Cauliflower (P1)	1 acre	3.0	>	1.0													
Cucumber	"	2.5	>		1.0									1.0			
Cucumber (P11)	"	0.5	>											1.0			
Tomato	"	5.0	>			1.0											
Lettuce (P4)	"	1.5	>				1.0										
Pumpkin	"	4.0	>					1.0					1.0				
Rhubarb	"	1.0	>						1.0								
Soleil d'Ors	"	0.67	>							1.0							
Lettuce (P8)	"	1.5	>								1.0						
Carrot	"	0.5	>									1.0					
Cabbage	"	2.0	>												1.0		
Cauliflower (P13)	"	1.0	>													1.0	
Glasshouse:	3000 sq.ft	1.0	>														1.0

matrix contains the activity gross margins.

All disposal activities have been omitted from table 3.12 to save space, as have those freehold and leased land restraints found to be dominated. The two dominated labour restraints are included, however, since these provide information not covered elsewhere.

3.8 The Solution [14/]

3.8.1 The cropping programme [15/]

Table 3.13 lists all real activities included in the linear programme solution, the months over which each activity occupies the land and the production level of each activity.

3.8.2 Comparison of the linear programme solution with the grower's plan

3.8.2.1 Comparison of activity levels

Table 3.14 contains the levels of all activities in the optimum plan and the grower's plan, profits from both plans, and indicates the differences in production levels and farm profits for both plans.

Of the crops grown on leased land, only the tomato activity (P3) is included in both plans at the same level of 5.0 acres. The planting of cucumber (P2) has decreased from 2.50 acres in the grower's plan to 2.00 acres in the optimum plan, and cauliflower (P1), lettuce (P4) and

---

14. The solution was obtained using an I.B.M. 1620-II computer and the I.B.M. 1620-1311 Linear Programming System.
15. Discussion of this initial solution with the grower led to the derivation of further plans (section 3.9). Since one of these latter plans was adopted by the grower, the initial solution will be discussed only briefly.

Table 3.13 The Cropping Programme

Activity	Land Use	Activity Level
In leased land:		
P1 : cauliflower	$September_1$ - $December_2$	2.35 acres
P2 : cucumber	$November_1$ - $April_2$	2.00 acres
P3 : tomato	$September_1$ - $July_2$	5.00 acres
P4 : lettuce	$June_1$ - $October_2$	1.50 acres
P5 : pumpkin	$November_1$ - $April_2$	1.65 acres
In freehold land:		
P6 : rhubarb	perennial	1.00 acre
P7 : Soleil d'Ors	perennial	0.67 acre
P9 : carrot	$June_2$ - $December_2$	0.50 acre
P10 : pumpkin	$November_1$ - $April_2$	0.33 acre
P11 : cucumber	$December_1$ - $April_2$	0.50 acre
P12 : cabbage	$August_1$ - $November_2$	0.50 acre
P13 : cauliflower	May_1 - $December_2$	1.00 acre
P14 : glasshouse cucumber		825 plants
Farm profit (pre-tax) : $11,368.72		

Note: Pre-tax farm profit $= \sum_j c_j x_j - Q$

where c_j = gross margin of activity P_j
x_j = level of activity P_j
Q = overhead costs.

Table 3.14 Comparison of Activity Levels and Farm Profits Between the Optimum and the Grower's Plan

Activity	Optimum Plan	Grower's Plan	Difference
P1 : cauliflower	2.35 acres	2.00 acres	+ 0.35
P2 : cucumber	2.00 acres	2.50 acres	- 0.50
P3 : tomato	5.00 acres	5.00 acres	-
P4 : lettuce	1.50 acres	1.00 acre	+ 0.50
P5 : pumpkin	1.65 acres	1.50 acres	+ 0.15
P6 : rhubarb	1.00 acre	1.00 acre	-
P7 : Soleil d'Ors	0.67 acre	0.67 acre	-
P9 : carrot	0.50 acre	-	+ 0.50
P10 : pumpkin	0.33 acre	2.33 acres	- 2.00
P11 : cucumber	0.50 acre	-	+ 0.50
P12 : cabbage	0.50 acre	-	+ 0.50
P13 : cauliflower	1.00 acre	-	+ 1.00
P14 : glasshouse cucumber	825 plants	825 plants	-
Farm profit (pre-tax)	\$11,368.72	\$10,736.86	+ \$631.86

Note: A positive entry in the fourth column indicates that the level of an activity in the optimum plan is greater than in the grower's plan.

pumpkin (P5) are included in the optimum plan at levels exceeding those in the grower's plan by 0.35, 0.50 and 0.15 acre respectively.

The grower's plan includes only three crops on the freehold land, as against seven on this land in the optimum plan. Rhubarb (P6) and

Soleil d'Ors (P7) are included in both plans at the same levels, although the area of pumpkin (P10) in the optimum plan is 2.0 acres below that in the grower's plan. Thus the total pumpkin acreage in the optimum plan is 1.98, compared with 3.83 acres of pumpkin in the grower's plan.

The cucumber (P11) activity is included only in the optimum plan (at 0.50 acre) so that the total cucumber acreage in both plans is similar, at 2.50 acres. Three other activities, carrot (P9), cabbage (P12) and cauliflower (P13) are included only in the optimum plan.

The glasshouse cucumber activity is at the maximum level in both plans.

3.8.2.2 Comparison of profits

By adopting the linear programme solution rather than his proposed plan the grower can expect an increase in pre-tax profits of $631.86, which is 5.9 percent of the pre-tax profits from his proposed plan.

3.8.3 Stability of the optimum plan

Table 3.15 includes the upper and lower gross margin limits.[16]

Of the basic activities it can be seen that tomato, lettuce (P4) and glasshouse cucumber are the most stable with regard to changes in their gross margins, although both of the cucumber activities, rhubarb, Soleil d'Ors and carrot may also be considered stable components of the cropping programme.

---

16. Should the gross margin of a basic activity equal one of the limits, the $z_j - c_j$ value for some non-basic activity will be zero and it may therefore enter the basis without reducing profits. Also, should the gross margin of a non-basic activity increase by the amount of its marginal opportunity cost, the $z_j - c_j$ value of that activity would become zero so that it could be included in the basis without reducing profits. See Chapter 2, section 2.1.11.

Table 3.15 Gross Margin Stability Limits ($)

Activity	Unit	Lower Limit	Gross Margin in Plan	Upper Limit
Basic Activities:				
P1 : cauliflower	1 acre	465.60	576.80	599.25
P2 : cucumber	1 acre	465.23	1361.52	1615.59
P3 : tomato	1 acre	301.45	3570.80	infinity
P4 : lettuce	1 acre	0	485.60	infinity
P5 : pumpkin	1 acre	443.15	465.60	576.80
P6 : rhubarb	1 acre	461.58	1247.15	infinity
P7 : Soleil d'Ors	1 acre	456.09	563.45	infinity
P9 : carrot	1 acre	499.98	743.32	infinity
P10 : pumpkin	1 acre	427.71	465.60	478.69
P11 : cucumber	1 acre	956.12	1210.19	infinity
P12 : cabbage	1 acre	164.13	418.20	456.09
P13 : cauliflower	1 acre	530.47	543.56	infinity
P14 : glasshouse cucumber	825 plants	27.43	521.55	infinity
Non-basic activity:				
P8 : lettuce	1 acre		413.60	517.05

The five remaining real activities in the solution form relatively unstable elements since at least one or other of the gross margin limits is quite likely to occur as a result of unforeseen fluctuations in prices and/or yields.

The lettuce activity (P8) may be profitably included in the cropping programme should prices or yields increase sufficiently to allow its gross margin to exceed the upper limit.

3.8.4 Unused resources

3.8.4.1 Land

The quantity of land unused (in disposal) during each time period is given in table 3.16.

Table 3.16 Land in Disposal

Disposal Activity	Level in Plan (acres)
Freehold land:	
September - October	0.33
January - April	1.50
May - $June_1$	1.33
$June_2$ - July	0.83
August	0.33
Leased land:	
September - October	4.65
November	1.00
$December_2$	0.17
January - April	0.35
May	4.00
June - July	4.00
August	7.50

All freehold land is used during November and December, and all leased land is cropped during $December_1$ so the corresponding disposal activities will not be present in the basis. At least 1.33 acres of freehold land is left unused from January until June[1] and this land could either be left fallow, or sown in a greencrop which the grower considered to be a sound management practice. Thus all freehold land would be sown in a greencrop once every three years, since the disposal level of 1.33 acres is one-third of the freehold land supply of four acres.

During May, June and July four acres of leased land are in disposal, and from August until the end of October (when the lease terminates) 7.50 acres of leased land are unused since 1.50 acres of lettuce (P4) is the only crop occupying the land over this period.

3.8.4.2 Labour

Table 3.17 includes the level of all labour disposal activities in the optimum plan, with some labour being unused during each period except the first half of December.

Table 3.17 Labour in Disposal

Month	Disposal Level (hours)	
	First half-month	Second half-month
January	93.86	208.25
February	155.35	139.50
March	231.00	263.00
April	298.42	227.41
May	352.69	119.50
June	26.70	377.99
July	47.42	135.36
August	188.86	228.87
September	13.35	153.74
October	121.71	145.53
November	254.07	192.73
December	-	76.88

3.8.5 Value of resources [17/]

3.8.5.1 Land

Table 3.18 gives the values imputed to the effective land restraints. The value of freehold land during November is \$418.20 per acre and that of freehold land during December is \$37.89 per acre. Since the grower must increase the supply of this land in every month of the year if he buys extra land, the value of an additional acre of freehold land is equal to the sum of the values imputed to November and December land, that is, \$456.09 per acre.

Table 3.18 Land Shadow Prices

Resource	Unit	Shadow Price
R2 : Freehold land - November	1 acre	\$418.20
R3 : Freehold land - December	"	\$37.89
R10 : Leased land - $December_1$	"	\$456.09

The value of leased land during $December_1$ is also \$456.09 per acre, and it will pay the grower to lease additional land (providing it is available during the first half of December) if the rent is less that \$456 per acre.

---

17. It is recalled that only 'scarce' resources have economic value - should part or all of a resource supply be unused the resource is 'free' and has no value, since additional units of that resource would not be used and would not, therefore, add to production. (See Chapter 2, section 2.1.9).
Also, a shadow price will remain unchanged only until the resource supply has been expanded to a certain limit (See Chapter 2, section 2.1.11.2). Unfortunately, the I.B.M. 1620-1311 L.P. System does not give these limits.

3.8.5.2 Labour

The entire labour supply is used only during the first half of December. The value of this labour is $1.22 per hour, this being the value at the margin of the increase in output which would result from hiring extra labour during this period.

3.8.5.3 Cropping limits

If a cropping limit restraint is effective, the level of that crop included in the optimum plan will be equal to the maximum level as defined by the grower, and a value will be imputed to the appropriate restraint indicating the increment in profit which would result by marginally relaxing the restraint. Such values are presented in table 3.19. All cropping limit restraints are effective except those imposed on the acreage of cauliflower (P1), pumpkin, lettuce (P8) and cabbage.

Table 3.19 Cropping Limit Shadow Prices

Restraint	Unit	Shadow Price
R41 : cucumber limit	1 acre	$896.29
R42 : cucumber (P11) limit	"	$254.07
R43 : tomato limit	"	$3269.35
R44 : lettuce (P4) limit	"	$485.60
R46 : rhubarb limit	"	$785.57
R47 : Soleil d'Ors limit	"	$107.36
R49 : carrot limit	"	$243.34
R51 : cauliflower (P13) limit	"	$13.09
R52 : glasshouse cucumber limit	825 plants	$494.12

The greatest profit increases will result by relaxing the tomato, cucumber, rhubarb or glasshouse cucumber cropping limits. (Should it be decided to expand the level of the latter activity, however, an extra glasshouse will be required, which means that other factors such as the availability of capital and returns on capital invested elsewhere must be considered).

3.8.5.3.1. The tomato cropping limit

The tomato cropping limit merits special consideration since its shadow price of \$3269.35 per acre is more than three times as great as the next highest shadow price. Thus it is apparent that the grower should give considerable attention to the possibility of increasing the size of the tomato crop beyond the limit of five acres which he imposed when the linear programming model was constructed.

3.9 A Parametric Solution

3.9.1 The reason for computing alternative plans

Although the grower had stated that five acres of the tomato activity was the maximum level he would be prepared to handle, further plans were computed, making the tomato cropping limit less restrictive to determine whether the large profit increases that appeared likely would persuade the grower to adopt a plan which included more than five acres of tomatoes.

3.9.2 Parametric linear programming

Parametric solutions are obtained when a price, resource supply or input-output coefficient, which is constant for any given linear programme, is allowed to vary from problem to problem. Since the tomato cropping limit is to be varied from problem to problem, the technique is similar to variable resource programming.[18]

The shadow price of the tomato cropping restraint will remain constant for restraints between five acres and some upper limit.[19] For any restraint between these limits the corresponding optimum solution can be found with reference to the final simplex tableau.[20] Once the restraint exceeds the upper limit, however, a new optimum basis will need to be computed.

The whole series of optimum solutions can be obtained with the minimum of computational effort by setting the 'supply' of the tomato cropping restraint in a new problem fractionally above the upper limit to increases in the resource supply from the preceding solution, and so continuing until all resource supplies of interest have been covered. The I.B.M. 1620-1311 L.P. system does not, however, give the limits over which shadow prices remain constant, so the parametric solution was obtained by increasing the tomato cropping limit from 5.0 acres to 5.1, 5.2 acres and so on, the critical plans being found by inspecting the shadow price of the tomato cropping limit in each plan and

---

18. Heady and Candler, op.cit., Chapter 7.
19. See Chapter 2, section 2.1.11.2.
20. The coefficients in the tomato cropping limit disposal activity column of the final simplex tableau indicate the changes in the level of basic activities which would result from a one-unit increase in the level of disposal (a positive coefficient indicates that the level of a basic activity would decrease, and a negative coefficient indicates that the level of a basic activity would increase). By reversing all signs in this column, the changes in the levels of basic activities resulting from a one-unit increase in the tomato cropping limit are indicated.

thus ascertaining the points where the shadow price changes.[21]

3.9.3 Activity levels in the parametric solution

Table 3.20 contains the farm plans obtained by varying the tomato cropping limit. The second column of the table contains the optimum solution to the initial linear programming problem which has already been discussed in section 3.8. The remaining columns give the levels of real activities and farm profits for each of the six critical plans.

Figure 3.2 gives a graphical presentation of the parametric solution by plotting the levels of basic real activities vertically above the tomato cropping limit of the plan in question for those activities whose levels vary in the parametric solution. Since linear programming is being employed, points in figure 3.2 may be joined by straight lines so as to represent the continuous nature of the parametric solution. The rate of change of an activity level and farm profits remains constant between successive critical plans so that once these plans are known the optimum solution for any tomato cropping limit (between 5.00 and 6.48 acres) may be read directly from figure 3.2.

3.9.4 Resource values in the parametric solution

The marginal value products of scarce resources, plus the marginal opportunity costs of non-basic real activities, are given

---

21. If the shadow price was found to change between, say, 5.0 and 5.1 acres, the limit was then set at 5.05 acres and so on to find the exact point at which the change occurred. Also, the shadow price may change more than once in one-tenth of an acre. The discrete points at which the shadow prices changed were thus found to an accuracy of three decimal places.

Table 3.20 The Parametric Solution - Activity Levels (acres) and Farm Profits

Activity	Initial Solution	First Critical Plan	Second Critical Plan	Third Critical Plan	Fourth Critical Plan	Fifth Critical Plan	Sixth Critical Plan
P1 : cauliflower	2.35	2.24	2.51	2.52	2.58	2.58	2.58
P2 : cucumber	2.00	2.00	2.00	2.00	2.00	2.00	2.00
P3 : tomato	5.00	5.59	6.28	6.29	6.44	6.46	6.48
P4 : lettuce	1.50	1.50	1.50	1.50	1.48	1.49	1.50
P5 : pumpkin	1.65	1.41	0.72	0.71	0.56	0.54	0.52
P6 : rhubarb	1.00	1.00	1.00	1.00	0.98	0.93	0.88
P7 : Soleil d'Ors	0.67	0.67	0.67	0.66	-	-	-
P9 : carrot	0.50	0.50	0.50	0.50	0.50	0.50	0.50
P10 : pumpkin	0.33	0.34	1.05	1.06	2.00	2.07	2.12
P11 : cucumber	0.50	0.50	0.50	0.50	0.50	0.50	0.50
P12 : cabbage	0.50	0.50	0.50	0.50	0.50	0.50	0.50
P13 : cauliflower	1.00	0.99	0.29	0.27	0.02	-	-
P14 : glasshouse cucumber	825 plants	825 plants	825 plants	825 plants	825 plants	825 plants	825 plants
Farm profits ($) (pre-tax)	11,368.72	13,297.54	15,543.81	15,575.01	15,970.19	15,998.95	16,021.06

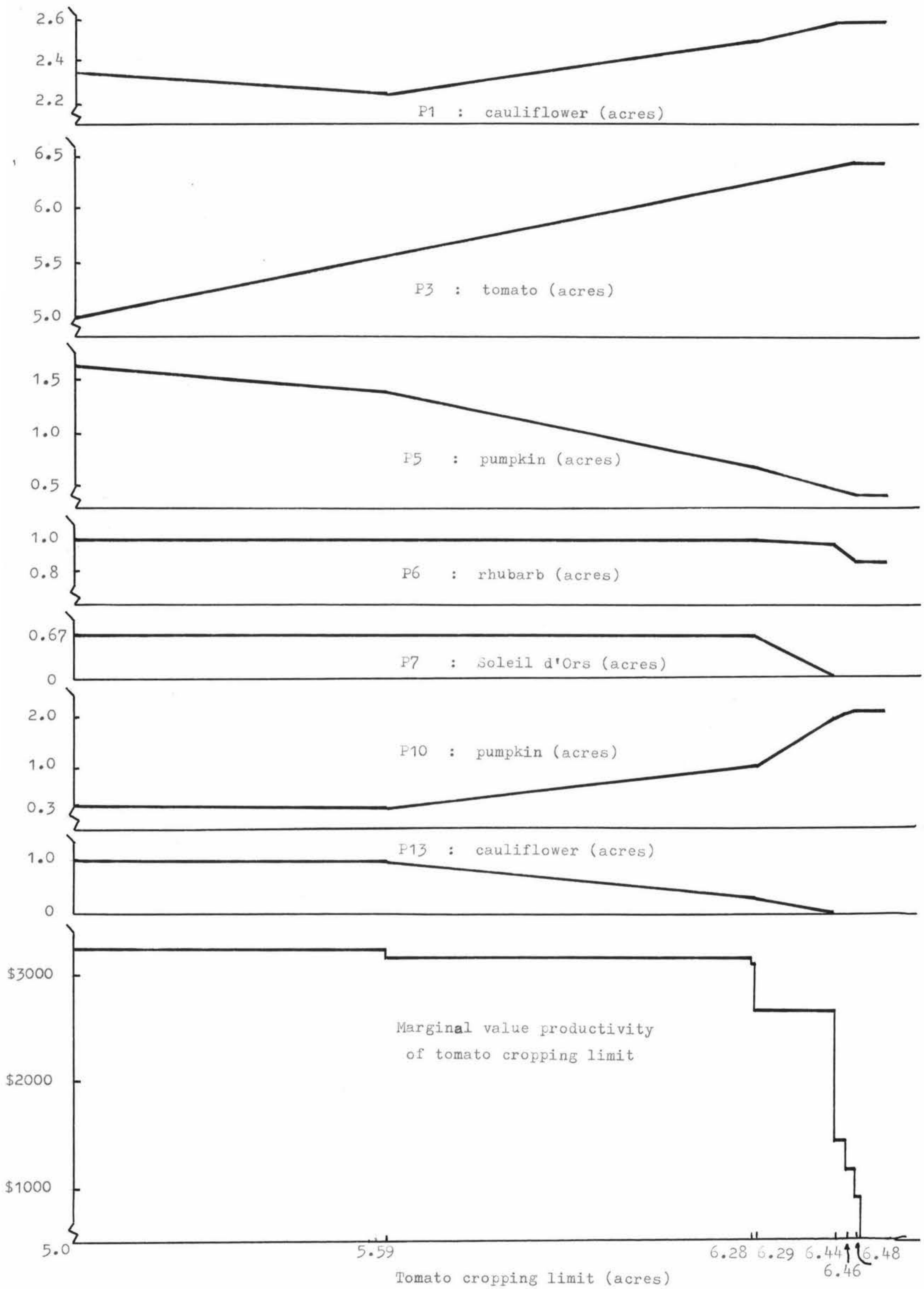

Figure 3.2 The Parametric Solution

for the initial plan and the six critical plans of the parametric solution in table 3.21. It should be noted that the marginal value product of the tomato cropping limit, which remains constant between any two consecutive plans, decreases from one critical plan to the next. Hence a "stepped" marginal value product curve is obtained in figure 3.2, rather than the smooth curves which are encountered in textbook presentations of production theory. Both curves, however, obey the law of diminishing returns to a variable factor.

3.9.5 Supplementary, complementary and competitive relationships within the parametric solution

Supplementary, complementary and competitive relationships between activities are reflected in the final simplex tableau.[22] Two activities with positive coefficients in the same row of the matrix require the same resource and are competitive with respect to that resource. If one activity has a positive coefficient and another has a zero entry in the same row, (or one with a zero and one with a negative coefficient, or both with negative coefficients in the same row), the activities will be supplementary with respect to the resource in that the level of one activity will have no effect on the level of the other. Where one activity has a positive coefficient and the other a negative coefficient in the same row, the activities are complementary. This is a one way relationship, however, since the level of the activity with the positive coefficient can be increased by increases in the level of the activity with the negative coefficient, but not vice versa.

---

22. Heady and Candler, op.cit., pp.214-215.

Table 3.21 — The Parametric Solution - Value of Scarce Resources ($)

Resource	Initial Solution	First Critical Plan	Second Critical Plan	Third Critical Plan	Fourth Critical Plan	Fifth Critical Plan	Sixth Critical Plan
Freehold land:							
November	418.20	418.20	418.20	418.20	418.20	418.20	418.20
December	37.89	35.97	36.01	36.09	36.29	1.96	1.96
Leased land:							
December$_1$	456.09	431.72	432.26	433.27	435.72	-	-
January - April	-	22.45	21.96	21.02	18.76	420.16	420.16
Labour:							
May$_2$	-	-	-	5.46	18.62	19.03	22.24
June$_1$	-	-	2.86	2.86	2.86	2.86	-
December$_1$	1.22	1.47	1.46	1.45	1.43	5.83	5.83
Cropping Limits:							
cucumber	896.29	896.36	896.36	896.36	896.35	897.67	897.67
cucumber (P11)	254.07	251.48	251.54	251.65	251.91	205.69	205.69
tomato	3269.35	3255.91	3196.15	2690.21	1469.84	1192.09	953.58
lettuce (P4)	485.60	485.60	-	-	-	-	485.60
rhubarb	785.57	786.39	670.55	474.03	-	-	-
Soleil d'Ors	107.36	109.28	109.24	-	-	-	-
carrot	243.34	236.40	236.55	236.84	237.53	113.42	113.42
cauliflower (P13)	13.09	-	-	-	-	-	-
glasshouse cucumber	494.12	488.58	488.70	488.93	489.49	390.46	390.46
Shadow prices of omitted crops:							
P8 : lettuce	103.45	113.84	113.61	113.18	112.14	297.87	297.87
P7 : Soleil d'Ors	-	-	-	-	263.50	237.24	301.58
P13 : cauliflower	-	-	-	-	-	234.19	234.22

Note: Once the tomato cropping limit reaches 6.49 acres it ceases to be effective and its shadow price becomes zero, since any further relaxation of the restraint will be allocated to disposal.

Thus any two activities may be complementary, supplementary and competitive to each other with respect to different resources. The effective relationship, which will depend upon the relative scarcity of resources, is immediately apparent from figure 3.2. Here, if two activity curves both have positive slopes, the activities will be effectively complementary over the relevant range of the tomato cropping limit. Likewise, two activities will be effectively supplementary if one curve has a positive slope and the other a zero slope, and finally two activities will be effectively competitive if one curve has a positive slope and the other a negative slope.[23]

With reference to figure 3.2 and table 3.21, it can be seen that as the tomato cropping limit is increased above five acres, the tomato activity is competitive with cauliflower (P1 and P13) and pumpkin (P5), complementary with respect to pumpkin (P10) and supplementary with respect to all other real activities in the basis. This process continues until the tomato cropping limit reaches 5.59 acres, when all leased land is used from January to April (in table 3.26 this resource is seen to be 'free' in the initial solution, but "scarce" in the first critical plan).

The tomato activity then becomes complementary with respect to cauliflower (P1) and pumpkin (P10) and competitive with the pumpkin (P5)

---

23. The ratio of the slopes of these curves will measure the marginal rate of substitution between the two activities. (Heady and Candler, op.cit., p.242). For example, the marginal rate of substitution of tomato (P3) for cauliflower (P1), between tomato cropping limits of 5.00 and 5.59 acres, is given as:

$$\text{M.R.S. P3 for P1} = \frac{\text{slope of P1 curve}}{\text{slope of P3 curve}} = -0.186$$

Since the marginal rate of substitution is negative, the two activities must be effectively competitive.

and cauliflower (P13) activities. The next resource to become scarce is $June_1$ labour, the entire supply being utilised once the tomato cropping limit reaches 6.28 acres. When the tomato cropping limit reaches 6.29 acres, May_2 labour becomes scarce and the third critical plan is obtained. Similarly, the fourth and fifth plans are obtained when the Soleil d'Ors and cauliflower (P13) activities, respectively, leave the basis, and the sixth critical plan is obtained when the lettuce (P4) crop maximum restraint becomes effective.

3.10 Adoption by the Grower of a Linear Programme Solution

3.10.1 Introduction

After each of the seven linear programme solutions had been discussed with the grower and compared with his own plan, the grower considered the second critical plan (with 6.28 acres of the tomato activity) to be the most attractive and said that he would be prepared to put this plan into operation in the coming season. He gave two main reasons for choosing this plan rather than any other. Firstly, pre-tax farm profits were over $4,800 above those from his own plan, (which represents an increase in pre-tax profits of almost 45 percent), and secondly, since the grower did not want to reduce his planting of Soleil d'Ors, the fourth, fifth and sixth critical plans (which exclude this activity) were unacceptable.

Although the second and third critical plans are almost identical (pre-tax profits from the latter are only $31 above those of the former plan) the grower chose the second critical plan since <u>some</u> labour is left in disposal during May_2, whereas this resource is fully employed in the third critical plan.

The remainder of section 3.10 will discuss the adopted plan in detail, and further comments made by the grower on the linear programming solutions will be given in section 3.12.

3.10.2 Comparison of the adopted plan with the grower's plan

3.10.2.1 Comparison of activity levels

Table 3.22 gives the level of all real activities in the adopted plan and the months over which they occupy the land, and a comparison of activity levels and farm profits between the adopted plan and the grower's plan is provided by table 3.23.

Table 3.22 The Adopted Cropping Programme

Activity	Land Use	Activity Level
In leased land:		
P1 : cauliflower	September_1 - December_2	2.51 acres
P2 : cucumber	November_1 - April_2	2.00 acres
P3 : tomato	September_1 - July_2	6.28 acres
P4 : lettuce	June_1 - October_2	1.50 acres
P5 : pumpkin	November_1 - April_2	0.72 acre
In freehold land:		
P6 : rhubarb	perennial	1.00 acre
P7 : Soleil d'Ors	perennial	0.67 acre
P9 : carrot	June_2 - December_2	0.50 acre
P10 : pumpkin	November_1 - April_2	1.05 acres
P11 : cucumber	December_1 - April_2	0.50 acre
P12 : cabbage	August_1 - November_2	0.50 acre
P13 : cauliflower	May_1 - December_2	0.29 acre
P14 : glasshouse cucumber		825 plants
Farm profit (pre tax) : $15,543.81		

Table 3.23 Comparison of Activity Levels and Farm Profits Between the Adopted and the Grower's Plan

Activity	Adopted Plan	Grower's Plan	Difference
P1 : cauliflower	2.51 acres	2.00 acres	+ 0.51
P2 : cucumber	2.00 acres	2.50 acres	- 0.50
P3 : tomato	6.28 acres	5.00 acres	+ 1.28
P4 : lettuce	1.50 acres	1.00 acre	+ 0.50
P5 : pumpkin	0.72 acre	1.50 acres	- 0.78
P6 : rhubarb	1.00 acre	1.00 acre	-
P7 : Soleil d'Ors	0.67 acre	0.67 acre	-
P9 : carrot	0.50 acre	-	+ 0.50
P10 : pumpkin	1.05 acres	2.33 acres	- 1.28
P11 : cucumber	0.50 acre	-	+ 0.50
P12 : cabbage	0.50 acre	-	+ 0.50
P13 : cauliflower	0.29 acre	-	+ 0.29
P14 : glasshouse cucumber	825 plants	825 plants	-
Farm Profit (pre tax)	$15,543.81	$10,736.86	+ $4,806.95

Of those crops grown on leased land, cauliflower, tomato and lettuce are included in the adopted plan at levels exceeding those in the grower's plan. Both cucumber and pumpkin, however, are included in the former plan at a lower level than in the grower's plan. Whereas the grower's plan included only rhubarb, Soleil d'Ors and pumpkin on freehold land, the adopted plan also contains the carrot, cucumber, cabbage and cauliflower activities. Rhubarb and Soleil d'Ors are at the same (maximum) level in both plans, although the acreage of pumpkin (P10) is considerably below that in the grower's plan.

Both plans include the maximum 2.50 acres of cucumbers, although pumpkin acreages have been reduced from 3.83 acres in the grower's plan to 1.77 acres in the proposed plan. The glasshouse cucumber activity is included in both plans at the maximum level.

3.10.2.2 Comparison of profits

By adopting the second critical plan in place of his earlier-proposed plan, the grower can expect his pre-tax farm profits to increase by over $4,800, or to almost 45 percent above the level of pre-tax profits from his own plan.

3.10.2.3 Comparison of resource requirements

The land requirements of both plans are compared in table 3.24 and the labour requirements are shown in table 3.25.

The adopted plan makes greater use of the land resources than does the grower's plan, using less freehold land during January - April only, and less leased land only in November. Whereas the requirement of the grower's plan for freehold land is at a minimum from May until the end of October, the adopted plan makes greater use of the land over each of these months due to the inclusion of some winter and spring crops (namely cauliflower (P13), carrot and cabbage).

Both plans make full use of the leased land from December until the end of April. The adopted plan has a greater requirement for leased land during every other month with the exception of November, due to increased plantings of tomato, cauliflower (P1) and lettuce (P4).

The adopted plan requires more labour than does the grower's plan during each half-month period with the exception of May_1. The increased labour requirement from January until the end of May is due almost entirely to the high labour requirements of the tomato activity

Table 3.24 Land Requirements of the Adopted and the Grower's Plan (acres)

Resource	Resource Supply	Adopted Plan Requirement	Grower's Plan Requirement	Difference
Freehold land:				
September - October	4.00	2.96	1.67	+ 1.29
November	4.00	4.00	4.00	-
December	4.00	4.00	4.00	-
January - April	4.00	3.22	4.00	- 0.78
May - $June_1$	4.00	1.96	1.67	+ 0.29
$June_2$ - July	4.00	2.46	1.67	+ 0.79
August	4.00	2.96	1.67	+ 1.29
Leased land:				
September - October	9.00	5.02	4.00	+ 1.02
November	9.00	7.74	8.00	- 0.26
$December_1$	9.00	9.00	9.00	-
$December_2$	9.00	9.00	9.00	-
January - April	9.00	9.00	9.00	-
May	9.00	6.28	5.00	+ 1.28
June - July	9.00	5.90	4.50	+ 1.40
August	9.00	1.50	1.00	+ 0.50

over this period, and the increased labour requirements over the remainder of the year are a result of the increased levels of cauliflower (P1) and lettuce (P4), and the inclusion of other winter and spring crops (cauliflower (P13), carrot and cabbage) in the adopted plan. In all, the adopted plan requires that 76 percent of the total labour supply be

Table 3.25 Labour Requirements of the Adopted and the Grower's Plan (hours)

Labour Resource	Labour Supply	Adopted Plan Requirement	Grower's Plan Requirement	Difference
$January_1$	362.00	322.27	279.73	+ 42.54
$January_2$	544.00	408.97	337.25	+ 71.72
$February_1$	706.00	676.35	565.25	+ 111.10
$February_2$	706.00	686.31	586.75	+ 99.56
$March_1$	795.00	677.92	563.75	+ 114.17
$March_2$	795.00	635.94	524.75	+ 111.19
$April_1$	795.00	599.49	480.43	+ 119.06
$April_2$	795.00	664.83	593.42	+ 71.41
May_1	795.00	507.51	549.60	- 42.09
May_2	633.00	632.21	513.40	+ 118.81
$June_1$	427.00	427.00	315.30	+ 111.70
$June_2$	427.00	50.91	43.02	+ 7.89
$July_1$	329.00	313.56	266.68	+ 46.88
$July_2$	329.00	212.46	191.78	+ 20.68
$August_1$	329.00	136.46	128.18	+ 8.28
$August_2$	329.00	75.28	71.12	+ 4.16
$September_1$	329.00	318.72	197.71	+ 121.01
$September_2$	329.00	175.09	145.96	+ 29.13
$October_1$	427.00	315.21	259.96	+ 55.25
$October_2$	427.00	282.59	206.50	+ 76.09
$November_1$	492.00	230.44	200.12	+ 30.32
$November_2$	492.00	273.81	193.50	+ 80.31
$December_1$	492.00	492.00	387.62	+ 104.38
$December_2$	422.00	365.87	292.64	+ 73.23
Total	12,506.00	9,481.20	7,894.42	+ 1,586.78

employed on the production and harvesting of crops, compared with only 65 percent in the grower's plan.

3.10.3 Stability of the adopted plan

Table 3.26 contains the gross margin limits for all basic real activities, as well as the gross margin below which the non-basic real activity cannot be profitably included in the basis. From these limits, the price and yield limits are derived and set out in tables 3.27 and 3.28 respectively.[24] (Neither rhubarb nor Soleil d'Ors appear in table 3.28 since yields from these crops increase from year to year. However, as prices for these activities were assumed to remain constant from year to year, the price limits could be calculated).

3.10.3.1 Stable components of the plan

Eight of the thirteen real activities in the solution can be considered stable components of the plan as far as likely price and yield deviations from the estimated levels are concerned. Of these activities, the most stable are tomato, lettuce (P4) and glasshouse cucumber. Both cucumber activities are relatively insensitive to price and yield fluctuations since prices are fixed under contract, and yields can be expected between 5.0 and 15.5 tons per acre on leased land, and above 9.6 tons per acre on freehold land. The rhubarb, Soleil d'Ors and carrot activities are insensitive to any upward price

---

24. The price limits are calculated on the assumption that yields remain constant, and the yield limits are calculated on the assumption that prices remain constant:

i.e. $$\frac{\text{gross margin limit} + \text{variable costs}}{\text{price}} = \text{yield limit}$$

and $$\frac{\text{gross margin limit} + \text{variable costs}}{\text{yield}} = \text{price limit}$$

Table 3.26 Gross Margin Stability Limits ($) - Adopted Plan

Activity	Unit	Lower Limit	Gross Margin in Plan	Upper Limit
Basic activities:				
P1 : cauliflower	1 acre	144.54	576.80	598.76
P2 : cucumber	1 acre	465.16	1361.52	1613.06
P3 : tomato	1 acre	374.66	3570.80	infinity
P4 : lettuce	1 acre	0	485.60	3296.49
P5 : pumpkin	1 acre	443.65	465.60	1361.96
P6 : rhubarb	1 acre	576.61	1247.15	infinity
P7 : Soleil d'Ors	1 acre	454.21	563.45	infinity
P9 : carrot	1 acre	506.77	743.32	infinity
P10 : pumpkin	1 acre	434.19	465.60	478.41
P11 : cucumber	1 acre	958.65	1210.19	infinity
P12 : cabbage	1 acre	166.66	418.20	454.21
P13 : cauliflower	1 acre	530.75	543.56	775.84
P14 : glasshouse cucumber	825 plants	32.85	521.55	infinity
Non-basic activity:				
P8 : lettuce	1 acre		413.60	527.21

fluctuations and it is reasonable to expect the prices of these activities to be at least $1.03 per case, $0.16 per dozen flowers and $1.11 per case respectively.

Table 3.27 Price Limits ($) - Adopted Plan

Activity	Lower Price Limit	Price in Plan	Upper Price Limit
Basic activities:			
P1 : cauliflower	0.43	1.20 per case	1.24
P2 : cucumber	41.19	109.10 per ton	128.16
P3 : tomato	0.28	1.39 per case	infinity
P4 : lettuce	0	1.06 per case	5.74
P5 : pumpkin	6.13	6.40 per sack	17.60
P6 : rhubarb	1.03	1.96 per case	infinity
P7 : Soleil d'Ors	0.16	0.21 per dozen	infinity
P9 : carrot	1.11	1.58 per case	infinity
P10 : pumpkin	6.01	6.40 per sack	6.56
P11 : cucumber	87.96	109.10 per ton	infinity
P12 : cabbage	0.38	0.80 per case	0.86
P13 : cauliflower	1.12	1.14 per case	1.55
P14 : glasshouse cucumber	0.03	0.11 per fruit	infinity
Non-basic activity:			
P8 : lettuce		0.94 per case	1.13

3.10.3.2 Unstable components of the plan

The five remaining real activities are unstable in that the price or yield deviations from those originally estimated, necessary to cause a change in the basis, are quite likely to occur.

Cauliflower (P1) and cabbage are sensitive to upward price and yield fluctuations only, since the lower limits would not normally be expected.

Pumpkin (P5) and cauliflower (P13) are sensitive only to downward price and yield movements, whereas pumpkin (P10) is the most unstable component of the plan, its present level becoming sub-optimal for small price and yield changes in either direction.

Table 3.28 Yield Limits - Adopted Plan

Activity	Lower Yield Limit	Yield in Plan	Upper Yield Limit
Basic activities:			
P1 : cauliflower	203.78	564 cases/acre	582.3
P2 : cucumber	5.0	13.2 tons/acre	15.5
P3 : tomato	580.6	2880 cases/acre	infinity
P4 : lettuce	0	600 cases/acre	3251.8
P5 : pumpkin	76.6	80 sacks/acre	220.1
P9 : carrot	354.3	504 cases/acre	infinity
P10 : pumpkin	75.1	80 sacks/acre	82.0
P11 : cucumber	9.6	11.9 tons/acre	infinity
P12 : cabbage	285.6	600 cases/acre	645.0
P13 : cauliflower	552.8	564 cases/acre	767.8
P14 : glasshouse cucumber	1.8	7 fruit/plant	infinity
Non-basic activity:			
P8 : lettuce		600 cases/acre	720.9

3.10.3.3 Crop which may become profitable

Lettuce (P8) may be profitably included in the basis should its price rise above $1.13 per case, or its yield rise above 721 cases per acre. Only the price increase, however, is likely to occur.

3.10.4 Unused resources

3.10.4.1 Land

The levels of all land disposal activities in the basis are given in table 3.29, with the level of disposal being equal to the supply of a resource less the plan's requirement for that resource. Freehold land is in disposal during every month with the exception of November and December, and at least one acre of this land may be sown with a green-crop from May until October. It would then be possible for all four acres of freehold land to be sown into greencrop once in every four years, which should be sufficient to maintain soil structure and fertility at an adequate level.

Table 3.29 Land in Disposal - Adopted Plan

Disposal Activity	Level in Plan (acres)
Freehold land:	
September - October	1.04
January - April	0.78
May - $June_1$	2.04
$June_2$ - July	1.50
August	1.04
Leased land:	
September - October	3.98
November	1.26
$December_2$	0
May	2.72
June - July	3.11
August	7.50

Note: The solution is degenerate in that even although all leased land is planted during $December_2$, any additions to the supply of this resource would be allocated to disposal since the $December_2$ leased land disposal activity is included in the basis at a zero level. See Gass, Saul I., "Linear Programming", McGraw-Hill Book Company, Inc., 1958, pp.39 and 46.

Almost four acres of leased land are left fallow during September and October, and over one acre remains fallow during November. By May, the cucumber (P2) and pumpkin (P5) crops have been harvested, leaving 2.72 acres in fallow and leased land disposal then increases during June, July and August over which time the tomato plantings are gradually removed.

3.10.4.2 Labour

The levels of all labour disposal activities in the basis are given in table 3.30, which indicates that labour is in disposal during each period with the exception of $June_1$ and $December_1$. However, February, May_2, $July_1$ and $September_1$ are periods during which the labour supply is utilised almost to capacity.

Table 3.30 Labour in Disposal - Adopted Plan

Month	Disposal Level (hours)	
	First half-month	Second half-month
January	39.73	135.03
February	29.65	19.69
March	117.08	159.06
April	195.51	130.17
May	287.49	0.79
June	-	376.09
July	15.44	116.54
August	192.54	253.72
September	10.28	153.91
October	111.79	144.41
November	261.56	218.19
December	-	56.13

3.10.4.2.1 Overhead labour

When the labour restraints were discussed in section 3.3.4, it was mentioned that the labour requirements of work of an overhead nature[25] had not been taken into account and that no provision had been made to ensure that labour was available for this work. It was also assumed that holidays would be taken during slack months.

It would have been necessary to ensure that sufficient labour was in disposal to satisfy the requirements of overhead work and holidays, had the grower considered the labour in disposal to be inadequate for these requirements. The total labour in disposal, however, amounted to 24 percent of the available supply, this being considered ample for all overhead and holiday requirements.

3.10.4.2.2 Replanting perennial crops

The input-output coefficients for labour of the two perennial crops do not include any planting requirement.[26] The labour requirements for any replanting of perennials must, then, be taken from labour in disposal.

The adopted plan requires that half an acre of rhubarb be planted since the existing bed is only half an acre in area. Also, since half the Soleil d'Ors planting is seven years old and due for replacement, one-third of an acre must be removed and replanted.

---

25. Overhead labour is required for such tasks as the repair and maintenance of machinery, buildings and glasshouse, repairing cases, cutting hedges, purchasing supplies and supervising labour.
26. See section 3.6.3. When the intertemporal programming model of Chapter 4 was being constructed, it became obvious to the author that an intertemporal model could easily have been constructed to handle the problems posed by the perennial crops of the present model.

It was estimated that 14 hours of labour would be required in November to replant half an acre of rhubarb, 70 hours would be needed during October and November to plough out and clean the seven-year-old Soleil d'Ors bulbs and a further 20 hours would be required over the second half of January to prepare the land and plant one-third of an acre with the Soleil d'Ors activity.

There would appear to be ample labour in disposal to supply these replanting requirements. (If insufficient labour should have been in disposal, the grower considered that he could still "make do", for example by working overtime, since replanting occurs only every fifth year for rhubarb and every seventh year for Soleil d'Ors.

3.10.5 Value of resources

The values imputed to the scarce resources of the adopted plan will not be discussed here, since these values are given in table 3.21.

3.11 Increased Profits and Taxation

Thus far, the impact of taxation on increased farm profits has not been considered. Although the grower adopted a plan which increased pre-tax income by 45 percent without examining the taxation implications of this increase (since the increase in tax-free profits would appear to be considerable), it would not have been unreasonable for him to enquire as to what proportion of the increased earnings will be paid in tax.

The first row of table 3.31 gives net farm profits from each plan. Since these profits are net of all tax-deductible expenditures,[27/]

27. These include all normal farm expenses such as the variable cost components of the activity gross margins. Overhead costs (such as depreciation, repairs and maintenance, and labour costs) are also deductible. (See "Farmers' Tax Guide", Inland Revenue Department, September 1966).

they represent taxable farm income. Social security and income tax payments were calculated[28] and by comparing each plan with the grower's plan, the increase in tax payments could be expressed as a percentage of the increase in pre-tax income.

For each of the seven linear programme solutions, about two-thirds of any extra profit obtained is paid out as tax and only one-third of the increased profits is retained by the grower. The present taxation system may therefore provide a reason for the non-adoption of the linear programme results, but only if a grower thought that the increase in tax-free profits was not sufficient reward for any extra labour and managerial input involved.

Even after allowing for the effects of taxation, the second critical plan which was adopted by the grower should see his tax-free farm profits increase by over 24 percent (as compared with the 45 percent increase in pre-tax profits), which means an increase in tax-free income of $1,562.

---

28. Tax calculations were made from the Inland Revenue Department's provisional tax tables accompanying the 1967 I.R.3 return. Income tax exemptions applicable are:

Personal exemption	$936
Life insurance	$120
Dependent wife	$312
Two dependent children	$312
Total income tax exemptions	$1680

Table 3.31 Impact of Taxation on Increased Profits

	Grower's Proposed Plan	LINEAR PROGRAMME SOLUTIONS: Initial Solution	First Critical Plan	Second Critical Plan	Third Critical Plan	Fourth Critical Plan	Fifth Critical Plan	Sixth Critical Plan
Net pre-tax profit ($)	10,736.86	11,368.72	13,297.54	15,543.81	15,575.01	15,970.19	15,998.95	16,021.06
Social security and income tax ($)	4,318.79	4,745.29	6,047.25	7,563.49	7,584.55	7,851.28	7,870.70	7,885.63
Tax-free profits ($)	6,418.07	6,623.43	7,250.29	7,980.32	7,990.46	8,118.91	8,128.25	8,135.43
Increase in pre-tax profits ($)		631.86	2,560.68	4,806.95	4,838.15	5,233.33	5,262.09	5,284.20
Increase in tax payments ($)		426.50	1,728.46	3,244.70	3,265.76	3,532.49	3,551.91	3,566.84
Extra tax as % extra profits		67.5	67.5	67.5	67.5	67.5	67.5	67.5
% increase in pre-tax profits		5.9	23.8	44.8	45.1	48.7	49.0	49.2
% increase in tax-free profits		3.2	13.0	24.3	24.5	26.5	26.6	26.8

3.12 Grower's Comments on the Programming Solutions

3.12.1 Physical plans

When presented with the seven linear programme solutions the grower believed each plan to be feasible, although he stated that the exclusion of Soleil d'Ors from the last three cropping programmes made these plans unattractive to him. He thought the first four solutions, though, to be 'sound and sensible' cropping programmes.

The grower was interested to see the carrot and cabbage activities included up to the limit in all plans, and appeared to place preference on plans which provided additional winter employment for his permanent staff. The inclusion of a greater acreage of the cauliflower crops in the computed plans than in the grower's proposed plan met with his approval since this would also provide additional winter work.

Although the grower had proposed to plant all 2.5 acres of cucumber on leased land, he was not concerned that only two acres of cucumber should be grown on leased land, and 0.5 acres on freehold land in each of the optimum plans.

One aspect of all the recommended plans which gave the grower a little concern was the large reduction in the acreage of the pumpkin activities from that in his own plan. He would prefer a larger acreage of this crop since it can be stored and sold during the winter months, this having the effect of evening-out his income during the year. After further discussion, however, the grower stated that this was not a really important requirement and he saw more sense in the optimum plans which provide a greater level of profits than his own plan, although income receipts may be unevenly distributed over the year.

The grower had placed a limit of 1.5 acres on the lettuce (P4) activity, but once he found this crop had been included in all plans at, or almost up to, the limit, he said that a cauliflower crop on the leased land may be a more suitable crop for the time of year, since lettuce was a very risky crop to establish and slow to make growth during the winter months. Although he was uncertain of the best course to follow, he suggested that he may reduce the lettuce (P4) crop to 0.75 acre, with a further 0.75 acre planted in spring cauliflower. (Such modifications to the computed plans are discussed further in the following sections).

3.12.2 Resource disposal levels

The grower realised the importance of the land and labour disposal information to his management. He thought the data relating to labour in disposal would be most useful in identifying those periods in which labour was likely to be in over-supply so that he could organise jobs of an overhead nature to coincide with these periods and therefore make better use of his labour supply and avoid labour "bottlenecks". The grower also believed that the labour disposal figures would be valuable when deciding on the timing of holidays, it being fortunate that August, the month during which he prefers to take his holidays, has one of the highest levels of labour disposal for each of the optimum plans.

The grower also believed that much useful information could be gained by studying the land and labour disposal tables of an optimum plan together. These tables would give him an idea as to what quantity of the various land and labour resources were left unused during each month of the year, and he could then perhaps think of further production activities which would take up some of the 'slack' in resource use. For example, he thought it may be possible to cultivate one acre of cabbage which would occupy the leased land from mid-September to mid-November, being harvested in time for the third tomato crop to be planted in $December_1$. Also, although he had stated earlier that leased land was not as suitable for winter crops as his own land, he believed that cauliflower could be grown on leased land after the tomato crops were finished, although the heavy requirement for $September_1$ labour in all plans could prove to be a limiting factor.

Although some freehold land was left in disposal in all plans, the grower decided that it would be wisest to either leave this land fallow or plant it with a greencrop. Should sufficient freehold land be in disposal during the right months, however, he could transfer some at least of the lettuce (P4) planting from the leased land to the more suitable freehold land. (This comment was made with reference to the adopted plan in which at least 1.04 acres of freehold land are unused from June until October when this lettuce activity occupies the land. The grower said he could plant one acre of lettuce in the freehold land, leaving the remaining half acre of this activity on the leased land).

New activities were not considered for further programming since the grower was quite satisfied with the production possibilities already defined and mentioned that new activities would probably only be decided upon as the season progressed. He thought it best to leave the plans with some 'slack' in resource use since a cropping programme could easily be disrupted or delayed due to unforseen circumstances such as unsuitable weather.

3.12.3 Shadow prices

The grower showed great interest in the marginal value products of the scarce resources and their interpretation. The crop maxima shadow prices of the initial solution were discussed at some length with the grower to point out the potential profit increases to be gained from relaxing some of the limits which he had earlier imposed.

His interest focussed on the three highest shadow prices, those of the tomato, rhubarb and total cucumber cropping limits. It was already clear to him that tomato was his most profitable crop but he also considered the production costs and labour requirements of this crop to be very high, so had limited the tomato activity to a maximum of five acres. He was, however, keen to see what the plans with more tomatoes 'looked like', whether or not labour did limit this expansion, and if so, how soon. Thus the parametric solution was derived.

The grower said that over the years he had gained the impression that cucumber and rhubarb were his next most profitable crops, as was indicated by their shadow prices. When questioned about the possibility of growing more cucumbers the grower replied that the processing firms were prepared to offer the existing prices for further contracts which could be obtained with no trouble at all. However, he expressed no desire to cultivate more than the present 2.5 acres of cucumbers since he considered them to be an "unpleasant crop to work with". In fact the grower said that he would always consider increasing profits by extending his tomato planting rather than the cucumber crop.

The relatively high shadow price imputed to the rhubarb cropping limit further strengthened the grower's desire to increase the acreage of this crop. However, as he had doubled the size of the rhubarb crop (from half to one acre) only recently, he planned to wait for perhaps two years for the latest planting to become established before increasing the acreage of this crop further. One advantage the grower saw in extending the rhubarb acreage was that it would provide further work during the winter months when labour disposal generally is greatest.

3.12.4 Further comments on the adopted plan

The grower expected the second critical plan, which shows an increase of $4,175 on pre-tax profits from the initial solution, to be much more sensitive to price changes than the initial plan. Comparing the gross margin limits of these plans, however, shows the second critical plan to be just as stable, if not more so, than the initial solution. Thus, it was a most rewarding discovery that profits could be increased substantially without increasing the sensitivity of the plan to unforeseen changes in prices or yields.

The labour disposal levels for the adopted plan show that it would probably not be possible to include an acre of cabbage on the leased land between mid-September and mid-November as had earlier been suggested by the grower, due to the heavy labour requirement of the tomato activity. However, he was seriously thinking of transferring part of the lettuce (P4) crop to his own land and planting a cauliflower crop on the leased land, should sufficient labour be available.

The grower believed that his present labour force would be sufficient to allow the operation of the adopted plan to flow smoothly with no labour bottlenecks. The author pointed out that such bottlenecks may occur (due, for example, to an exceptionally heavy tomato crop) since the level of labour disposal is less than 40 hours per week in eight half month periods (see table 3.30). This did not worry the grower, who stated that labour in the district was easy to obtain at the present time and he expected this situation to continue. He also mentioned that the ease of obtaining extra casual labour may lead him to actually include the early cabbage planting on the leased land, as well as a cauliflower planting on the same land following the tomatoes. He said that he would leave the cropping programme as computed, though, and would not decide on any additional crops until the time of planting was approached. This means he can see 'how the plan is actually working' and whether or not unavoidable delays or interruptions have occurred.

CHAPTER 4

INVESTMENT IN PERENNIAL CROPS - A CAPITAL BUDGETING, INTERTEMPORAL LINEAR PROGRAMME

4.1 Introduction

4.1.1 The reason for an intertemporal approach [1]

To illustrate the use of linear programming in formulating cropping programmes for process vegetable growers, contact was made with a Heretaunga Plains farmer. The farmer outlined his crop contracts for the coming season and emphasized that he had very little choice in determining his cropping programme. Nevertheless, a linear programme model was constructed and solved, the solution being very similar to the grower's plan simply because the acreage of most crops had been set under contract with the processing company.

The grower explained that he was more concerned, not with trying to alter the present season's cropping programme, but in budgeting his funds over future years to enable him to make new plantings of perennial crops, such as apples and asparagus, and to gradually phase out some of the contract vegetable crops.

It then became apparent that the farmer would obtain more benefit from a study which gave the optimum development programme, rather than

---

1. Intertemporal problems are those concerned with the allocation of resources, such as available funds, over several periods of time.

the one-season linear programme which had already been solved.

Thus an intertemporal, capital budgeting linear programme was constructed to obtain the optimum farm development programme.

A development period, or planning horizon, of six years was chosen, beginning in the Spring of 1968. Data for the model was collected during late 1967 - early 1968.

4.1.2 Brief review of intertemporal programming in farm management

Techniques commonly used to obtain solutions to capital budgeting, farm development problems include computing the present value of the expected flow of future net returns, the internal rate of return from that flow, or the minimum payback period for the relevant investment alternatives.[2/]

The use of such techniques is limited, however, since many types of agricultural investments are interdependent, or they may have multiple uses so that difficulty is experienced in determining net revenue flows. Linear programming was then employed to handle problems such as interdependence between, and multiple uses of, investment activities. Such applications were generally designed to maximise the present value of future incomes over some planning period.[3/ 4/] Candler[5/] then suggested that it would be equivalent but simpler to build a model which would maximise income at the end of the planning period,

---

2. Gunn, H.J. and Hardaker, J.B., "Long-term Farm Planning - A Re-examination of Principles and Methods", Journal of Agricultural Economics, vol.18, pp.271-278, 1967.
3. Loftsgard, Laurel D., and Heady, Earl O., "Application of Dynamic Programming Models for Optimum Farm and Home Plans", Journal of Farm Economics, vol.41, pp.51-62, 1959.
4. Dean, Gerald W., and De Benedictis, Michele, "A Model of Economic Development for Peasant Farms in Southern Italy", Journal of Farm Economics, vol.46, pp.295-312, 1964.
5. Candler, Wilfred, "Reflections on 'Dynamic Programming Models'", Journal of Farm Economics, vol.42, pp.920-926, 1960.

including the return from an investment activity which would use surplus cash. Such a model would therefore reduce the chance of logical or transcription errors occuring, since it would not be necessary to discount all future costs and revenues to the beginning of the development period.

A model which maximised the net worth of the farm business over a planning period, rather than the present value of some expected income stream, was proposed by Cocks [6] and his model was developed as a special case of the Hicksian discounting model of multi-period optimisation.

The intertemporal model of the present chapter is based upon that proposed by Candler [7] in which the farm firm's overall objective function to be maximised is a weighted sum of net tax-free cash available to the firm at the end of the planning period and the value of assets owned by the firm at the end of the planning period. [8]

Present day attempts to make intertemporal programming models more closely related to real-life situations include the treatment of indivisibilities in investment opportunities as integer programming problems, [9] [10] and the incorporation of price and cost variability into a quadratic capital budgeting programme. [11]

---

6. Cocks, K.D., "Capital Accumulation and Hicksian Models", Farm Economist, vol.10, pp.458-465, 1965.
7. Personal communication.
8. Two further components of a firm's objective function suggested by Candler are the maximisation of dividends paid to shareholders and the minimisation of the probability of financial collapse of the firm during the planning period.
9. Weingartner, Martin H., "Mathematical Programming and the Analysis of Capital Budgeting Problems", Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1963.
10. Colyer, Dale, "A Capital Budgeting, Mixed Integer, Temporal Programming Model", Canadian Journal of Agricultural Economics, vol.16, pp.1-7, 1968.
11. Candler, personal communication.

4.1.3 An outline of the intertemporal programming matrix

The basic matrix for the intertemporal linear programme is similar to a block-diagonal matrix, with resources and activities repeated for each year of the development period.

A schematic representation of the matrix is given in table 4.1. The sub-matrices a to f are somewhat similar, although not identical, and give the matrix its block-diagonal appearance. Activities of one year may also require resources in later years. For example, sub-matrix g_1 contains the requirements of first-year activities for resources in the second year. That is, an apple activity, say, planted in year 1, will require resources and supply cash in later years. Likewise, sub-matrix j contains the requirements of activities initiated in year 2 for resources in year 3.

The prices (c_j values) attached to all activities are zero with the exception of the final cash and final assets activities, which have prices (or weights) of λ_1 and λ_2 respectively.

Sub-matrices a, g and h are presented in Appendix A.3, tables A.6, A.7 and A.8, and construction of the matrix in general is discussed in subsequent sections.

4.2 Description of the Holding

4.2.1 Location and size

The holding is situated on the Heretaunga Plains, between Napier and Hastings. It comprises four separate properties with a total area of 192.5 acres.

Table 4.1 Schematic Representation of the Basic Matrix

	c_j	0...					...0	λ_1	λ_2
Restraints	B	Year 1 Activities	Year 2 Activities	Year 3 Activities	Year 4 Activities	Year 5 Activities	Year 6 Activities	Final Cash	Final Assets
Year 1		a							
Year 2		g_1	b						
Year 3		g_2	j	c					
Year 4		g_3			d				
Year 5		h_1				e			
Year 6		h_2					f		
Final cash								+1	
Final assets									+1

4.2.2 Present cropping practice

The contracts obtained by the grower over the past two years are given in table 4.2. It can be noted that the contract for a given crop can vary from one year to the next, and the grower mentioned that the processing company could also change the contracts at any time during the year. Thus, a plan computed at the beginning of a year may require modification if some contracts are altered.

Table 4.2 Process Crop Contracts

Crop	Contracts 1966/67	Contracts 1967/68
Carrot	50 tons	60 tons
Potato	70 tons	70 tons
Asparagus	24 acres	24 acres
Beetroot	7 acres	7 acres
Broad bean	6 acres	5 acres
Green bean	42 acres	27 acres
Pea	40 acres	19 acres
Peach	25 acres	25 acres
Tomato	24 acres	21 acres

In some years, it has been possible to double-crop peas and green beans in the same land. During 1966/67, two acres of the green bean contract were required by the processing company to be an early crop, but it was possible to sow the remaining 40 acres of green beans in the land occupied by peas, once the latter crop had been harvested.[12]

12. Double-cropping means that two crops are grown in the same land in one year, which, in accordance with the definition of rotations adopted in this chapter, can be referred to as a one-year rotation. Thus a two-year rotation means that one year must separate plantings of a given crop in the same land, a three-year rotation means that two years must separate plantings of a given crop in the same land, and so on.

In 1967/68, however, the entire green bean contract was for an early crop so that it was not possible to double-crop the pea land with green beans. Also, the grower has been informed by the processing company that the double-cropping of peas and green beans will not be possible during 1968/69 (the first year of the planning period).

Good crop husbandry requires that a greencrop, comprising a rye-corn-ryegrass mixture, be sown following the harvesting of beetroot, green beans, peas and tomatoes. This greencrop is grazed with hoggets over the winter months.

As well as the crops grown under contract to the processing company, two crops for the fresh market and ryegrass for seed production have been grown during the past two years. In 1966/67, these included five acres of mangolds, six acres of ryegrass (for seed production) which is also winter-grazed with hoggets, and 17 acres of kumaras. (Six acres of the kumara crop were double-cropped with the broad bean crop, with kumaras following broad beans in the same land). During the 1967/68 season, five acres of mangolds and 24 acres of kumaras were grown, with five acres of the kumara crop double-cropped with broad beans.

New plantings of asparagus and apples have been made over recent years and at present these comprise four acres of asparagus in its third year (which is in addition to the 24 acres of established asparagus described in section 4.2.3) and 30 acres of apples in their sixth year. These recent plantings will hereafter be referred to as the young asparagus and young apple crops.

4.2.3 Future cropping practice

The grower intends to increase his asparagus planting to about 40 acres and sees no difficulty in expanding the contract. The existing

24 acres of old asparagus are in their 27th year and are continually being removed and replaced with new plantings.

Concerning the tree crops, the grower intends to retain the peach planting at 25 acres or perhaps reduce it slightly. He believes apples to be potentially the most profitable of all his crops and intends to expand the planting to a maximum of 60 acres.

4.2.3.1 Treatment of the young apple and young asparagus crops in the model

These crops have only recently been planted and will not be removed over the six years of the development period. Hence it was not necessary to include these crops as activities in the programming model since they will remain at a constant level over the planning period.

In order to correctly state the resource (for example, cash and labour) requirements and total revenue of any solution to the problem, however, it was necessary to include the resource requirements and revenue contributions of these crops in the programming computations.[13]

4.2.4 Intercropping

Five crops, carrot, potato, beetroot, kumara and mangold, may be intercropped amongst one- or two-year-old apple plantings. A greencrop would not be sown to follow intercropped beetroot, however.

4.2.5 Soil types

The soil type on two of the four properties is a clay loam. This land has a fairly high water table and flooding may occur, the most recent flooding being three years ago. The soil type of the two remaining

---

13. See sections 4.3.1, 4.3.2 and 4.3.6.3.

properties is a well-drained sandy loam.

4.2.6 Irrigation

Irrigation of all crops is necessary during the summer months, especially those crops grown on the sandy loam soils.

4.2.7 Disease problems

One disease which is causing the grower some concern is Sclerotinia and the incidence of this disease in tomato crops has been on the increase each year. Both tomatoes and green beans were infected during the past season. The grower expects problems in attempting to control Sclerotinia by means of crop rotation since peas, green beans and tomatoes (which are all hosts to the disease) form his largest contracts and insufficient land would be available to rotate these crops with the smaller acreages of beetroot, carrot, broad beans and mangolds which appear to be resistant to Sclerotinia attack.

Thus one reason why the grower intends to replace some of his annual vegetable crops with asparagus and apples is to help avoid future problems with this disease.

4.3 The Restraints[14/]

4.3.1 Land

The total area of the property is 192.5 acres of which 3.0 acres

14. The following notation for restraints, activities, resource supplies and input-output coefficients will be used throughout the present chapter: $R_{i,k}$ = the i^{th} restraint in year k; $k = 1,\ldots,6$;
$P_{j,k}$ = the j^{th} activity in year k; $k = 1,\ldots,6$;
$b_{i,k}$ = the supply of the i^{th} resource in year k; $k = 1,\ldots,6$; and
$a_{ij,k}$ = the per unit requirement of the j^{th} activity for the i^{th} restraint in year k; $k = 1,\ldots,6$.

is taken up by buildings and wasteland and 34.0 acres are occupied by the young apple and asparagus plantings, leaving a total of 155.5 acres available to the cropping activities of the intertemporal programme. However, 24.0 acres of this land are occupied by old asparagus and 25.0 acres are occupied by peaches, so only 106.5 acres are available for either annual crops or new plantings of perennial crops at the beginning of the development period.

The land restraints are given in table 4.3.

Table 4.3 Land Restraints

Restraint			Resource Supply (acres)	
			(k = 1)	(k = 2,...,6)
$R_{2,k}$	:	peach land	25.0	0
$R_{3,k}$	:	asparagus land	24.0	0
$R_{4,k}$	:	annual cropland	106.5	0
$R_{5,k}$	:	perennial cropland	0	0
$R_{6,k}$	:	intercropped land	0	0
$R_{7,k}$	:	cropland transfer control	106.5	106.5

The peach land and asparagus land restraints ensure that the already-established peach and old asparagus crops can be, at most, 25.0 and 24.0 acres in area respectively during each year of the planning period. These crops may be removed, however, and the land transferred to other uses.

The annual cropland restraint supplies land (106.5 acres at the beginning of the development period) to annual crops, a fallow activity, and an annual cropland transfer activity which supplies land to perennial crops.

The perennial cropland restraint makes land available to new plantings of apples and asparagus, the initial supply being zero since land is transferred to this restraint by way of an annual cropland transfer activity.[15]

The intercropped land restraint allows new plantings of apples to be intercropped with some annual crops and has a zero initial supply since this land is supplied by the apple activities.

The cropland transfer control restraint was necessary to ensure that no more than the 106.5 acres of annual cropland was transferred to perennial crops over the development period.[16]

All land restraints, with the exception of the cropland transfer control which has a resource supply of 106.5 acres in each of the six years, have a resource supply of zero in each of years two to six ($k = 2,\ldots,6$) of the development period, since land is transferred from one year to the next via cropping or land transfer activities.

4.3.2 Labour

To keep the number of restraints included in the model down to manageable proportions, the annual labour supply has been divided into four three-month periods, as follows:

Spring	:	September - November
Summer	:	December - February

---

15. See section 4.4.6.1.
16. However, more than 106.5 acres may be transferred to perennials since the established peach and asparagus plantings may be dug out. This land then must be cropped for two consecutive years before it can be replanted with perennials. The inclusion of this requirement in the matrix will be explained in section 4.5.2.

Autumn : March - May, and

Winter : June - August.

The permanent [17] labour force comprises five men, each of whom works 48 hours per week in September, 55 per week from October until the end of April, 50 per week during the first half of May and 44 per week for the remainder of May until August. The owner's time has not been included in the labour supply as he is fully occupied in organising and managing the enterprise. Twelve women are employed from October until the first half of May but only three women are employed for the remainder of the year, all working 30 hours per week.

Thus the total supply of permanent labour during each period in each of the six years of the development period is:

Spring : 6934 hours,

Summer : 8256 hours,

Autumn : 7498 hours,

Winter : 4032 hours.

The total number of hours spent on farm and machinery maintenance over the past season was obtained from the grower's records. This information was considered by the grower to be an acceptable estimate of the labour requirements of such maintenance work for each year of the development period. The available supply of labour in each season was therefore calculated by deducting the requirements of maintenance work,

---

17. Labour has been divided into three categories in the model, permanent, contract and hired labour. Contract labour (paid, for example, on a per-acre or per-case-harvested basis) is included as a variable cost for the activity concerned, and hired labour activities are included in the model so that labour in excess of the permanent and contract supply may be hired if its marginal value product exceeds the going wage rate.

as well as the labour requirements of the young apple and asparagus plantings, from the total supply.

Table 4.4 gives the labour required by maintenance work and the young apple and asparagus crops, and table 4.5 gives the labour restraints and their supplies.

4.3.3 Crops grown under contract for processing

Since acceptance of a contract requires the grower to crop a given acreage, it was necessary to decide whether the process crops should be treated as being grown under a definite contract (in which case the restraints would be equalities), whether no cropping limit should be placed on them, or whether the acreage cropped may be less than, but not greater than, some pre-determined level.

It was not possible to define the size of the contracts in future years since these may be changed by the processing company during a season and are almost certain (for some crops at least) to vary from year to year. Also, as apple and asparagus plantings are made in future years, the grower would have to accept contracts smaller than those of the past (1967/68) season since less land would be available for contract cropping. Furthermore, although the grower saw no difficulty in obtaining contracts smaller than those of the past season, he thought it would be very difficult to obtain contracts greater than these.

It was decided, therefore, to set an upper limit on the size of the contracts, with such limits equal to the contracts of the 1967/68 year. That is, the grower believed it was realistic to assume that in each year of the development period he could grow less than, but not more than, the acreages of process crops as defined by the 1967/68 contracts. (An exception is asparagus, the contract for which could be increased to a maximum of 40 acres).

Table 4.4 Labour Requirements of Maintenance Work and the Young Apple and Asparagus Crops

Year (k)	Maintenance				Young Apples (30 acres)				Young Asparagus (4 acres)			
	Spring	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter
1	249	249	249	318	465	4128	2259	2616	8	8	3	3
2	249	249	249	318	495	5115	3336	3204	242	59	3	3
3	249	249	249	318	495	4602	4413	3789	409	102	3	3
4	249	249	249	318	495	5589	5487	4497	542	136	3	3
5	249	249	249	318	495	7107	7641	5088	591	149	3	3
6	249	249	249	318	495	8091	8721	5214	641	162	3	3

Table 4.5 Labour Restraints

Restraint			Resource Supply (hours)					
			k = 1	k = 2	k = 3	k = 4	k = 5	k = 6
$R_{8,k}$	:	Spring labour	6212	5948	5781	5648	5599	5549
$R_{9,k}$	:	Summer labour	3871	2833	3303	2282	751	-246
$R_{10,k}$	:	Autumn labour	4987	3910	2833	1759	-395	-1475
$R_{11,k}$	:	Winter labour	1095	507	-78	-786	-1377	-1503

Note: Negative resource supplies simply indicate that the requirements of maintenance work and the young apple and asparagus plantings will exceed the present permanent labour supply, so additional labour must be hired.

The process crop restraints are given in table 4.6.

Table 4.6 Process Crop Restraints

Restraint			Resource Supply (k = 1,...,6)
$R_{12,k}$	:	beetroot	7 acres
$R_{13,k}$	:	potato	70 tons
$R_{14,k}$	:	asparagus	36 acres
$R_{15,k}$	:	carrot	60 tons
$R_{16,k}$	:	tomato	21 acres
$R_{17,k}$	:	green bean	27 acres
$R_{18,k}$	:	pea	19 acres
$R_{19,k}$	:	broad bean	5 acres

Note: Although the maximum contract acreage of asparagus is 40 acres, the effective asparagus cropping restraint in table 4.6 is 36 acres since the young asparagus (four acres) is not included in the model as an activity. (See section 4.2.3.1).

4.3.4 Restrictions on fresh market crops

The grower placed upper limits on the acreage of mangolds, kumara, ryegrass sown for seed production, and apples to be planted in any one year, as well as a limit to the total apple plantings over the six-year period.

Should he grow more than eight acres of mangolds or 24 acres of kumaras, the grower expected marketing problems to arise, with extra quantities only being sold at prices considered unacceptable by the grower. Also, the grower considered that if he were to expand the production of kumaras, he would have to purchase bulk-handling equipment (which he did not wish to do) in order to lower production costs and remain competitive with other suppliers.

Since the grower is prepared to handle up to 60 acres of apples, only 30 acres may be planted over the six-year period as a similar acreage has already been planted. In addition, the grower did not want to plant more than 10 acres of apples in any one year (even if sufficient cash should be available) and considered this a precaution against future uncertainties (for example flooding, or an unforeseen fall in apple prices).

Table 4.7 gives the restrictions on the acreage of fresh market crops.

4.3.5 Rotation restraints

For reasons of soil fertility and disease control, the processing company requires that a certain number of years elapse between plantings of some crops in the same land. A five-year rotation is required for both tomatoes and potatoes (that is, four years must separate plantings of tomatoes and/or potatoes on the same land), a four-year rotation is required for carrots, and plantings of peas must satisfy a three-year rotation as must green bean plantings. Therefore the maximum combined acreage of tomatoes and potatoes must not exceed one-fifth of the available cropland, carrot plantings must not exceed one-quarter, peas must not exceed one-third, and green bean plantings must not exceed one-third of the available cropland. Since the processing company is not permitting double-cropping between peas and green beans during the first year of the development period, it has been assumed that such double-cropping is not possible during any future year.

Table 4.7 Fresh Market Crop Restraints

Restraint			Resource Supply
$R_{20,k}$	:	kumara	24 acres (k = 1,...,6)
$R_{21,k}$	:	mangold	8 acres (k = 1,...,6)
$R_{22,k}$	:	ryegrass	12 acres (k = 1,...,6)
$R_{23,1}$	:	apple plantings in year 1	10 acres
$R_{23,2}$	:	apple plantings in year 2	10 acres
$R_{23,3}$	:	apple plantings in year 3	10 acres
$R_{23,4}$	:	apple plantings in year 4	10 acres
$R_{23,5}$	:	apple plantings in year 5	10 acres
$R_{23,6}$	:	apple plantings in year 6	10 acres
$R_{39,6}$	:	total apples planted at end of year 6	30 acres

Table 4.8 contains the rotation restraints. The resource supply at the beginning of the planning period is 106.5 acres (this being the initial supply of annual cropland), but is zero for every other year since cropland is transferred to these restraints via other activities.[18]

18. See section 4.5.6 for a discussion of the coefficients which make these rotation restraints effective.

Table 4.8 Rotation Restraints

Restraint			Resource Supply	
			(k = 1)	(k = 2,...,6)
$R_{24,k}$	:	tomato & potato	106.5 acres	0
$R_{25,k}$	:	carrot	106.5 acres	0
$R_{26,k}$	:	pea	106.5 acres	0
$R_{27,k}$	:	green bean	106.5 acres	0

4.3.6 Cash and taxation restraints

4.3.6.1 Treatment of cash flows in the model

Before discussing this group of restraints, the incorporation into the model of the various components of the cash flow between years will be presented.

The supply of tax-free cash available to the grower at the beginning of the development period (net of his personal drawings) forms an upper limit to total variable production costs of that year.

All cash from the sale of produce is assumed to be received at the end of the year, and such (pre-tax) cash receipts will include the gross revenue from the activities in the model, the net revenue from those crops which are assumed to remain at a constant level throughout the development period, and interest earned on any cash deposited in the bank. (Such bank deposits will equal the supply of tax-free cash at the beginning of the year less total variable costs of that year).

Next, taxation payments must be calculated on assessable farm income, where the latter equals (pre-tax) cash receipts less total variable costs and tax-deductible overhead costs. Then, assessable farm income less taxation payments gives net tax-free farm income.

Finally, to obtain the supply of tax-free cash available at the beginning of the second year, an amount equal to :

cash saved over the first year, plus

total variable costs of the first year,[19] plus

depreciation allowances for the first year,[20] less

personal drawings for the second year,

must be added to net tax-free farm income.

This process continues throughout the six years of the development period, with the supply of tax-free cash available at the end of year six (that is, the beginning of year seven), entering the objective function to be maximised along with the value of assets at the end of year six.[21]

Thus in order to understand the derivation of the resource supplies for the cash and taxation restraints, reference must be made to tax-free cash available at the beginning of the six-year period, net revenue from the young apple and asparagus crops, withdrawals of tax-free cash, tax-deductible overhead costs, total tax-deductible expenditures and taxation limits.

4.3.6.2 Tax-free cash available at the beginning of the development period

The grower estimated that he would have access to $27,000 tax-free cash (over and above his personal cash requirements of the first

---

19. Total variable costs must be added back into the supply of tax-free cash since such costs have been deducted twice, once when the cash payments were made at the beginning of the year, and again when taxation payments were calculated at the end of the year.
20. Although tax-deductible overhead costs were subtracted from pre-tax cash receipts when assessable income was calculated, only the cash overhead payments need be withdrawn from the cash flow, and depreciation allowances are the only tax-deductible overhead costs which do not require payment in cash.
21. Section 4.1.2 outlines the nature of the objective function in intertemporal programming models, and that of the present model is discussed in more detail in section 4.6.

year) at the beginning of the planning period.[22] This entire sum would be available, then, to finance the running expenses of the holding, or to invest in perennial crops. Of this sum $12,000 was in the form of a bank overdraft, but was treated as a permanent bank loan in the model, with the remainder made up of accumulated savings.

The model was not designed to allow repayment of the loan over the development period and interest charges on the entire amount borrowed have been included in overhead costs (table 4.9). However, Bank activities allow excess cash to be saved and thus earn interest, so that in any year when the amount saved is less than $12,000, net interest payments are made on the difference between the total amount borrowed and the amount of cash banked. (Such an annual interest calculation approximates the real situation, since interest charges on a bank overdraft are calculated on a daily balance). When the amount of cash banked exceeds $12,000, interest is received on the net amount of cash saved, that is, total savings less total borrowings.

4.3.6.3 Net revenue from the young apple and asparagus crops

To obtain total pre-tax cash receipts in any year, the net revenue

---

22. Since all cash payments are assumed made at the beginning of the year, the entire $27,000 must be treated as being available at that time. In practice though, cash is paid out and received continually. This problem may be lessened by allowing cash transactions to be made several times during a year, which is similar to deciding whether any other resource (e.g. labour) is to be divided into several (monthly labour) resources, or aggregated into a single (annual labour) resource.

(gross revenue less variable costs) from these crops[23] must be added into the programming matrix. This was achieved by setting the supply of the 'pre-tax cash receipts - end of year k' restraint equal to the net revenue from the young apple and asparagus plantings in year k, for k = 1,...,6.

4.3.6.4 Withdrawals of tax-free cash

At the beginning of each of the second to sixth years, allowance was made in the model for the grower to withdraw an amount from tax-free cash:

(i) sufficient to meet his personal requirements over the coming year (personal drawings), and

(ii) sufficient to allow him to meet the cash overhead payments of the past season.[24]

The grower (a single man) stated that $2,500 tax-free cash would be adequate to meet his personal requirements in each year of the planning period. In addition, he required $20,000 to be available at the beginning of the fifth year, to be invested in an apple packing shed

---

23. See Appendix A.2, where the net revenue from apples and asparagus is presented in detail. Pre-tax net revenue from these crops will equal gross revenue less variable costs only, since their labour requirements and labour costs have already been included in the model. (Labour costs have been accounted for since the labour requirements have been deducted from the appropriate supply of permanent labour (section 4.3.2), and the costs of the total permanent labour supply are included in the overhead costs (table 4.9). Should these crops require more than the supply of some labour resource, the resource supply will become negative (as in table 4.5) and labour must be hired to make any solution feasible).
24. Depreciation allowances were not included in the cash requirements of overhead costs, as explained in section 4.3.6.1 and footnote 20.

which he considered would be necessary at that time to handle the requirements of his present 30 acre planting of apples.[25] The shed would be designed, though, to have adequate capacity to handle up to the maximum 60 acres of apples.

The supply of each tax-free cash resource for year's two to six was thus equal to the negative value of the sum of all withdrawals for the year in question (with such withdrawals including the cash required for the packing shed at the beginning of the fifth year).

4.3.6.5 Tax-deductible overhead costs

For simplicity, the present overhead costs of the holding were assumed to remain constant for each year of the planning period. However, allowance was made for depreciation on the packing shed which is to be built in the fifth year.[26]

The tax-deductible overhead costs for each of the six years are given in table 4.9.

Table 4.9 Tax-Deductible Overhead Costs

Overhead Item	Overhead Cost
Wages of permanent staff (k = 1,...,6)	$19,688
Repairs, maintenance and sundry (k = 1,...,6)	$8,702
Lease and interest charges (k = 1,...,6)	$8,326
Depreciation (k = 1,...,4)	$2,310
Depreciation (k = 5)	$6,110
Depreciation (k = 6)	$3,850

25. The cost of the packing shed includes $10,000 for the building itself and $10,000 for equipment (for example a grader and bulk-handling equipment).
26. This included 20% special depreciation on the shed to be claimed in year 5; special depreciation on plant, spread over five years; ordinary depreciation of 2% (constant value) on the shed; and 10% ordinary depreciation (diminishing value) on plant.

Therefore, the resource supply of the 'tax deductions' restraint in any year is the sum of the tax-deductible overhead costs of that year, from table 4.9.

4.3.6.6 Total tax-deductible expenditures

These will include the variable costs of the enterprise as well as the tax-deductible overhead costs of the above section,[27] and their incorporation into the matrix will be described in section 4.5.7.1.

4.3.6.7 Taxation limits

To allow the programme to closely approximate actual progressive taxation, marginal tax rates were assumed constant over some range of taxable income, then rising to a higher constant tax rate for some higher range of taxable income, and so on. For example, in accordance with present tax legislation the first $208 of income is exempt from all taxation, as is the first $60 of interest earned (interest is earned in the model through Bank activities) giving a total of $268 free of tax. (The assumption is made that at least $60 in interest is received each year; that is, the farmer is assumed to always have a credit bank balance of at least $1,000). Also, $718 of assessable income will pay only Social Security Tax, this sum including the $936 personal exemption and an estimate of $50 for Life Insurance, less the $268 which is exempt from all tax.

In all, ten marginal tax rates were calculated to approximate, as closely as possible, actual tax payments.[28]

---

27. "Farmers' Tax Guide", Inland Revenue Department, September 1966.
28. More detailed discussion of the treatment of taxation in the matrix is to be found in section 4.5.7., where the tax input-output coefficients are described.

Table 4.10 Cash and Taxation Restraints

Restraint		Resource Supply ($)					
		k = 1	k = 2	k = 3	k = 4	k = 5	k = 6
$R_{1,k}$	tax-free cash, beginning year k	27000	-39216	-39216	-39216	-59216	-39216
$R_{28,k}$	pre-tax cash receipts, end of year k	8748.28	14554.10	20620.94	26704.12	38977.22	44939.18
$R_{29,k}$	tax deductions, year k	39026	39026	39026	39026	42826	40566
$R_{30,k}$	tax limit 1:k	268	268	268	268	268	268
$R_{31,k}$	tax limit 2:k	718	718	718	718	718	718
$R_{32,k}$	tax limit 3:k	1000	1000	1000	1000	1000	1000
$R_{33,k}$	tax limit 4:k	800	800	800	800	800	800
$R_{34,k}$	tax limit 5:k	1000	1000	1000	1000	1000	1000
$R_{35,k}$	tax limit 6:k	1000	1000	1000	1000	1000	1000
$R_{36,k}$	tax limit 7:k	1000	1000	1000	1000	1000	1000
$R_{37,k}$	tax limit 8:k	1200	1200	1200	1200	1200	1200
$R_{38,k}$	tax limit 9:k	1200	1200	1200	1200	1200	1200

4.3.6.8 Summary of cash and taxation restraints

All cash and taxation restraints and resource supplies included in the model are presented in table 4.10.

4.3.7 Objective function accounting restraints

The final two restraints in the model, given in table 4.11, account for all tax-free cash at the end of the planning period and the value of all assets (that is, perennial crops planted over the planning period) at the end of the planning period. Final tax-free cash and final assets may then enter the objective function via final tax-free cash and final assets activities.[29]

The final tax-free cash accounting restraint ($R_{40,6}$) has a supply equal to the overhead costs of the sixth year which must be paid out of tax-free cash, but does not include the owner's personal drawings for the year *following* the end of the planning period.

Table 4.11 Objective Function Accounting Restraints

Restraint	Resource Supply
$R_{40,6}$: final tax-free cash	- \$36,716
$R_{41,6}$: final assets	0

4.4 The Activities

4.4.1 Introduction

All activities included in the intertemporal programming matrix can be divided into, and will be discussed under, the following nine

29. The objective function is discussed in detail in section 4.6.

groupings:

(i) intercropped annual crops,

(ii) annual crops,

(iii) existing perennial plantings,

(iv) new plantings of perennials,

(v) land transfer activities,

(vi) activities to hire labour,

(vii) taxation and tax-deductible expenditures,

(viii) bank activities, and

(ix) final tax-free cash and final assets.

4.4.2 Intercropping activities

The five annual crops which may be planted amongst new apple plantings are beetroot, carrot, kumara, mangold and potato. Husbandry of these crops is identical to that of the corresponding annual activities of section 4.4.3, except that a greencrop is not sown following intercropped beetroot.

The five intercropped activities are:

$P_{1,k}$; $k = 1,\ldots,6$: intercropped beetroot,

$P_{2,k}$; $k = 1,\ldots,6$: intercropped carrot,

$P_{3,k}$; $k = 1,\ldots,6$: intercropped kumara,

$P_{4,k}$; $k = 1,\ldots,6$: intercropped mangold, and

$P_{5,k}$; $k = 1,\ldots,6$: intercropped potato.

4.4.3 Annual cropping activities

4.4.3.1 Tomato ($P_{6,k}$; $k = 1,\ldots,6$)

The tomato activities consist of one planting made during September.

Contract labour is employed to harvest the crop and once completed (during March) the land is sown into greencrop for winter grazing by hoggets.

A five- or six-year rotation is required and applies to tomatoes and/or potatoes.

4.4.3.2 Green bean ($P_{7,k}$; k = 1,...,6)

These activities may comprise three to four successive crops, sown over the October - December period.[30] The processing company harvests the beans and transports them to the factory, after which the land will be sown into greencrop during April. The processing company requires that a three-year rotation be practised.

4.4.3.3 Beetroot ($P_{8,k}$; k = 1,...,6)

Beetroot is sown during September and October and consists of from two to four crops. Sowing dates must be a week to 10 days apart to allow crop-thinning (a labour-intensive operation) to be performed without the occurrence of labour bottlenecks. The crop is harvested by hand, after which the land is sown into greencrop during March.

4.4.3.4 Potato ($P_{9,k}$; k = 1,...,6)

These activities comprise one planting only, and must follow the same rotation requirement as the tomato activities.

---

30. Sowing dates for process crops may be known only about ten days in advance. If a contract consists of more than one crop, the processing company will indicate what proportion of the contract is to be sown on a certain date. This applies also to harvesting in that the company will indicate when it wants a crop harvested, so as to maintain a constant flow of vegetables into the factory.

4.4.3.5 Pea ($P_{10,k}$; $k = 1,\ldots,6$)

Up to three pea crops may be sown anytime from August until October as required by the processing company. The company harvests the crop between December and February and transports the peas to the factory. As with green beans, a three-year rotation is necessary.

4.4.3.6 Carrot ($P_{11,k}$; $k = 1,\ldots,6$)

The carrot activities consist of a single crop sown in October. The carrots are harvested when required by the processing company, which may be anytime between May and September. Carrots would normally be planted in land previously occupied by greencrop, and a four-year rotation is required.

4.4.3.7 Broad bean - kumara ($P_{12,k}$; $k = 1,\ldots,6$)

These activities are one-year rotations, with kumaras being planted after the harvesting of broad beans. The broad bean crop consists of a single sowing in either May or June. The beans are harvested during October and November and followed within a week with kumaras, which in turn are harvested the following May, placed in storage, and bagged and sold from June until early August.

4.4.3.8 Kumara ($P_{13,k}$; $k = 1,\ldots,6$)

These activities are planted during September, earlier than the kumara crop which follows broad beans. Harvesting is completed by the end of May and the kumaras are then stored, to be bagged and sold from June until early August.

4.4.3.9 Mangold ($P_{14,k}$; k = 1,...,6)

Mangold crops are sown from September and lifted in July. The crop is then left on the paddock to be collected by the purchasers.

4.4.3.10 Ryegrass ($P_{15,k}$; k = 1,...,6)

Ryegrass is sown by contractor, and is grazed with hoggets (eight sheep per acre) from June until October, when the paddocks are closed for seed production. Mowing and threshing of the crop is also carried out by a contractor.

4.4.4 Existing perennial plantings

4.4.4.1 Old asparagus ($P_{16,k}$; k = 1,...,6)

The inclusion of these activities allows the model to determine the optimum replacement pattern for the established ('old') asparagus beds, which are in their 27th year at the beginning of the development period. The grower is at present planning to replace some of these plantings each year, and in fact the optimum age at which to replace the asparagus is at the end of its 28th year. (See section 4.5.8.2.3).

4.4.4.2 Old peach ($P_{17,k}$; k = 1,...,6)

The old peach activities were included in the model since the grower wanted to know whether or not he should replace some or all of the peach planting with some other crop/s.

The existing 25 acres of peaches are not due for replacement during the six-year planning period, since the trees are at present in their seventh year compared with an optimum replacement time of the end of their 18th year (section 4.5.8.2.3). Also, no provision is made in the model for new peach plantings since the grower is satisfied with the present acreage.

4.4.5 New plantings of perennials

4.4.5.1 Asparagus ($P_{18,k}$; k = 1,...,6)

New asparagus plantings may be made in any year, as long as the total planting of both 'new' (activity $P_{18,k}$) and 'old' (activity $P_{16,k}$) asparagus does not exceed the cropping limit of 36 acres. Generally, the first harvest is three years after planting when 0.5 tons per acre may be obtained. Yields then increase annually to over 2.0 tons per acre which should be maintained until about 15 years from planting, after which time yields may gradually fall off.

4.4.5.2 Apple ($P_{19,k}$; k = 1,...,6)

The grower intends to plant only apples on Malling-Merton 106 rootstock (that is, semi-dwarf trees), trained to a single-leader system. The trees are planted intensively, with an average of 200 trees per acre. Although the grower could name a number of varieties to plant (especially Granny Smith, Red Dougherty and Hawkes Bay Red Delicious) it was practicable to include only one apple activity in each year.[31]

Although little data is available on the cropping habits of semi-dwarf apples under New Zealand growing conditions, it is reasonable to assume that a yield of 100 bushels per acre will be obtained three years after planting, increasing to 2000 bushels per acre after 12 years. Apart from biennial-bearing fluctuations, such a yield should be maintained until the trees are about 35 years of age, beyond which yields would probably commence to decrease.

---

31. It is conceptually possible, however, to include an activity for each apple variety, each of which may have different yields, prices and/or labour requirements.

4.4.6 Land transfer activities

4.4.6.1 Annual cropland transfer ($P_{20,k}$; k = 1,...,6)

These activities are included to provide land for new perennial plantings, by transferring annual cropland to perennial cropland.

4.4.6.2 Asparagus land transfer ($P_{21,k}$; k = 1,...,6)

Since the old asparagus plantings may be removed during the planning period, these activities are designed to transfer any vacated asparagus land to annual cropland, as this land must be cropped for at least two years before being replanted with perennial crops.

4.4.6.3 Peach land transfer ($P_{22,k}$; k = 1,...,6)

This group of activities serves a similar purpose as those of the preceding section except that they allow vacated peach land to be transferred to annual cropland.

4.4.6.4 Fallow ($P_{23,k}$; k = 1,...,5)

These activities ensure that any land which may be left fallow in any year will be available for cropping the following year. A fallow activity was not required for year six since any land left fallow in that year would be represented by a land disposal activity appearing in the basis.

4.4.7 Activities to hire labour

Four such activities are included in the model for each year of the planning period, their purpose being to allow additional labour to be hired as necessary.

The hired labour activities are:

$P_{24,k}$; k = 1,...,6 : hire Spring labour

$P_{25,k}$; k = 1,...,6 : hire Summer labour

$P_{26,k}$; k = 1,...,6 : hire Autumn labour

$P_{27,k}$; k = 1,...,6 : hire Winter labour.

4.4.8 Taxation and tax-deductible expenditures

4.4.8.1 Taxation transfer activities

Ten taxation transfer activities are included for each year, allowing the computation of progressive taxation payments to be closely approximated with 10 different marginal tax rates. For example, when only Social Security Tax is being paid (a marginal tax rate of 0.075) the appropriate taxation transfer activity would take \$1 (up to a certain limit) from assessable income at the end of one year, and deliver \$0.925 into the supply of tax-free cash at the beginning of the next year.

The tax transfer activities are:

$P_{28,k}$; k = 1,...,6 : tax transfer 1:k ;

$P_{29,k}$; k = 1,...,6 : tax transfer 2:k ;

.

.

.

$P_{37,k}$; k = 1,...,6 : tax transfer 10:k.

4.4.8.2 Tax-deductions transfer ($P_{38,k}$; k = 1,...,6)

Since all tax-deductible expenditures must be subtracted from pre-tax cash receipts before taxation is calculated, the tax-deductions transfer activities remove from pre-tax cash receipts in one year, a sum of cash equal to the tax-deductible expenditures of that year, and place a similar quantity of cash into the supply of tax-free cash at the beginning of the following year. (Cash overhead payments and personal drawings are then deducted from the supply of tax-free cash, as explained in sections 4.3.6.4 and 4.5.7.1).

4.4.9 Bank activities ($P_{39,k}$; k = 1,...,6)

These represent investment activities which will allow surplus cash to earn interest at six percent. Hence the programme can either save money at a six percent rate of return or reinvest money in the holding at a rate of return greater than six percent.

4.4.10 Final tax-free cash ($P_{40,6}$) and final assets ($P_{41,6}$)

These activities appear only once in the matrix and allow the supply of the final tax-free cash and final assets restraints to enter the objective function, so allowing the programme to maximise the value of tax-free cash and assets at the end of the planning period.

4.5 Input-Output Coefficients

4.5.1 Introduction

This section will discuss the input-output coefficients of the basic matrix corresponding to each group of restraints as presented in section 4.3. To make the presentation as clear as possible, it will be necessary to present condensed sections of the basic matrix.

4.5.2 Land restraints

The input-output coefficients associated with the various land restraints are indicated in table 4.12. (All activities and restraints in this table are measured in units of one acre). Only 'year one' activities need be shown, since 'year two' activities (for example) will have similar resource requirements in years two, three,... etc. as the 'year one' activities have for years one, two,... etc.

All intercropped activities have a +1 coefficient for intercropped land for the year in which they are planted. The annual cropping activities and the fallow activity have a +1 coefficient in the annual cropland row for the appropriate year and also have a -1 coefficient for annual cropland in the following year, since these activities supply this land to the cropping and fallow activities of the next year.

The old asparagus activity of the first year has a +1 coefficient for the asparagus land of year one, and a -1 coefficient in the asparagus land row for the following year thus making asparagus land available in that year. Likewise, each unit of the old peach activity (not shown in table 4.12) will require one acre of peach land in one year and supply one acre of peach land in the following year.

Each unit of the apple activity of year one requires one acre of perennial cropland in that year, but supplies three-quarters of an acre for intercropping over two years, (the remaining quarter-acre being occupied by the trees). All land to be planted with perennials must be supplied through the annual cropland transfer activities. Thus this transfer activity in the first year can increase the supply of perennial cropland in that year, but also reduces by a similar amount (through the +1 coefficient in the cropland transfer control row of all future years) the quantity of annual cropland which remains for possible transfer to

Table 4.12 — Land : Input-Output Coefficients

Restraints	Supply		First-year Activities						Second-year Activities	
			Intercrop Activities P_1 to P_5	Annual Crop and Fallow Activities P_6 to P_{15}, P_{23}	Old Asparagus P_{16}	Apple P_{19}	Annual Cropland Transfer P_{20}	Asparagus Land Transfer P_{21}	Intercrop Activities P_1 to P_5	etc ...
First year:										
R_3 : asparagus land	24.0	≥			+1			+1		
R_4 : annual cropland	106.5	≥		+1			+1	-1		
R_5 : perennial cropland	0	≥				+1	-1			
R_6 : intercropping land	0	≥	+1			-0.75				
R_7 : cropland transfer control	106.5	≥					+1			
Second year:										
R_3 : asparagus land	0	≥			-1					
R_4 : annual cropland	0	≥		-1						
R_5 : perennial cropland	0	≥								
R_6 : intercropping land	0	≥				-0.75			+1	
R_7 : cropland transfer control	106.5	≥					+1			
Third to sixth years:										
R_7 : cropland transfer control	106.5	≥					+1	-1		

perennial crops.

The 'year one' asparagus land transfer activity reduces the supply of asparagus land in that year by transferring it to annual cropland. However, since annual cropland must be cropped for at least two years before it may be replanted in perennials, each unit of the 'year one' asparagus land transfer activity will add to the supply of the cropland transfer control restraints only from the third year. The peach land transfer activity serves a similar function, and is not included in table 4.12.

4.5.3 Labour restraints

The grower's records for 1966/67 included the nature of operations performed on all crops, the timing of these operations and the quantity of labour involved. The labour requirements per acre of each annual cropping activity were derived from these records, being adjusted where necessary to compensate for any difference between that season's yields and those estimated for the planning period. Because of the uncertainty surrounding planting and harvesting dates of some process crops (and therefore all intervening operations), the timing of operations for these crops was assumed to be the same as in 1966/67.

Although the grower could also provide estimates of the labour requirements of asparagus and peaches, his lack of experience with semi-dwarf intensive apple plantings meant that labour requirements for each year of this crop's life had to be estimated after discussion with orchardists and fruit research workers in the district.[32/]

---

32. The labour input-output coefficients covered such operations as land preparation (prior to planting as well as subsequent cultivations), planting, sowing a greencrop, applying fertiliser, pruning, spraying, thinning, harvesting and packing, and making cases or cartons.

Labour would also be required if the established asparagus and peach plantings were to be removed, so each asparagus land transfer activity requires autumn labour and each peach land transfer requires labour during the winter months.

The labour-hire activities (which are measured in 10-hour units) may <u>supply</u> labour in each time period and thus have coefficients of -10 in the appropriate labour resource rows.

Table 4.13 summarises the labour input-output coefficients of the intertemporal programme.

4.5.4 Process crop restraints

Process crops have a requirement coefficient of +1 for the supplies of the appropriate process crop restraints where these are defined in acres. For example, the 'asparagus limit' restraint for any one year will restrict the total acreage of new asparagus plantings in that year, any remaining old asparagus, plus all plantings of new asparagus in the preceding years of the planning period, to a maximum of 36 acres. Hence the 'asparagus limit' restraint of year six is given by:

$$36 \text{ acres}(b_{14,6}) \geqslant \sum_{k=1}^{6} 1.0x_{18,k} + 1.0x_{16,6} \quad ; \qquad (4\text{-}1)$$

where $x_{18,k}$ are the production levels (acres) of activities $P_{18,k}$ (new asparagus plantings) ; and

$x_{16,6}$ is the level of the old asparagus activity (acres) in year six.

Carrot and potato crops are usually measured in tons rather than acres, and the extent to which an acre of these activities will reduce the supply of the respective cropping limit restraints will equal the crop yield in tons per acre.

Table 4.13 — Labour : Input-Output Coefficients $(k = 1,\ldots,6)$

Restraint	Supply (hours)		Activities Requiring Labour $P_{1,k}$ to $P_{19,k}$ $P_{21,k}$; $P_{22,k}$	Hire Spring Labour $P_{24,k}$	Hire Summer Labour $P_{25,k}$	Hire Autumn Labour $P_{26,k}$	Hire Winter Labour $P_{27,k}$
$R_{8,k}$: Spring labour	$b_{8,k}$	$\geq$	$+a_{8j,k}$	-10			
$R_{9,k}$: Summer labour	$b_{9,k}$	$\geq$	$+a_{9j,k}$		-10		
$R_{10,k}$: Autumn labour	$b_{10,k}$	$\geq$	$+a_{10j,k}$			-10	
$R_{11,k}$: Winter labour	$b_{11,k}$	$\geq$	$+a_{11j,k}$				-10

4.5.5 Fresh market crop restraints

All fresh market crops have an input-output coefficient of +1 for the appropriate cropping limit restraints. For example apple plantings, as well as being restricted to no more than 10 acres in any year, must not exceed 30 acres over the entire planning period. This condition is ensured by:

$$10 \text{ acres}(b_{23,k}) \geqslant 1.0x_{19,k} \quad ; \quad k = 1,\ldots,6 \; ; \quad \text{and}$$

$$30 \text{ acres}(b_{39,6}) \geqslant \sum_{k=1}^{6} 1.0x_{19,k} \; ; \qquad (4\text{-}2)$$

where $x_{19,k}$ are the production levels (acres) of the apple activities in each of the k years.

4.5.6 Rotation restraints

The input-output coefficients found in the rotation restraints are set out in table 4.14, in which all activities are measured in one-acre units.

The supplies of the rotation restraints are initially equal to 106.5 acres <u>plus</u> the acreage of land transferred to annual cropping via the asparagus and peach land transfer activities <u>less</u> the quantity of land transferred to perennials through the annual cropland transfer activity. Then to conform to (for example) the rotational requirement that all tomato and potato plantings in any year be no more than one-fifth of the total area of annual cropland, a coefficient of +5 is required by the tomato and both potato activities in the 'tomato-potato rotation limit' row. Similarly, no more than one-quarter of annual cropland may be planted in carrots, no more than one-third planted in pea crops and no more than one-third sown with green beans.

<u>All</u> annual cropping and fallow activities in any year will <u>supply</u>

Table 4.14 Rotation Restraints : Input-Output Coefficients

Restraints	Supply		First-year Activities											Second-year Activities...	
			Intercrop Carrot P_2	Intercrop Potato P_5	Tomato P_6	Green Bean P_7	Potato P_9	Pea P_{10}	Carrot P_{11}	Other Annual Crop and Fallow Activities P_{12} to P_{15} P_8, P_{23}	Apple P_{19}	Annual Cropland Transfer P_{20}	Peach Land Transfer P_{22}	Intercrop Carrot P_2	etc
First year:															
R_{24} : tomato-potato rotation	106.5	$\geq$		+5	+5		+5				-0.75	+1	-1		
R_{25} : carrot rotation	106.5	$\geq$	+4						+4		-0.75	+1	-1		
R_{26} : pea rotation	106.5	$\geq$						+3				+1	-1		
R_{27} : green bean rotation	106.5	$\geq$				+3						+1	-1		
Second year:															
R_{24} : tomato-potato rotation	0	$\geq$			-1	-1	-1	-1	-1	-1	-0.75				
R_{25} : carrot rotation	0	$\geq$			-1	-1	-1	-1	-1	-1	-0.75			+4	
R_{26} : pea rotation	0	$\geq$			-1	-1	-1	-1	-1	-1					
R_{27} : green bean rotation	0	$\geq$			-1	-1	-1	-1	-1	-1					
Third year:															
etc															
.															
.															
.															

land to the rotation restraints of the following year through negative coefficients.

Since each acre of the apple activities planted contributes three-quarters of an acre for intercropping over two years, this land must be added to the total land available for carrots and potatoes, since both of these activities may also be intercropped.[33]

4.5.7 Cash and taxation restraints

4.5.7.1 General description of coefficients

Table 4.15 describes the inclusion of the cash and taxation input-output coefficients in the programming model. Activities contributing to the cash flow (such as annual and perennial crops) are measured in units of one acre, while all other activities and restraints of table 4.15 are measured in one-dollar units. Only first-year activities and restraints are shown since the matrix construction is similar for all years of the planning period.

All activities requiring tax-free cash (for example, for payment of variable costs) will have a positive coefficient in the tax-free cash row, and perennial activities will also have coefficients (in the same column) in the tax-free cash rows of future years. These activities will supply cash (hence the negative coefficient) to pre-tax cash receipts, which is then transferred through the tax transfer activities to the tax-free cash row of the following year.[34]

The bank activity can transfer tax-free cash from one year to the next, as well as paying a six percent rate of return into pre-tax cash

---

33. Strictly, the intercropped land added to the tomato-potato rotation restraint can only be used by potatoes. However, the author felt that such an approximation was preferable to increasing the number of restraints in the model.
34. All activity prices, yields, gross revenues and variable costs are presented in Appendix A.

Table 4.15 Cash and Taxation Restraints : Input-Output Coefficients

Restraints	Supply		First-year Activities														Second-year Activities ...	
			Activities Contributing to Cash Flow P_1 to P_{19}	Tax Transfer Activities										Tax Deductions Transfer	Bank		Activities Contributing to Cash Flow P_1 to P_{19}	etc ...
				1:1 P_{28}	2:1 P_{29}	3:1 P_{30}	4:1 P_{31}	5:1 P_{32}	6:1 P_{33}	7:1 P_{34}	8:1 P_{35}	9:1 P_{36}	10:1 P_{37}	P_{38}	P_{39}			
First year:																		
R_1 : tax-free cash	27000	$\geq$	$+a_{1j,1}$												+1			
R_{28} : pre-tax cash receipts	8748.28	$\geq$	$-a_{28j,1}$	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	-0.06			
R_{29} : tax deductions	39026	$\geq$	$-a_{29j,1} = -a_{1j,1}$											+1				
R_{30} : tax limit 1:1	268	$\geq$		+1														
R_{31} : tax limit 2:1	718	$\geq$			+1													
R_{32} : tax limit 3:1	1000	$\geq$				+1												
R_{33} : tax limit 4:1	800	$\geq$					+1											
R_{34} : tax limit 5:1	1000	$\geq$						+1										
R_{35} : tax limit 6:1	1000	$\geq$							+1									
R_{36} : tax limit 7:1	1000	$\geq$								+1								
R_{37} : tax limit 8:1	1200	$\geq$									+1							
R_{38} : tax limit 9:1	1200	$\geq$										+1						
Second year:																		
R_1 : tax-free cash	-39216	$\geq$	$(+a_{1j,2})$	-1	-0.925	-0.775	-0.712	-0.637	-0.575	-0.513	-0.443	-0.368	-0.325	-1	-1		$+a_{1j,2}$	
R_{28} : pre-tax cash receipts	14554.10	$\geq$	$(-a_{28j,2})$														$-a_{28j,2}$	
etc																		
.																		
.																		
.																		

receipts (with the first $60 of interest free of tax, as explained in section 4.3.6.7).

The tax-deduction transfer activity will transfer to tax-free cash in year two an amount equal to the total tax-deductible expenditures of the first year, which are equal to tax-deductible overhead costs ($39,026 in the first year) plus the variable costs of the first-year activities.

From the supply of tax-free cash at the beginning of year two, $39,216 must be withdrawn to cover the cash overhead payments of year one and the grower's personal drawings for year two, to obtain the total quantity of tax-free cash available to the second-year activities.

4.5.7.2 Derivation of marginal tax rates[35]

The first $268 of income, comprising the $208 special exemption and the first $60 of interest receipts, are free of both Income and Social Security Tax.

A total of $986, being the $936 personal exemption and $50 life insurance, (the farm owner is a single man), is exempt from Income Tax, and since the first $268 of income is free of all tax, Social Security Tax only is paid on $986 - $268, or the next $718 of income.[36]

The next $1000 of income pays Income Tax[37] at the rate of $0.15 in

---

35. These were calculated with reference to "Farming as a Business - Farm Taxation", Farm Economics Section, Department of Agriculture.

36. The author mistakenly included the entire $986 in the model as paying Social Security Tax only, instead of the correct value of $718. He did not consider the small error this would cause to tax calculations as justifying the considerable computational effort which would be required to resolve the intertemporal programme.

37. A rebate was not deducted from tax payments, since rebates may vary from year to year and would have a negligible effect on the development programme since total rebates cannot exceed $200 per year.

the dollar, and Social Security Tax at the rate of $0.075 in the dollar, giving a total marginal tax rate of $0.225 per dollar. Therefore the taxation transfer activity paying tax at this rate would deliver $0.775 into tax-free cash for each dollar taxed.

Table 4.16 gives the marginal tax rates of which the above is an example, plus the rates at which the taxation transfer activities deliver cash to the tax-free cash rows.

Table 4.16 Marginal Tax Rates

Assessable Income (pre-tax cash receipts less all tax-deductible expenditures)	Income Tax on Each Dollar	Social Security Tax on Each Dollar	Total Marginal Tax Rate on Each Dollar	Proportion of Each Dollar Remaining After Tax
Exemptions: $268	free of all	tax	-	1.000
$718	-	0.075	0.075	0.925
Taxable balance:				
$1 - $1000	0.150	0.075	0.225	0.775
$1001 - $1800	0.213	0.075	0.288	0.712
$1801 - $2800	0.288	0.075	0.363	0.637
$2801 - $3800	0.350	0.075	0.425	0.575
$3801 - $4800	0.412	0.075	0.487	0.513
$4801 - $6000	0.482	0.075	0.557	0.443
$6001 - $7200	0.557	0.075	0.632	0.368
Over $7200	0.600	0.075	0.675	0.325

4.5.8 Objective function accounting restraints

4.5.8.1 General description of coefficients

Table 4.17 gives a summarised account of the coefficients to be found in the final tax-free cash and final assets rows of the matrix.

Table 4.17 Objective Function Accounting Restraints : Input-Output Coefficients

Restraints	Supply		Asparagus Year 1 $P_{18,1}$	...	Asparagus Year 6 $P_{18,6}$	Apple Year 1 $P_{19,1}$	...	Apple Year 6 $P_{19,6}$	Old Asparagus Year 6 $P_{16,6}$	Old Peach Year 6 $P_{17,6}$	Tax Transfer 1:6 $P_{28,6}$	...	Tax Transfer 10:6 $P_{37,6}$	Tax Deductions Transfer Year 6 $P_{38,6}$	Bank Year 6 $P_{39,6}$	Final Tax-Free Cash $P_{40,6}$	Final Assets $P_{41,6}$
$R_{40,6}$: final tax-free cash	-36716	$\geq$									-1	...	-0.325	-1	-1	+1	
$R_{41,6}$: final assets	0	$\geq$	-4126	...	-3389	-15524	...	-11246	-3123	-4944							+1

All activities are measured in units of one dollar except for the perennial activities which are defined in one-acre units.

The tax transfer, tax-deductions transfer and bank activities of year six (the final year of the planning period) all provide cash to the final tax-free cash row. After the withdrawal of an amount sufficient to pay the year six cash overhead payments ($36,716), all tax-free cash available at the end of the planning period is transferred to the objective function via the final tax-free cash activity.

All perennial crops (apple, asparagus and peach) in existence on the holding at the end of the planning period are assumed to have a value (as assets) which reflects their future income-generating ability. The negative of such asset values are entered as coefficients in the final assets row and the total value of assets (perennial crops only) at the end of the planning period is transferred into the objective function through the final assets activity.

4.5.8.2 Derivation of asset values

4.5.8.2.1 Introduction

Either a positive or normative approach can be adopted when assets are to be valued. In the former approach, all assets would be valued in accordance with their present market value. The latter approach would assign assets a value equal to the present value of the future income stream from the asset.

Because of the difficulty in differentiating between the present market value of crops of different ages, a normative approach to asset valuation was adopted.

4.5.8.2.2 The procedure adopted

The perennial crops in existence at the end of the planning period were assigned an asset value equal to the present value of future net revenue discounted from infinity, with crop replacement at the optimum time. Hence the next step was to determine the optimum age at which to replace each of the three perennial crops.

4.5.8.2.3 Optimum replacement times[38]

The optimum time at which to replace a perennial crop is when the annual net revenue (marginal net revenue) from the present crop becomes equal to the amortized present value (or annuity value) of the net revenue from the following crop. Equivalently, the optimum replacement age will be given by the year in which the annuity value of net revenue obtained from the crop reaches a maximum.

Net revenue in any year from a perennial crop is equal to:

$$NR_n = Y_n - a_{n-1}i - b_n - c_n , \qquad (4\text{-}3)$$

where Y_n = gross revenue ($ per acre) in year n,

$a_{n-1}i$ = the interest to be compounded on any unpaid balance of the establishing cost at the beginning of year n,

b_n = the annual costs ($ per acre) in year n, and

c_1 = the planting cost ($ per acre) applicable in the first year only.

---

38. Faris, J. Edwin, "Analytical Techniques Used in Determining the Optimum Replacement Pattern", Journal of Farm Economics, vol.42, pp.755-766, 1960.

The amortized present value ($ per acre) in any year n will be given by:

$$\text{Amortized PV}_n = \left[\sum_{k=1}^{n} \frac{Y_k - a_{k-1}i - b_k - c_k}{(1+r)^k}\right]\left[\frac{r(1+r)^n}{(1+r)^n - 1}\right], \quad (4\text{-}4)$$

where $a_o = 0$;

and the year in which the amortized present value of net revenue reaches a maximum can be found by inspection. Since the 'mathematical optimum' replacement age need not occur at the end of a year, amortized present value will reach a maximum when this value is most nearly equal to the annual net revenue for that year.

The optimum replacement times were determined as described above, using the net revenue data of Appendix A.2 and an interest rate of six percent.

The optimum time to replace the perennial crops, assuming that price, cost and yield assumptions hold good in future years, are:

(i) semi-dwarf apple : at the end of the forty-ninth year;

(ii) peach : at the end of the eighteenth year; and

(iii) asparagus : at the end of the twenty-eighth year.

Such replacement times appear to be consistent with those observed in practice (assuming good crop husbandry). Semi-dwarf apple trees may be expected to be replaced somewhere between the 40^{th} and 50^{th} years, whereas peach trees require replacement much sooner due to the onset of diseases such as Silver Leaf. Asparagus crops could be replaced anytime between the 15^{th} and 30^{th} years, depending inter alia on the soil type. (The asparagus crops of the present chapter are on a soil type considered excellent for asparagus and a replacement age of 28 years is not unreasonable).

4.5.8.2.4 Calculation of asset values

Asset values, assumed to be equal to the present value of future net revenues discounted from infinity, with crop replacement at the optimum time, are derived from:

$$AV_{jk} = \frac{R_{m+1}}{(1+r)} + \frac{R_{m+2}}{(1+r)^2} + \dots + \frac{R_{m+n}}{(1+r)^n} + \frac{\frac{A}{r}}{(1+r)^{n+1}} , \qquad (4\text{-}5)$$

where AV_{jk} = asset value (\$ per acre) of activity j planted in year k,

R = net revenue (\$ per acre) from the crop in a particular year,

m = age in years of the crop at the end of the planning period,

$m+n$ = optimum replacement age, in years,

A = the maximum amortized present value (\$ per acre) of the crop, and

$\frac{A}{r}$ = the capitalised value of net revenue from future plantings of the crop at the end of year m+n.

All asset values, included in table 4.18, were calculated using the net revenue data of Appendix A.2 and an interest rate of six percent.

Table 4.18 Asset Values ($ per acre) of Perennial Crops

Year of Planting	Perennial Activity			
(k)	Old Asparagus $P_{16,6}$	Old Peach $P_{17,6}$	Asparagus $P_{18,k}$	Apple $P_{19,k}$
1			4,126	15,524
2			4,112	14,676
3			4,032	13,723
4			3,834	12,829
5			3,605	11,985
6			3,389	11,246
'Old' plantings	3,123	4,944		

Note: Since the old asparagus is in its 27th year at the beginning of the planning period, it would be 32 years old at the end of the six years (providing it had not been completely replaced beforehand) compared with an optimum replacement age of 28 years. The asset value of this activity was therefore set equal to the capitalized value of the maximum annuity.

4.6 The Objective Function

Although the objectives of firms may be many and varied, it is reasonable to assume that the following two conflicting objectives are important components of the farm firm's overall objective function:

(i) maximisation of tax-free cash available to the firm at the end of the planning period, and

(ii) maximisation of the value of assets owned by the firm at the end of the planning period.

(It should be remembered that the model also allows for withdrawals of tax-free cash as required by the grower in each year).

Hence, the objective function of the intertemporal linear programme includes only two non-zero entries, and is represented schematically in table 4.19.

Table 4.19 The Objective Function

Restraints	$c_j \rightarrow$		0 ... 0	λ_1	λ_2
	B		All Other Activities $P_{1,k}$ ... $P_{39,k}$	Final Tax-Free Cash $P_{40,6}$	Final Assets $P_{41,6}$
$R_{40,6}$: final tax-free cash	-$36,716	$\geq$	$(-a_{ij,k})$	+1	
$R_{41,6}$: final assets	0	$\geq$	$(-a_{ij,k})$		+1

Notes: 1. Only activities $P_{28,6}$ to $P_{39,6}$ will have negative coefficients in the final tax-free cash row.
2. Only the perennial activities of table 4.18 will have negative coefficients in the final assets row.

The objective may be stated as:

$$\text{maximise } Z = \lambda_1 x_{40,6} + \lambda_2 x_{41,6} , \qquad (4\text{-}6)$$

where $x_{40,6}$ is the level ($) of activity $P_{40,6}$ (final tax-free cash), and

$x_{41,6}$ is the level ($) of activity $P_{41,6}$ (final assets).

If the grower is unable to specify unique values of λ_1 and λ_2, a whole series of "efficient" capital budgeting programmes may be generated by varying the values of these parameters.[39]

4.7 The Basic Matrix

The structure of the initial linear programming matrix was outlined in section 4.1.3. Since the matrix includes a total of 231 restraints (rows) and 235 real activities (columns) it is not practicable to present the entire matrix in the thesis. However, parts of the matrix (as indicated in section 4.1.3) are presented in Appendix A, section A.3.

4.8 The Solution

4.8.1 A parametric solution

Parametric techniques were employed to obtain a series of "efficient" capital budgeting programmes by assigning the final tax-free cash activity a constant price of $1.000 and reducing the price of the final assets activity from an initial value of $2.000.[40] The solution to the initial programme showed that the basis would remain unchanged until the final assets price fell below $0.447. Therefore to obtain the second "efficient" capital budgeting programme the final assets price was set at $0.446, and so on. The farm manager considered that all tax-free cash to final assets relative prices of interest would be covered when the final assets price

---

39. In fact, a parametric solution to the problem was obtained (see section 4.8.1) by setting $\lambda_1 = 1$ for all problems and varying the value of λ_2 (the price of assets relative to tax-free cash).
40. A price of $2.000 for the final assets activity indicates that the farm operator values assets twice as much as tax-free cash or considers $1 of assets to be equivalent to $2 tax-free cash.

had been reduced to approximately one-third of the tax-free cash price, and computations were therefore terminated at that point. The parametric solutions obtained vary only in the rate at which the old asparagus is replaced with new plantings, and in the value of the objective function.

Table 4.20 Price Limits ($) of the Final Assets Activity

Plan	Price of Final Assets Activity	Lower Price Limit	Limiting Variable	Upper Price Limit	Limiting Variable
1	2.000	0.447	Asparagus land $transfer_3$	2.150	Asparagus land $transfer_1$
2	0.446	0.369	Annual cropland $transfer_5$	0.447	Hire Spring $labour_5$
3	0.368	0.357	Annual cropland $transfer_6$	0.369	Asparagus land $transfer_3$
4	0.356	0.346	$Asparagus_4$	0.357	Spring labour $disposal_4$
5	0.345	0.339	Asparagus land $transfer_3$	0.346	$Asparagus_6$

Note: Subscripts attached to activities denote the year: e.g. $asparagus_4$ is the asparagus activity of year four.

Table 4.20 gives the upper and lower limits of the final assets price,[41] plus the activities which would enter the basis should these limits be exceeded. For example, when the price of the final assets activity is reduced from $0.447 to $0.446, the asparagus land transfer activity of year 3 will enter the basis and Plan 2 of the parametric solution is obtained.

41. See Chapter 2, section 2.1.11.1.

Table 4.21 Crop Activity Levels (acres) Which Vary in the Parametric Solution

Activity	Plan 1	Plan 2	Plan 3	Plan 4	Plan 5
$P_{16,2}$: old asparagus$_2$	-	7.0	6.8	14.8	23.1
$P_{18,2}$: asparagus$_2$	24.0	17.0	17.2	9.2	0.9
$P_{16,3}$: old asparagus$_3$	-	-	6.8	14.8	23.1
$P_{18,3}$: asparagus$_3$	-	7.0	-	-	-
$P_{16,4}$: old asparagus$_4$	-	-	6.8	14.8	15.0
$P_{18,4}$: asparagus$_4$	-	-	-	-	8.1
$P_{16,5}$: old asparagus$_5$	-	-	-	4.7	9.3
$P_{18,5}$: asparagus$_5$	-	-	6.8	10.1	5.7
$P_{16,6}$: old asparagus$_6$	-	-	-	-	9.3
$P_{18,6}$: asparagus$_6$	-	-	-	4.7	-

Note: These activity levels will remain constant for all prices of the final assets activity between the upper and lower limits for the appropriate plan (as given in table 4.20). For example, 24.0 acres of asparagus will be planted in the second year, for any price of the final assets activity between \$0.447 and \$2.150.

When presented with the five plans the farm manager considered Plan 4 to be the most suitable, with asparagus replacement continuing until the sixth year. The following sections will therefore discuss only this plan in detail, and the level of activities in the other four plans can be found by reference to both Plan 4 (section 4.8.2) and table 4.21 (which presents the levels of only those activities which differ from one plan to another).

Table 4.22 contains the value of the objective function for each of the five parametric plans. The value of tax-free cash available at the end of the planning period increases, and the value of final assets decreases, as the price of the final assets activity relative to that of the final tax-free cash activity is reduced.

Table 4.22 Values of the Objective Function in the Parametric Solution

Plan	Price of Final Assets Relative to Final Cash ($)	Components of Objective Function		
		Value of Final Tax-Free Cash ($)	Value of Final Assets ($)	Total ($)
1	2.000	77,812	1,422,060	1,499,872
2	0.446	78,064	316,868	394,932
3	0.368	79,131	260,388	339,519
4	0.356	80,949	250,085	331,034
5	0.345	82,957	240,357	323,314

Note: It should be remembered that 'assets' as used in the present model refer only to plantings of perennial crops, and do not include the value of fixed assets such as land, buildings and machinery, or assets acquired according to a pre-determined pattern, such as the apple packing shed.

The value of final assets is a weighted value, and will only equal the present value of future income from perennial crops when this weight is equal to unity. Thus for Plan 1, the weight (or price) attached to the final assets activity is $2.000, so that the present value (at the end of the planning period) of future income from perennial crops will be $711,030. The corresponding value for Plan 5 is $696,687, or the value of the final assets activity divided by the 'price' of this activity.

4.8.2 Activity levels:Plan 4

4.8.2.1 Intercropped and annual cropping activities

Table 4.23 gives the levels of all intercropped and annual cropping activities in the optimum solution for each year of the planning period.

Table 4.23 Intercropped and Annual Cropping Activities (acres) - Plan 4

Activity	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
$P_{1,k}$: intercrop beetroot	7.0	7.0	7.0	7.0	-	-
$P_{3,k}$: intercrop kumara	-	-	8.0	-	-	-
$P_{4,k}$: intercrop mangold	0.5	8.0	-	0.5	-	-
$P_{6,k}$: tomato	18.4	17.9	15.9	14.4	12.9	12.9
$P_{7,k}$: green bean	27.0	24.8	21.0	15.0	9.0	9.0
$P_{8,k}$: beetroot	-	-	-	-	7.0	7.0
$P_{10,k}$: pea	8.5	4.2	-	-	-	-
$P_{11,k}$: carrot	3.6	3.6	3.6	3.6	3.6	3.6
$P_{12,k}$: broad bean - kumara	-	5.0	5.0	5.0	5.0	5.0
$P_{13,k}$: kumara	19.5	19.0	11.0	19.0	19.0	19.0
$P_{14,k}$: mangold	7.5	-	8.0	7.5	8.0	8.0

Note: All tomato (P6), green bean (P7), beetroot (P8) and pea (P10) crops are followed with a greencrop grazed by hoggets.

The beetroot, mangold and carrot crops are at the maximum level in each of the six years, with part or all of the beetroot and mangold crops being intercropped amongst young apple plantings in some years.

The broad bean - kumara activity, although omitted from the first year's cropping programme, is at the maximum level of five acres in all succeeding years and total kumara plantings are at the maximum of 24 acres in each of the latter five years. The acreages of tomatoes and green beans are reduced each year until the fifth when they become stable at 12.9 and 9.0 acres respectively. The pea activity is included for the first two years only, whilst the potato and ryegrass activities have been excluded from the cropping programmes of all years.

4.8.2.2 Perennial crops and land transfer activities

Table 4.24 contains the acreages of new perennial crops to be planted, that of the existing perennial crops,[42] and the level of the land transfer activities in each year of the planning period.

Table 4.24 Perennial Crops and Land Transfer Activities (acres) - Plan 4

Activity	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
$P_{16,k}$: old asparagus	24.0	14.8	14.8	14.8	4.7	-
$P_{17,k}$: old peach	25.0	25.0	25.0	25.0	25.0	25.0
$P_{18,k}$: asparagus	12.0	9.2	-	-	10.1	4.7
$P_{19,k}$: apple	10.0	10.0	10.0	-	-	-
$P_{20,k}$: annual cropland transfer	22.0	19.2	10.0	-	10.1	4.7
$P_{21,k}$: asparagus land transfer	-	9.2	-	-	10.1	4.7

In the first year 10.0 acres of apples and 12.0 acres of asparagus are planted, which requires 22.0 acres of land to be transferred to the

42. These include four acres of young asparagus and 30 acres of young apples, (see section 4.2.2) in addition to the old peach and old asparagus activities.

perennial cropland resource by the annual cropland transfer activity. In the second year 9.2 acres of the old asparagus activity are dug out and this land is transferred by the asparagus land transfer activity to the year two supply of annual cropland. New plantings of perennials made in the second year are asparagus (9.2 acres) and apples (10.0 acres), which requires 19.2 acres of land to be transferred to perennial cropland by the year two annual cropland transfer activity.

A further 10.0 acres of apples are planted in the third year, reflecting the high potential profit from semi-intensive apple plantings. (The 30.0 acres are planted in the shortest time possible, even though each dollar of future income from this activity, discounted to the end of the planning period, is given a value of only 35.6 cents in the objective function).[43]

In the fifth year, 10.1 acres of the old asparagus activity are removed and a similar acreage of asparagus is planted. Finally, the remaining 4.7 acres of old asparagus are removed and replaced during the sixth year. Thus all of the old asparagus activity has been removed during the planning period, with the maximum asparagus area of 36.0 acres present in each year.

4.8.2.3 Activities to hire labour

The requirements of the optimum programme for labour are such that labour must be hired to supplement the permanent staff in each season of the development period, with the exceptions of the Spring seasons of year's one to five. Table 4.25 indicates the number of hours of labour

---

43. That is, the price of the final assets activity is $0.356, compared with a price of $1.000 for the final tax-free cash activity.

which must be hired at different stages of the planning period.

Table 4.25 Labour to be Hired (hours) - Plan 4

Activity	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
$P_{24,k}$: hire Spring labour	-	-	-	-	-	398
$P_{25,k}$: hire Summer labour	3,676	5,101	4,849	6,479	8,605	10,781
$P_{26,k}$: hire Autumn labour	757	2,845	3,941	5,100	7,398	9,031
$P_{27,k}$: hire Winter labour	4,392	5,887	6,881	7,375	8,695	9,553

Generally, seasonal labour requirements increase from one year to the next. The greatest increase over the six-year period is for labour in Autumn, with one extra man per week required during the Autumn of year one (assuming a work rate of 50 hours per week), increasing to 14 extra men per week in the sixth year. This is mainly for fruit harvesting, the labour requirements of which rise from year to year as production increases. There are also considerable increases in the requirements for labour during Summer and Winter due to fruit crop thinning, fruit harvesting and tree pruning, the labour requirements of which increase as the trees approach full production.

4.8.2.4 Cash flows and taxation

The cash flows of the optimum programme are set out, year by year, in table 4.26. The tax-free cash available at the beginning of a year is used to meet the variable costs of the cropping programme for that year, with any balance invested (by way of the Bank activities) at a six percent rate of return. Tax-deductible expenditures (which comprise both variable

Table 4.26 Cash Flows and Taxation ($) - Plan 4

Item	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Tax-free cash at beginning of year	27,000	33,530	41,974	52,757	45,218	61,548
Variable costs	27,000	31,792	31,873	33,620	39,136	44,823
Cash banked	-	1,738	10,101	19,137	6,082	16,725
Pre-tax cash receipts	79,087	89,767	97,045	103,956	113,487	125,619
Tax-deductible expenditures:						
Variable costs	27,000	31,792	31,873	33,620	39,136	44,823
Overhead costs	39,026	39,026	39,026	39,026	42,826	40,566
Total deductions	66,026	70,818	70,899	72,646	81,962	85,389
Assessable farm income	13,061	18,949	26,146	31,310	31,525	40,230
Tax payments	6,341	10,315	15,173	18,659	18,805	24,679
Tax-free cash receipts	72,746	79,452	81,872	85,297	94,682	100,940
Less total cash withdrawals	39,216	39,216	39,216	59,216	39,216	36,716
Plus cash in bank	-	1,738	10,101	19,137	6,082	16,725
Gives tax-free cash available at beginning of next year	33,530	41,974	52,757	45,218	61,548	80,949

Notes:

1. Total cash withdrawals equal the sum of cash overhead payments and the owner's drawings, plus $20,000 for the packing shed at the end of year four.
2. The assumption was made (when taxation was discussed in section 4.3.6.7) that the farmer would always have a credit bank balance of $1000. This assumption is violated only in the first year, but the resulting inaccuracy in the tax calculations would be negligible.

and overhead costs) are deducted from pre-tax cash receipts to obtain assessable farm income. Tax payments are then calculated and deducted from pre-tax cash receipts to obtain tax-free cash receipts. Tax-free cash available at the beginning of the following year is calculated by deducting the cash components of overhead costs and other withdrawals of tax-free cash from, and adding the amount of tax-free cash in the Bank to, tax-free cash receipts.

It can be noted from table 4.26 that $80,949 of tax-free cash is available at the end of the planning period to cover production costs and cash withdrawals in the following year, (and also to contribute towards repayment of the farm manager's bank loan).

The net cash surplus from each year's cropping programme can be determined by deducting tax payments and the farmer's drawings (which include $2,500 per annum, plus $20,000 at the end of the fourth year) from assessable farm income. The farmer's cash surplus has increased from $4,220 at the end of the first year, to $13,051 at the end of the sixth year.

The available supply of tax-free cash will increase from one year to the next by an amount equal to the cash surplus plus depreciation costs (the latter are not covered in the model by cash withdrawals). For example, tax-free cash available at the beginning of the second year is $6,530 greater than that available at the beginning of the development period. This cash increase is equal to the cash surplus of the first year ($4,220) plus year one depreciation costs ($2,310, as in table 4.9).

4.8.3 Value of the objective function; Plan 4

At the end of the six-year period, the programme has generated \$80,949 of tax-free cash, and the present value (at the end of the sixth year) of future net revenue from the perennial crops planted during the planning period is \$702,487. However, since one dollar's worth of 'assets' is valued in Plan 4 at 35.6 cents, the value of the objective function is:

tax-free cash + final assets

= (\$80,949 x \$1) + (\$702,487 x \$0.356)

= \$331,034 (4-7)

4.8.4 Resources in disposal: Plan 4

Table 4.27 includes the levels of all disposal activities in the optimum basis.

Since the cropland transfer control restraint has an initial supply of 106.5 acres and 22.0 acres of annual cropland are transferred to perennials in the first year, only 84.5 acres of annual cropland remain to be transferred to perennials. Hence the level of the corresponding disposal activity represents the supply of annual cropland which remains available for perennial plantings.[44]

Potatoes and ryegrass for seed are not grown in any year and the corresponding disposal levels are equal to the potato and ryegrass cropping limits respectively. The disposal levels of the other crops indicate by how much the cropping limit exceeds the actual acreage cropped.

---

44. The level of the cropland transfer control disposal activity increases by 9.2 acres from year three to year four since this represents the 9.2 acres of asparagus which was removed in year two, sown with annual crops for two years, and then became available for perennial plantings in the fourth year.

Table 4.27 Resources in Disposal - Plan 4

Disposal Activity	Unit	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
$R_{7,k}$: cropland transfer control	1 acre	84.5	65.3	55.3	64.5	54.4	49.7
$R_{8,k}$: Spring labour	1 hour	413.6	1097.1	883.7	-	-	-
$R_{13,k}$: potato limit	1 ton	70.0	70.0	70.0	70.0	70.0	70.0
$R_{16,k}$: tomato limit	1 acre	2.6	3.1	5.1	6.6	8.1	8.1
$R_{17,k}$: green bean limit	1 acre	-	2.2	6.0	12.0	18.0	18.0
$R_{18,k}$: pea limit	1 acre	10.5	14.8	19.0	19.0	19.0	19.0
$R_{19,k}$: broad bean limit	1 acre	5.0	-	-	-	-	-
$R_{20,k}$: kumara limit	1 acre	4.5	-	-	-	-	-
$R_{22,k}$: ryegrass limit	1 acre	12.0	12.0	12.0	12.0	12.0	12.0
$R_{23,4}$: apple limit, year 4	1 acre				10.0		
$R_{23,5}$: apple limit year 5	1 acre					10.0	
$R_{23,6}$: apple limit year 6	1 acre						10.0
$R_{25,k}$: rotation limit on carrots	1 acre	77.6	75.1	65.1	57.6	50.1	50.1
$R_{26,k}$: rotation limit on peas	1 acre	59.1	62.0	64.5	64.5	64.5	64.5
$R_{27,k}$: rotation limit on green beans	1 acre	3.5	-	1.5	19.5	37.5	37.5

Note: The disposal activities in the basis are those corresponding to the above restraints.

The rotation limit disposal activities have the following interpretation, using the rotation limit on green beans as an example. The supply of this restraint in year one will equal 106.5 acres less 22.0 acres transferred to perennial crops, or 84.5 acres, and the green bean crop cannot, therefore, exceed one-third of 84.5 acres. However the green bean crop in the first year (27.0 acres) is one-third of 81.0 acres, which leaves 3.5 acres of the green bean rotation limit in disposal.

4.8.5 Value of resources: Plan 4

4.8.5.1 Introduction

Many of the shadow prices have little or no practical meaning as far as the management of the holding is concerned. For example, the value imputed to the peach land restraint of year one is of limited use since the grower is not interested in buying more land planted with seven-year-old peaches. Likewise, it is not practical to speak of increasing the supply of intercropping land or perennial cropland in any year. What *is* meaningful to the farm operator is increasing the supply of *annual cropland* which may then be transferred to perennial crops and perhaps intercropped, if this is the optimal use for such extra land.

The shadow prices of the pre-tax cash receipts, tax deductions and tax limits restraints are also of little value unless the farmer can receive income from non-farm sources, or has not fully described his tax-deductible expenditures. Varying the tax limits is beyond the control of the farmer, since taxation is a matter of *law*. However, the grower would be interested in knowing by how much an extra dollar of tax-free cash at the beginning of any year would increase tax-free cash available at the end of the planning period.

The shadow prices are also somewhat different to the marginal value products obtained in Chapter 3, since they refer to tax-free gains, and often refer to a period of time greater than one year.

All shadow prices of interest are presented in the following sections.

4.8.5.2 Annual cropland restraints

The values imputed to the annual cropland restraints for each year are:

year 1	:	\$285.11 per acre,
year 2	:	\$247.49 " " ,
year 3	:	\$214.91 " " ,
year 4	:	\$156.51 " " ,
year 5	:	\$99.36 " ", and
year 6	:	\$43.94 " ".

For example, should the grower obtain an extra acre[45/] of land at the beginning of the planning period, the value of the objective function would increase by \$285.11.[46/]

4.8.5.3 Labour restraints

The values imputed to the effective labour restraints are given in table 4.28.

---

45. Unit increases in crops will be referred to throughout the discussion, although the limit to which a shadow price will remain constant may well be less than one unit of the activity.
46. Providing the final simplex tableau could be obtained, the annual cropland (year one) disposal activity would indicate exactly how the \$285.11 was earned. It should also be noted that increasing the supply of annual cropland in the first year also increases the supply available in all other years. (The I.B.M. 1620-1311 L.P. System does not provide the final simplex tableau, and neither does it give the limits over which the shadow prices remain constant).

Table 4.28 Labour Shadow Prices ($ per hour) - Plan 4

Restraint	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
$R_{8,k}$: Spring labour	-	-	-	0.01	0.05	0.25
$R_{9,k}$: Summer labour	0.44	0.27	0.26	0.26	0.25	0.25
$R_{10,k}$: Autumn labour	0.44	0.27	0.26	0.26	0.25	0.25
$R_{11,k}$: Winter labour	0.44	0.27	0.26	0.26	0.25	0.25

Should the grower add one hour of labour to his permanent staff during the Summer of year one, the value of the objective function would increase by 44 cents. However, should he obtain this labour in the sixth year the objective function value would increase by only 25 cents.

4.8.5.4 Process and fresh market crop restraints

Table 4.29 gives the values imputed to the effective process and fresh market crop acreage restraints.

The values imputed to the asparagus cropping limits, except that of the sixth year, have little practical meaning since asparagus is a perennial crop and the contract would not normally be increased one year and reduced the next, as can the contracts for annual crops.

Should the grower have the opportunity to increase the acreage of some process crops, beetroot and carrot would be the most profitable to expand, although increasing the kumara cropping limit generally appears even more attractive.[47] Finally, it would appear that making

---

47. However, the grower mentioned earlier that marketing difficulties were to be expected if a greater acreage of kumara was grown. (See section 4.3.4).

further plantings of apples over the six year period would be approximately four times more profitable than making further plantings of asparagus.

Table 4.29 Shadow Prices ($ per acre) of Process and Market Crop Restraints - Plan 4

Restraint	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
$R_{12,k}$: beetroot	48.83	81.95	53.91	52.10	45.57	24.37
$R_{14,k}$: asparagus	-	-	-	-	-	1,112.34
$R_{15,k}$: carrot	32.53	80.91	52.89	51.85	50.68	48.85
$R_{17,k}$: green bean	16.35	-	-	-	-	-
$R_{19,k}$: broad bean	-	30.55	29.97	29.61	30.17	35.09
$R_{20,k}$: kumara	-	85.82	57.70	56.26	53.39	43.59
$R_{21,k}$: mangold	43.84	57.46	29.89	28.63	24.40	5.97
$R_{39,6}$: total apple						4,369.56

Note: The shadow prices of the carrot cropping limit restraints have been converted to a per acre basis (by multiplying by the yield of carrots in tons per acre) to allow a more convenient comparison with the other crops.

4.8.5.5 Tax-free cash restraints

This group of shadow prices is most interesting since it indicates the extra tax-free cash the farmer would obtain at the end of the planning period for an extra dollar of tax-free cash invested in the holding (or banked) at any stage during the planning period.

The values imputed to the 'tax-free cash - beginning of year k' restraints are:

beginning of year 1 : $1.35

" " year 2 : $1.10

beginning of year 3 : \$1.08
" " year 4 : \$1.06
" " year 5 : \$1.04, and
" " year 6 : \$1.02.

For example, an extra dollar of tax-free cash available at the beginning of the development period would increase the value of the objective function (which includes tax-free cash available at the end of the sixth year) by \$1.35.

Since some cash is banked during each year except the first (see table 4.26) any extra cash made available in these years would also be banked, so that the supply of tax-free cash available at the beginning of the first year is the only tax-free cash restraint which limits the development programme.

This is also indicated by the above shadow prices, which show a tax-free simple rate of return of 2% per annum from the second year. Since the maximum tax rate in the model leaves 32.5 cents for every dollar taxed (that is, approximately one-third), only 2% (one third) of the 6% interest earned by any extra cash in the Bank activity would remain after tax.

4.8.6 Opportunity costs of growing excluded crops:Plan 4

The marginal opportunity costs of including non-basic cropping activities in the optimum programme are given in table 4.30.

Many non-basic cropping activities may be forced into the optimum programme (should the grower desire this) causing only a small reduction in the value of the objective function. Crops such as carrot and beet-root may be either intercropped or not with very little effect on the value of the objective function. Including potatoes, ryegrass for seed or peas in the programme has a slightly larger effect on the objective function. For example, should the grower wish to plant an acre of potatoes in the second year, the objective function would be reduced by over $43. Such changes in the objective function are still small, however, and it would appear that the grower may be able to alter his cropping programme (if contracts change, for example) with little effect on the overall profitability of the development programme.[48]

Table 4.30 Marginal Opportunity Costs ($) of Growing Non-Basic Crops - Plan 4

Activity	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
$P_{2,k}$: intercrop carrot	0	0.28	0.28	0.27	1.65	1.69
$P_{3,k}$: intercrop kumara	0	-	-	0	1.38	1.43
$P_{4,k}$: intercrop mangold	-	-	0	-	1.38	1.43
$P_{5,k}$: intercrop potato	12.81	43.39	42.56	41.67	41.86	39.21
$P_{8,k}$: beetroot	3.54	1.45	1.42	1.40	-	-
$P_{9,k}$: potato	12.81	43.39	42.56	41.67	40.48	37.78
$P_{10,k}$: pea	-	-	26.47	25.98	25.55	25.37
$P_{12,k}$: broad bean - kumara	9.22	-	-	-	-	-
$P_{15,k}$: ryegrass	5.63	6.26	32.61	31.99	31.35	30.63
$P_{16,k}$: old asparagus	-	-	-	-	-	2.61
$P_{18,k}$: asparagus	-	-	-	4.01	-	-
$P_{19,k}$: apple	-	-	-	-	206.85	437.28

Note: Some crops which were not included in the farm plans also do not appear in the above table, since such crops were included in the basis at a zero level. Examples are intercropped kumara (year two), asparagus (year three) and apple (year four).

48. The opportunity costs apply only when one crop is forced into the basis at a time and also remain constant only up to some limit. Hence if many contracts change, or one changes by an amount greater than the limit, a new optimum basis would need be computed.

Should the farm manager decide to retain an acre of old asparagus for cropping in the sixth year (instead of digging it out), the value of the objective function would fall by only $2.61. Or should he plant an acre of apples in year five rather than an earlier year, the objective function will decrease by over $200, this being the difference (at the end of the development period) between the asset value and cash earnings of the new planting and the replaced (earlier) planting.

4.9 Effect of Varying the Length of the Planning Horizon

Since capital budgeting involves the planning of cash allocations over a series of time periods, the farm manager may want to know how far into the future such capital budgeting should be carried.

To find the effect of varying the length of the planning horizon, the programme, with a price on the final assets activity of $0.356 (that is, Plan 4 of section 4.8) was shortened from a six-year planning period to first a five-year and then a four-year planning horizon.

Candler has suggested[49] that the planning horizon should be extended until investment decisions in the <u>first</u> time period become insensitive to further extensions of the planning horizon. The author, however, found that although the <u>first</u> year's cropping programmes for the four-year and five-year planning horizon models were similar to that of the six-year model, the cropping programmes in the <u>following</u> years differed.

Such differences between the four-year, five-year and six-year planning horizon models affected only the rate at which the old asparagus activity was replaced. In the four-year horizon model, the only asparagus replanted

---

49. Personal communication.

is 0.9 acre in the third year. When the planning horizon is extended to five years, 0.9 acre of asparagus is replanted in year three and 8.1 acres in year four. (And as discussed in section 4.8, the six-year horizon model replaces 9.2 acres of asparagus in year two, with no new plantings in either the third or fourth years).

To summarise then, if the farm operator considered a planning horizon of four years, the old asparagus activity would comprise 23.1 acres at the end of that period. If he budgeted his funds over five years the old asparagus activity would comprise 15.0 acres at the end of the fourth year, and if he planned over a six-year period, the level of this activity would be 14.8 acres at the end of the fourth year.[50/]

4.10 Discussion on the Programming Results

4.10.1 Physical plans and problems with changed contracts

The farm manager stated that Plan 4 (discussed in section 4.8) was the most suitable since it contained the asparagus replacement pattern most suitable to him. Apart from this, other aspects of the cropping programme which appealed to the grower were that the beetroot, carrot and broad bean crops were included up to the limit of the cropping restraints, and that potatoes and ryegrass (for seed) had been excluded in each year. Although he agreed with a gradual reduction in the tomato crop, the grower would prefer no tomatoes at all in the sixth year since harvesting of this crop would clash with apple harvesting and problems

---

50. The author intended to extend the planning horizon beyond six years, but unfortunately a lack of time prevented any further Masterate research with the model.

associated with the supervision of labour were foreseen. He would therefore prefer to replace the 12.9 acres of tomatoes in year six with green beans.

The strongest complaint the farmer had against following a set programme was that although he would <u>like</u> to put Plan 4 into operation, future changes in contracts would make the optimum programme infeasible. For example, the contract for green beans had been changed from 27 to 30 acres, that for peas from 19 to 31 acres, and that for carrots from 60 to 35 tons, while the model was being set up and solved.

It is not difficult, though, to recognise that in situations where contracts (or any other restraints, prices, costs, yields or input-output coefficients) are changed, linear programming can become a most useful analytical tool in that the impact of the change can be readily determined by solution of the altered problem and the new optimum solution returned to the farm operator with a minimum of delay.

Since the model was formulated, lower-than-expected market realisations from the kumara and mangold crops had persuaded the grower to reduce the acreage of these crops in the coming season from 24 to 12 acres, and eight to five acres, respectively.

Plan 4 requires that the acreage of peas falls to 4.2 acres (in year two) and that of green beans to 9.0 acres (in year's five and six). In practice, the acreage of these crops would need to be at least 10 acres to make mechanical harvesting worthwhile. The grower mentioned that the plan could be modified by replacing tomatoes with green beans in year's five and six, bringing the size of the latter crop to almost 22 acres in each of those years.

A problem, which could easily have been avoided, arose when the grower mentioned that apples could not be planted until the third year since trees on Malling-Merton 106 rootstocks had to be ordered two to three years in advance. (Had the author been aware of this earlier, the model would not have included apple activities in either of the first two years).

4.10.2 Resource use and disposal levels

Although the calculation within the programme of labour requirements was necessary for purposes of cash budgeting, the grower thought that such information would be of little use to his management. Seasonal labour requirements were interesting to him in that they gave some idea of the size of the labour force which would be required in future years, although problems due to aggregation of labour into seasonal periods meant this information was of little use in planning future labour requirements. The aggregation problem can be overcome simply by dividing the labour supply into shorter periods (at the expense of a much larger model), but even so, the grower decided that monthly labour requirements may not be of much application in his management since he finds it difficult to plan ahead as regards labour requirements. This is because planting and harvesting dates for process crops are not known at the beginning of the season, let alone for future years. The grower may have as little as 10 days to obtain adequate labour (and machinery) for the sowing or harvesting of a crop and under these conditions bottlenecks are likely to occur at any time. Also, labour planning involves finding the correct type of labour (as well as a sufficient quantity) and the grower thought the greatest labour problem over future years would not be obtaining sufficient labour, but obtaining sufficient skilled labour.

4.10.3 Problems associated with the variability of input-output coefficients

The grower was concerned that variability in the cost and gross revenue coefficients could upset a programme budgeted over a number of years. Although the total cost, gross revenue and cash surplus figures for each year of the development period seemed reasonable to the grower, he mentioned that a fall in apple prices (for example) could make the optimum programme infeasible (unless further funds could be borrowed). The grower was also concerned that variable cost items tend to increase from year to year, producing a 'price - cost' squeeze. Such a possibility could be insured against to some extent by including in the model lower than expected prices, higher than expected costs, and provision for cash withdrawals in excess of those thought likely to be required. Also, variable costs may be increased by a constant proportion each year to reflect, say, likely rises in a 'farm costs' index.

Another method of overcoming price and cost variability is to include such variability in the capital budgeting programme by treating prices and costs as random variables, as proposed by Candler.[51/] Further research in this direction is required, however, before such a possibility becomes practicable.

4.10.4 Value of resources

Although the grower showed interest in the values imputed to the process crop restraints, discussion with the processing company indicated that even though extension of the beetroot and carrot contract limits appeared profitable on the farm (and the grower would like to see

51. Personal communication.

increases in these contracts) such changes could not be met by the company. Hence, the inability to alter contracts to suit the wishes of a grower reduces somewhat the practical usefulness of their shadow prices.

The grower believed that budgets indicating how extra land would be cropped (with such information available from the optimum simplex tableau, or variable resource programming) would be of great use providing he was contemplating buying more land. However, he believed he was more likely to sell land in the future, rather than purchase land. The reasons for this are firstly, should he experience difficulty in obtaining finance in the future he may sell land, and secondly, he considers that a smaller holding growing mainly tree crops and asparagus will provide an adequate income without the managerial problems associated with other process crops.

Similar budgets indicating how extra tax-free cash should be (optimally) invested would also be most useful in farm management if the possibility of obtaining such funds in the future becomes likely.

4.10.5 Summary

The model was constructed so as to represent as nearly as possible the 'real life' management problems of the farm operator. That some success was achieved was evident, since the optimum solutions appeared reasonable and feasible to the grower and he said he would <u>like</u> to follow such a programme. This was not possible though since the 'real world' problems of variations in prices, costs and cropping contracts were <u>not</u> incorporated in the linear programming model.

It has been mentioned that these problems could be overcome to a certain degree by using lower-than-expected prices and higher-than-expected costs, or resolving the programme after contracts have been altered.

Future research into nonlinear programming methods may allow the incorporation of price and cost variation into the model itself, with the hope of providing farm plans which may be of somewhat more use in the field of practical application.

CHAPTER 5

QUADRATIC PROGRAMMING AND PROFIT MAXIMISATION

5.1 Introduction

The behaviour of many farm firms, such as the horticultural holding of Chapter 3, can be realistically approximated as perfectly competitive in both factor and product markets. The fresh vegetable industry, for example, comprises a large number of relatively small holdings and any single producer is but one of many sellers. Under such conditions it may be assumed that individual firms cannot affect the price they receive by changing output levels, and are thus price-takers, the demand conditions facing such firms being perfectly elastic. In factor markets, the larger the number of buyers, the less can any one firm influence the price it pays for a factor by changing the quantity it uses. Hence farm firms may also be 'near-perfect' competitors in factor markets.

Although perfect competition is an 'extreme' market structure which rarely exists, many farm firms may be realistically treated, as indicated above, as perfect competitors when profit-maximisation is being studied. Linear programming may then be employed to determine profit-maximising behaviour since it has been assumed that the market price of any product is independent of the amount produced, and that marginal costs of production are unaffected by the quantity of resources purchased.[1/]

For some nursery firms, however, such conditions of perfect competition cannot be assumed since changes in output levels may influence the price received by the firm. If so, the demand conditions facing the firm will be less than perfectly elastic.

The objective function:

$$Z = \sum_j c_j x_j \quad \text{a maximum,}$$

will be linear providing the c_j (price) values are independent of the x_j (output) levels (that is, if perfect competition is assumed to exist in product markets). For the imperfectly competitive situation indicated for a nursery firm, price <u>is</u> a function of the level of output:

$$c_j = f(x_j) ,$$

since the price which the firm can charge if it hopes to sell its entire output may decrease as the quantity produced increases. In this case the objective function becomes:

$$Z = \sum_j \left[f(x_j)\right] x_j \quad \text{a maximum;}$$

which is <u>nonlinear</u>.

The assumption is made in the present chapter that quantity demanded is a linear function of price so that the objective function is of quadratic form.[2/] Such an objective function can then be maximised,

---

1. Situations of falling average revenue and increasing average costs may be approximated by linear programming - see, for example, Candler, Wilfred and Musgrave, Warren F., "A Practical Approach to the Profit Maximisation Problems in Farm Management", <u>op.cit</u>. Should the supply function for a factor be downward sloping, however, the convexity assumption of linear programming would be violated.
2. For a firm producing a single product x, sold at a price p, total revenue will equal px. If demand for x is linear and the price-output function is expressed as $p = a-bx$, total revenue may be re-written as $(a-bx)x = ax-bx^2$ which is a quadratic function.

subject to linear production restraints, by quadratic programming.[3]

5.2 The Approach Adopted

Since nurseries generally produce a large number of different plant types, a small nursery was required so that production possibilities could be adequately represented in the quadratic programming matrix. Difficulty was experienced, however, in locating a small nursery which could provide sufficient information to allow estimation of the demand functions.

Although such information was obtainable from a larger Taranaki nursery, its output included over 1,300 different types of plants so that formulation of the optimum output for the firm as a whole would be beyond the capability of available computers. Data was therefore collected from the nurseryman relating only to a small number of plants and was used to construct a hypothetical nursery.

Attention was concentrated on that section of the Taranaki nursery which produced glasshouse-propagated plants and in order to reduce the problem to a manageable size, nineteen plants propagated during Spring or Summer were chosen as activities for the quadratic programme.

The supply of resources on the hypothetical nursery was set equal to the total requirements of the nineteen activities for the various nursery resources when produced at the levels of the last season. The end result was a nursery, which although hypothetical, was based on 'real-life' data and comparable in size to several nurseries which are to be found throughout New Zealand.

---

3. Quadratic programming is discussed in Chapter 2, section 2.2, to which references will be given as necessary.

The quadratic programme was formulated and solved, then, not to assist the nurseryman to increase his profits, but rather to illustrate how such production situations may be handled by quadratic programming.

5.3 The Restraints

5.3.1 Glasshouse restraints

5.3.1.1 Propagating-house (restraints R1 - R13)

Each of the nineteen activities makes use of the heated propagating-house at some stage during the period from mid-July until the end of February, and it was necessary to treat the propagating-house as a separate production restraint over 13 time periods.[4]

The greatest number of plants occupying the propagating-house for last season's output of the nineteen activities from the Taranaki nursery was 13,460, during the first half of November. Therefore the capacity of the propagating-house on the hypothetical nursery was assumed to be 14,000 plants. Since these restraints are measured in units of 100 plants they will have a supply of 140 units, and the propagating-house restraints are:

R1 : $July_2$
R2 : $August_1$
R3 : $August_2$
R4 : $September_1$
R5 : $September_2$

---

4. This treatment of a resource which provides services over a period of time is no different from the division of the land and labour resources into various time periods, as in Chapter 3.

R6 : $October_1$

R7 : $October_2$

R8 : $November_1$

R9 : $November_2$

R10 : $December_1$

R11 : $December_2$

R12 : January

R13 : February ,

where the subscripts 1 or 2 denote the first or second-half of a month respectively. Restraint R1, for example, ensures that the propagating-house is required to hold no more than 140 hundred plants during the second half of July.

5.3.1.2 Growing-house (restraints R14-R26)

Following propagation the plants are generally transferred from the heated propagating-house to the unheated growing-house for a further period, and the growing-house was found to impose a possible restraint on production during 13 periods of time. To allow last season's output of the nineteen plant types to be just feasible the capacity of the growing-house would need to be 9,420 plants, in which case the glass-house would be fully occupied during the second half of November. The capacity of the growing-house on the hypothetical nursery was therefore assumed to be 10,000 plants, and the growing-house restraints are:

R14 : $August_2$

R15 : $September_1$

R16 : $September_2$

R17 : $October_1$

R18 : $October_2$

R19 : November_1
R20 : November_2
R21 : December_1
R22 : December_2
R23 : January
R24 : February_1
R25 : February_2
R26 : March - April.

Each of these restraints is measured in units of 100 plants and has a supply of 100 units. Thus R14, for example, ensures that only 100 hundred plants, at most, occupy the growing-house during the second half of August.

5.3.1.3 Shade-house (restraints R27 - R36)[5]

After removal from the glasshouses, many of the plant types require a period of hardening in the shade-house prior to their being either sold or planted in the open ground to make further growth.

Last season's output of the nineteen activities would have required a shade-house with a capacity of 8,790 plants, in which case it would be completely utilised during the second half of December. The capacity of the shade-house was set equal to 9,000 plants, and it provides possible restraints on production over 10 time periods as follows:

R27 : September
R28 : October - November_1

---

5. Strictly, the shade-house (or lathhouse) is not a glasshouse, but a structure covered with strips of some material (such as wood) to provide partial protection from sunlight. For convenience, the shade-house has been grouped with the two glasshouses in the discussion.

R29 : $November_2$

R30 : $December_1$

R31 : $December_2$

R32 : $January_1$

R33 : $January_2$

R34 : $February_1$

R35 : $February_2$

R36 : $March_1$

Each restraint is measured in units of 100 plants and has a resource supply of 90 units.

5.3.2 Labour (restraints R37 - R41)

The total labour supply is represented by five restraints, with the supply of labour over the Spring - Summer propagating season divided into three, two-month periods, and the labour supply of the remaining six months of the year divided into two periods of three months each.

Once the total labour requirements of the nineteen activities at last season's output levels had been determined, it appeared that a labour supply of the nurseryman working 44 hours per week and one woman working 25 hours per week would have been sufficient in the previous season. Furthermore, such a labour force represented a typical staff for nurseries of similar size to the hypothetical firm.

For every eight hours worked by the nursery manager, it was assumed that one and a half hours would be taken up on such jobs as cleaning sheds and glasshouses, washing containers, watering and spraying plants in the glasshouses and attending to ventilation and heating. After making provision for this overhead labour, the nurseryman would supply only 35.75 hours of labour per week to the total labour available to the activities of the quadratic programme.

The five labour restraints and resource supplies are to be found in table 5.1.

Table 5.1 Labour Restraints

Labour Restraint	Unit	Resource Supply
R37 : August - September	1 hour	526
R38 : October - November	1 hour	526
R39 : December - January	1 hour	526
R40 : February - April	1 hour	789
R41 : May - July	1 hour	789

Note: One month was taken as 4.33 weeks in determining labour supplies.

5.3.3 Land (restraint R42)

The area of land required by the nineteen activities at the production levels of the past season was 1.10 acres, or approximately 5,400 square yards, which is reasonable for a nursery of similar size to the hypothetical holding. Therefore R42 is a land restraint with a supply of 5,400 square yards.[6]

5.3.4 Summary of resource supplies on the hypothetical nursery

The resources included in the quadratic programme are a heated propagating-house with a capacity of 14,000 plants, an unheated glasshouse with a capacity of 10,000 plants and a shade-house which can hold

6. This land supply refers to cultivated land only, and does not include any land occupied by glasshouses or other buildings, outdoor frames or sawdust beds.

up to 9,000 plants. The labour force consists of the manager plus one woman, and just over one acre of land is available.

These resources were assumed to limit production of all plant types sooner than would other resources such as soils and containers, a soil sterilization unit, stool-beds to provide cutting material, outdoor frames and work sheds. Hence the latter group of resources were assumed to be in more than adequate supply on the hypothetical nursery so did not require inclusion as restraints in the programming matrix.

5.4 The Activities

5.4.1 Telopea speciosissima (P1)

Seed is sown in trays towards the end of July and placed in the propagating-house to germinate. The trays are then transferred to the growing-house for a fortnight, and then to the shade-house. By the end of September the young plants will be sufficiently hardened to be planted in the open ground.[7] The plants are lifted during the following August and replanted at a wider spacing where they remain for one year, being wrenched, lifted and sold in the following winter.

5.4.2 Acacia baileyana (P2)

Acacia is raised from seed sown in August. After one month in the propagating-house the plants are set out in three-inch diameter containers and moved to the growing-house. After four to six weeks

---

7. The plants are spaced two inches apart with 15 inches between rows, this form of planting being referred to as long-row bedding.

they are repotted into four-inch containers and plunged in open beds of sawdust, to be sold from March onwards.

5.4.3 Passiflora 'Crackerjack' (P3)

Stock plants chosen from the previous season's output are held in the propagating-house from August, to provide cutting material. The cuttings normally form roots within three weeks, when they are planted into tubes. (These young plants, as well as the stock plants, provide cutting material for the next batch of cuttings so that propagation is a continuous process until the output target is reached). The plants are repotted into four-inch containers and shifted to the growing-house for initial hardening. The plants are transferred to outdoor frames for final hardening and sold from November onwards.

5.4.4 Banksia grandis (P4)

This activity is raised from seed sown during the second half of August. After one month in the propagating-house the seedtrays are transferred to the growing-house for two weeks, and then to the shade-house where hardening-off is completed by mid-November. The plants are then set through polythene in the open ground where they remain for about 20 months, being sold during the second winter.

5.4.5 Photinia robusta (P5)

Cuttings are made in September, set in trays, and kept in the propagating-house until mid-November. They are then shifted into the growing-house, and a fortnight later into the shade-house where they remain until the end of December. The plants are then set in fumigated raised beds where they remain until the following November when the plants are lifted and set through polythene into the open ground. They remain in this position for about 20 months, being sold during the second winter.

5.4.6 Eucalyptus ficifolia (P6)

The husbandry requirements of this activity are similar to those of Acacia (P2) except that all operations are carried out one month later. Seed is sown in September and kept in the propagating-house until mid-October, when the plants are potted into three-inch containers and transferred to the growing-house where they remain until the end of November. The plants are then repotted into four-inch containers and plunged in sawdust beds, to be sold from April onwards.

5.4.7 Stachyurus praecox (P7)

Stachyurus is raised from cuttings made in October. They remain in the propagating-house until mid-November, are transferred to the growing-house for a fortnight, and are then shifted into the shade-house for December. The plants are set out in fumigated beds during January where they remain until the end of September. At this time they are placed in long-row beds until the following August, and set out in the open ground for a further year before the plants are sold.

5.4.8 Cistus purpureus 'Brilliancy' (P8)

Cuttings are made in October and kept in the propagating-house for one month. They are then potted into tubes and transferred to the growing-house, after which they are shifted to the shade-house for the first half of December. The plants are then potted into four-inch containers and plunged in sawdust beds, ready for sale.

5.4.9 Protea cynaroides (P9)

Protea cuttings are made in October and remain in the propagating-house until mid-December, when they are potted and shifted to the growing-house. Hardening-off is completed in the shade-house during January. The plants are put into four-inch containers during February and plunged in sawdust beds until the end of September, when they are set out (through polythene) in the open ground. The plants are ready for sale during the following winter.

5.4.10 Tibouchina grandiflora (P10)

The cuttings, made towards the end of October, remain in the propagating-house for four weeks, and are then placed in containers and shifted to the growing-house. In mid-December the plants are transferred to the shade-house where they are held until the following spring. The plants are set in long-row beds from October through to August, and then planted out for one year. During the following winter the plants are sold.

5.4.11 Azalea indica 'Salmonea' (P11)

Cuttings are made towards the end of October and kept in the propagating-house for two months, after which they are transferred to the shade-house. The plants are set in fumigated beds during February where they remain until the following November, at which time they are set (through polythene) into the open ground. During the second winter after being planted out, they are ready for sale.

5.4.12 Viburnum japonicum (P12)

Viburnums are raised from cuttings made during November. The cuttings are transferred from the propagating-house to the growing-house in mid-December, and then to the shade-house by the beginning of January. A fortnight later the plants are set in fumigated beds where they remain until the end of September. The plants are placed in long-row beds in October and then planted out during August, being ready for sale one year later.

5.4.13 Rhododendron 'Christmas Cheer' (P13)

Cuttings are made in November and remain in the propagating-house until the end of February. The plants are then transplanted into containers and shifted to the growing-house for March and April. They are then transferred to the shade-house where the plants remain until mid-November when they are planted (through polythene) in the open ground. About 18 to 20 months later the plants are ready for sale.

5.4.14 Weigela florida variegata (P14) and Forsythia 'Karlsax' (P15)

Cuttings of both these plants are made in November and by the beginning of December are ready to be transferred to the growing-house. The plants are hardened in the shade-house from mid-December until mid-January and then set in fumigated beds until the end of September. They are set out in long-row beds during October, planted out in the open ground during the following August, and are sold one year later.

5.4.15 Azalea occidentalis (P16)

Cuttings are made in mid-November and held in the propagating-house until the end of January, after which they are transplanted into tubes and held for one month in the growing-house. They are then plunged in sawdust beds until October, when they are set in long-row beds for a period of one year. In the following October the plants are set in the open ground for a further two winters before being ready for sale.

5.4.16 Magnolia stellata (P17)

The cuttings, made in mid-December, remain in the propagating-house until the end of January when they are shifted to the growing-house. The plants are transplanted into tubes during late February and March and held in the growing-house until the end of April. The tubes are plunged in open beds until the end of September, the plants then being set out (through polythene) in the open ground, where they remain for two winters before being sold.

5.4.17 Callicarpa dichotoma (P18) and Hypericum 'Hidcote Gold' (P19)

Cuttings of both these activities are made in January and kept in the propagating-house for one month, after which they are transferred to the growing-house for the first half of February. The plants are then shifted into the shade-house for one month and in mid-March are set in fumigated beds where they remain until mid-November. The plants are then placed in long-row beds, and in the following October they are planted in the open ground until the following winter, when they will be ready for sale.

5.5 Input-Output Coefficients

5.5.1 Problems of multi-period production

As indicated by the previous section, production of most plants extends over a period exceeding one year. Any programming, then, should take into account:

(i) the resource supplies and resource requirements of a production plan over a number of years, and

(ii) the resource requirements over present and future time periods of those plants whose production was initiated before the programming was carried out.

Such a situation could be represented in an intertemporal programming approach with resources and activities included for each time period, where resources required by those activities already in production would be deducted from the appropriate resource supply. Such an approach, however, would have yielded a matrix too large for the available computation facilities,[8] so the following alternative was adopted.

For those activities with a production period greater than one year, the total quantity of resources required over the production period was included in the input-output coefficients per unit of that activity, and one unit of an activity which is produced over, say, three years, would include plants in their first, second and third years of growth.[9]

This method has the advantage over an intertemporal approach in that a much smaller matrix is required to obtain the optimum production plan, but has the disadvantage that the most profitable progression from the present plan to the optimum plan will not be indicated to the nursery manager. (It may take up to four years for the optimum plan to be fully operational, since only after four years will one-year, two-year, three-year and four-year-old plants be present in the nursery).

---

8. Activity P16 has the longest production period, almost four years elapsing from propagation until the plants are sold. Therefore a period of four years would need to be covered in an intertemporal model with the number of restraints and activities increased four-fold.
9. Any plan will then be feasible since sufficient resources will be available to the one-, two- and three-year-old plants, each of which will be present in any one year.

5.5.2 Glasshouse restraints

One unit of any activity (with the exception of Passiflora) will require one unit of each of the appropriate glasshouse resources since both activities and glasshouse restraints are defined in units of 100 plants and the total output of an activity will occupy a glasshouse at the one time. The input-output coefficients will all be equal to +1.00 and, for example, the requirement that the propagating-house contains no more than 140 hundred plants during the second half of December is met by the inequality:

$$1.00x_{11} + 1.00x_{13} + 1.00x_{16} + 1.00x_{17} \leqslant 140,$$

since Azalea indica (P11), Rhododendron (P13), Azalea occidentalis (P16) and Magnolia (P17) are the only plants requiring the propagating-house during this period.

Unlike other activities, the total output of Passiflora will not occupy a glasshouse all at the one time since individual plants are removed and replaced over a period of months. For example, of last season's total Passiflora output of 4,200 plants, only 300 occupied the propagating-house during August, and 600 during September. In other words, for every 100 Passiflora plants produced, seven will occupy the propagating-house during August and 14 during September. Since this activity is measured in a unit of 100 plants, the input-output coefficients will be 0.07 and 0.14 respectively, and the remaining glasshouse coefficients for this activity were calculated similarly.

5.5.3 Labour restraints

The nursery manager provided estimates of the labour requirements of the various operations such as making and setting cuttings, planting-out and preparing the plants for sale, which allowed the labour coefficients to be derived. An example is given in table 5.2, which indicates the labour required in any one year by the one-, two- and three-year-old plants of the Photinia activity.

Table 5.2 Labour Input - Photinia (P5)

Labour Restraint	Operation	Labour Requirement (hours/100 plants)
First-year plants:		
August - September	Make cuttings	1.00
October - November	Shift to growing-house	0.10
" "	Shift to shade-house	0.10
December - January	Transport to, and plant in beds	0.43
Second-year plants:		
October - November	Lift from beds	0.33
" "	Transport to, and plant in open ground	1.17
Third-year plants:		
October - November	Hand-weed and spray	0.24
December - January	" "	0.24
February - April	" "	0.36
May - July	Wrench plants	0.07
" "	Prepare plants for sale	4.00

The requirements were then summed over each labour restraint to obtain the labour input-output coefficients which are (per 100 Photinia plants):

labour required during August - September = 1.00 hours

" " " October - November = 1.94 "

labour required during December - January = 0.67 hours

" " " February - April = 0.36 "

" " " May - July = 4.07 "

The labour input-output coefficients for the remaining 18 activities were calculated in a similar manner.

5.5.4 Land restraint

The land input-output coefficients must include the total land requirements of each age of plant included within an activity. For example in any one season first-year plants of an activity may be planted in raised beds, while second-year and third-year plants may be in long-row beds and planted through polythene respectively.

Table 5.3 gives the area of land occupied by 100 plants for various planting methods, from which the land input-output coefficients can be derived.

Table 5.3 Planting Methods and Land Requirements

Planting Method	Plant Spacing	Land Required by 100 plants
Fumigated raised beds	2" apart in 6" rows	1.04 sq. yds
Long-row beds	2" apart in 15" rows	3.55 " "
Planted in open ground	9" apart in 30" rows	20.83 " "
Planted through polythene	9" x 9", 4 rows per bed	10.42 " "
Planted through polythene	12" x 12", 3 rows per bed	18.52 " "

Note: Strictly, all the above planting methods are made in the open ground (the land resource of section 5.33), and the third planting method is a normal field spacing of nursery stock in their final stage of production.

Husbandry of the Callicarpa activity, for example, requires that first-year plants be in raised beds from $March_2$ to $November_1$, second-year plants be in long-row beds over the $November_2$ to $September_2$ period and third-year plants occupy the open ground from October until the following winter. Summing these land requirements, then, gives a land input-output coefficient of 25.42 square yards per unit of the Callicarpa activity.

Acacia (P2), Passiflora (P3), Eucalyptus (P6) and Cistus (P8) require no land since these plants remain in containers until they are sold.

5.5.5 Dominated restraints

Once all input-output coefficients had been derived the restraints were tested for dominance[10] so that the quadratic programming matrix could be reduced to the minimum number of rows. The following restraints, found to be dominated, were excluded from the model:

(i) propagating-house; restraints R1, R3, R5, R10 and R13,

(ii) growing-house ; restraints R14, R15, R18, R23 and R25, and

(iii) shade-house ; restraints R29, R33, R34 and R36.

5.6 Matrix of Activity Resource Requirements

Each row of table 5.4 represents a production restraint and each column includes the per-unit requirements of an activity for the various resources. Thus resource supplies are entered in the B column and the

---

10. As in Chapter 3, section 3.6.5.

Table 5.4

Activity Resource Requirements and Resource Supplies

Restraints	Unit	B	Relationship	Telopea P1	Acacia P2	Passiflora P3	Banksia P4	Photinia P5	Eucalyptus P6	Stachyurus P7	Cistus P8	Protea P9	Tibouchina P10	Azalea ind. P11	Viburnum P12	Rhododendron P13	Weigela P14	Forsythia P15	Azalea occid. P16	Magnolia P17	Callicarpa P18	Hypericum P19
Propagating-house:																						
August 1	100 plants	140	≥	1.00	1.00	0.07																
September 1	" "	140	≥			0.14	1.00	1.00	1.00													
October 1	" "	140	≥			0.29		1.00	1.00	1.00	1.00	1.00										
October 2	" "	140	≥			0.29		1.00		1.00	1.00	1.00	1.00	1.00								
November 1	" "	140	≥			0.50		1.00		1.00		1.00	1.00	1.00	1.00	1.00	1.00	1.00				
November 2	" "	140	≥			0.50						1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00			
December 2	" "	140	≥											1.00		1.00			1.00	1.00		
January	" "	140	≥													1.00			1.00	1.00	1.00	1.00
Growing-house:																						
September 2	100 plants	100	≥		1.00	0.07	1.00															
October 1	" "	100	≥		1.00	0.21																
November 1	" "	100	≥			0.50			1.00		1.00											
November 2	" "	100	≥			0.38		1.00	1.00	1.00	1.00											
December 1	" "	100	≥			0.63							1.00				1.00	1.00				
December 2	" "	100	≥			0.63						1.00			1.00							
February 1	" "	100	≥			0.13													1.00	1.00	1.00	1.00
March - April	" "	100	≥													1.00				1.00		
Shade-house:																						
September	100 plants	90	≥	1.00												1.00						
October-November 1	" "	90	≥				1.00									1.00						
December 1	" "	90	≥			0.25		1.00		1.00	1.00											
December 2	" "	90	≥			0.25		1.00		1.00			1.00				1.00	1.00				
January 1	" "	90	≥			0.25						1.00		1.00	1.00		1.00	1.00				
February 2	" "	90	≥			0.25															1.00	1.00
Labour:																						
August-September	1 hour	526	≥	1.04	0.61	0.25	0.23	1.00	0.13	0.84			0.84		0.84		0.84	0.84				
October-November	" "	526	≥	0.60	2.10	0.97	1.67	1.94	0.60	1.70	1.60	2.37	1.60	2.41	1.60	2.08	1.60	1.60	2.94	1.41	1.44	1.44
December-January	" "	526	≥	0.10		0.42	0.40	0.67	2.10	0.70	2.20	0.90	0.80	0.34	0.80	0.24	0.80	0.80	0.60	1.24	1.10	1.10
February-April	" "	789	≥	0.20	0.88	1.56	0.80	0.36	0.88	0.20	0.88	2.45	0.20	0.86	0.20	1.13	0.20	0.20	2.90	0.96	0.90	0.90
May - July	" "	789	≥	4.15	0.88		4.07	4.07	0.88	4.07	0.88	4.07	4.07	4.07	4.07	4.17	4.07	4.07	4.07	5.17	4.07	4.07
Land:	1 sq. yard	5400	≥	24.38			20.84	38.08		25.42		10.42	24.38	21.89	25.42	37.04	25.42	25.42	45.21	37.04	25.42	25.42

Notes:
1. Activities P2, P3, P6 and P8 are produced and sold within one year and are measured in units of 100 plants.
2. Activities P1, P4 and P9 are produced over two years and one unit of these activities includes 100 one-year-old plants and 100 two-year-old plants.
3. Activities P5, P7, P10 to P15 and P17 to P19 are produced over three years and one unit of these activities includes 100 one-year, 100 two-year and 100 three-year-old plants.
4. Activity P16 is produced over four years and hence one unit includes 100 one-year, 100 two-year, 100 three-year and 100 four-year-old plants.

input-output coefficients form the body of the matrix.

5.7 The Objective Function

5.7.1 Introduction

Linear demand functions are estimated for all plant types included in the model. Average variable costs (assumed to remain constant over all levels of output[11]) are then deducted from the demand functions to give average net revenue functions. The latter are then incorporated into a total net revenue objective function.

5.7.2 Estimation of the demand functions

A more reliable estimate of the linear demand functions could have been made had accurate elasticity coefficients been available.[12]

The lack of such data meant that to obtain some indication of the slope of the demand curves, two sets of data had to be used - the past season's outputs and prices, and the nurseryman's estimate of how much price would need to be lowered in order to sell (say) an extra 100 plants. The nurseryman was also asked if the quantity sold of some plants would affect the demand for others, but found this question difficult to answer. Because of this measurement problem, it was necessary to assume that cross-effects did not exist.[13]

---

11. Hence the nurseryman is assumed to be a perfect competitor in factor markets - factor prices are unaffected by the quantity of a factor purchased. However, falling or rising average costs may easily be incorporated into a quadratic programming problem providing the cost functions are linear. See Hadley, G., "Nonlinear and Dynamic Programming", op.cit., pp.243-244.
12. See Louwes, S.L., Boot, J.C.G., and Wage, S., "A Quadratic Programming Approach to the Optimal Use of Milk in the Netherlands", Journal of Farm Economics, vol.49, pp.309-317, 1963.
13. Such an assumption is probably unrealistic, since the wide range of ornamental trees and shrubs available would indicate a priori that significant substitution relationships would be present. This is also suggested by the high own-price elasticity coefficients mentioned in a note to table 5.5.

The nurseryman considered that he could sell as much as he produced of nine of the nineteen activities, so the demand curves for these plant types would be horizontal. For the remaining activities he found it necessary to think in terms of a quite large increase in output (in most cases an increase of 500 plants above last season's output levels) in order to estimate the new prices at which the entire output of these activities could be sold. As an illustration, the following describes the estimation of the linear demand function for the Acacia (P2) activity.

During the past season, 1000 plants were sold to the wholesale trade at a price of 55 cents each. The nurseryman then considered that if output was increased to 1,500 plants he would need to lower his price to 45 cents in order to sell the extra 500 plants. That is, he would charge 55 cents for as long as possible (for the first 1,000 plants) and then reduce the price to sell the remaining plants. Thus the quantity - price co-ordinate of 1,500 plants and 45 cents is not a point on the demand curve because only part of this output is sold at a price of 45 cents. Instead, the average revenue obtained from the sale of 1,500 plants needed to be calculated to give the second co-ordinate of the demand (average revenue) curve.

This demand function was then estimated as:[14/]

---

14. The equation of a linear demand function is given by:

$$x - x_1 = \frac{x_2 - x_1}{p_2 - p_1}(p - p_1) \quad ,$$

where p_1 and x_1 is one set of price-quantity co-ordinates, and p_2 and x_2 is the other.

In the example, $p_1 = \$55$

$x_1 = 10$ hundred plants

$p_2 = \$51.6667$ (average revenue from the sale of 1,500 plants), and

$x_2 = 15$ hundred plants.

$$x = 92.50 - 1.50p \quad ,$$

where x = the number of Acacia plants sold, in hundreds, and

p = the wholesale price per 100 Acacia plants.

The linear demand functions[15] for all nineteen activities are to be found in table 5.5.

5.7.3 Variable costs

Although some variable costs, such as spraying and marketing materials, are relatively easy to determine on a 'per plant' basis, others are more difficult. Examples are the costs of potting media (soils), pots, tubes and trays, and the maintenance of stool-beds which provide cutting materials. Apart from being only a very small cost per plant, these resources provide services over a number of years (for example, the same soils are used continuously) and some arbitrary accounting procedure would need to be adopted to allocate these costs over the appropriate period of time. Exclusion of these costs from the objective function would not affect the optimum allocation of resources to any marked extent, since their total would probably remain almost constant even though the level of activities may differ from those of the past season. (That is, even should the optimum plan differ widely from the nurseryman's plan, about the same number of plants would be produced, so both plans would require similar quantities of soils and containers, etc).

---

15. The functions as presented in table 5.5 are inverse demand functions, since they express $p = f(x)$ rather than $x = f(p)$. Transposition of the demand functions was necessary because the problem must be solved in terms of the x values (as some prices are assumed to remain constant and hence are already known). The inverse demand functions will be referred to, though, simply as demand functions.

Table 5.5 The Demand (Average Revenue) Functions

Activity	Demand Function ($p = a - bx$)
P1 : Telopea	$p_1 = 81.78 - 0.1220x_1$
P2 : Acacia	$p_2 = 61.67 - 0.6667x_2$
P3 : Passiflora	$p_3 = 55.00$
P4 : Banksia	$p_4 = 78.00$
P5 : Photinia	$p_5 = 60.00$
P6 : Eucalyptus	$p_6 = 56.13 - 0.5732x_6$
P7 : Stachyurus	$p_7 = 65.00$
P8 : Cistus	$p_8 = 60.25 - 0.9524x_8$
P9 : Protea	$p_9 = 100.00$
P10 : Tibouchina	$p_{10} = 55.00$
P11 : Azalea indica	$p_{11} = 71.96 - 0.5357x_{11}$
P12 : Viburnum	$p_{12} = 64.67 - 1.3333x_{12}$
P13 : Rhododendron	$p_{13} = 125.65 - 0.8696x_{13}$
P14 : Weigela	$p_{14} = 54.32 - 1.8000x_{14}$
P15 : Forsythia	$p_{15} = 73.01 - 1.3986x_{15}$
P16 : Azalea occidentalis	$p_{16} = 130.00$
P17 : Magnolia	$p_{17} = 110.00$
P18 : Callicarpa	$p_{18} = 60.00$
P19 : Hypericum	$p_{19} = 62.86 - 0.8276x_{19}$

Note: Own-price demand elasticities (E) may be derived from the linear demand functions as:

$$E = \frac{dx}{dp} \cdot \frac{p}{x} .$$

Using the means of the two quantity-price co-ordinates, the estimated demand functions imply elasticity coefficients of between -3.42 and -7.16 for eight of the 10 'downward-sloping' functions. The two remaining coefficients are somewhat higher, being -9.21 (for Viburnum) and -17.62 (for Telopea). Such high coefficients would appear reasonable *a priori*, especially those between -3 and -7. This is because purchasers are faced with a wide range of ornamental trees and shrubs from which to choose, and this type of plant is considered a luxury rather than a necessity, both of these factors suggesting a highly elastic demand.

The variable costs per 100 plants, which are assumed to apply no matter what quantity of the various plants are produced, are given in table 5.6.

Table 5.6 Variable Costs ($ per 100 plants)

Activity	Soil-Fumigating Materials	Weed, Pest and Disease Control	Marketing Materials	Total
P1 : Telopea		0.04	3.00	3.04
P2 : Acacia		0.02	2.48	2.50
P3 : Passiflora		0.02	2.48	2.50
P4 : Banksia		0.46	3.00	3.46
P5 : Photinia	0.09	0.81	3.00	3.90
P6 : Eucalyptus		0.02	2.48	2.50
P7 : Stachyurus	0.09	0.04	3.00	3.13
P8 : Cistus		0.02	2.48	2.50
P9 : Protea		0.46	3.00	3.46
P10 : Tibouchina		0.04	3.00	3.04
P11 : Azalea indica	0.09	0.46	3.00	3.55
P12 : Viburnum	0.09	0.04	3.00	3.13
P13 : Rhododendron		0.81	3.00	3.81
P14 : Weigela	0.09	0.04	3.00	3.13
P15 : Forsythia	0.09	0.04	3.00	3.13
P16 : Azalea occid.		0.06	3.00	3.06
P17 : Magnolia		0.81	3.00	3.81
P18 : Callicarpa	0.09	0.04	3.00	3.13
P19 : Hypericum	0.09	0.04	3.00	3.13

Note: All plants in the glasshouses are regularly sprayed with a fungicide mixture, whilst those planted in the open ground receive two to four applications per year of fungicide and/or insecticide mixtures. Weeds are controlled either by spraying (two to four applications per year) or by planting through polythene film.

5.7.4 Total net revenue objective function

The demand functions, which relate quantity sold with wholesale price, may be regarded as average gross revenue functions. To obtain net revenue, the nursery manager must deduct his variable costs from the gross revenue received from the sale of plants. That is,

average net revenue = average gross revenue - average variable costs.

As an example, average net revenue (dollars per 100 plants) from Acacia (activity P2) is given by:

$$(61.67 - 0.6667x_2) - 2.50$$
$$= 59.17 - 0.6667x_2$$

Hence the total net revenue from the sale of Acacias (dollars per 100 plants) is equal to average net revenue times output, or:

$$(59.17 - 0.6667x_2)x_2$$
$$= 59.17x_2 - 0.6667x_2^2.$$

Similar calculations were carried out for all activities and a total net revenue function of the form:

$$\text{total net revenue} = \sum_{j=1}^{19}\left[a_j x_j - b_j x_j^2\right]$$

was derived.[16]

---

16. Thus the objective function is of the form:

$$Z = \underline{a}\underline{x}' + \underline{x}B\underline{x}'$$

with matrix B negative semidefinite (since the value of b in some demand functions is zero, see table 5.5). The quadratic programming algorithm explained in Chapter 2, section 2.2.6, which requires the matrix B to be negative definite, can only be used if all demand curves are 'downward sloping'.

5.7.5 Overhead costs

Since the programme concerns a hypothetical nursery, little would be achieved by including hypothetical overhead costs in the objective function, since such costs play no role in the short-run decision making of the firm. The overhead costs would include, however, such items as depreciation, repairs and maintenance on glasshouses, sheds and equipment, wages of permanent staff, administration expenses and land rents, as well as those variable costs which are difficult to allocate amongst output, such as the costs of cutting material and re-usable plant containers, soils and fertiliser, and electric power used for heating the propagating house.

5.8 The Solution[17]

5.8.1 Introduction

The reader is referred to section 2.2.7 of Chapter 2, where an economic interpretation is presented of the values given by the final matrix of the quadratic programming problem. The four important groups of data obtained are the operation levels of basic activities, the marginal opportunity costs of including non-basic activities in the basis, the marginal revenue products of scarce resources and the quantities of resources which remain unused in the optimum solution.

---

17. The solution was obtained at Purdue University (Indiana) using a programme written by Professor W.V. Candler, who was the author's supervisor at that time.

5.8.2 Comparison of the optimum plan and the nurseryman's plan

5.8.2.1 Production levels and prices

It will be apparent from study of the objective function (section 5.7) that a solution to the quadratic programme will be given in terms of the x_j values. The price which the nurseryman can charge in order to sell the entire output of some activity P_j is found by substituting the appropriate value of x_j into the demand function for that activity.

For example, the value of x_2 (the production level of the Acacia activity) in the optimum basis is 26.40 hundred plants. To obtain the maximum price which the nurseryman can charge, this value is substituted into the demand function for Acacia as follows:

$$\begin{aligned} p_2 &= 61.67 - 0.6667x_2 \\ &= 61.67 - 0.6667(26.40) \\ &= 44.07 \end{aligned}$$

Therefore profit maximisation requires *inter alia* the production of 2,640 Acacia plants sold at a price of 44 cents each.

The prices and production levels of activities in the optimum solution are given in table 5.7, as well as the corresponding values for last season's production of the nineteen activities on the Taranaki nursery.[18]

The prices to be charged for six of the nine real activities in the optimum solution are similar to those of the nurseryman's plan.

---

18. Last season's production levels and prices for the nineteen activities on the Taranaki nursery will hereafter be referred to as the proposed management plan of the hypothetical nurseryman.

Table 5.7 Output and Prices for the Optimum and the Nurseryman's Plan

Activity			Optimum Solution		Nurseryman's Plan	
			Output (no. of plants)	Price ($/plant)	Output (no. of plants)	Price ($/plant)
P1	:	Telopea	2,974	0.78	3,100	0.78
P2	:	Acacia	2,640	0.44	1,000	0.55
P3	:	Passiflora	16,000	0.55	4,000	0.55
P4	:	Banksia	5,659	0.78	200	0.78
P5	:	Photinia	-	-	5,100	0.60
P6	:	Eucalyptus	1,405	0.48	1,070	0.50
P7	:	Stachyurus	-	-	1,200	0.65
P8	:	Cistus	595	0.55	550	0.55
P9	:	Protea	-	-	220	1.00
P10	:	Tibouchina	-	-	320	0.55
P11	:	Azalea indica	-	-	1,300	0.65
P12	:	Viburnum	-	-	350	0.60
P13	:	Rhododendron	904	1.18	1,800	1.10
P14	:	Weigela	-	-	240	0.50
P15	:	Forsythia	-	-	930	0.60
P16	:	Azalea occidentalis	3,769	1.30	300	1.30
P17	:	Magnolia	3,933	1.10	1,275	1.10
P18	:	Callicarpa	-	-	200	0.60
P19	:	Hypericum	-	-	950	0.55

Notes:
1. Activity P9 was included in the optimum solution, but at a zero level.
2. Although the total output of Passiflora last season was 4,200 plants (see section 5.5.2), only 4,000 were sold, the remainder being kept as stock plants to provide cutting materials.

It was assumed when the demand functions were estimated that four of these six activities could be produced at any level with no effect on the price to be charged. Hence the output of Passiflora (P3) has increased from 4,000 plants in the nurseryman's plan to 16,000 plants in the optimum plan with price remaining constant at $0.55 per plant, the output of Banksia (P4) has been increased from 200 to 5,659 plants with price constant at $0.78 per plant, output of Azalea occidentalis (P16) has increased from 300 to 3,769 plants at a constant price of $1.30 each, and the output of Magnolia (P17) has increased from 1,275 plants to 3,933 plants in the optimum solution, with price remaining at $1.10 per plant.

The differences between the output levels of two further activities in both the optimum and nurseryman's plans are sufficiently small to have had a negligible effect on prices. Thus the output of Telopea (P1) has been reduced from 3,100 plants in the nurseryman's plan to 2,974 in the optimum plan with price rising from $0.78 to $0.7815 per plant, and the output of Cistus (P8) has increased from 550 plants to 595 plants in the optimum plan with a corresponding fall in price of from $0.55 to $0.5458 per plant.

The prices to be charged for both Acacia (P2) and Eucalyptus (P6) have fallen, the former from $0.55 in the nurseryman's plan to $0.44 in the optimum solution (with output increasing from 1,000 to 2,640 plants), and the price of the Eucalyptus activity has fallen from $0.50 to $0.48 per plant with output increasing from 1,070 to 1,405 plants.

The Rhododendron (P13) price is higher in the optimum solution than in the nurseryman's plan, a rise from $1.10 to $1.18 per plant made possible by a reduction in output from 1,800 to 904 plants.

5.8.2.2 Comparison of net revenue

The total net revenue (total gross revenue less total variable costs) from the optimum solution is \$26,890.90 compared with a total net revenue from the nurseryman's plan of \$15,768.89. Thus quadratic programming has allowed the (hypothetical) nurseryman to increase his net revenue by over 70 percent.

5.8.2.3 Comparison of resource requirements

5.8.2.3.1 Glasshouse restraints

Table 5.8 shows the number of plants which will be occupying the propagating-house at various times in both plans, table 5.9 the number of plants occupying the growing-house, and table 5.10 gives similar data for the shade-house.

Table 5.8 Propagating-House Requirements of the Optimum and the Nurseryman's Plan (in numbers of plants)

Propagating-House Restraint	House Capacity	Optimum Plan	Nurseryman's Plan	Difference
R1 : July_2	14,000	2,974	3,100	- 126
R2 : August_1	14,000	6,750	4,384	+ 2,366
R3 : August_2	14,000	9,435	1,484	+ 7,951
R4 : September_1	14,000	9,352	6,942	+ 2,410
R5 : September_2	14,000	3,693	6,742	- 3,049
R6 : October_1	14,000	6,576	9,284	- 2,708
R7 : October_2	14,000	5,171	9,834	- 4,663
R8 : November_1	14,000	8,904	13,460	- 4,556
R9 : November_2	14,000	12,673	7,460	+ 5,213
R10 : December_1	14,000	4,673	3,970	+ 703
R11 : December_2	14,000	8,606	4,675	+ 3,931
R12 : January	14,000	8,606	4,525	+ 4,081
R13 : February	14,000	904	1,800	- 896

Note: A positive entry in the final column indicates that the optimum plan has a greater requirement for a resource than has the nurseryman's plan.

In neither plan is the propagating-house fully utilised, although it is almost filled to capacity during $November_2$ in the optimum plan, and during $November_1$ for the nurseryman's plan.

Table 5.9 Growing-House Requirements of the Optimum and the Nurseryman's Plan (in numbers of plants)

Growing-House Restraint	House Capacity	Optimum Plan	Nurseryman's Plan	Difference
R14 : $August_2$	10,000	2,974	3,100	- 126
R15 : $September_1$	10,000	3,776	1,284	+ 2,492
R16 : $September_2$	10,000	9,434	1,484	+ 7,950
R17 : $October_1$	10,000	6,064	1,856	+ 4,208
R18 : $October_2$	10,000	4,829	1,926	+ 2,903
R19 : $November_1$	10,000	10,000	3,620	+ 6,380
R20 : $November_2$	10,000	8,000	9,420	- 1,420
R21 : $December_1$	10,000	10,000	3,990	+ 6,010
R22 : $December_2$	10,000	10,000	3,070	+ 6,930
R23 : January	10,000	6,000	1,500	+ 4,500
R24 : $February_1$	10,000	9,702	3,225	+ 6,477
R25 : $February_2$	10,000	9,702	2,075	+ 7,627
R26 : March-April	10,000	4,837	3,075	+ 1,762

The growing-house is more restrictive on production, containing, for the optimum plan, the maximum of 10,000 plants during $November_1$ and all of December, as well as being almost filled to capacity during $September_2$ and February. The growing-house is generally under-utilised by the nurseryman's plan, being filled almost to capacity only during $November_2$.

Table 5.10 Shade-House Requirements of the Optimum and the Nurseryman's Plan (in numbers of plants)

Shade-House Restraint	House Capacity	Optimum Plan	Nurseryman's Plan	Difference
R27 : September	9,000	3,878	4,900	- 1,022
R28 : October - November$_1$	9,000	6,563	2,000	+ 4,563
R29 : November$_2$	9,000	2,000	500	+ 1,500
R30 : December$_1$	9,000	4,595	7,850	- 3,255
R31 : December$_2$	9,000	4,000	8,790	- 4,790
R32 : January$_1$	9,000	4,000	4,040	- 40
R33 : January$_2$	9,000	4,000	2,520	+ 1,480
R34 : February$_1$	9,000	4,000	1,000	+ 3,000
R35 : February$_2$	9,000	4,000	2,150	+ 1,850
R36 : March$_1$	9,000	2,000	1,650	+ 350

Part of the shade-house remains unused at all times in both plans, although this resource is almost used to capacity in December$_2$ by the nurseryman's plan.

Table 5.11 Land and Labour Requirements of the Optimum and the Nurseryman's Plan

Resource	Resource Supply	Optimum Plan	Nurseryman's Plan	Difference
R37 : labour (Aug.-Sept.)	526 hours	101.9 hours	125.3 hours	- 23.4
R38 : labour (Oct.-Nov.)	526 "	526.0 "	363.1 "	+ 162.9
R39 : labour (Dec.-Jan.)	526 "	209.0 "	153.5 "	+ 55.5
R40 : labour (Feb.-April)	789 "	498.9 "	185.9 "	+ 313.0
R41 : labour (May-July)	789 "	789.0 "	753.0 "	+ 36.0
R42 : land	5,400 sq.yds	5,400.0 sq.yds	5,383.4 sq.yds	+ 16.6

5.8.2.3.2 Labour and land restraints

The optimum plan makes use of all labour during the months of October, November, May, June and July, and requires more labour than the nurseryman's plan during all months except August and September, as indicated by table 5.11.

The optimum plan also requires the entire supply of land, with the nurseryman's plan using almost all of the land supply and nearly all of the labour available during May, June and July.

5.8.3 Resources in disposal

Disposal levels are not presented separately since they can be easily obtained from the four preceding tables by deducting the optimum plan's requirement for a resource from the total supply of that resource.

5.8.4 Value of resources

The optimum plan requires that the total supply of three growing-house restraints, two labour restraints and the land restraint be fully used, and as scarce resources their values (or shadow prices) are given by the solution.[19] Thus the values imputed to the six effective restraints are contained in table 5.12.

The marginal revenue products[20] of the two effective labour restraints are much higher than the going wage rate for labour (which

---

19. The shadow prices are the partial derivatives of the objective function with respect to the resource in question, and since the objective function is nonlinear, they will not remain constant up to some specified limit as is the case with linear programming.
20. Shadow prices may be interpreted as marginal value products in linear programming due to the underlying (perfect competition) assumption of constant marginal revenue. In the present model, where imperfections in market competition exist, marginal revenue need not be constant and the term marginal value product is replaced by marginal revenue product.

could be reasonably estimated as $1.00 per hour) so the nurseryman would be advised to hire extra labour. For example, an extra hour of labour hired during October and November will increase the value of the objective function by $8.89, compared with the cost of such labour of $1.00.

The other shadow prices may be interpreted in a similar manner. For profit maximisation, though, extra units of a resource should only be hired providing its marginal revenue product is greater than the cost of the resource unit.

Table 5.12 Value of Resources

Scarce Resource	Unit	Marginal Revenue Product
R19 : growing-house ($November_1$)	100 plants	$26.89
R21 : growing-house ($December_1$)	100 plants	$15.34
R22 : growing-house ($December_2$)	100 plants	$33.36
R38 : labour (October-November)	1 hour	$8.89
R41 : labour (May - July)	1 hour	$6.03
R42 : land	1 square yard	$1.69

5.8.5 The cost of including non-basic activities in the optimum plan

Nine real activities are excluded from the basis since their marginal revenue products (partial derivatives of the objective function with respect to the non-basic activities) are negative. Hence inclusion of any of these activities into the optimum solution will cause a reduction in the value of the objective function. The marginal opportunity costs of including non-basic activities in the optimum plan are given by table 5.13. For example, if the nurseryman wanted to include 100 Photinia plants in the optimum plan the value of the objective function would fall by almost $50, this being the reduction in profits from reducing the level(s) of some basic activity or activities so as to free resources and make the production of 100 Photinia plants possible.

Table 5.13 Marginal Opportunity Costs

Non-Basic Activity	Marginal Opportunity Cost (per 100 plants)
P5 : Photinia	$49.92
P7 : Stachyurus	$20.66
P10 : Tibouchina	$43.27
P11 : Azalea indica	$14.48
P12 : Viburnum	$53.46
P14 : Weigela	$45.79
P15 : Forsythia	$27.10
P18 : Callicarpa	$23.35
P19 : Hypericum	$20.49

5.9 Comments on the Quadratic Programming Model and Results

5.9.1 Extension of the model to represent actual situations

Since the model has dealt only with a hypothetical situation it was not possible to discuss the results with the nurseryman with a view to adoption of the plan. A few comments were obtained, however, on additions to the model which may be necessary in practical applications. Firstly, the model included only plants propagated during the Spring and Summer (so as to reduce the number of restraints). As a result, the glasshouses and shade-house are unoccupied for about five months of the year. In practice, plants are propagated all year round so that the glasshouses will always be occupied. Hence a 'real life' model would differ from that of the present chapter in that activities would be included to allow plants to be propagated at any time of the year, and additional restraints would be necessary to represent the availability of glasshouses over the entire year.

It may be necessary to differentiate between different types of labour, since some operations (for example making cuttings) are often carried out by women, whilst others (for example soil sterilisation and spraying) usually make use of male labour.

The amounts of cutting material available from the existing stool-beds are likely to form additional restrictions in a 'real life' application. Such restraints would simply take the form of an upper bound on the production level of the activity in question, and the values which could be imputed to such restraints would indicate to the nurseryman the extent to which profits would rise if he increased the number of stock plants in the stoolbed.

5.9.2 Incorporation of demand cross-effects into the quadratic programme

Although the assumption was made that cross-effects did not exist in this case, the high price elasticity coefficients (indicated in a note to table 5.5) would suggest that such an assumption is unrealistic and that significant substitution relationships may exist. Such cross-effects may be easily included in the quadratic programme however, provided that estimates of the cross-elasticities of demand are available.

For example, the demand function for Azalea indica (P11) yields an own-price elasticity coefficient of -7.14 at the means of the data values. For the purposes of this illustration, it will be assumed that the following cross-elasticities (which indicate the responsiveness of demand for Azalea indica to changes in the prices of some other plants) have been derived:

cross-elasticity with respect to Viburnum (P12) = + 2.0,

" " " " " Rhododendron (P13) = - 0.5,

" " " " " Forsythia (P15) = + 1.5.

(Such coefficients would indicate that both Viburnum and Forsythia are substitutable for Azalea, but Azalea is complementary with Rhododendron. The latter may occur, since Azaleas and Rhododendrons are often planted together).

The inverse demand function for Azalea indica, originally

$$p_{11} = 71.96 - 0.5357x_{11} \text{ , becomes}$$

$$p_{11} = 77.03 - 0.5357x_{11} - 0.4041x_{12} + 0.0356x_{13} - 0.3282x_{15} \text{ ,}$$

when the cross-elasticities are included in the estimation of the demand function.[21]

Therefore the disposal activity y_{11}[22] which would have been incorporated in the initial simplex tableau as:

$$-71.96 = -1.0714x_{11} - (A'\underline{\lambda}') + y_{11}$$

using the original demand function, would become:

$$-77.03 = -1.0714x_{11} - 0.4041x_{12} + 0.0356x_{13} - 0.3282x_{15} - (A'\underline{\lambda}') + y_{11}$$

when the cross-effects are included in the demand functions.

5.9.3 Problems associated with the demand curves

Apart from the problem of estimating the demand curves as accurately as possible, the profit-maximising point on a linear demand curve may be so

---

21. See Louwes, Boot and Wage, op.cit., p.312.
22. See Chapter 2, equation (2-27).

far removed from the price-quantity co-ordinate for the past season's output as to give an unrealistic price estimate. This problem could be lessened by restricting the solution to only a specified range of the demand curves, although it would not be known whether such restrictions are necessary until the problem is solved. Fortunately, price changes in the present model were relatively small and the new prices quite reasonable.

5.9.4 Shadow prices and resource disposal levels

As was found with the linear programming models of earlier chapters, considerable interest is shown in the values of resources, the cost of including non-basic activities in the solution, and the quantities of resources left unused. Such data can allow the nurseryman to plan further additions to his resource supplies, or perhaps the consideration of new activities to make use of unused resources.

CHAPTER 6

RISK PROGRAMMING AND FRESH VEGETABLE PRODUCTION

6.1 Introduction

Since market prices of fresh vegetables may be expected to fluctuate markedly even from day to day, many fresh vegetable producers often attempt to reduce the consequent fluctuations in their incomes by cultivating crops which may be expected to realise, on average, a rather low but stable income. However, some growers are less averse to risk and may wish to cultivate crops whose prices may vary greatly, in the hope of receiving a high but fluctuating income.

Risk programming[1/] was considered a most appropriate technique with which to derive management plans under the above conditions with both market auction prices and crop yields subject to stochastic variability.[2/]

---

1. Chapter 2, section 2.3, discusses the risk programming concept and the formulation of risk aversion problems as quadratic programmes.
2. Variation in yields from the expected levels will cause variations in the labour input required for harvesting the crops, so that stochastic variability will also exist in the labour input-output coefficients. Such variability in the input-output coefficients may also exist for other reasons (Chapter 2, section 2.3.2) but all such error terms attached to the estimated input-output coefficients are not taken into account by the present model.
Heady and Candler, op.cit, p.557, mention that concentrating on income variability alone will simplify the risk aversion problem, but this approach is used since (a) errors associated with income variability are the only errors on which reliable data is available, and (b) discussion with the grower indicated that, as far as he was concerned, variability in income is the major component of risk in fresh vegetable production.

A grower who wishes to insure against income variability to some extent may select a plan with only a moderate level of expected income but also a low income variability. A grower who places less importance on income stability may select a plan with a high expected income but also a high degree of income variability.

A risk programming model was constructed for a Wanganui fresh vegetable producer and the solution was to have been obtained at Purdue University, U.S.A., using a programme written by Professor W.V. Candler, who was the author's supervisor when the model was formulated. Unfortunately, due to delays in correspondence and to a lesser extent computational difficulties, the results were not available in time to include in the thesis.[3]

Hence the present chapter will discuss the formulation of the risk programming problem, dealing only briefly with those aspects, such as a description of activities, restraints and input-output coefficients, which are similar to a linear programming model. Finally, the maximum risk solution will be presented and compared with the cropping programme proposed by the grower.

6.2 The Restraints

6.2.1 Land (restraints R1 and R2)

The total area of the holding is 96 acres of which 90 acres are available for cropping, with the remaining land occupied by buildings,

---

3. The results, when available, will be published in a Discussion Paper by the Department of Agricultural Economics and Farm Management, Massey University.

paths and headlands. In addition to his own land, the grower intends to lease 19 acres of land from a nearby farmer.

Since the grower's choice of production activities permits little opportunity to crop land more than once during a year, the land restraints did not require division into time periods of less than one year (as was necessary for the Otaki horticultural holding of Chapter 3). Thus the land resources can be represented adequately by two restraints, one for freehold land and one for leased land. The restraints and their supplies are:

R1 : freehold land - 90 acres;

R2 : leased land - 19 acres.

6.2.2 Labour availability (restraints R3 - R14)

The permanent labour supply consists of the owner and two men. One man works 48 hours per week and the others 60 hours per week, giving a total labour supply of 727 hours per month.[4]

As in past seasons, the grower does not intend to crop the entire 90 acre block of freehold land during the coming season. His policy is to crop as much land as his existing labour force will allow, with all remaining freehold land sown into grass by a contractor and grazed with sheep.

Thus from the total labour supply, the requirements of the sheep were deducted to give the supply of labour available to the vegetable crops.[5] No deduction, however, was made for holidays or overhead work,

---

4. 1 month = 4.33 weeks.
5. It has been assumed that any plan will have sufficient land in disposal to carry 300 sheep. (No labour needs to be supplied to sow the green-crop since this is carried out by a contractor). Alternatively, the problem could have been formulated including a forage activity, but this was considered unnecessary since the past experience of the grower indicated that sufficient land to carry 300 sheep would always be available.

so that sufficient labour must remain in disposal to cover these requirements if a plan is to be feasible.

The labour restraints and supplies are given in table 6.1.

Table 6.1 Labour Restraints and Supplies (hours)

Labour Restraint		Total Supply	Sheep Requirement	Supply for Crops
R3 :	January	727	17	710
R4 :	February	727	17	710
R5 :	March	727	17	710
R6 :	April	727	17	710
R7 :	May	727	17	710
R8 :	June	727	17	710
R9 :	July	727	17	710
R10 :	August	727	81	646
R11 :	September	727	97	630
R12 :	October	727	33	694
R13 :	November	727	65	662
R14 :	December	727	33	694

6.2.3 Rotations and cropping limits (restraints R15 - R24)

Crop rotation is practised only on the freehold land, since leased land is ploughed out of pasture and cropped for only one year.

Of the 90 acres of freehold land, 30 acres are suitable for all crops with the exception of Spring carrots, 40 acres are suitable for all crops including Spring carrots, and the remaining 20 acres are suitable only for the Summer Cucurbita crops as this land cannot be cultivated during Winter. Thus all 90 acres are suitable for Cucurbita crops, 70 acres are suitable for Brassica crops and 40 acres are suitable for Spring carrots and parsnips.

The grower requires a three-year rotation for Cucurbita crops and Brassica crops, and a two-year rotation for carrots and parsnips. Therefore no more than 30 acres (that is, one-third of 90 acres) may be sown with Cucurbita crops, no more than 23 acres may be planted in Brassica crops, and no more than 20 acres may be sown with Spring carrots and parsnips.

The grower's past marketing experience led him to impose further limits on the acreages of individual crops, and all cropping restraints are given in table 6.2.

Table 6.2 Cropping Restraints

Restraint	Supply (acres)
R15 : Cucurbita crops (on freehold land)	30
R16 : butternut pumpkin	15
R17 : carrot and parsnip (on freehold land)	20
R18 : Brassica crops	23
R19 : Winter Brassica crops	10
R20 : Spring Brassica crops	10
R21 : cauliflower	15
R22 : cabbage	15
R23 : Winter lettuce	5
R24 : Spring lettuce	5

6.3 The Activities

6.3.1 Production on freehold and leased land

Of the eleven crops included in the risk programme, five (Spring carrot, parsnip and the three Cucurbita crops) may be grown on either the freehold or leased land, with crop management being identical on either land block.

A brief description of the activities is given in following sections.

6.3.2 Spring carrot (P1 and P12) and parsnip (P2 and P13)

Three sowings are made, in April, August and September. Harvesting is continual from mid-November until the end of March, over which time the grower augments his labour force with three female workers.

6.3.3 Crown pumpkin (P3 and P14)

The seed is drilled in October, and the entire crop harvested in April when it is necessary to hire two extra men. The crop is stored and marketed from mid-April until the end of August.

6.3.4 Buttercup pumpkin (P4 and P15) and butternut pumpkin (P5 and P16)

Crop husbandry is similar to, but commences a fortnight before, that of crown pumpkin. Buttercup and butternut pumpkins do not keep as well as crown pumpkin in store, so are harvested and marketed continuously from February until April.

6.3.5 Brassica activities

6.3.5.1 Winter cauliflower (P6) and Winter cabbage (P8)

Six plantings are made, from February until the end of May. Both crops are harvested from April until August.

6.3.5.2 Spring cauliflower (P7) and Spring cabbage (P9)

Three plantings are made, in June, July and August. Harvesting extends from September until the end of November.

6.3.6 Winter lettuce (P10)

One planting is made during April, and harvesting is from mid-July to the end of August.

6.3.7 Spring lettuce (P11)

Three plantings are made, two in July and one in August. The crops are harvested successively, from September until the end of December.

6.4 Input-Output Coefficients[6]

Derivation of the input-output coefficients will not be discussed, since calculation of the coefficients is similar to that described in Chapter 3, section 3.6.

Table 6.3 contains the matrix of resource supplies and input-output coefficients, where each row of the matrix represents a restraint and each column contains the requirements per acre of an activity for each resource.

---

6. This section refers to the requirements of the production activities for the supplies of the various resources. Other coefficients of the quadratic programming matrix, such as net revenue variances and covariances, are discussed in later sections.

Table 6.3

Activity Resource Requirements and Resource Supplies

Restraints	unit ↓ / unit →	B	Relationship	Spring Carrot P1 1 acre	Parsnip P2 1 acre	Crown Pumpkin P3 1 acre	Buttercup Pumpkin P4 1 acre	Butternut Pumpkin P5 1 acre	Winter Cauliflower P6 1 acre	Spring Cauliflower P7 1 acre	Winter Cabbage P8 1 acre	Spring Cabbage P9 1 acre	Winter Lettuce P10 1 acre	Spring Lettuce P11 1 acre	Spring Carrot P12 1 acre	Parsnip P13 1 acre	Crown Pumpkin P14 1 acre	Buttercup Pumpkin P15 1 acre	Butternut Pumpkin P16 1 acre
R1 : Freehold land	1 acre	90	≯	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0					
R2 : Leased land	"	19	≯												1.0	1.0	1.0	1.0	1.0
R3 : Labour - January	1 hour	710	≯	25.2	25.2	15.8			7.5	6.9	7.5	6.9			25.2	25.2	15.8		
R4 : February	"	710	≯	25.2	25.2		6.0	6.0	7.4		7.4				25.2	25.2		6.0	6.0
R5 : March	"	710	≯	25.2	25.2		6.0	6.0	14.2		14.2		2.5		25.2	25.2		6.0	6.0
R6 : April	"	710	≯	4.1	4.1	24.0	6.0	6.0	21.4		28.0		50.1		4.1	4.1	24.0	6.0	6.0
R7 : May	"	710	≯	1.5	1.5	2.0	6.0	6.0	27.0	1.2	33.6	1.2	24.0		1.5	1.5	2.0	6.0	6.0
R8 : June	"	710	≯	0.2	0.2	2.0			14.2	14.7	20.8	14.7	4.0	2.5	0.2	0.2	2.0		
R9 : July	"	710	≯	1.9	1.9	2.0			13.4	14.3	20.0	14.3	24.0	34.2	1.9	1.9	2.0		
R10 : August	"	646	≯	4.6	4.6	2.0			13.4	13.0	20.0	13.0	42.5	37.8	4.6	4.6	2.0		
R11 : September	"	630	≯	3.4	3.4	6.1	5.1	5.1		23.7		34.8		20.6	3.4	3.4	6.1	5.1	5.1
R12 : October	"	694	≯	1.5	1.5	11.8	11.3	11.3		22.2		33.3		20.0	1.5	1.5	11.8	11.3	11.3
R13 : November	"	662	≯	12.8	12.8	1.6	16.9	16.9		22.2		33.3		19.2	12.8	12.8	1.6	16.9	16.9
R14 : December	"	694	≯	25.2	25.2	14.0	14.0	14.0							25.2	25.2	14.0	14.0	14.0
R15 : Cropping limits - Cucurbita crops	1 acre	30	≯			1.0	1.0	1.0											
R16 : butternut pumpkin	"	15	≯					1.0											1.0
R17 : carrot (P1) and parsnip (P2)	"	20	≯	1.0	1.0														
R18 : Brassica crops	"	23	≯						1.0	1.0	1.0	1.0							
R19 : Winter Brassica	"	10	≯						1.0		1.0								
R20 : Spring Brassica	"	10	≯							1.0		1.0							
R21 : cauliflower	"	15	≯						1.0	1.0									
R22 : cabbage	"	15	≯								1.0	1.0							
R23 : Winter lettuce	"	5	≯										1.0						
R24 : Spring lettuce	"	5	≯											1.0					

6.5 The Objective Function

6.5.1 Introduction

The objective function of a risk programming problem is designed to allow expected income to be maximised for any level of income variance,[7/] and may be written:

$$Z = \phi\underline{\mu}\underline{x}' + \underline{x}B\underline{x}' \text{ a maximum.} \quad (6\text{-}1)$$

Information on prices received and crop yields for the three years 1963/64, 1964/65 and 1965/66 was available from the grower, but that of the most recent year (1966/67) was incomplete when the model was formulated since not all crops had been harvested.[8/] By deducting variable costs from the gross revenue of each activity over the three years, the net revenue from each crop in each year was obtained. The mean (expected) net revenues (that is, the vector $\underline{\mu}$ in 6-1) and the matrix of net revenue variances and covariances (the negative of B in 6-1) for all activities were then calculated, allowing the expected (mean) income and income variance of any farm plan to be derived.

6.5.2 Activity prices and yields

All auctioneering firms' Account Sales documents for the three years had been kept by the grower and since all produce had been sold to auction firms, these provided accurate data on the number of cases of any

7. See Chapter 2, section 2.3.5.
8. Variable costs, with the exception of the cost of containers which will vary in proportion to the size of the crop, have been assumed constant from year to year. This appears reasonable since the effects of likely increases in costs causing variations in net revenue can be expected to be much less than those resulting from crop yield and price fluctuations.

crop sold over each year and the average price per case received. The grower was only required to estimate the number of acres of each crop grown so that data on the number of cases sold could be converted to the marketable yield per acre.

Table 6.4 Wholesale Price and Quantity Sold - Crown Pumpkin

	1964		1965		1966	
Half-month	Quantity Sold (cases)	Average Price ($/case)	Quantity Sold (cases)	Average Price ($/case)	Quantity Sold (cases)	Average Price ($/case)
$April_1$	6	0.742	-	-	-	-
$April_2$	141	1.296	233	0.948	22	1.084
May_1	440	1.150	521	0.892	99	1.008
May_2	252	1.258	234	0.544	279	0.862
$June_1$	180	1.108	138	0.648	177	0.856
$June_2$	197	1.034	223	0.836	195	0.818
$July_1$	244	1.026	323	1.008	183	0.624
$July_2$	321	1.490	346	1.068	120	1.076
$August_1$	194	1.668	157	0.736	180	1.660
$August_2$	119	2.536	56	0.600	82	1.394

Note: The subscripts 1 and 2 denote the first and second half of a month, respectively.

Table 6.4 illustrates the type of data obtained for all crops. In this table, the quantity sold and price received for crown pumpkin is given by half-month periods.

Next, the total quantity sold in any one year was divided by the acreage of the crop grown to give the yield per acre. To obtain the average price received by the grower, it was necessary to deduct the wholesaler's commission of 10 percent of the value of all produce sold, from the average price received over the year. These calculations are shown for crown pumpkin, together with the gross revenue figures, in table 6.5.

Table 6.5 Prices, Yields and Gross Revenue - Crown Pumpkin

Year	Quantity Sold (cases)	Acreage	Cases Marketed (per acre)	Average Price (per case)	Average Price Less Commission ($ per case)	Gross Revenue ($ per acre)
1963/64	2094	6	349	$1.322	$1.190	$415.32
1964/65	2231	12	186	$0.866	$0.780	$145.08
1965/66	1337	10	134	$0.996	$0.896	$120.06

Notes:
1. Quantities sold in each year are totalled from table 6.4.
2. Average price in each year is the average of the half-month prices of table 6.4 weighted by the quantity sold in each half-month period.
3. Gross revenue is the average price net of the wholesaler's commission multiplied by the number of cases marketed per acre.

A complete account of quantities sold, acreages planted[9] and yields from all activities in each of the three years, is to be found in Appendix B, table B.1. Data did not exist for Winter lettuce (1964/65), parsnip (1965/66) and butternut pumpkin (1963/64) since these crops were not grown.

---

9. Unfortunately, the grower was unable to give reliable estimates of the acreage of both Winter and Spring Brassica and lettuce crops grown in past years, although he could give the total acreage of Brassica and lettuce. It was necessary, therefore, to aggregate Winter and Spring crops of Brassica and lettuce when determining the yield per acre, so that for these activities the yield from both the Winter and Spring crops will be the same.

All yield, average wholesale price (net of commission), and gross revenue data for all activities in each of the three years is given in Appendix B, table B.2

6.5.3 Variable costs

Only one component of the activity variable costs has been assumed to have associated stochastic error terms. This item is the cost of containers in which produce is marketed, since the cost will be directly related to the actual quantity sold. All other items have been costed at 1967 prices.

The variable costs for each activity in each of the three years are to be found in Appendix B, table B.3.

6.5.4 Activity net revenues

Activity net revenues in each of the three years were found by deducting each year's variable costs from the corresponding year's gross revenue. For example, the net revenue per acre from crown pumpkin in each of the three years is derived in table 6.6.

Table 6.6 Net Revenue ($ per acre) - Crown Pumpkin

Year	Gross Revenue	Variable Costs	Net Revenue
1963/64	415.32	86.44	328.88
1964/65	145.08	78.28	66.80
1965/66	120.06	75.68	44.38

Note: The large reduction in net revenue in 1964/65 and 1965/66 was due to high storage losses.

The net revenues received from each activity over the past three years are given in table 6.7, along with the mean (expected) net revenues.

Table 6.7 Activity Net Revenues ($ per acre)

Activity	Net Revenue			
	1963/64	1964/65	1965/66	Mean
P1 & P12 : Spring carrot	664.91	469.11	1047.81	727.26
P2 & P13 : parsnip	334.60	282.40	no crop	308.50
P3 & P14 : crown pumpkin	328.88	66.80	44.38	146.68
P4 & P15 : buttercup pumpkin	416.06	183.72	318.46	306.08
P5 & P16 : butternut pumpkin	no crop	202.46	350.86	276.66
P6 : Winter cauliflower	457.60	546.68	712.78	572.36
P7 : Spring cauliflower	401.10	460.86	587.68	483.22
P8 : Winter cabbage	471.82	392.48	653.84	506.04
P9 : Spring cabbage	439.38	343.76	662.86	482.00
P10: Winter lettuce	931.18	no crop	2015.40	1473.30
P11: Spring lettuce	1072.18	736.54	471.74	760.16

6.5.5 Net revenue variances and covariances[10/]

6.5.5.1 The variance of activity net revenues

An unbiassed estimate of the population variance computed from a random sample is given by:[11/]

$$\sigma_i^2 = \frac{\sum_k (c_i - \mu_i)^2}{n-1} \qquad (6\text{-}2)$$

10. The measurement of variance and covariance, and the use of such measures as an index of risk, are discussed in Chapter 2, sections 2.3.4 and 2.3.5.
11. See Chapter 2, equation (2-34).

For example, the variance of crown pumpkin net revenue over the three years is equal to:

$$\frac{(328.88 - 146.68)^2 + (66.80 - 146.68)^2 + (44.38 - 146.68)^2}{3-1}$$

$= 25{,}021.52$

The variance of net revenue for each activity was calculated similarly (see table 6.8). Since the units of variance are squared deviations, the net revenue standard deviations 12/ are also presented in table 6.8. This measure refers to the same units in which the deviations occur (that is, dollars per acre). 13/

Table 6.8 Variance of Activity Net Revenues

Activity	Mean Net Revenue ($ per acre)	Variance	Standard Deviation ($ per acre)
P1 & P12 : carrot	727.26	86,617.52	294.31
P2 & P13 : parsnip	308.50	1,362.44	36.92
P3 & P14 : crown pumpkin	146.68	25,021.52	158.18
P4 & P15 : buttercup pumpkin	306.08	13,610.40	116.66
P5 & P16 : butternut pumpkin	276.66	11,011.28	104.93
P6 : Winter cauliflower	572.36	16,773.56	129.51
P7 : Spring cauliflower	483.22	9,077.76	95.28
P8 : Winter cabbage	506.04	17,955.88	134.00
P9 : Spring cabbage	482.00	26,818.56	163.76
P10: Winter lettuce	1473.30	587,766.52	766.66
P11: Spring lettuce	760.16	96,550.28	300.92

12. The standard deviation of a set of observations is the square root of their variance.
13. Also, if the observations are normally distributed, 95 percent of all net revenue observations should fall within plus or minus two standard deviations from the mean net revenue.

6.5.5.2 The covariance of activity net revenues

Covariance indicates how closely two variables, in this case the net revenues from two activities, move together and is given by:[14]

$$\sigma_{ij} = \frac{\sum_k (c_i - \mu_i)(c_j - \mu_j)}{n-1} \qquad (6\text{-}3)$$

As an example, the covariance of Winter cabbage and Winter cauliflower net revenues over the three years under review is given by:

$$\begin{bmatrix} (471.82 - 506.04)(457.60 - 572.36) \\ + (392.48 - 506.04)(546.68 - 572.36) \\ + (653.84 - 506.04)(712.78 - 572.36) \end{bmatrix} \begin{bmatrix} \frac{1}{3-1} \end{bmatrix}$$

$$= 13{,}798.72$$

The positive covariance estimate indicates that net revenue from Winter cabbage and Winter cauliflower tends to move in the same direction from year to year.

Covariance estimates could not be obtained from equation (6-3) when one of the activities was either parsnip, butternut pumpkin or Winter lettuce, since these crops were grown during only two of the three years. Hence the net revenue covariances involving these crops were estimated as follows:

By definition : $\sigma_{ij} = r_{ij}\, \sigma_i\, \sigma_j$ (6-4)

where r_{ij} is the coefficient of correlation between the net revenue from activities i and j, and

σ_i and σ_j are the net revenue standard deviations for activities i and j respectively.

---

14. See Chapter 2, equation (2-35).

$$\text{Therefore,} \quad \overset{\text{estimated}}{\sigma_{ij}} = \bar{r}_{ij}\,\sigma_i\,\sigma_j \qquad (6\text{-}5)$$

where $\bar{r}_{ij}$ is the mean value of all net revenue correlation coefficients which could be calculated (for $i \neq j$),

was used to derive the covariance of net revenue for all pairs of activities which include at least one of the parsnip, butternut pumpkin or Winter lettuce activities.

6.5.5.3 The variance - covariance matrix

The matrix of all net revenue variance and covariance estimates is to be found in Appendix B, table B.4.

6.5.6 Overhead costs

The overhead costs of the holding (with the exception of wages paid to the permanent staff which were costed at the 1967 rates) were taken from the grower's 1966/67 financial accounts, and are presented in table 6.9. The wages item does not include the owner's drawings which are assumed to be withdrawn from farm profits.

Table 6.9 Overhead Costs ($)

Item	Cost
Wages of permanent staff	4,217
Rent and rates	1,486
Depreciation	886
Repairs and maintenance	458
Motor expenses	494
Sundry	184
Total overhead cost	7,725

6.6 Construction of the Initial Risk Programming Matrix

The risk programming matrix is similar to the initial simplex tableau for quadratic programming described in Chapter 2, page 23, table 2.1. The $-c_1$ to $-c_n$ coefficients of the B column are the negative values of the expected net revenues from each activity (see table 6.7). The remaining (b_1 to b_m) coefficients of the B column are the resource supplies of section 6.2.

The sub-matrix A of table 2.1 would contain the coefficients of table 6.3, and the negative of the transpose of A forms the sub-matrix -A' of the quadratic programming tableau.

The sub-matrix 2B of table 2.1 contains coefficients derived from the variance - covariance matrix. Firstly, the complete variance - covariance matrix is obtained by repeating the north-east section of the matrix in the opposite section. Secondly, all coefficients are multiplied by -1,[15] and thirdly, all coefficients in the north-west to south-east diagonal are multiplied by +2.[16] The resulting matrix may then be inserted as sub-matrix 2B in the quadratic programming simplex tableau.

The whole series of preferred plans[17] is then found by using a parametric quadratic programming algorithm[18] and varying the weight (or

---

15. The objective function is

$$Z = \phi \underline{\mu x}' + \underline{x} B^* \underline{x}', \text{ where } B^* = -B, \text{ and B is the variance - covariance matrix.}$$

See Chapter 2, section 2.3.5.

16. This is equivalent to partially differentiating the lagrangian objective function with respect to the elements of $\underline{x}$. See Chapter 2, section 2.2.4.

17. That is, the plans corresponding to maximum expected income for each level of income variance, or all points on the curve OA of figure 2.2, Chapter 2 (page 29).

18. See Chapter 2, page 33, footnote 37.

risk-aversion parameter) applied to the c_1 to c_n values of the B column.

6.7 The Maximum Risk Solution

6.7.1 Introduction

It was mentioned in section 6.1 that difficulties arose when the risk programming problem was being solved and that the results were not available at the time of writing.

However, the plan which maximised expected income with complete indifference to risk was obtained by treating the problem as a linear programme with the expected net revenues in the objective function. This plan will now be discussed and compared with the grower's proposed plan.

6.7.2 Comparison of the maximum risk solution with the grower's plan

6.7.2.1 Comparison of activity levels

Table 6.10 gives the levels of activities in both plans. The grower has included the Spring carrot, buttercup pumpkin and butternut pumpkin activities on the leased land, whereas 0.10 acre of Spring carrots is the only leased land cropping in the optimum plan. The maximum risk plan includes a greater acreage of both Winter and Spring cauliflower, but a smaller acreage of Spring cabbage and none of the Winter cabbage activity, compared with the grower's plan. The maximum risk solution also requires the production of a greater quantity of both Winter and Spring lettuce than does the grower's plan.

Table 6.10 Comparison of Activity Levels Between the Maximum Risk and the Grower's Plan

Activity	Level in Maximum Risk Solution (acres)	Level in Grower's Plan (acres)	Difference (acres)
P1 : Spring carrot	20.00	8.00	+ 12.00
P2 : parsnip	-	2.00	- 2.00
P3 : crown pumpkin	4.81	4.50	+ 0.31
P5 : buttercup pumpkin	8.58	-	+ 8.58
P6 : Winter cauliflower	9.82	6.00	+ 3.82
P7 : Spring cauliflower	5.18	4.00	+ 1.18
P8 : Winter cabbage	-	4.00	- 4.00
P9 : Spring cabbage	2.61	3.00	- 0.39
P10 : Winter lettuce	5.00	2.00	+ 3.00
P11 : Spring lettuce	2.61	1.00	+ 1.61
P12 : Spring carrot	0.10	10.00	- 9.90
P15 : buttercup pumpkin	-	7.00	- 7.00
P16 : butternut pumpkin	-	2.00	- 2.00

Note: A positive entry in the fourth column indicates that the level of an activity in the maximum risk solution is greater than that in the grower's plan.

6.7.2.2 Expected net revenue and net revenue variance

Table 6.11 gives the expected net revenue, the variance and standard deviation of net revenue and the limit below which net revenue can be expected to fall in only one year in 40.[19]

Table 6.11 Expected Net Revenue

	Maximum Risk Solution	Grower's Plan
Expected net revenue	$36,681.38	$29,607.58
Net revenue variance	79,729.12	61,887.30
Net revenue standard deviation	$8,929.11	$7,866.85
Lower net revenue confidence limit	$18,823.16	$13,873.88

Note: Variance is measured in units of 1,000.

Should the grower adopt the maximum risk plan, he can expect, in the long run, an average net revenue of $36,681.38, which exceeds the average net revenue from the grower's plan by $7,073.80.

However, the average net revenue may not be realised in each year, and actual realised net revenue may fall somewhere between[20] $18,823.16 and $54,539.60 in 19 out of 20 years.[21] Therefore, in only one year in 40 is realised net revenue expected to fall below $18,823.16.

---

19. Assuming that annual net revenues are normally distributed, only 5 percent of observations will fall outside plus or minus two standard deviations from the mean value. Therefore only $2\frac{1}{2}$ percent (or 1 in 40) will fall below the mean value less twice the standard deviation.
20. Assuming that past years' prices and yields give an accurate estimate of future price and yield variability.
21. That is, the average net revenue plus or minus twice the standard deviation of net revenue.

By deducting the overhead costs of the holding ($7,725, from table 6.9) from average net revenue, an estimate of average (pre-tax) farm profits is obtained. Such farm profits amount to $28,956.38 for the maximum risk plan, or 32.3 percent greater than average farm profits of the grower's plan.

6.7.2.3 Comparison of resource requirements

The requirements of both plans for the land and labour resources are compared in table 6.12. In the maximum risk solution all crops except 0.1 acre of Spring carrots are grown on the freehold land, whereas the grower intended to crop all 19.0 acres of the leased land.

The maximum risk solution requires more labour than does the grower's plan during 10 months of the year, but it is interesting to note that the former plan uses 83 percent of the available labour supply compared with 76 percent required by the grower's plan. In other words, the increase in labour input required to put the optimum plan into operation is small when compared with the expected increase in average (pre-tax) farm profits of $7,074.

Labour disposal in the optimum plan is greatest during May, June, July, September and October, and the grower believed that sufficient labour would be available (after meeting the requirements of the cropping activities) for all overhead work.

6.7.3 Value of resources

The shadow prices imputed to the scarce resources of the maximum risk solution are given in table 6.13. The value imputed to the Winter lettuce cropping limit of $689.26 per acre reflects the high profitability of this crop, but growing extra Winter lettuce may also be expected to increase income variability and hence the risk attached to the consequent cropping programme. (See table 6.8, which indicates the high net revenue variance of this activity).

Table 6.12 Resource Requirements of the Maximum Risk and the Grower's Plan

Resource	Supply	Maximum Risk Plan	Grower's Plan	Difference
Land (acres):				
R1 : freehold land	90.0	58.6	34.5	+ 24.1
R2 : leased land	19.0	0.1	19.0	- 18.9
Labour (hours):				
R3 : January	710.0	710.0	698.4	+ 11.6
R4 : February	710.0	630.7	632.0	- 1.3
R5 : March	710.0	710.0	705.0	+ 5.0
R6 : April	710.0	710.0	584.6	+ 125.4
R7 : May	710.0	485.8	445.8	+ 40.0
R8 : June	710.0	294.2	294.8	- 0.6
R9 : July	710.0	500.0	389.7	+ 110.3
R10 : August	646.0	646.0	475.2	+ 170.8
R11 : September	630.0	408.8	361.2	+ 47.6
R12 : October	694.0	438.0	393.5	+ 44.5
R13 : November	662.0	662.0	623.2	+ 38.8
R14 : December	694.0	694.0	693.0	+ 1.0

The high labour shadow prices indicate that the grower should consider hiring extra labour. For example, should he add an extra man to his permanent staff, the necessary alterations to the labour resource supplies can be made and the new optimum solution obtained.

Table 6.13 Value of Resources

Restraint	Shadow Price ($)
Labour:	
R3 : January	0.60 per hour
R5 : March	16.58 " "
R6 : April	1.26 " "
R10 : August	15.99 " "
R13 : November	8.11 " "
R14 : December	4.43 " "
Cropping limits:	
R17 : carrot (P1) and parsnip (P2)	0
R21 : cauliflower	91.20 per acre
R23 : Winter lettuce	689.26 " "

6.7.4 Marginal opportunity costs

The extent to which expected net revenue would fall should non-basic activities be included in the cropping programme, is given in table 6.14.

Table 6.14 Marginal Opportunity Costs

Non-Basic Activity	Shadow Price ($ per acre)
P2 : parsnip	418.76
P5 : butternut pumpkin	29.42
P8 : Winter cabbage	88.96
P13 : parsnip	418.76
P14 : crown pumpkin	0
P15 : buttercup pumpkin	0
P16 : butternut pumpkin	29.42

Including the crown pumpkin (P14) and buttercup pumpkin (P15) activities in the plan would have no effect on expected net revenue since these crops (which are sown in leased land) would simply replace the similar crops grown in freehold land.

Should the grower include an acre of parsnip in the plan, expected net revenue would fall by \$418.76, irrespective of whether the parsnip crop is grown on the freehold or leased land. (The low net revenue variance of the parsnip activity, however, would indicate *a priori* that such a substitution would also reduce the net revenue variance of the cropping programme).

6.8 Summary of the Risk Programming Model

The real life production situation of a fresh vegetable grower has been represented as a risk programming model. This approach was adopted, rather than treating the model as a linear programme (that is, similar to the model of Chapter 3) since the grower paid particular attention to the variability of prices and yields when he chose a cropping programme. He realised, for example, that parsnip was a 'safe' but low income crop, whereas Winter lettuce could provide a high level of income but was also subject to a high degree of risk.

The author intended to provide the grower with the whole set of preferred plans, each minimising income variance for each level of expected income. It was unfortunate that the results could not be obtained in time to be included in the thesis, since the grower's reaction to the set of risk-minimising plans could have indicated the extent to which risk programming could be usefully employed in providing management advice to fresh vegetable producers.

The author would emphasise, however, that the problems encountered in the risk programming study were purely 'mechanical'. Risk programming algorithms are available and once a computer programme is available in New Zealand, risk programming can be studied to a greater extent than has thus far been possible.

CHAPTER 7

SUMMARY OF RESULTS AND COMMENTS ON MATHEMATICAL PROGRAMMING IN HORTICULTURE

7.1 Introduction

Two linear programming and two quadratic programming studies have been discussed in some detail, in order to illustrate the application of mathematical programming techniques to different horticultural production situations. The following sections will draw together and summarise the various ideas developed in the thesis.

7.2 The Linear Programming Applications

The linear programming study of Chapter 3 provided a series of plans for an Otaki horticulturalist. The grower soon became familiar with the more important programming concepts and had little difficulty in understanding the basic matrix and interpreting the results. That he was satisfied with the type of information provided is obvious from his comments on the solution (section 3.12). He believed that a linear programming model gave a more complete description and analysis of the management of his holding than would simpler techniques such as gross margins analysis and budgeting.

The intertemporal linear programme of Chapter 4 was designed to analyse a more complex production situation, with cropping planned several years ahead. The grower found the results of great interest but emphasised that the uncertainty surrounding such variables as prices and incomes in future time periods, made it difficult to closely follow a pre-determined development programme. Therefore research to incorporate risk into an intertemporal programming model would appear to be worthwhile (section 4.10.3).

Intertemporal programming would also be a useful aid to management when planning the development of orchards. Possible cropping activities could include different varieties of, say, apples, pears, peaches and plums, plus any annual crops which could be intercropped amongst young trees. The capacity of the grading and packing facilities would be an important restraint in orchard development models. Such restraints would represent the maximum quantity of fruit which could be handled during, say, every fortnight of the harvesting season, since different types and varieties of fruit may be harvested at different times. While the collection of input-output data for tree crops is often difficult, close co-operation between the farm adviser, the orchardist and fruit research workers should enable the compilation of acceptable estimates.

Both of the linear programming studies have demonstrated parametric techniques. The supply of a resource was varied in the model of Chapter 3, and in Chapter 4 a price was varied. It is hoped that as a result, many of the underlying economic principles of production such as competitive and complementary relationships between cropping activities, and a diminishing marginal value product of a factor as the factor supply is increased, have been adequately illustrated. A good understanding of these principles, which is often not obtained from the use of simpler planning techniques, is necessary for a clear understanding of profit-maximising behaviour.

7.3 The Quadratic Programming Applications

The existence of imperfectly competitive elements in some horticultural product markets could provide many instances where quadratic programming techniques (as demonstrated in Chapter 5) would be more appropriate than a linear programming approach. Vegetable growers, as well as nurserymen, may find that marketing increasing quantities has a depressing effect upon prices, especially since large holdings are becoming more numerous and replacing, to some extent, the traditional small family unit.

It is hoped that the risk programming model of Chapter 6, although incomplete, has indicated the likely role of this technique in management advisory work, especially for growers who sell their produce through an auction system. Many growers would prefer to avoid excessive price fluctuations so as to guard against the likelihood of low incomes. Risk programming, although doing nothing to reduce such price fluctuations (and hence the resulting income fluctuations), does provide growers with cropping plans for which the likelihood of income fluctuations has been minimised. Therefore, by following a risk-minimising programme, a grower will know that there is no other safer cropping programme which will provide him with the same level of expected income (assuming, of course, that the problem has been correctly specified and that the probabilities of certain prices occuring in the future are the same as observed in the past).

The author sees an important place for quadratic programming (both risk programming and programming under conditions of imperfect competition) in horticultural advisory work in New Zealand. Such techniques will become practicable once a computing routine is available locally, and work in this direction is proceeding at Massey University.

7.4 The Potential of Mathematical Programming in the New Zealand Horticultural Advisory Service

7.4.1 The general use of programming techniques

It is important that the adviser formulating the programming model has a sound knowledge of modern horticultural production techniques. If the adviser confronts some specific technique with which he is unfamiliar, then the programming model should be constructed in conjunction with a person who can provide the relevant technical knowledge. Also, it may be possible to detect obvious inefficiencies in resource use (for example, unsuitable fertiliser or spray programmes) and hence exclude such inefficient techniques from the programming model. (Of course, many different crop husbandry techniques may be included in the model, allowing the profit-maximising technique to be determined. The size of the model would be reduced, though, if some of these techniques were seen as 'obviously inefficient').

Although programming requires the collection of a considerable amount of data, once several models have been constructed for horticulturalists in a specific region, much technical data (such as input-output coefficients) will have been collected and may be drawn on when programming similar holdings in the future (provided that such input-output data is modified in response to any technological advances).

Aggregated linear programming models[1] could be of particular importance in horticulture. From such models, the aggregate ('normative') supply function for some crop in a specified region can be determined, indicating the quantities of the crop which should be supplied at various prices. For example, should contracts be offered for vegetable crops in the Manawatu, an aggregated programming model would indicate to the processing company the price which would have to be paid to growers in order to obtain some specified quantity of produce.

7.4.2 The use of farm records

The need for accurate and detailed farm records is becoming more and more apparent to horticultural producers. Two of the growers contacted in the present study had only just begun to keep detailed production and marketing records, but programming models which provided realistic solutions could still be constructed. Had more comprehensive records been available, however, the time required to collect data would have been considerably reduced, and farm management advisers should therefore encourage growers to keep adequate records.

Farm records should include the prices received for all produce sold, crop yields and acreages, the costs of all inputs purchased and the quantities of such inputs used on each crop, and the dates of all operations carried out on the crops plus the hours of labour and tractor usage required. Provided with such data, the farm adviser may be able to construct a programming model in a relatively short period of time.

---

1. Day, Richard H., "On Aggregating Linear Programming Models of Production", _Journal of Farm Economics_, vol. 45, pp.797-813, 1963.

7.4.3 The role of market intelligence

Although risk programming has been suggested as a useful technique to reduce the possibility of violently fluctuating incomes, such a technique does nothing to reduce the fluctuations in market prices which may invalidate linear programming solutions. Market intelligence services, by providing data on quantities of crops available for marketing and prices received, may enable a more orderly planning of production with a consequent reduction in the possibility of unforeseen price fluctuations. Under such conditions, price-prediction would be somewhat less hazardous than at present. A market intelligence service, then, publishing a report to growers say every three months, should provide the following type of information:[2]

(i) acreages of crops sown over the past three months;

(ii) acreages of crops which growers intend to plant during the coming three months;

(iii) quantities of crops marketed during the past three months and prices received; and

(iv) quantities of crops which growers intend to harvest during the coming three months and an estimate of market prices.

Also, price prediction can be expected to become more accurate once data has been collected over a period of years, and the estimation of prices in response to a forecast level of output should be made more accurate than at present.

---

2. An excellent example of a market intelligence service is given by Vegetable Situation, published quarterly by the Economic Research Service, United States Department of Agriculture.

7.5 Conclusions

Having constructed various programming models and discussed the resulting solutions with horticulturalists, the author believes that mathematical programming can play an important role in the New Zealand Horticultural Advisory Service. Fresh vegetable producers, process vegetable producers, nurserymen or orchardists could be provided with cropping programmes (or farm development programmes) to assist them in their decision making.

The likelihood of inaccurate estimates (of prices or yields, for example) or an incorrect specification of the problem causing the programmed results to become invalid should not be overlooked, however. In this respect, farm advisers should encourage growers to keep farm records suitable for programming purposes.

The need for a market intelligence service for New Zealand horticulturalists has been indicated on past occasions,[3] and the existence of such a service would do much to reduce the possibility of unforeseen price fluctuations, and hence give growers more confidence in following a recommended cropping programme.

---

3. Enting, L.M., Philpott, B.P., and Ridler, D., A Report On The Economic Position Of The Fresh Vegetable Industry In New Zealand, New Zealand Vegetable and Produce Growers' Federation (Inc.), May 1965, pp.28, 86-87.

APPENDIX A

DATA RELATING TO THE INTERTEMPORAL LINEAR PROGRAMME OF CHAPTER 4

A.1 Price, Yield and Variable Cost Estimates for Annual Crops

Table A.1 gives the price and yield estimates for annual crops, the product of which indicates the contribution of a crop to pre-tax cash receipts in any year. (It was necessary to estimate prices only for the three fresh market crops, since prices of all other annual crops are fixed under contract). For those crops which are followed with a greencrop, the net revenue (sales value less purchase value) from the grazed hoggets has been added to gross revenue. (The cost of the greencrop itself has been incorporated into the variable costs of the annual crops).

Table A.2 includes the variable production costs for the annual crops, these figures representing the requirements of the annual crops for tax-free cash in any year.

Table A.1 Gross Revenue From Annual Crops

Activity		Price ($)	Yield (per acre)	Net Revenue from Hoggets ($/acre)	Gross Revenue ($/acre)
$P_{1,k}$	: intercrop beetroot	40.00 per ton	12.50 tons	-	500.00
$P_{2,k}$	: intercrop carrot	34.00 per ton	16.70 tons	-	567.80
$P_{3,k}$	: intercrop kumara	168.00 per ton	4.50 tons	-	756.00
$P_{4,k}$	: intercrop mangold	4.50 per ton	75.00 tons	-	337.50
$P_{5,k}$	: intercrop potato	44.00 per ton	10.00 tons	-	440.00
$P_{6,k}$	: tomato	32.00 per ton	22.00 tons	5.44	709.44
$P_{7,k}$	: green bean	65.34 per ton	3.50 tons	5.44	234.13
$P_{8,k}$	: beetroot	40.00 per ton	12.50 tons	5.44	505.44
$P_{9,k}$	: potato	44.00 per ton	10.00 tons	-	440.00
$P_{10,k}$	: pea	-	-	5.44	105.44
$P_{11,k}$	: carrot	34.00 per ton	16.70 tons	-	567.80
$P_{12,k}$	: broad bean -	70.00 per ton	5.00 tons	-	1030.40
	kumara	168.00 per ton	4.05 tons	-	
$P_{13,k}$	: kumara	168.00 per ton	4.50 tons	-	756.00
$P_{14,k}$	: mangold	4.50 per ton	75.00 tons	-	337.50
$P_{15,k}$	: ryegrass	1.80 per bushel	43.70 bushels	5.44	84.10

Notes:
1. Gross revenue = (price x yield) + net revenue from hoggets.
2. The grower believed that a gross revenue from the pea crop of $100 per acre would be usual. The price paid is not fixed, but depends upon the maturity of the peas when harvested.

Table A.2 — Variable Costs ($ per acre) of Annual Crops

Activity	Seed/ Plants	Fertiliser	Spray	Tractor	Cartage	Contract Labour	Greencrop	Sundry	Total
$P_{1,k}$: intercrop beetroot	9.42	11.42	-	12.48	2.82	12.28	-	-	48.42
$P_{2,k}$: intercrop carrot, $P_{11,k}$: and carrot	13.50	12.50	25.00	19.42	33.34	123.00	-	-	226.76
$P_{3,k}$: intercrop kumara, $P_{13,k}$: and kumara	103.88	7.06	-	18.66	6.00	3.52	-	51.74	190.86
$P_{4,k}$: intercrop mangold, $P_{14,k}$: and mangold	9.20	28.00	-	10.32	-	8.00	-	-	55.52
$P_{5,k}$: intercrop potato, $P_{9,k}$: and potato	86.58	24.00	20.00	18.70	2.00	9.00	-	6.66	166.94
$P_{6,k}$: tomato	47.50	27.76	38.34	16.14	19.38	148.70	7.24	-	305.06
$P_{7,k}$: green bean	20.96	13.72	12.90	8.42	-	6.34	7.24	-	69.58
$P_{8,k}$: beetroot	9.42	11.42	-	12.48	2.82	12.28	7.24	-	55.66
$P_{10,k}$: pea	13.54	2.86	-	2.04	-	14.12	7.24	-	39.80
$P_{12,k}$: broad bean -	{ 13.34	10.00	30.66	9.06	3.90	97.34	-	- }	
kumara	103.88	7.06	-	18.66	5.40	3.52		46.57 }	349.39
$P_{15,k}$: ryegrass	4.50	-	-	-	3.34	12.66	-	19.00	39.50

Notes: 1. The greencrop cost includes $3.74 per acre for seed, and $3.50 per acre for contract services.
2. Sundry items include the cost of sacks and twine for potato and kumara, and the seed-dressing costs of the ryegrass.

A.2 Price, Yield and Variable Cost Estimates for Perennial Crops and their Optimum Replacement

A.2.1 Apple

Prices paid to growers are in the form of a guaranteed price set by the Apple and Pear Prices Authority, and is estimated to cover the total production costs of the unwrapped fruit and case. In addition, growers are paid for wire and labels used in packing the fruit, plus an extra payment for all fruit exported. Although the price actually paid varies between varieties and grades of fruit, it was assumed that the grower would receive, in each of the six years of the development programme, a price of $1.51 per case, which is equal to that guaranteed for the 1968 season. Since the grower has planted (and intends to plant in the future) the preferred, higher-priced apple varieties such a price assumption may be conservative but was thought prudent since considerable uncertainty surrounds the future marketing prospects for apples, with prices likely to fall rather than rise.

Apple variable costs include such expenses as planting costs, spray materials, fertiliser, greencrop seed, wire used to shape the trees, cases and other packing materials, and tractor and sprayer running costs.

Such variable costs represent the requirement of apples in a particular year for tax-free cash, whilst multiplication of price and yield gives gross revenue, or the contribution of apples to pre-tax cash in any year.

When the optimum replacement time (and asset values) were calculated for apples (or perennial crops in general), it was necessary to include labour costs with the other variable costs. (Labour costs are included in the six years of the programme as either part of overhead costs, or as hired labour).

Table A.3 includes annual yields, gross revenues, variable costs and labour costs (the latter valued at 72 cents per hour) and net revenues. The final column of the table gives the amortized present values, the maximum of which indicates the optimum replacement age.

A.2.2. Asparagus

The average price received by the grower over the past three seasons was $236.66 per ton. Multiplication by the yield per acre in any year gives the gross revenue from the crop, which is contributed to the pre-tax cash supply of that year.

Variable costs are mainly for weedicides (and plants in the initial year) and represent the crop's requirement for tax-free cash.

Labour costs are included in the analysis to determine the optimum replacement age, these being equal to the sum of the annual labour requirements per acre times the wage rate of 72 cents per hour. Table A.4 presents gross revenue, variable costs and the optimum replacement data.

Table A.3 Revenue, Costs and Optimum Replacement: Apples

Year	Yield (bushels/acre)	Gross Revenue ($/acre)	Variable Costs ($/acre)	Labour Costs ($/acre)	Interest on Unpaid Establishment Costs ($/acre)	Net Revenue ($/acre)	Amortized Present Value ($/acre)
1	-	-	121.52	49.10	-	-170.62	-
2	-	-	32.04	21.60	10.24	-63.88	-
3	-	-	63.87	47.38	13.46	-124.71	-
4	100	151.00	147.44	109.08	20.13	-125.65	-
5	150	226.50	176.02	153.86	26.46	-129.84	-
6	400	604.00	312.18	227.23	32.66	31.93	-
7	600	906.00	436.43	291.60	28.79	149.18	-
8	800	1208.00	558.29	319.18	18.11	312.42	-
9	1000	1510.00	667.22	385.63	-	457.15	-
10	1400	2114.00	866.85	487.94	-	759.21	-
11	1600	2416.00	974.85	540.50	-	900.65	-
12	1800	2718.00	1082.85	582.05	-	1053.10	-
13-35	2000	3020.00	1211.85	620.64	-	1187.51	-
36	1900	2869.00	1153.41	599.83	-	1115.76	-
37	1900	2869.00	1153.41	599.83	-	1115.76	-
38	1800	2718.00	1082.85	579.02	-	1056.13	-
39	1800	2718.00	1082.85	579.02	-	1056.13	-
40	1700	2567.00	1044.49	558.22	-	964.29	-
41	1700	2567.00	1044.49	558.22	-	964.29	-
42	1600	2416.00	974.85	537.48	-	903.67	-
43	1600	2416.00	974.85	537.48	-	903.67	-
44	1500	2265.00	935.56	516.67	-	812.77	
45	1500	2265.00	935.56	516.67	-	812.77	
46	1400	2114.00	866.85	495.86	-	751.29	628.24
47	1400	2114.00	866.85	495.86	-	751.29	628.77
48	1300	1963.00	826.64	475.06	-	661.30	628.87
49	1300	1963.00	826.64	475.06	-	661.30	628.99
50	1200	1812.00	772.17	454.25	-	585.58	628.84

Notes:
1. Year 1 is the year of planting.
2. Amortized present values are given only for 46-year-old to 50-year-old plantings.

Table A.4 Revenue, Costs and Optimum Replacement: Asparagus

Year	Yield (tons/acre)	Gross Revenue ($/acre)	Variable Costs ($/acre)	Labour Costs ($/acre)	Interest on Unpaid Establishment Costs ($/acre)	Net Revenue ($/acre)	Amortized Present Value ($/acre)
1	-	-	101.00	14.47	-	-115.47	-
2	-	-	1.58	4.40	6.93	-12.91	-
3	-	-	1.58	4.04	7.29	-12.91	-
4	0.50	118.33	1.58	77.16	7.62	31.97	-
5	1.20	283.99	1.58	115.18	5.25	161.98	-
6	1.60	378.66	23.48	123.23	-	231.95	-
7	1.75	414.16	23.48	134.46	-	256.22	-
8	1.90	449.65	23.48	145.69	-	280.48	-
9	2.00	473.32	23.48	153.18	-	296.66	-
10	2.05	485.15	23.48	156.92	-	304.75	-
11	2.10	496.99	23.48	160.67	-	312.84	-
12	2.15	508.82	23.48	164.41	-	320.93	-
13	2.10	496.99	23.48	160.67	-	312.84	-
14	2.05	485.15	23.48	156.92	-	304.75	-
15	2.00	473.32	23.48	153.18	-	296.66	-
16	1.95	461.49	23.48	149.44	-	288.57	-
17	1.90	449.65	23.48	145.69	-	280.48	-
18	1.85	437.82	23.48	141.95	-	272.39	-
19	1.80	425.99	23.48	138.20	-	264.31	-
20	1.75	414.16	23.48	134.46	-	256.22	-
21	1.70	402.32	23.48	130.72	-	248.12	-
22	1.65	390.49	23.48	126.97	-	240.04	-
23	1.60	378.66	23.48	123.23	-	231.95	-
24	1.55	366.82	23.48	119.48	-	223.86	-
25	1.50	354.99	23.48	115.74	-	215.77	186.75
26	1.45	343.16	23.48	112.00	-	207.68	187.11
27	1.40	331.32	23.48	108.25	-	199.59	187.30
28	1.35	319.49	23.48	104.51	-	191.50	187.36
29	1.30	307.66	23.48	100.76	-	183.42	187.30

Notes:
1. Year 1 is the year of planting.
2. Amortized present values are given only for 25-year-old to 29-year-old plantings.

A.2.3 Peach

The present contract price for process peaches is three or four dollars per 120 pounds, depending on the quality of the fruit. The grower estimated that 90 per cent of the crop would be sold at the higher price.

Table A.5 contains the gross revenue data (contributions to pre-tax cash), variable costs (requirements for tax-free cash), labour costs (the total annual labour requirements times the wage rate), and the net revenues and amortized present values.

Table A.5 Revenue, Costs and Optimum Replacement:Peaches

Year	Yield (tons/acre)	Gross Revenue ($/acre)	Variable Costs ($/acre)	Labour Costs ($/acre)	Interest on Unpaid Establishment Costs ($/acre)	Net Revenue ($/acre)	Amortized Present Value ($/acre)
1	-	-	100.00	27.50	-	-127.50	-
2	-	-	58.00	30.02	7.65	-95.67	-
3	-	-	68.00	42.70	12.93	-123.63	-
4	0.50	36.40	84.00	57.38	19.57	-124.55	-
5	3.25	236.60	124.00	93.38	25.87	-6.65	-
6	6.50	473.20	148.00	143.71	24.72	156.77	-
7	10.50	764.40	180.00	171.94	13.83	398.63	-
8	13.00	946.40	186.00	199.15	-	561.25	-
9	15.00	1092.00	190.00	219.53	-	682.47	-
10	15.00	1092.00	190.00	219.53	-	682.47	-
11	14.00	1019.20	188.00	212.76	-	618.44	-
12	13.00	946.40	186.00	213.55	-	546.85	-
13	13.00	946.40	186.00	213.55	-	546.85	-
14	12.00	873.60	184.00	210.02	-	479.58	-
15	12.00	873.60	184.00	210.02	-	479.58	-
16	11.00	800.80	182.00	206.86	-	411.94	250.25
17	10.00	728.00	180.00	203.69	-	344.31	253.55
18	9.00	655.20	178.00	200.52	-	276.68	254.33
19	8.00	582.40	176.00	197.35	-	209.05	253.00

Notes:
1. Year 1 is the year of planting.
2. Amortized present values are given only for 16-year-old to 19-year-old plantings.

A.3 Part of the Basic Matrix

Table 4.1 of Chapter 4 gives a schematic representation of the basic matrix. The sub-matrices a, g and h of that table are presented as tables A.6, A.7 and A.8 respectively.

Table A.6 is that section of the basic matrix comprising first-year activities and first-year restraints. All non-zero coefficients of first-year activities and second-, third- and fourth-year restraints are given in table A.7, and table A.8 contains the coefficients of the first-year activities and the fifth- and sixth-year restraints.

The sections of table 4.1 labelled b, c, d, e and f, contain coefficients identical to those of table A.6 except those of the old peach and old asparagus activities, since as these activities increase in age costs, yields and labour requirements may change from year to year.

The remainder of the matrix can be completed using the coefficients of tables A.7 and A.8, plus the asset value coefficients of Chapter 4, table 4.18. For example, the sub-matrix containing second-year activities and third-, fourth- and fifth-year restraints will have identical coefficients to those of table A.7. Finally, all coefficients of the B column may be obtained from Chapter 4, section 4.3.

Table A.6

Sub-Matrix a

Restraints	unit → ↓	B	Relationship	P1,1 Intercrop Beetroot 1 acre	P2,1 Intercrop Carrot 1 acre	P3,1 Intercrop Kumara 1 acre	P4,1 Intercrop Mangold 1 acre	P5,1 Intercrop Potato 1 acre	P6,1 Tomato 1 acre	P7,1 Green Bean 1 acre	P8,1 Beetroot 1 acre	P9,1 Potato 1 acre	P10,1 Pea 1 acre	P11,1 Carrot 1 acre	P12,1 Broad Bean - Kumara 1 acre	P13,1 Kumara 1 acre	P14,1 Mangold 1 acre	P15,1 Ryegrass 1 acre
c_j → ($)				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R1,1 : tax-free cash, beginning year 1	$1	27000	≥	48.42	226.76	190.86	55.52	166.94	305.06	69.58	55.66	166.94	39.80	226.76	349.39	190.86	55.52	39.50
R2,1 : peach land	1 acre	25	≥															
R3,1 : asparagus land	"	24	≥															
R4,1 : annual cropland	"	106.5	≥						1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
R5,1 : perennial cropland	"	0	≥															
R6,1 : intercropped land	"	0	≥	1.00	1.00	1.00	1.00	1.00										
R7,1 : cropland transfer control	"	106.5	≥															
R8,1 : Spring labour	1 hour	6212	≥	102.05	5.25	44.74	90.60	6.50	16.09	0.95	102.39	6.50	2.51	5.25	17.28	44.74	90.60	0.34
R9,1 : Summer labour	"	3871	≥	204.69	12.75	68.35	12.20	10.00	51.99	22.11	204.69	10.00	2.69	12.75	101.41	68.35	12.20	1.08
R10,1 : Autumn labour	"	4987	≥		1.75	158.70	13.60	68.67	50.18	9.23	2.18	68.67	1.78	1.75	156.75	158.70	13.60	
R11,1 : Winter labour	"	1095	≥		35.20	100.80	47.50	24.50	0.30	0.70	0.30	24.50	1.03	35.20	95.03	100.80	47.50	0.24
R12,1 : beetroot limit	1 acre	7	≥	1.00							1.00							
R13,1 : potato limit	1 ton	70	≥					10.00				10.00						
R14,1 : asparagus limit	1 acre	36	≥															
R15,1 : carrot limit	1 ton	60	≥		16.70									16.70				
R16,1 : tomato limit	1 acre	21	≥						1.00									
R17,1 : green bean limit	"	27	≥							1.00								
R18,1 : pea limit	"	19	≥										1.00					
R19,1 : broad bean limit	"	5	≥												1.00			
R20,1 : kumara limit	"	24	≥			1.00									1.00	1.00		
R21,1 : mangold limit	"	8	≥				1.00										1.00	
R22,1 : ryegrass limit	"	12	≥															1.00
R23,1 : apple limit, year 1	"	10	≥															
R24,1 : tomato-potato rotation limit	"	106.5	≥					5.00	5.00			5.00						
R25,1 : carrot rotation limit	"	106.5	≥		4.00									4.00				
R26,1 : pea rotation limit	"	106.5	≥										3.00					
R27,1 : green bean rotation limit	"	106.5	≥							3.00								
R28,1 : pre-tax cash receipts, end year 1	$1	8748.28	≥	-500.00	-567.80	-756.00	-337.50	-440.00	-709.44	-234.13	-505.44	-440.00	-105.44	-567.80	-1030.40	-756.00	-337.50	-84.10
R29,1 : tax deductions	"	39026	≥	-48.42	-226.76	-190.86	-55.52	-166.94	-305.06	-69.58	-55.66	-166.94	-39.80	-226.76	-349.39	-190.86	-55.52	-39.50
R30,1 : tax limit 1:1	"	268	≥															
R31,1 : tax limit 2:1	"	718	≥															
R32,1 : tax limit 3:1	"	1000	≥															
R33,1 : tax limit 4:1	"	800	≥															
R34,1 : tax limit 5:1	"	1000	≥															
R35,1 : tax limit 6:1	"	1000	≥															
R36,1 : tax limit 7:1	"	1000	≥															
R37,1 : tax limit 8:1	"	1200	≥															
R38,1 : tax limit 9:1	"	1200	≥															

Restraints	P16,1 Old Asparagus 1 acre	P17,1 Old Peach 1 acre	P18,1 Asparagus 1 acre	P19,1 Apple 1 acre	P20,1 Annual Cropland Transfer 1 acre	P21,1 Asparagus Land Transfer 1 acre	P22,1 Peach Land Transfer 1 acre	P23,1 Fallow 1 acre	P24,1 Hire Spring Labour 10 hours	P25,1 Hire Summer Labour 10 hours	P26,1 Hire Autumn Labour 10 hours	P27,1 Hire Winter Labour 10 hours
c_j → ($)	0	0	0	0	0	0	0	0	0	0	0	0
R1,1 : tax-free cash, beginning year 1	23.48	180.00	101.00	121.52		3.06			7.20	7.20	7.20	7.20
R2,1 : peach land		1.00					1.00					
R3,1 : asparagus land	1.00					1.00						
R4,1 : annual cropland					1.00	-1.00	-1.00	1.00				
R5,1 : perennial cropland			1.00	1.00	-1.00							
R6,1 : intercropped land				-0.75								
R7,1 : cropland transfer control					1.00							
R8,1 : Spring labour	118.89	10.80							-10.00			
R9,1 : Summer labour	29.85	93.70								-10.00		
R10,1 : Autumn labour	0.86	50.90	4.00			7.00					-10.00	
R11,1 : Winter labour	0.75	83.40	16.10	68.20			15.00					-10.00
R12,1 : beetroot limit												
R13,1 : potato limit												
R14,1 : asparagus limit	1.00		1.00									
R15,1 : carrot limit												
R16,1 : tomato limit												
R17,1 : green bean limit												
R18,1 : pea limit												
R19,1 : broad bean limit												
R20,1 : kumara limit												
R21,1 : mangold limit												
R22,1 : ryegrass limit												
R23,1 : apple limit, year 1												
R24,1 : tomato-potato rotation limit				-0.75	1.00	-1.00	-1.00					
R25,1 : carrot rotation limit				-0.75	1.00	-1.00	-1.00					
R26,1 : pea rotation limit					1.00	-1.00	-1.00					
R27,1 : green bean rotation limit					1.00	-1.00	-1.00					
R28,1 : pre-tax cash receipts, end year 1	-331.32	-764.40										
R29,1 : tax deductions	-23.48	-180.00	-101.00	-121.52		-3.06			-7.20	-7.20	-7.20	-7.20
R30,1 : tax limit 1:1												
R31,1 : tax limit 2:1												
R32,1 : tax limit 3:1												
R33,1 : tax limit 4:1												
R34,1 : tax limit 5:1												
R35,1 : tax limit 6:1												
R36,1 : tax limit 7:1												
R37,1 : tax limit 8:1												
R38,1 : tax limit 9:1												

Restraints	P28,1 Tax Transfer 1:1 $1	P29,1 Tax Transfer 2:1 $1	P30,1 Tax Transfer 3:1 $1	P31,1 Tax Transfer 4:1 $1	P32,1 Tax Transfer 5:1 $1	P33,1 Tax Transfer 6:1 $1	P34,1 Tax Transfer 7:1 $1	P35,1 Tax Transfer 8:1 $1	P36,1 Tax Transfer 9:1 $1	P37,1 Tax Transfer 10:1 $1	P38,1 Tax Deductions Transfer $1	P39,1 Bank $1
c_j → ($)	0	0	0	0	0	0	0	0	0	0	0	0
R1,1 : tax-free cash, beginning year 1												1.00
R2,1 : peach land												
R3,1 : asparagus land												
R4,1 : annual cropland												
R5,1 : perennial cropland												
R6,1 : intercropped land												
R7,1 : cropland transfer control												
R8,1 : Spring labour												
R9,1 : Summer labour												
R10,1 : Autumn labour												
R11,1 : Winter labour												
R12,1 : beetroot limit												
R13,1 : potato limit												
R14,1 : asparagus limit												
R15,1 : carrot limit												
R16,1 : tomato limit												
R17,1 : green bean limit												
R18,1 : pea limit												
R19,1 : broad bean limit												
R20,1 : kumara limit												
R21,1 : mangold limit												
R22,1 : ryegrass limit												
R23,1 : apple limit, year 1												
R24,1 : tomato-potato rotation limit												
R25,1 : carrot rotation limit												
R26,1 : pea rotation limit												
R27,1 : green bean rotation limit												
R28,1 : pre-tax cash receipts, end year 1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-0.06
R29,1 : tax deductions											1.00	
R30,1 : tax limit 1:1	1.00											
R31,1 : tax limit 2:1		1.00										
R32,1 : tax limit 3:1			1.00									
R33,1 : tax limit 4:1				1.00								
R34,1 : tax limit 5:1					1.00							
R35,1 : tax limit 6:1						1.00						
R36,1 : tax limit 7:1							1.00					
R37,1 : tax limit 8:1								1.00				
R38,1 : tax limit 9:1									1.00			

Table A.7

Sub-Matrix g

Restraints	unit	B	Relationship	Tomato P6,1 1 acre	Green Bean P7,1 1 acre	Beetroot P8,1 1 acre	Potato P9,1 1 acre	Pea P10,1 1 acre	Carrot P11,1 1 acre	Broad Bean - Kumara P12,1 1 acre	Kumara P13,1 1 acre	Mangold P14,1 1 acre	Ryegrass P15,1 1 acre	Old Asparagus P16,1 1 acre	Old Peach P17,1 1 acre
R1,2 : tax-free cash, beginning year 2	$1	-39216	≥												
R2,2 : peach land	1 acre	0	≥												-1.00
R3,2 : asparagus land	"	0	≥											-1.00	
R4,2 : annual cropland	"	0	≥	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00		
R6,2 : intercropped land	"	0	≥												
R7,2 : cropland transfer control	"	106.5	≥												
R8,2 : Spring labour	1 hour	5948	≥												
R9,2 : Summer labour	"	2833	≥												
R10,2 : Autumn labour	"	3910	≥												
R11,2 : Winter labour	"	507	≥												
R14,2 : asparagus limit	1 acre	36	≥												
R24,2 : tomato-potato rotation limit	"	0	≥	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00		
R25,2 : carrot rotation limit	"	0	≥	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00		
R26,2 : pea rotation limit	"	0	≥	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00		
R27,2 : green bean rotation limit	"	0	≥	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00		
R29,2 : tax deductions	$1	39026	≥												
R1,3 : tax-free cash, beginning year 3	$1	-39216	≥												
R7,3 : cropland transfer control	1 acre	106.5	≥												
R8,3 : Spring labour	1 hour	5781	≥												
R9,3 : Summer labour	"	3303	≥												
R10,3 : Autumn labour	"	2833	≥												
R11,3 : Winter labour	"	-78	≥												
R14,3 : asparagus limit	1 acre	36	≥												
R29,3 : tax deductions	$1	39026	≥												
R1,4 : tax-free cash, beginning year 4	$1	-39216	≥												
R7,4 : cropland transfer control	1 acre	106.5	≥												
R8,4 : Spring labour	1 hour	5648	≥												
R9,4 : Summer labour	"	2282	≥												
R10,4 : Autumn labour	"	1759	≥												
R11,4 : Winter labour	"	-786	≥												
R14,4 : asparagus limit	1 acre	36	≥												
R28,4 : pre-tax cash receipts, end year 4	$1	26704.12	≥												
R29,4 : tax deductions	"	39026	≥												

Restraints	Asparagus P18,1 1 acre	Apple P19,1 1 acre	Annual Cropland Transfer P20,1 1 acre	Asparagus Land Transfer P21,1 1 acre	Peach Land Transfer P22,1 1 acre	Fallow P23,1 1 acre	Tax Transfer 1:1 P28,1 $1	Tax Transfer 2:1 P29,1 $1	Tax Transfer 3:1 P30,1 $1	Tax Transfer 4:1 P31,1 $1	Tax Transfer 5:1 P32,1 $1	Tax Transfer 6:1 P33,1 $1	Tax Transfer 7:1 P34,1 $1	Tax Transfer 8:1 P35,1 $1	Tax Transfer 9:1 P36,1 $1	Tax Transfer 10:1 P37,1 $1	Tax Deductions Transfer P38,1 $1	Bank P39,1 $1
R1,2	1.58	32.04					-1.00	-0.925	-0.775	-0.712	-0.637	-0.575	-0.513	-0.443	-0.368	-0.325	-1.00	-1.00
R2,2																		
R3,2																		
R4,2						-1.00												
R6,2		-0.75																
R7,2			1.00															
R8,2	3.50	12.00																
R9,2	1.00	4.50																
R10,2	0.86	3.50																
R11,2	0.75	10.00																
R14,2	1.00																	
R24,2		-0.75				-1.00												
R25,2		-0.75				-1.00												
R26,2						-1.00												
R27,2						-1.00												
R29,2	-1.58	-32.04																
R1,3	1.58	63.87																
R7,3			1.00	-1.00	-1.00													
R8,3	2.00	14.00																
R9,3	2.00	17.50																
R10,3	0.86	3.50																
R11,3	0.75	30.80																
R14,3	1.00																	
R29,3	-1.58	-63.87																
R1,4	1.58	147.44																
R7,4			1.00	-1.00	-1.00													
R8,4	60.38	15.50																
R9,4	14.76	67.30																
R10,4	0.86	21.50																
R11,4	0.75	47.20																
R14,4	1.00																	
R28,4	-118.33	-151.00																
R29,4	-1.58	-147.44																

Table A.8

Sub-Matrix h

Restraints	unit	B	Relationship	Asparagus P18,1 1 acre	Apple P19,1 1 acre	Annual Cropland Transfer P20,1 1 acre	Asparagus Land Transfer P21,1 1 acre	Peach Land Transfer P22,1 1 acre
R1,5 : tax-free cash, beginning year 5	$1	-59216	>	1.58	176.02			
R7,5 : cropland transfer control	1 acre	106.5	>			1.00	-1.00	-1.00
R8,5 : Spring labour	1 hour	5599	>	102.36	15.50			
R9,5 : Summer labour	"	751	>	25.58	100.30			
R10,5 : Autumn labour	"	-395	>	0.86	30.20			
R11,5 : Winter labour	"	-1377	>	0.75	67.70			
R14,5 : asparagus limit	1 acre	36	>	1.00				
R28,5 : pre-tax cash receipts, end year 5	$1	38977.22	>	-283.99	-226.50			
R29,5 : tax deductions	"	42826	>	-1.58	-176.02			
R1,6 : tax-free cash, beginning year 6	$1	-39216	>	23.48	312.18			
R7,6 : cropland transfer control	1 acre	106.5	>			1.00	-1.00	-1.00
R8,6 : Spring labour	1 hour	5549	>	135.43	15.50			
R9,6 : Summer labour	"	-246	>	34.11	137.60			
R10,6 : Autumn labour	"	-1475	>	0.86	75.30			
R11,6 : Winter labour	"	-1503	>	0.75	87.20			
R14,6 : asparagus limit	1 acre	36	>	1.00				
R28,6 : pre-tax cash receipts, end of year 6	$1	44939.18	>	-378.66	-604.00			
R29,6 : tax deductions	"	40566	>	-23.48	-312.18			
R39,6 : total apple plantings	1 acre	30	>		1.00			
R41,6 : final assets	$1	0	>		-15524			

APPENDIX B

DATA OF THE RISK PROGRAMMING MODEL OF CHAPTER 6

B.1 Description of the Data

This appendix includes, for all activities in each of the three years studied, data on the quantities of produce sold, acreages cropped and yields per acre (table B.1), and the realised gross revenues (table B.2). All activity variable costs are detailed in table B.3, and the variance and covariance of activity net revenues over the three year period are given in table B.4.

Table B.1 Quantities Sold, Acreages and Yields

Activity	Year	Quantity Sold (cases)	Acreage	Yield (cases/ acre)
P1 & P12 : Spring carrot	1963/64	1376	2.5	550
	1964/65	1264	2.5	506
	1965/66	6864	13.0	528
P2 & P13 : parsnip	1963/64	217	0.5	434
	1964/65	261	1.0	261
P3 & P14 : crown pumpkin	1963/64	2094	6.0	349
	1964/65	2231	12.0	186
	1965/66	1337	10.0	134
P4 & P15 : buttercup pumpkin	1963/64	77	0.2	385
	1964/65	789	2.5	316
	1965/66	1362	3.0	454
P5 & P16 : butternut pumpkin	1964/65	623	1.5	415
	1965/66	599	1.5	399
P6 & P7 : cauliflower	1963/64	1273	3.0	424
	1964/65	2207	6.0	368
	1965/66	2301	5.5	418
P8 & P9 : cabbage	1963/64	1055	2.0	528
	1964/65	1149	3.0	383
	1965/66	2046	3.75	546
P10 & P11: lettuce	1963/64	630	1.0	630
	1964/65	659	1.0	659
	1965/66	651	1.0	651

Table B.2 Average Net Wholesale Prices, Yields and Gross Revenues

Activity			Year	Average Net Price ($/case)	Yield (cases/acre)	Gross Revenue ($/acre)
P1 & P12	:	Spring carrot	1963/64	1.514	550	832.70
			1964/65	1.250	506	632.50
			1965/66	2.298	528	1213.34
P2 & P13	:	parsnip	1963/64	0.878	434	381.06
			1964/65	1.260	261	328.86
P3 & P14	:	crown pumpkin	1963/64	1.190	349	415.32
			1964/65	0.780	186	145.08
			1965/66	0.896	134	120.06
P4 & P15	:	buttercup pumpkin	1963/64	1.178	385	453.54
			1964/65	0.700	316	221.20
			1965/66	0.784	454	355.94
P5 & P16	:	butternut pumpkin	1964/65	0.576	415	237.04
			1965/66	0.966	399	385.44
P6	:	Winter cauliflower	1963/64	1.272	424	539.32
			1964/65	1.700	368	625.60
			1965/66	1.900	418	794.20
P7	:	Spring cauliflower	1963/64	1.138	424	482.52
			1964/65	1.466	368	539.48
			1965/66	1.600	418	668.80
P8	:	Winter cabbage	1963/64	1.062	528	560.74
			1964/65	1.238	383	474.16
			1965/66	1.362	546	743.66
P9	:	Spring cabbage	1963/64	1.000	528	528.00
			1964/65	1.110	383	425.14
			1965/66	1.378	546	752.38
P10	:	Winter lettuce	1963/64	1.646	630	1036.98
			1965/66	3.260	651	2122.26
P11	:	Spring lettuce	1963/64	1.838	630	1157.94
			1964/65	1.250	659	823.76
			1965/66	0.858	651	558.56

Table B.3 Variable Costs ($ per acre)

Activity	Seed	Fertiliser	Spray	Tractor	Contract	Containers			Total Variable Costs		
						1963/64	1964/65	1965/66	1963/64	1964/65	1965/66
P1 & P12 : Spring carrot	13.50	6.75	6.40	14.37	71.42	55.35	50.95	53.09	167.79	163.39	165.53
P2 & P13 : parsnip	7.50	6.76	6.40	14.38	11.42	-	-	-	46.46	46.46	no crop
P3 & P14 : crown pumpkin	13.20	20.00	3.34	8.44	24.00	17.46	9.30	6.70	86.44	78.28	75.68
P4 & P15 : buttercup pumpkin	5.70	20.00	3.34	8.44	-	-	-	-	37.48	37.48	37.48
P5 & P16 : butternut pumpkin	2.80	20.00	3.34	8.44	-	-	-	-	no crop	34.58	34.58
P6 : Winter cauliflower	10.50	20.00	3.88	14.72	11.42	21.20	18.40	20.90	81.72	78.92	81.42
P7 : Spring cauliflower	10.50	20.00	3.88	14.42	11.42	21.20	18.40	20.90	81.42	78.62	81.12
P8 : Winter cabbage	12.50	20.00	3.88	14.72	11.42	26.40	19.16	27.30	88.92	81.68	89.82
P9 : Spring cabbage	12.50	20.00	3.88	14.42	11.42	26.40	19.16	27.30	88.62	81.38	89.52
P10 : Winter lettuce	3.00	20.00	36.64	3.24	11.42	31.50	no crop	32.56	105.80	no crop	106.86
P11 : Spring lettuce	3.50	20.00	27.48	3.28	-	31.50	32.96	32.56	85.76	87.22	86.82

Note: Parsnip, buttercup and butternut crops are marketed in bushel cases which have a zero net cost - they are purchased at 10 cents each, but the grower receives a refund of 10 cents for each bushel case sent to the market. Cabbage, cauliflower, lettuce and crown pumpkin crops are marketed in banana cases which cost 15 cents each to purchase but after deducting the 10 cents refund, have a net cost of five cents. Carrots are marketed in plastic bags (containing 50 pounds) which cost $125.80 per thousand.

Table B.4 The Net Revenue Variance - Covariance Matrix ($)

Activity	Spring Carrot P1 & P12	Parsnip P2 & P13	Crown Pumpkin P3 & P14	Buttercup Pumpkin P4 & P15	Butternut Pumpkin P5 & P16	Winter Cauliflower P6	Spring Cauliflower P7	Winter Cabbage P8	Spring Cabbage P9	Winter Lettuce P10	Spring Lettuce P11
P1 & P12 : Spring carrot	86617.52	2107.92	-11765.28	14346.52	5991.48	29394.80	22186.00	39408.00	48152.96	43771.84	-52899.68
P2 & P13 : parsnip		1362.44	1132.96	835.56	751.64	927.68	682.44	959.76	1172.92	5491.20	2155.32
P3 & P14 : crown pumpkin			25021.52	14273.00	3220.28	-16611.48	-11931.20	-6141.84	-7612.36	23526.44	44121.08
P4 & P15 : buttercup pumpkin				13610.40	2375.00	-3870.36	-2501.20	5980.76	7233.40	17351.08	16817.76
P5 & P16 : butternut pumpkin					11011.28	2636.80	1939.76	2728.04	3333.88	15607.96	6126.24
P6 : Winter cauliflower						16773.56	12333.28	13798.72	16918.72	19263.76	-37850.40
P7 : Spring cauliflower							9077.76	10394.28	12741.84	14171.20	-27611.64
P8 : Winter cabbage								17955.88	21944.04	19930.08	-25311.76
P9 : Spring cabbage									26818.56	24356.36	-31098.36
P10: Winter lettuce										587766.52	44756.44
P11: Spring lettuce											96550.28

Note: Only half of the variance-covariance matrix needs to be presented since the covariance of, for example, Spring lettuce and Spring carrot net revenues, will be the same as that of Spring carrot and Spring lettuce net revenues.