Monotone iterates for nonlinear singularly perturbed convection-diffusion problems

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Sophie Pack

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Abstract

We are interested in monotone iterative algorithms for solving nonlinear singularly perturbed convection-diffusion problems. These problems arise in many physical phenomena. One of the most common sources of these problems is the linearization of Navier-Stokes equations with large Reynolds numbers, other sources include drift-diffusion equations of semi-conductor device modelling, financial modelling, modelling in mathematical biology, fluid dynamics and heat transport problems. Singularly perturbed convection-diffusion problems are characterized by thin areas of rapid change of solutions. Many of these problems can not be solved analytically but must instead be solved numerically. Classical numerical approaches for solving these problems do not always work and may show unsatisfactory behaviours. In this thesis, we focus on constructing monotone iterative methods for solving nonlinear singularly perturbed convection-diffusion problems. Monotone difference schemes have significant advantages: they guarantee that systems of algebraic equations based on such schemes are well-posed; the finite difference operators satisfy the discrete maximum principle.

We construct a uniform convergent difference scheme for solving a nonlinear singularly perturbed two-point boundary value problem of the convection-diffusion type with discontinuous data. The uniform convergence of this scheme is proven on arbitrary meshes. A monotone iterative method is applied to computing the nonlinear difference scheme.

In the past fifteen years, much interest has been shown in domain decomposition techniques for solving singularly perturbed convection-diffusion problems. In this thesis, we construct one- and two-level monotone domain decomposition algorithms based on the multiplicative and additive Schwarz algorithms. These algorithms are proven to converge to the exact solution of the problem.

We construct monotone relaxation methods by modifying the point and block ω-Jacobi

and successive underrelaxation methods. We prove that the point and block monotone relaxation methods converge to the exact solution of the problem.

We combine the monotone domain decomposition algorithms and relaxation methods to construct composite monotone domain decomposition algorithms. These algorithms are proven to converge to the exact solution of the problem.

Multigrid methods are generally accepted as fast efficient solvers. The standard multigrid method has been shown to be unsatisfactory when applied to singularly perturbed problems. We construct monotone multigrid methods for solving nonlinear singularly perturbed convection-diffusion problems. We prove that these methods converge to the exact solution of the problem.

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